ESTIMATION OF A DYNAMIC AUCTION GAME[1]

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This paper proposes an estimation method for a repeated auction game under the presence of capacity constraints. The estimation strategy is computationally simple as it does not require solving for the equilibrium of the game. It uses a two stage approach. In the first stage the distribution of bids conditional on state variables is estimated using data on bids, bidder characteristics, and contract characteristics. In the second stage, an expression of the expected sum of future profits based on the distribution of bids is obtained, and costs are inferred based on the first order condition of optimal bids.

We apply the estimation method to repeated highway construction procurement auctions in the state of California between May 1996 and May 1999. In this market, previously won uncompleted contracts reduce the probability of winning further contracts. We quantify the effect of intertemporal constraints on bidders' costs and on bids. Due to the intertemporal effect and also to bidder asymmetry, the auction can be inefficient. Based on the estimates of costs, we quantify efficiency losses.

KEYWORDS: Dynamic oligopoly, auctions, estimation.

1. INTRODUCTION

MOST OF THE LITERATURE on empirical estimation of auctions assumes a static auction setting. Paarsch (1992), Laffont, Ossard, and Vuong (1995), Guerre, Perrigne, and Vuong (2000), and others develop an empirical approach to quantify informational uncertainty in static auction games. Nevertheless, there is little empirical work on dynamic auction games[3] or dynamic oligopoly games.[4]

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[1]An earlier version of the paper was circulated under the title "Bidding Behavior in a Repeated Procurement Auction."

[3]Laffont and Robert (1999) and Donald, Paarsch, and Robert (2002) analyze finitely repeated auctions. Laffont and Robert consider a sequence of auctions in which, at each stage, an identical object is sold. Their model generates complex intra-day dynamics that are applied to data on eggplant auctions. Donald, Paarsch, and Robert consider a model in which a finite number of objects are sold in a sequence of ascending-price auctions. They estimate the model using data on the sales of Siberian timber-export permits.

[4]Pakes (1994) summarizes the literature on estimation in dynamic games.

This paper proposes an estimation method for dynamic auction games. The method is computationally simple as it does not require solving for the equilibrium strategies of the game. We apply the method to estimate a repeated first-price bidding game under the presence of capacity constraints.

We observe data on bids, contract characteristics, and bidders' state variables. The estimation problem is how to infer privately known costs. We propose an estimation method that infers costs based on the first order condition of optimal bids of the repeated bidding game. We assume that observed data are generated by equilibrium play and estimate the distribution of bids conditional on state variables. Our crucial idea is that the expected discounted sum of future profits, which enter the first order condition, can be written looking forward and depending entirely on the distribution of bidders' bid choices.[5] In particular, the resulting expression of the value function can be written as a linear equation that can be easily solved numerically. With the value function at hand, costs are inferred based on the first order condition.

Our approach builds partially on the two stage approach that Elyakime, Laffont, Loisel, and Vuong (1994), and Guerre, Perrigne, and Vuong (2000) develop for static models. These papers estimate the distribution of equilibrium actions based on bid data. In equilibrium, the distribution function estimates summarize bidders' beliefs and can be used to infer bidders' valuations based on the first order condition of optimal actions. The main contribution of our paper is to extend the estimation method to dynamic auction games.

In work in progress, Berry and Pakes (2001) consider a related estimation strategy based on the first order condition of dynamic games. The innovative feature of their approach is to consider an alternative representation of the value function in which the expected sum of future profits is replaced with a weighted sequence of future profit realizations. In dynamic auction games, this representation is less attractive because profits are not observed and cannot be expressed indirectly, from observed bids, without knowing the equilibrium bid functions.

We apply our method to repeated procurement auctions for highway paving contracts. In this setting, previously won uncompleted contracts may affect the ability to win further contracts. Two distinct effects may arise: First, since the duration of highway paving contracts is a number of months, winning a large contract may commit some of the bidder's machines and paving resources for the duration of the contract. Although rental of additional equipment is available, this may increase total cost. Second, an experience effect may arise, since supplying services on a large contract may give a bidder the necessary expertise

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[5]A related estimation strategy of the value function is employed by Hotz and Miller (1993). They approximate the value function with discrete choices using estimates of choice probabilities. Their framework differs from ours in a number of ways: First, they consider a single agent dynamic decision problem. Second, they restrict their attention to discrete actions. Finally, they do not model informational constraints.

to conduct further services. The expertise effect may lower the cost for future contracts.

The results of our estimation using highway procurement data show that capacity constraints do affect firms' bidding strategies. In particular, the cost of taking on an additional contract is increasing in backlog. The increase in costs resulting from a larger than average backlog seems to cancel out any cost-reducing expertise effects, if they exist. Based on the estimates of costs, we quantify efficiency losses.

Our estimation method requires that bidders completely understand the auction environment and that our estimates of winning probabilities correctly measure bidders' perceived odds of winning. Beyond some general regularity predictions, we are not able to assess the adequacy of our behavioral assumptions. The main reason is that our data do not contain information about bidders' assessments. In particular, we do not observe bidders' perceived odds of winning a contract, nor bidders' actual contract costs. More detailed information about ex post realized outcomes is observed in outer continental shelf auctions studied by Hendricks and Porter (1988) and Porter (1995) among others. Hendricks and Porter convincingly show that auction models provide accurate predictions of outcomes and that the behavioral assumptions of game theoretic models are satisfied.

The paper is organized as follows: Section 2 describes the bidding model. Section 3 introduces the estimation method. The section addresses several issues: first, how to estimate the distribution of bids conditional on bidders' state variables and contract characteristics; second, how to estimate the expected future discounted payoff of bidders based on estimates of the bid distribution function; third, how to infer costs using the first order condition of optimal bids in the repeated game. Finally, conditions are provided under which the bidding model's parameters are identified.

Section 4 describes the industry and the data. We examine data on highway procurement contracts in California. The descriptive data analysis suggests the presence of capacity constraints. Estimates of the probability of submitting a bid reveal that bidders with low backlog levels are about twice as likely to submit a bid than bidders with high backlog levels.

Section 5 presents the estimates of the bidding model. The distribution of costs exhibits the expected properties of capacity constraints. To illustrate the estimates, we evaluate the estimated bid and cost distributions at sample average characteristics. The distribution of bids and costs at low backlog values stochastically dominates (in the first order sense) the distribution at high backlog values. Moreover, increasing the backlog appears to monotonically decrease the discounted sum of future profits.

Section 6 reports the effect of backlog on the expected price paid by the auctioneer and on the expected auction return to individual bidders. In addition, we compare the observed outcome to a cost minimizing allocation of contracts. We conduct experiments to assess the magnitude of efficiency losses. Finally, Section 7 provides some discussion and concludes.

2. THE BIDDING MODEL

This section describes the bidding model. We begin with a general description of the game, the sequencing of events in the stage game, and the assumptions entering each event. We then describe the law of motion of the state variables and the payoffs to bidders. We conclude the section with some remarks on the existence of a value function representation of payoffs and the existence of Markovian equilibria of the game.

The bidding model that we take to the data has the following features: Time is discrete with an infinite horizon, $t = 1, 2, \ldots$. There are two types of bidders: regular and fringe bidders. Fringe bidders have a short life and exit in the period they entered.[6] Regular bidders stay in the game forever. The set of regular bidders is denoted by $\{1, \ldots, n_r\}$ and the set of fringe bidders in period t is denoted by $\{n_r + 1, \ldots, n^t\}$. The number of fringe bidders, $n^t - n_r$, may vary over time and can equal zero.

The stage game: In every period t the buyer offers a single contract for sale. The sale can be described by the following sequence of events: First, the characteristics of the contract are revealed to all bidders. Second, bidders learn their costs privately. Third, bidders may submit bids. Fourth, the buyer may award the contract to the low bidder, or reject all bids. The events in the stage game have the following features:

The contract characteristics, s_0^t, are drawn independently and identically from the exogenous probability distribution function, $F_0(.)$, with finite support S_0. Future contract characteristics are assumed to be unknown to bidders.[7] The contract characteristics, s_0^t, include the physical attributes of the contract such as the contract size and duration, as well as the total number of fringe bidders that are active in the period. We define $\overline{\tau}$ as the maximum contract duration of all contracts in S_0. Finally, there is a fixed (nonrandom) reserve price of the seller R^t, which is an upper bound on admissible bids.

Costs: Each bidder i learns her cost for the contract, c_i^t, after the contract characteristics are revealed. The cost is privately known and independently distributed conditional on state variables. The cost of a regular bidder i is drawn

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[6] We observe a number of bidders that submit a bid only once, or a small number of times. On the other hand, we observe a number of bidders that submit bids frequently. To account for this difference, we classify firms into two groups: "Regular" bidders, which are the largest 10 firms in dollar value won and with at least 80 bids submitted; and "Fringe" bidders, which are the remaining firms.

[7] In highway procurement auctions future projects are not known to bidders. Only a short period in advance do project descriptions become available. The length of the announcement period ranges between 4 and 10 weeks. Information on the specification of immediately upcoming projects could be incorporated into the model by including the information on upcoming contracts as additional state variables. However, due to the added computational burden of additional state variables, we decided to ignore this feature of the data, and assume that contract characteristics become known in the period of the letting.

from the continuous conditional distribution function $F(.|s_0^t, s_i^t, s_{-i}^t)$ with support $[\underline{C}(s_0^t, s_i^t, s_{-i}^t), \overline{C}(s_0^t, s_i^t, s_{-i}^t)]$, where $s_i^t \in \mathcal{S}_i$, with $\mathcal{S}_i$ finite, denotes a vector of state variables for bidder i and s_{-i}^t denotes the vector of other bidders' state variables $(s_1^t, \ldots, s_{i-1}^t, s_{i+1}^t, \ldots, s_n^t)$.[8] We sometimes use the symbol s^t to denote the vector of all regular bidders' state variables with $s^t \in \mathcal{S}$ and $\mathcal{S} = \times_{i=1}^n \mathcal{S}_i$ and write (s_i^t, s_{-i}^t) to indicate that the state vector is evaluated from bidder i's perspective. Bidder i's state vector s_i includes a list of the sizes of all uncompleted projects that she has won in the last $\overline{\tau}$ periods and the time left to complete each of them. Specifically, s_i records the remaining size in dollars and the number of days left until completion of each project previously won by bidder i.[9] We assume that both s_0^t and s^t are observable to all bidders *and* to the econometrician. We do not consider heterogeneity that is known to the bidders but not known to the econometrician. The distribution of costs has a continuous density function $f(.|s_0^t, s_i^t, s_{-i}^t)$. Similarly, the cost of a fringe bidder is drawn from a continuous distribution function $F_f(.|s_0^t, s^t)$ with associated continuous density function $f_f(.|s_0^t, s^t)$ and support $[\underline{C}(s_0^t, s^t), R^t]$. The support assumption ensures that fringe bidders submit a bid.[10]

Bids: Each bidder may submit a bid, b, which is the price at which the bidder is willing to provide the service. All agents are risk neutral.

The buyer awards the contract to the bidder with the low bid at a price equal to her bid. Ties are resolved with a flip of a coin. The buyer may decide to reject all bids if the low bid is marginal. We model the buyer's rejection decision as a secret and nonstrategic[11] reservation price b_s^t drawn from the exogenous continuous distribution function $G_0(.|s_0)$ with support $[\underline{R}(s_0), R^t]$ and continuous probability density function $g_0(.|s_0)$. Bids above b_s^t are rejected. The continuity of G_0 is a technical assumption. It implies that the seller's rejection decision is

[8]An alternative two step process in which costs are learned is considered in Hendricks, Pinske, and Porter (2003). Hendricks, Pinske, and Porter assume that bidders decide, initially, whether to acquire private information or not, and only bidders that decide to acquire information will potentially participate in the auction. This approach appears reasonable for auction environments where it can be rather costly to obtain an estimate about value of the item. Our data do not contain information on information acquisition decisions that would allow us to identify such a two step model. In addition, the two step approach appears less plausible for highway paving jobs in which most bidders have a relatively accurate assessment about the opportunity cost of using their own paving resources. The cost uncertainty in highway bidding arises because the bidder's opportunity costs are not known to other bidders.

[9]The list consists of $\overline{\tau}$ entries, one for each of the last $\overline{\tau}$ periods. The entry in the lth position in the list consists of the size and the remaining time until completion of the project won l periods ago. If no contract was won l periods ago, then the entry in position l is empty.

[10]Alternatively, the cost distribution of fringe bidders can be modelled as a conditional cost distribution, conditional on drawing a cost below R^t, and taking into account the potential number of fringe bidders in the period.

[11]The secret reserve price does not affect the model conceptually, as it can be viewed as another bidder with a known bid distribution function. For an analysis of strategic secret reserve prices, see Elyakime, Laffont, Loisel, and Vuong (1994).

continuous at the announced reserve price R^t. In our application, we assume a uniform density for g_0.[12]

Conditional independence of contract characteristics and cost realizations is the crucial assumption in the data generating process. It permits us to adopt the framework of Markov dynamic decision processes. The role of conditional independence is discussed further in Rust (1994).

The transition function of the state variable, $\omega : S_0 \times S \times \{0, 1, 2, \dots, \max(n^t)\} \rightarrow S$, is a deterministic function of the contract characteristics, the state variables, and the identity of the low bidder—where identity 0 indicates that nobody won the auction (and the seller retained the contract).[13] As described above, the bidders' state variables include the sizes of all the projects left to do and the time remaining until their completion. Unfortunately, the data do not include information on the pattern in which projects are completed over time, but only the contract scheduled completion date. We assume that, at every point of time, an equal share of the project is completed and that contracts are completed as planned.[14] Let the ith component of the transition function ω consist of the list of the sizes and remaining times of all the project left to do for bidder i. It can be written as

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\omega_i(s_0, s, j) = \begin{cases} \left((z_0, \tau_0), \left(\dfrac{\max(\tau_i^l - 1, 0)}{\tau_i^l} \cdot z_i^l, \max(\tau_i^l - 1, 0) \right)_{l=1}^{\bar{\tau}-1} \right) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } j = i, \\[2mm] \left((0, 0), \left(\dfrac{\max(\tau_i^l - 1, 0)}{\tau_i^l} \cdot z_i^l, \max(\tau_i^l - 1, 0) \right)_{l=1}^{\bar{\tau}-1} \right) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise,} \end{cases}
$$

where z_i^l denotes the size of the project won by bidder i exactly l periods ago, and τ_i^l its remaining time until completion. Given the linearity assumption regarding the completion of projects, for each project l, the size carried over to next period, z_i^{l+1}, equals $(\max(\tau_i^l - 1, 0)/\tau_i^l) \cdot z_i^l$, and the remaining time until completion τ_i^{l+1} equals $\max(\tau_i^l - 1, 0)$. If the contract at period t is won by bidder i, it is added at the beginning of the list of projects with the initial size and time until completion taken from the contract's characteristics $(z_0, \tau_0) \in s_0$. If

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[12]According to conversations with Caltrans, the secret reserve price is an integral feature of our application. However, it is hardly ever observed. We observe no rejected bids below R^t in our data (and no bids are exactly equal to R^t). Further, 99 percent of winning bids are less than $0.85 \cdot R^t$. The lack of data makes the estimation of the reserve price distribution function G_0 difficult.

[13]Note that the identity of the low bidder can be deduced from the vector of submitted bids. Thus, an equivalent formulation of the state's transition function uses the vector of bids by all bidders (including the seller's reserve price) instead of the identity of the low bidder.

[14]Section 8-1.07, "Liquidated Damages," of the Standard Specifications (1999) of the Department of Transportation of California specifies that unjustified delays may come at a high cost to the contractor. The cost of delays for contracts during our sample period ranges between hundreds of dollars to thousands of dollars per day.

the contract at period t is not won by bidder i, then a vector $(0, 0)$ is added at the beginning of the list of projects.

We introduce a common discount factor parameter, $\beta \in (0, 1)$, that measures firms' patience with regard to future profits.

We consider *sequential equilibria* and restrict our attention to *symmetric Markovian strategies*.[15] Markovian strategies do not depend on time and in the subsequent analysis we omit the time superscript whenever possible. A strategy for bidder i is a function of bidder i's cost, the contract's characteristics, and her own and her competitors' states, $b(c_i, s_0, s_i, s_{-i})$. Let $\mathbf{b}_{-i}$ denote a strategy profile of other bidders.

Payoff of a regular bidder: The discounted sum of future expected payoffs for bidder i can be written in value function form as

$$W_i(s, s_0, c_i, \mathbf{b}_{-i}) = \max_b \Big[[b - c_i] \Pr(i \text{ wins} | b, s_0, s_i, s_{-i}, \mathbf{b}_{-i})$$

$$+ \beta \sum_{j=0}^n \Pr(j \text{ wins} | b, s_0, s_i, s_{-i}, \mathbf{b}_{-i})$$

$$\times E_0 \int W_i\big(\omega(s_0, s, j), s_0', c_i', \mathbf{b}_{-i}\big)$$

$$\times f\big(c_i' | s_0', \omega_i(s_0, s, j), \omega_{-i}(s_0, s, j)\big) dc_i' \Big],$$

where $\Pr(i \text{ wins} | b, s_0, s_i, s_{-i}, \mathbf{b}_{-i})$ denotes the probability that bidder i with state s_i wins contract s_0 given the strategy $\mathbf{b}_{-i}$ and the state s_{-i} by other bidders; E_0 denotes the expectation operator with respect to the contract characteristics s_0'; and $j = 0$ indicates that nobody won the auction. We assume that the strategy profile $\mathbf{b}_{-i}$ is such that $\Pr(j \text{ wins} | b, s_0, s_i, s_{-i}, \mathbf{b}_{-i})$ is continuous in b. Notice that, due to the reserve price rule, any bid exceeding the reserve price will be rejected. Therefore, without loss of generality, we can assume that bids exceeding the upper bound of the reserve price distribution equal b_0, with $b_0 > \max(R^t)$, and restrict attention to the compact bid space $[0, \max(R^t)] \cup b_0$. The assumption of symmetric cost distribution functions conditional on state variables and symmetric bidding strategies conditional on state variables implies that the value function can be written as

$$W(s_i, s_{-i}, s_0, c_i, \mathbf{b}_{-i}) = W_i(s, s_0, c_i, \mathbf{b}_{-i}) \quad \text{for all } i$$

for some function $W(.)$. For notational convenience, however, we will frequently use the symbol W_i to denote the value function W evaluated at the state $(s_i, s_{-i}, s_0, c_i, \mathbf{b}_{-i})$.

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[15]See Maskin and Tirole (1994) and Maskin and Tirole (2001) for expositions on Markov perfect equilibria.

Existence of $W_i(s, s_0, c_i, \mathbf{b}_{-i})$ follows from Bhattacharya and Majumdar (1989). To see this, notice that our assumptions imply that the period payoff function,

$$[b - c_i]\Pr(i \text{ wins}|b, s_0, s_i, s_{-i}, \mathbf{b}_{-i}),$$

is continuous in b, c_i, s_0, s_i, and s_{-i}, and bounded by $\max(R^t - \min \underline{C}((s_0^t, s_i^t, s_{-i}^t)))$. Moreover, the bid space, $[0, \max(R^t)] \cup b_0$, is compact; the state space, S, and the contract space, S_0, are finite; and the cost space $[\underline{C}(s_0^t, s_i^t, s_{-i}^t), \overline{C}(s_0^t, s_i^t, s_{-i}^t)]$ is compact. Furthermore, $\Pr(j \text{ wins}|b, s_0, s_i, s_{-i}, \mathbf{b}_{-i})$ is continuous in b, s_0, s_i, and s_{-i} for all j. Finally, since the state space is finite, $F(.|s_0, s_i, s_{-i})$ is weakly continuous in s_0, s_i, and s_{-i}.[16] Thus, the infinite-horizon dynamic programming problem of bidder i satisfies the necessary conditions of Theorem 3.2 in Bhattacharya and Majumdar (1989), which guarantees the existence of a unique solution to the value function, $W_i(s, s_0, c_i, \mathbf{b}_{-i})$.

In the subsequent analysis, we sometimes use the ex ante value function, which is defined as the value function evaluated before contract characteristics and bidder i's cost are known. We can write the ex ante value function as

$$V_i(s, \mathbf{b}_{-i}) = E_0 \int W_i(s, s_0, c, \mathbf{b}_{-i}) f(c|s_0, s_i, s_{-i}) dc,$$

where E_0 denotes the expectation operator with respect to contract characteristics. For expositional simplicity, from now on we suppress the dependence on the bidding strategies of other bidders in the value function. The above value function equation leads to the following recursive equation for V_i:

$$(2.1) \quad V_i(s) = E_0 \Bigg[\int \max_b \bigg\{ [b - c]\Pr(i \text{ wins}|b, s_0, s_i, s_{-i})$$

$$+ \beta \sum_{j=0}^{n} \Pr(j \text{ wins}|b, s_0, s_i, s_{-i}) V_i(\omega(s_0, s, j)) \bigg\}$$

$$\times f(c|s_0, s_i, s_{-i}) dc \Bigg].$$

Payoff of a fringe bidder: For a fringe bidder, the ex ante payoff is equal to the ex ante expected period payoff:

$$E_0 \Bigg[\int \max_b \big\{ [b - c]\Pr(i \text{ fringe wins}|b, s_0, s) \big\} f_f(c|s_0, s) dc \Bigg].$$

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[16]If D is a metric space, then $\mathcal{P}(D)$ denotes the set of all probability measures on the Borel sigmafield $\mathcal{B}(D)$ of D. A function F on a metric space D_1 into $\mathcal{P}(D)$ is *weakly continuous* if the sequence of probability measures $F(y_n)$ converge weakly to $F(y)$ when $y_n \to y$ in D_1.

The role of our earlier assumptions on the behavior of fringe bidders can be illustrated at this point: Fringe bidders exit in the period they entered and, therefore, they assign no value to the future. The assumption that the cost of fringe bidders is contained in the interval $[\underline{C}, R^t]$ guarantees that fringe bidders find it profitable to submit a bid in the period they enter. In contrast, note that our model implies that a regular bidder may not submit a bid in a given period. This may arise either because the cost realization exceeds the upper bound of the reserve price, or because the cost of being constrained in the future outweighs the current period gain of winning the contract.

Existence of an equilibrium: Jofre-Bonet and Pesendorfer (1999) establish existence of an equilibrium in an example with exponential distributed costs. The example has the feature that bid functions are linear in cost realizations. While a general existence theorem is beyond the scope of this paper, we briefly outline how recent results in the literature may be applied to achieve this goal: The existence of an equilibrium in the period game for any continuation value follows from Jackson, Simon, Swinkels, and Zame (2002). They show existence of a (possibly mixed strategy) equilibrium if payoffs are affine in outcomes, where an outcome is a (possibly vector valued) function of the submitted bids. In our case, the outcome consists of a vector of winning probabilities. Furthermore, they show that under our assumption of independent costs, the equilibrium bids are strict monotone in costs and a pure strategy equilibrium exists.[17]

An equilibrium of our dynamic game requires that the equilibrium payoff in the period game be consistent with the continuation payoff, which can be verified using a fixed point theorem. Theorem 2 in Jackson et al. (2002) implies the upper hemi-continuity of the equilibrium payoff correspondence.[18] Uniqueness in asymmetric first price auctions for a number of environments covering special cases of our model is shown in Maskin and Riley (1996).[19] Provided uniqueness holds, the Brouwer fixed point theorem yields a consistent value function and thus a dynamic equilibrium. In cases in which the uniqueness results do not apply, existence of a consistent continuation value can be obtained by introducing an independently and identically distributed public random variable. Following Maskin and Tirole (1994), the bidding strategy can be augmented to be a function of the realization of the public random variable. Doing so, convexifies the correspondence of equilibrium payoffs. Kakutani's fixed point theorem then yields the existence of a consistent value function and, thus, a dynamic equilibrium.

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[17]Therefore, our restriction to consider the continuous distribution function of low competitors' bids does not affect the existence question.

[18]See Lebrun (2002) for a related result in asymmetric first-price auctions.

[19]These cases include the general two bidder asymmetric first-price auction and the multiple bidder asymmetric first-price auction with different support of the cost distribution function and identical payoff functions. The condition is satisfied if the continuation values only shift the support of the cost distribution function and if the continuation value in case an opponent wins is independent of the identity of the winning opponent.

Bid distribution function: We denote the distribution function of equilibrium bids of bidder i with state (s_0, s_i, s_{-i}) by $G(.|s_0, s_i, s_{-i})$, and the associated derivative at points less than or equal to R by $g(.|s_0, s_i, s_{-i})$. We denote the distribution function of equilibrium bids by a fringe bidder with state (s_0, s) by $G_f(.|s_0, s)$ and the associated density function by $g_f(.|s_0, s)$. We sometimes suppress the f subscript in the distribution function of fringe bids and the 0 subscript in the seller's reserve price distribution. Instead, in an abuse of notation and in order to simplify the expressions involved, we also write $G(.|s_0, s_i, s_{-i})$ where i takes values 0 and f. Doing so, the probability $\Pr(i \text{ wins}|s_0, s_i, s_{-i})$ can be written as $\prod_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})]$ where the product is taken over regular bidders, fringe bidders, and the seller. The ex-ante probability of the event that bidder i wins with a bid less than or equal to b is denoted by $G^{(i)}(b|s_0, s)$, and the associated derivative for b less than or equal to R is given by $\prod_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})]g(b|s_0, s_i, s_{-i})$.

3. ESTIMATION METHOD

This section describes the estimation method: Section 3.1 explains the estimation approach, which is based on the first order condition of optimally chosen bids. The first order condition provides an explicit function of the cost in terms of the submitted bid, the distribution of bids, and the value function. We show that the value function, which enters the first order condition, can be represented in the form of a recursive equation involving the equilibrium bid distribution function. Therefore, with an estimate of the bid distribution function we can infer costs by using the first order condition of optimal bids. Section 3.2 establishes a condition under which the distribution function of privately known costs is identified. Section 3.3 describes in detail our parametric specification of the bid distribution function.

3.1. *Estimation Approach*

We observe data on bids, contract characteristics, and bidders' state variables. Our goal is to infer privately known costs. We propose a computationally simple estimation method that does not require solving for the equilibrium bid functions. The method requires the assumption that observed bids are generated by equilibrium play and satisfy the first order condition of optimal bids.

Estimation methods based on the first order condition in static games are well known in the literature. Elyakime, Laffont, Loisel, and Vuong (1994) estimate the beliefs about the equilibrium play of agents based on data on bids. The cost realization is then inferred from the first order condition. We extend the estimation method to dynamic bidding games.

First order condition: Let $\phi(.)$ denote the unobserved cost associated with a bid, which is a function of the bid, b, the contract characteristics, s_0, the state vector, (s_i, s_{-i}), and the value function V_i. Let

$$h(.|s_0, s_i, s_{-i}) = \frac{g(.|s_0, s_i, s_{-i})}{1 - G(.|s_0, s_i, s_{-i})}$$

denote the hazard function of bids submitted by bidder i when the state equals (s_0, s_i, s_{-i}). The first order condition for optimal bids yields the following equation (see the Appendix for a more detailed description) for privately known costs, ϕ:

$$(3.1) \quad \phi(b|s_0, s_i, s_{-i}, h, V_i)$$

$$= b - \frac{1}{\sum_{j \neq i} h(b|s_0, s_j, s_{-j})}$$

$$+ \beta \sum_{j \neq i} \frac{h(b|s_0, s_j, s_{-j})}{\sum_{l \neq i} h(b|s_0, s_l, s_{-l})} \big[V_i(\omega(s_0, s, i)) - V_i(\omega(s_0, s, j)) \big].$$

Equation (3.1) provides an explicit expression of the privately known cost that involves the bid; the hazard function of bids, h; and, the value function, V_i. Equation (3.1) states that the cost equals the bid minus a mark-down. The mark-down has two parts: The first part accounts for the level of competition in the current period. The second part accounts for the incremental effect on the future discounted profit if firm i wins the contract instead of another firm. The first order condition for fringe bids is of the same form as equation (3.1). However, fringe bidders assign no value to the future, $V_f = 0$, and the second term in equation (3.1) vanishes for fringe bids.

The first order condition provides an explicit function of the cost for a submitted bid, $b \leq R$. If a bid is not submitted, $b > R$, then the monotone relationship (3.1) implies a lower bound for the cost, $c \geq \phi(R|s_0, s_i, s_{-i}, h, V_i)$. Notice that the behavior of function (3.1) at the boundary R differs from the static first price auction. As in the static model, at R the first mark-down vanishes due to the secret reserve price and the presence of fringe bidders.[20] However, in the dynamic model the second mark-down remains. Thus, in general a bid need not equal costs at the reserve price R. The magnitude of the mark-down at the reserve price depends on the incremental effect on the future discounted profit if firm i wins the contract instead of another firm.

In order to infer the distribution of costs, we need estimators for the functions appearing in the right-hand side of (3.1). These functions are: the transition function of the state, the bid hazard function, the secret reserve price hazard function, the discount factor, and the value function. First, as described

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[20]On 97.5% of contracts at least one fringe bidder is present. As described above, fringe bidders submit a bid less than R.

before, in our auction model, the transition function of the state is a given function. Second, an estimator of the bid distribution function, and thus of the hazard function, can be directly obtained from the data on bids, contract characteristics, and state variables. We describe the details of our estimator in Section 3.3. We do not estimate a distribution function for the secret reserve price due to the lack of data on rejected bids. Instead, we assume a uniform distribution and examine how sensitive the estimates are to variations in the support of the secret reserve price distribution function. Third, we choose the discount factor, and we examine how sensitive the estimates are to variations in the discount factor. Finally, the definition of the value function is given in equation (2.1). However, observe that the expression of the value function in equation (2.1) involves cost variables that are unobserved, and decisions by multiple agents, which are endogenous. In the next paragraphs, we explain how we overcome the problems posed by this latter observation and obtain an approximation of the value function.

The value function: The key idea of our method is to notice and use the fact that the distribution of equilibrium bids determines the discounted sum of expected future profits. Thus, there is a representation of the value function in terms of the distribution of bids only. The following proposition states that the value function can be represented as a recursive equation involving the bid distribution function. The proof of Proposition 1 is given in the Appendix.

PROPOSITION 1:

$$(3.2) \qquad V_i(s) = E_0 \Bigg\{ \int_{\underline{b}}^R \frac{1}{\sum_{j \neq i} h(.|s_0, s_j, s_{-j})} dG^{(i)}(.|s_0, s)$$

$$+ \beta \sum_{j=0, j \neq i}^n \Bigg[\Pr(j \text{ wins}|s_0, s_i, s_{-i})$$

$$+ \int_{\underline{b}}^R \frac{h(.|s_0, s_i, s_{-i})}{\sum_{l \neq i} h(.|s_0, s_l, s_{-l})} dG^{(j)}(.|s_0, s) \Bigg]$$

$$\times V_i(\omega(s_0, s, j)) \Bigg\}.$$

The value function representation in Proposition 1 has two parts: The first part accounts for bidder i's current expected profits. The second part accounts for bidder i's sum of discounted expected future payoffs. Notice that the terms in square brackets in (3.2.) can be interpreted as probabilities, as they sum to one, since

$$\Pr(i \text{ wins}|s_0, s_i, s_{-i}) = \sum_{j \neq i} \int_{\underline{b}}^R \frac{h(.|s_0, s_i, s_{-i})}{\sum_{l \neq i} h(.|s_0, s_l, s_{-l})} dG^{(j)}(.|s_0, s).$$

The proof of the proposition is based on two observations: First, we may write the probability of winning as a function of the distribution of bids by other bidders, ignoring dependence of other bidders' bids on cost draws. Thus, each bidder dynamic game is reduced to a single agent dynamic decision problem where each bidder maximizes the discounted sum of future payoffs taking as given the equilibrium bid distribution associated with other bidders. Still, this single agent dynamic decision problem does involve bidder i's privately known costs, which are not observed. Our second observation is that the first order condition of optimal bids gives an explicit expression of bidder i's unknown costs in terms of her equilibrium bids and the equilibrium bids distribution. Substituting this expression into the value function yields an expression involving the distribution and the density of equilibrium bids only.

Approximation of the value function: Numerical methods can be used to approximate the value function based on Proposition 1, which we describe next. The assumption of a symmetric Markovian strategy space imposes a number of restrictions on the shape of the value function, which facilitates the approximation. In particular, Pakes (1994) shows that the number of coefficients in a Jth order polynomial approximation does not increase as the number of bidders increases. The reason is that symmetric Markovian strategies require that bidders with the same state follow the same bidding strategy. Thus, we can exchange the state of two competing bidders without affecting the value of bidder i's expected future payoffs. As has been shown by Pakes, in a Jth order polynomial approximation of the value function, the polynomial coefficients associated with the state variable of a competitor j, s_j, are identical for all $j \neq i$, or, in other words, their identities do not matter. In Section 5, we discuss in detail the variables entering the value function approximation. In our case, the dimensionality of the approximation is reduced to a four dimensional problem for each bidder.

Numerical methods to approximate the value function based on equation (3.2) are discussed in more detail in Judd (1998). We briefly summarize the method we use: We select a grid of state vectors $\hat{S} = (s^1, \ldots, s^m)$ by drawing 50 states from the distribution of observed states. We numerically solve equation (3.2) for every bidder on this grid. We restrict the range of the transition function ω to $\hat{S}$ by defining a transition function $\hat{\omega}(s_0, s, j) = \{s \in \hat{S} | s \text{ is closest to } \omega(s_0, s, j)\}$. For every point $s \in \hat{S}$ we calculate the expectations on the right-hand side of equation (3.2). Specifically, we numerically evaluate the expected current period payoff,

$$A_i(s) = E_0 \left\{ \int_{\underline{b}}^{R} \frac{1}{\sum_{j \neq i} h(.|s_0, s_j, s_{-j})} dG^{(i)}(.|s_0, s) \right\},$$

and the $1 \times m$ vector of transition probabilities of the events that the states $(s^1, \ldots, s^m)$ are reached when bidder j wins the contract,

$$B_{ij}(s) = E_0 \left\{ \int_{\underline{b}}^{R} \left[1 + \frac{h(.|s_0, s_i, s_{-i})}{\sum_{l \neq i} h(.|s_0, s_l, s_{-l})} \right] dG^{(j)}(.|s_0, s) \right.$$

$$\left. \times (1_{\{\hat{\omega}(s_0, s, j) = s^1\}}, \ldots, 1_{\{\hat{\omega}(s_0, s, j) = s^m\}}) \right\}.$$

In both expressions, the first expectation is with respect to contract characteristics. We evaluate the expectation with respect to contract characteristics in the following way: We select a set of contract characteristics, $\hat{S}_0$, by randomly drawing 50 contracts from the set of observed contracts. Since contract characteristics are modelled as independent and identical draws from the set $\hat{S}_0$, we can approximate the expectation with respect to contract characteristics by using the sample average. We evaluate the expectation with respect to the bid distribution function by numerical integration using the estimated derivatives $d\hat{G}^{(i)}$ and $d\hat{G}^{(j)}$. The specification of the bid distribution function is described in the next section. Using the symbol A_i for current period payoff and B_{ij} for the transition probabilities, the value function is given by the equation,

$$(3.3) \qquad V_i(s) = A_i(s) + \beta \sum_{j \neq i} B_{ij}(s) V_i,$$

where V_i denotes the vector $(V_i(s^1), \ldots, V_i(s^m))'$. We can rewrite the value function in matrix notation as: $[I - \beta B_i] V_i = A_i$, where I denotes the m-dimensional identity matrix, B_i denotes the $m \times m$ transition matrix given by $[\sum_{j \neq i} B_{ij}(s^1), \ldots, \sum_{j \neq i} B_{ij}(s^m)]'$, and A_i denotes the vector $(A_i(s^1), \ldots, A_i(s^m))'$. The value function can be expressed as

$$V_i = [I - \beta B_i]^{-1} A_i.$$

To evaluate the value function for points $s \in \mathcal{S}$, possibly outside the grid $\hat{S}$, we approximate the function with a quadratic polynomial.

Estimation of the cost distribution function: With the distribution function of bids at hand, the parameters of the bidding model can be readily inferred. To see how we infer the distribution of costs, notice the following: First, there is a relationship between the distribution function of costs and the distribution function of bids given by $F(c|s_0, s_i, s_{-i}) = G(b(c, s_0, s_i, s_{-i})|s_0, s_i, s_{-i})$. Second, the inverse of the bid function conditional on state variables, $c = \phi(b|s_0, s_i, s_{-i})$, is given in equation (3.1). Thus, using these two relationships, we can specify our estimator of the cost distribution function as

$$F(c|s_0, s_i, s_{-i}) = \int_{\{b|\phi(b|s_0, s_i, s_{-i}) \leq c\}} dG(.|s_0, s_i, s_{-i}),$$

for $c \leq \phi(R(s_0)|s_0, s_i, s_{-i})$.[21] Provided ϕ is invertible, the estimator can be written as $F(c|s_0, s_i, s_{-i}) = G(\phi^{-1}(c|s_0, s_i, s_{-i})|s_0, s_i, s_{-i})$. Standard errors of estimates are calculated using the delta method.

A nice feature of the auction model is that the discount factor and the distribution of costs are the only unknown parameters in the model. However, observe that our estimation method extends to models in which the per period payoff function may depend on additional parameters. For example, the Hotz and Miller (1993) estimator can be applied.[22]

Next, we examine whether the distribution function of costs can be identified.

3.2. *Identification*

This subsection addresses the question of under which conditions we can infer the parameters of our model. There are two related and somewhat opposing results in the literature: Guerre, Perrigne, and Vuong (2000) have established that, in the static first price auction, the distribution of costs is identified. On the other hand, Rust (1994) finds that the primitives in a dynamic decision problem with an unknown per-period payoff function are not identified. Proposition 2 below establishes that our model lies in between those two extremes. First, observe that, in our setting, the (nonparametric) identification problem consists in determining if the two underlying primitives, which are the distribution of costs $F(.|.)$ and the discount factor β, are identified based on expressions (3.1) and (3.2), and a sufficient number of observations (b^t, s_0^t, s^t).

We assume that the bid observation b^t is drawn from the distribution function $G(.|s_0^t, s_i^t, s_{-i}^t)$, which is continuous and has the derivative $g(.|s_0^t, s_i^t, s_{-i}^t)$ on $[\underline{b}, R]$. Furthermore, the distribution function satisfies the property that function (3.1) is strict monotone for any β on $[\underline{b}, R]$.

PROPOSITION 2: (i) *Suppose* β *is fixed. Then, the cost distribution function* $F(.|.)$ *is identified on the interval* $[\underline{C}(.), \phi(R()|.)]$. (ii) *The pair* (β, F) *is not identified*.

The Proposition establishes that the truncated distribution of costs is identified for a discount factor β. The truncation occurs at the point where it is not profitable to submit a bid, that is for costs exceeding $\phi(R(s_0)|s_0, s_i, s_{-i})$. Thus, in contrast to Rust (1994) we obtain a partially positive answer in our model. A main reason for the apparently opposing result is that the reward function

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[21]Guerre, Perrigne, and Vuong (2000) propose an alternative two step estimator.

[22]Hotz and Miller define an iterative procedure for single agent dynamic decision problems in which the value function is calculated for a given parameter vector at every step of the iteration. A method of moments estimator is defined that compares the predicted and observed actions based on the first order condition. Although Hotz and Miller consider discrete choices, it can also be applied to continuous choices.

in our auction model is known, but not known in Rust's. Part (ii) establishes that the discount factor and the cost distribution function are not identified jointly. Thus, the condition in (i) of the Proposition is necessary. The nonidentification problem of the discount factor is similar to other dynamic estimation approaches; see Rust (1994).

3.3. *Bid Distribution Functions*

So far we have described the estimation procedure to infer costs and addressed the identification question. What is left to do is to specify our estimator of the distribution function of bids. This section describes the parametric specification of the bid distribution function of fringe and regular bidders. In addition, we discuss the restrictions on parameters implied by the bidding model and describe how we impose these restrictions on the estimation procedure.

At an early stage of the research, we adopted kernel methods to estimate the bid distribution functions as proposed by Guerre, Perrigne, and Vuong (2000). Our estimation results are summarized in Jofre-Bonet and Pesendorfer (2000). In this earlier work, we found that nonparametric approaches reduce greatly the number of covariates we could use, which, in our case, limited severely the ability of the model to capture the richness of the data. For this reason we decided to adopt a parametric framework.

The bid distribution function is not a primitive of the model. However, Jofre-Bonet and Pesendorfer (1999) provide the equilibrium mapping from costs into bids for a particular class of cost distribution functions. In this paper, we select a parameterization of the bid distribution function of regular bidders that contains that particular class of cost distribution functions as a special case. In other words, the example in Jofre-Bonet and Pesendorfer (1999) illustrates a class of cost distribution functions that satisfies our parametric assumptions.

Density function of regular bidders: We experimented with different specifications for the density function of bids by regular bidders and we obtained the best fit with a Weibull density function. Specifically, we define the density function of a monotone bid transformation as a Weibull density function. The transformation involves taking the logarithm of one plus the bid, $\ln(b+1)$. The lower endpoint of the Weibull density function is $\ln(\theta_3 + 1)$. In our specification, the parameters of the density are a function of the state variables and contract characteristics. By suppressing the parameters' dependence on these variables, we can write the density function as

$$g(b|\theta_1, \theta_2, \theta_3) = \frac{1}{b+1}\left[\frac{\theta_1[\ln(b+1) - \ln(\theta_3+1)]^{\theta_1-1}}{\theta_2^{\theta_1}}\right]$$
$$\times \exp\left(-\left(\frac{\ln(b+1) - \ln(\theta_3+1)}{\theta_2}\right)^{\theta_1}\right).$$

The support of bids for regular bidders is $[\theta_3, \infty)$.[23] The parameters of the distribution function consist of the constant θ_1, and two parameters, θ_2 and θ_3, that depend on the state variables as will be explained below.

The *density function of bids by fringe bidders* is specified as a beta density function. We can write the density function as:

$$g_f(b|\theta_3, \theta_4, \theta_5) = \frac{1}{R - \theta_3}\left(\frac{b - \theta_3}{R - \theta_3}\right)^{\theta_4 - 1}\left(\frac{R - b}{R - \theta_3}\right)^{\theta_5 - 1}\frac{1}{B(\theta_4, \theta_5)},$$

where the distributional parameter θ_4 is a constant; the distributional parameters, θ_3, θ_5, depend on the state variables as will be described below; R denotes the upper bound on the reserve price; the support of fringe bids is $[\theta_3, R]$; and, the function $B(\theta_4, \theta_5)$ denotes the beta function.[24]

Estimation procedure: The regularity conditions of maximum likelihood need not hold for auction models as has been shown by Donald and Paarsch (1993). In our case, a nonregularity arises since one parameter is the boundary of the support of bids. There is a substantial statistics literature including Smith (1985, 1994) and Harter and Moore (1965) studying properties of estimation methods of the support and shape parameters for the Weibull and beta distribution function. Smith (1985) shows that the maximum likelihood estimates of the parameters are consistent and efficient provided that $\theta_1, \theta_4 \geq 2$.[25] For the case $\theta_1, \theta_4 < 2$, Harter and Moore (1965), and Smith (1985, 1994) describe a two stage estimation procedure: In the first stage the lower bound is estimated using the sample minimum. In the second stage the observation involving the sample minimum is dropped and the shape and scale parameters are estimated using maximum likelihood. Smith (1994) considers a multi-dimensional lower bound and proposes a linear program estimator for the lower bound to be used in the first stage. Smith (1985, 1994) establish that the two stage estimation procedure is consistent for $\theta_1, \theta_4 > 1$, and also asymptotically efficient, provided the density does not vanish too fast at the lower bound, $1 \leq \theta_1, \theta_4 < 2$. The above results require that θ_1 and θ_4 be one dimensional, and allow θ_2, θ_3, and θ_5 to be multi-dimensional. The asymptotic results do not readily extend to the case in which θ_1 and θ_4 are multi-dimensional.

We estimate the model using both estimation procedures. Under the two stage estimation procedure, we found that θ_1 and θ_4 exceed 2 when θ_1 and θ_2 are left unrestricted. Therefore, only maximum likelihood is efficient. In the following sections, we report the estimation results using maximum likelihood estimation.

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[23]In principle, it is possible to estimate different supports of bid distributions for individual bidders. However, we restrict the supports to be identical. The main reason is that, empirically, with a small data sample, it is difficult to determine whether bidders have indeed different supports or not.

[24]$B(v, w) = \int_0^1 u^{v-1}(1 - u)^{w-1}du$.

[25]For $\theta_1, \theta_4 < 1$, the maximum likelihood estimators may be inconsistent.

Restrictions on the parameters: As mentioned, some of the distributional parameters are a function of state variables and contract characteristics. Nevertheless, there are a number of restrictions on the functional form of the parameters' dependence on the state variables. First, there are the restrictions that $\theta_2, \theta_5 > 0$, and that $\theta_1, \theta_4 \geq 2$. These restrictions ensure that the conditions for a probability density function are satisfied, that the monotonicity of the bid hazard functions holds, which is required from the bidding model,[26] and that we can apply the asymptotic efficiency result in Smith (1985). Moreover, we assume that bids are nonnegative. To impose these restrictions, we define $\theta_j = 2 + \exp \theta_{0j}$ for $j = 1, 4$, and $\theta_j = \exp \theta_{0j}$ for $j = 2, 3, 5$.

The second restriction is the symmetry of the bidding functions conditional on state variables. Bidders with the same state follow the same bidding strategy. Thus, the parameter entering the bid distribution of fringe firms, $\theta_{03}(s)$, $\theta_{05}(s)$, should not be affected by the order of elements in the vector s. Similarly, the order of elements in the vector s_{-i} does not matter for parameter θ_{02} either. We consider the following specification, which incorporates the described conditional symmetry restrictions:

$$\theta_{0j}(s_0, s_i, s_{-i}) = \gamma_{j,0} + \gamma_{j,1} s_0 + \gamma_{j,2} \cdot \sum_{l=1}^{n} \Psi(s_l, s_0) + \gamma_{j,3} \Psi(s_i, s_0)$$
$$\text{for } j = 2,$$

$$\theta_{0j}(s_0, s) = \gamma_{j,0} + \gamma_{j,1} s_0 + \gamma_{j,2} \cdot \sum_{l=1}^{n} \Psi(s_l, s_0) \quad \text{for } j = 3, 5,$$

where $\Psi(s_i, s_0)$ is a function that denotes the characteristics of a bidder with state s_i on contract s_0. In our application the characteristics Ψ include the bidder's distance to the contract location and the bidder's number of plants within the region of the contract.

Likelihood function: Finally, in order to estimate the density functions described above, we have to take into account that the bid data for regular bidders are censored. We only observe bids that are below the upper bound of the reserve price, R^t. Let o_i^t be a dummy variable that equals one if we observe a bid by bidder i on contract t, and zero otherwise. In an abuse of notation, we abbreviate the dependence of parameters on the state vector with superscripts, and write the parameter vector for regular bidders as θ^{it} where $\theta^{it} = (\theta_1, \theta_2(s_0^t, s_i^t, s_{-i}^t), \theta_3(s_0^t, s^t))$. Similarly, the parameter vector for fringe bidders we abbreviate as θ^t where θ^t denotes $(\theta_3(s_0^t, s^t), \theta_4, \theta_5(s_0^t, s^t))$. Doing

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[26] For $\theta_1, \theta_4 < 1$, the hazard of bids can be decreasing, which would violate the condition that equilibrium bids are monotone increasing in costs.

so, we may write the likelihood of regular and fringe bids conditional on the initial state s^0 as

$$L = \prod_t \left[\prod_{i=1}^{n_r} \left[g(b_i^t | \theta^{it}) \right]^{o_i^t} \left[1 - G(R^t | \theta^{it}) \right]^{1-o_i^t} \prod_{j=n_r+1}^{n^t} g_f(b_j^t | \theta^t) \right],$$

where b_i^t is the bid by regular bidder i on contract t, and b_j^t is the bid by fringe bidder j on contract t. We assume that the initial state s^0 is exogenous. We maximize the logarithm of the likelihood function. Billingsley (1961) establishes the asymptotic properties of maximum likelihood estimators for Markov processes, as the number of auctions T goes to infinity. The Markovian structure of the observed controlled process (s_0, s, b) follows directly from the assumption that the cost realizations are conditionally independently distributed, the bid functions are Markovian, and the assumption that the distribution of contract characteristics is exogenous. A closely related result is Theorem 3.3 in Rust (1994).

The next section describes the data and the industry. Section 5 reports the estimation results.

4. THE DATA AND INDUSTRY

In this section, we describe some characteristics of the highway construction industry with emphasis on California. We present our data and describe the awarding process for contracts. In addition, we report reduced form evidence on the effect of previously won and uncompleted contracts on bid submission and bid level decisions.

4.1. *The California Market*

According to the 1992 US Census of Construction Industries[27] a total of $35.3 billion was spent during 1992 on highway and street construction activities. In California, the total amount spent in highway construction was $2.7 billion, 93% of which was done by 896 establishments located in California. Transportation costs play an important role in this industry, and, therefore, we consider California as a market.

The data: Our data consist of California Department of Transportation (Caltrans) contract awards for highway and street construction made between December 1988 and May 1999.[28] Information on bids is available from May 1st, 1996 through May 31st, 1999. During the latter period, Caltrans advertised

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[27]U.S. Department of Commerce, Economics and Statistics Administration, Bureau of Census, 1992.

[28]We obtained our data from the California Department of Transportation. The Office of Engineers publishes the data on the web: *http://www.dot.ca.gov/hq/esc/oe/awards/bidsum.*

2,566 projects from which 2,207 were finally awarded, 343 cancelled or post-poned,[29] and 16 received no bids.

The bid data contain the following information on every project awarded: Bid Opening Date; Contract Number; Location; Reservation Price; Number of Working Days and the Engineers' Estimate. Additionally, the data provides the Name, the Address, the Amount of the Bid, and the Rank of the Bid for each of the bidding firms. In order to obtain a measure of past performance and maximum capacity of the firms active in our period of analysis, we complement the bid data with the Caltrans Contract Performance database. This source contains information on contracts awarded between December 1988 to May 1999. It provides the actual dollar amount received for the contract, the contract duration, and the identity of the contractor.

The awarding process: Contracts are awarded by the California Department of Transportation subject to Federal Acquisition Regulations and, therefore, the process is very similar to other states' procedures. The process can be described in three steps: First, the Caltrans' Headquarters Office Engineer announces a project that is going to be let and the invitation to submit bids starts. This period is called the Advertising period and its length ranges between 4 and 10 weeks, depending on the size or complexity of the project. Occasionally, the Advertising period will be reduced to expedite project scheduling. Second, potential bidders may collect bid proposals that explain the plans and specifications of the work required, i.e., the project's characteristics, terms, and identification number. Based on the proposal, bidders may submit a sealed bid. Bidders do not know who else submits a bid. For each bid, Caltrans checks that the bidding firm is among the firms that are qualified to do business with Caltrans.[30] Third, on the letting day, the bids received are unsealed and ranked. The project is awarded to the lowest bidder provided it is below the reserve price. The reserve price consists of a fixed nonrandom dollar amount (R^t), which is assigned prior to the bidding. Additionally, a secret reserve price can affect the outcome at the margin. In order to win the project, it is required that the lowest bidder fulfills certain responsibility criteria. The bid is accepted if all

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[29]According to the Federal Acquisition Regulation, part 14, a contract might be cancelled before opening if either the project is no longer needed or if the advertised contract characteristics become obsolete or inadequate and have to be revised. Other reasons to cancel are that all bids are either unreasonable or collusive or both. Cancellation can also occur if all reasonable bids belong to bidders that cannot prove to be responsible. Additionally, the awarding agency might postpone the opening bids if it believes that a large fraction of bids have been delayed in the mail or other disruptive circumstances interfered in the regular reception of bids.

[30]Prior to the bidding, potential bidders have to qualify for contractual work for the Department of Transportation. In addition, firms are required to deposit a predetermined amount of funds that have to be available. Receipt of funds clearance, permit issuance, and local agencies approvals are needed for the bid of a firm to be accepted. A submitted bid can be rejected if either it fails to conform to the essential requirements of the invitation for bids; or does not conform to the applicable specifications without having been authorized to do so; or fails to conform to the delivery schedule or permissible alternates stated in the invitation.

TABLE I
DESCRIPTIVE STATISTICS OF SELECTED VARIABLES

	Number of Observations	Mean	Standard Deviation	Minimum	Maximum
Number of Bidders	2223	4.63	2.46	0.00	19.00
Estimate[a]	2223	13.41	1.35	9.47	18.31
(Ranked1[b]−Estimate)/Estimate	2207	−0.04	0.22	−0.79	3.07
(Ranked2[b]−Ranked1)/Ranked1	2111	0.09	0.12	0.00	2.62
Backlog[c]	22230	0.00	1.00	−3.24	2.97

[a]Logarithm of the engineers' estimate.

[b]Ranked1 and Ranked2 are the winning bid and the bid ranked in second position, respectively.

[c]Backlog measures the $ value of previously won uncompleted contracts. It is standardized by subtracting the bidder specific mean and dividing by the bidder specific standard deviation.

computations and cost imputations are considered correct. After each letting, a list of all bids and their rankings is announced and made accessible to the public. The winning firm is awarded the project no more than 30 days after the letting date.

The highway paving industry has already been studied by a number of authors. Porter and Zona (1993) and Feinstein, Block, and Nold (1985) study issues of bidder collusion. Bajari (1997) studies asymmetry between bidders. He estimates a static bidding model based on a numerical calculation of equilibrium bid functions.

4.2. *Descriptive Statistics*

Between May 1st of 1996 and May 31st of 1999, the Caltrans awarded 2,207 contracts. The total value of the contracts was $4,661.73 million.

Contracts are offered for sale on a frequent and regular basis. Typically, there are several letting dates per week. According to our data, the average duration between letting dates equals 2.96 days. Several contracts may be offered on a given letting date and contracts are offered in 11 distinct regions of the state.

Table I reports that on average there were 4.63 bidders per contract, ranging from 0 to 19 bidders across contracts. A total of 10,289 bids were received for these contracts and 16 contracts received no bids.[31] Table II reveals that a

[31]A total of 1,466 submitted bids, or 12% of all bid observations, violate the reserve price requirements. We exclude these bids from the analysis. These bids may have been submitted erroneously. Alternatively, bidders may have expected that the reserve price rule would not be enforced. According to conversations with Caltrans, it is indeed possible that the reserve price is altered ex post. Nevertheless, our data do not include information on bids below the reserve price being rejected, or bids above the reserve price being accepted. The lack of data points that fall into either of these two categories suggests that the probability of these events is low. In our analysis we assume that the reserve price rule is binding.

TABLE II
DESCRIPTIVE STATISTICS OF SELECTED VARIABLES BY NUMBER OF BIDDERS

Number of Bidders:	All	0	1	2	3	4	5	6	7–8	9–19
# Observations:	2223	16	96	285	393	432	356	237	251	157
Estimate[a]										
Mean	13.41		13.14	13.47	13.49	13.32	13.42	13.55	13.48	13.12
Standard Deviation	1.35		1.04	1.27	1.33	1.29	1.35	1.59	1.47	1.19
(Ranked1[b]−Estimate)/Estimate										
Mean	−0.04		0.11	0.03	−0.01	−0.04	−0.06	−0.09	−0.10	−0.14
Standard Deviation	0.22		0.36	0.29	0.21	0.20	0.19	0.16	0.20	0.16
(Ranked2[b]−Ranked1)/Ranked1										
Mean	0.09			0.14	0.11	0.09	0.08	0.06	0.07	0.06
Standard Deviation	0.12			0.11	0.19	0.10	0.09	0.06	0.07	0.07

[a]Logarithm of the engineers' estimate.

[b]Ranked1 and Ranked2 are the winning bid and the bid ranked in second position, respectively.

total of 96 contracts received one bid, 285 contracts received two, 393 contracts received three bidders, and so on.

Table II illustrates that in highway procurement competition informational asymmetries may be important. As the number of bidders increases, the relative difference between the low bid and the Caltrans estimate falls. The low bid is 11% above the estimate when there is one bidder, and the low bid falls to 14% below the estimate when there are nine or more bidders.

Money left on the table measures the difference between the low and second lowest bid. As expected, the difference declines as the number of bidders increases. However, it does not approach zero. When there are nine or more bidders, the money left on the table is about 6% of the low bid, which suggests that the magnitude of informational asymmetries may be quite large.

Fringe and regular bidders: In total, more than 500 bidders submit a bid at least once. Most of these bidders submit a bid only once, or only on a few occasions. For these bidders, the number of bid observations are too few to make inferences about their behavior in a repeated game setting. We classify these bidders as fringe bidders. On the other hand, there is a small number of bidders that submit bids regularly and win a substantial fraction of contracts. With "regular" bidders we denote the set of the largest 10 firms in dollar value won that submit a bid at least 80 times during the sample period.[32] The number of regular bidders per contract ranges from 0 to 4 and has an average of 0.53. Regular bidders win 25% of the total dollar value awarded and 17% of all contracts. For these 10 regular bidders, we supplement the data with information on the locations of their plants. Then, for each firm, we create a variable

[32]The estimation results and, in particular, the effect of backlog, remain very similar as we vary the definition of regular bidders.

called distance that measures the distance between the contract location and the firm's closest plant.

4.3. *The Effect of Backlog*

During our sample period, the average contract duration is 156 days. We define backlog as the amount of work measured in dollars that is left to do from previously won projects. The backlog variable is constructed in the following way: For every contract previously won, we calculate the amount of work measured in dollars that is left to do by taking the initial size of the contract and multiplying it by the fraction of time that is left until the project's completion date. For contracts that finished prior to the end of the sample period, we use the actual completion date of the contract. For contracts that did not finish by the end of the sample period, we use the planned completion date. Based on this calculation, we determine the total amount of work measured in dollars that is left to do at any given point in time. We standardize the backlog variable by subtracting the bidder specific mean (calculated using daily observations) and dividing this difference by the bidder specific standard deviation. The resulting backlog variable is a number that is comparable across bidders.[33] There is substantial variation in the backlog variable. On average on about 10% of the observations a regular bidder has almost no capacity committed at the letting day, while on about 5% of the observations, the firm is about two standard deviations above its average backlog.

The effect of backlog in a simple dynamic bidding game is described in Jofre-Bonet and Pesendorfer (1999). Using a theoretical model, they examine the case in which bidders with above average backlog levels, or capacity constrained bidders, have a higher cost, in the sense of first order stochastic dominance, than unconstrained bidders. A prediction from the bidding model is that constrained bidders bid less frequently and higher than unconstrained bidders. Alternatively, there may be benefits to performing several contracts simultaneously, which we may call expertise effect. Suppose the expertise effect lowers the cost of additional projects in the sense of first order stochastic dominance. In this latter case, a prediction of the bidding model is that bidders with high backlog bid more frequently and lower than bidders with low backlog. Next, we assess the presence of these opposing intertemporal effects by using a reduced form analysis.

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[33]We experimented with different definitions of the backlog variable. In particular, we also used a variable that measures the total backlog from previously won uncompleted contracts divided by the maximum dollar amount won during the sample period. The estimation results were very similar. We prefer the described specification because we do not have an accurate estimate of the maximum capacity. We also experimented with regional backlog variables. The regional effects appear less important perhaps because capacity and resources can be moved. Therefore, we report the results of the analysis when only the aggregate backlog level is used.

Reduced form estimates: Table III reports nine columns of estimation results for regular bidders. The first three columns report probit estimates of the decision to submit a bid. The fourth to sixth column report Tobit estimates of the bid level decision. The seventh to ninth column report Heckman estimates of the bid level decision. We observe bids only if they are below the reserve price. To apply the Tobit and Heckman analysis we consider a transformation of the bid. The dependent variable equals the reserve price minus the bid and is divided by the engineers' estimate. The dependent variable is negative if the bid is not observed, and it is positive if the bid is below the reserve price. Explanatory variables include contract specific characteristics, such as the estimate and the number of working days, and bidder specific characteristics such as the firm's size, measured as the number of plants in the region, the distance of the bidder's closest plant to the project location, and the bidder's backlog. For each regression, we report three sets of estimates: without any firm specific dummy variables, with a set of firm specific fixed effects, and with a set of firm specific backlog variables.

Backlog has a significant effect in all specifications. The sign of the coefficients suggests the presence of capacity constraints. The magnitude of the effect is substantial. An examination of the coefficients reveals that, on average, a constrained bidder is 50% less likely to submit a bid than an unconstrained bidder. An increase of the backlog from −1 to 1 increases the bid level between 2.5% and 7.6%.[34]

Asymmetries between bidders accounted for by observable variables are important. In addition to capacity effects, asymmetries between bidders due to location and size have significant effects. Distance to the project decreases the probability of submitting a bid and increases the bid level. The size of the bidder, measured by the number of plants within the region, increases the probability of submitting a bid. The effect of size on the bid level decision is negative in the Tobit model and not significant in the Heckman model.

Bidder heterogeneity not captured by our observables is also present. Firm specific fixed effects are included in columns two, five, and eight. In all three specifications we can reject the null of no significant firm fixed effects.[35] A possible explanation is that the paving technology between firms differs beyond what our measures of firm size, backlog, and location capture. Ideally, we would like to have additional variables measuring firm heterogeneity.

Bidder heterogeneity not captured by our observables may account for part of the backlog effect, which could bias our estimates. We test whether the coefficient of the backlog variable changes as we introduce firm specific fixed

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[34]Note that if a firm's backlog changes from −1 to +1, its committed capacity increases from one standard deviation below its average value to one standard deviation above it.

[35]Under the null the test statistic is a chi-squared random variable with nine degrees of freedom in the probit and it equals 366.75. In the Heckman model the test statistic is a chi-squared random variable with 18 degrees of freedom and it equals 388.12. For the Tobit model we constructed an F-test. The test statistic equals 35.35 with (9, 22215) degrees of freedom.

TABLE III
BID SUBSMISSION AND BID LEVEL DECISIONS

Estimation Method:	Probit[a]			Tobit[a]			Heckman[a]		
Dependent Variable:	Bid Submission			(R−Bid)/Estimate[b]			(R−Bid)/Estimate[b]		
Number of Observations:	22230	22230	22230	22230	22230	22230	22230	22230	22230
Chi-square:	1605.65	1984.17	1677.77	1518.99	1883.42	1589.23	420.41	444.64	420.44
Degrees of Freedom:	6	15	15	6	15	15	6	15	15
Log Likelihood:	−4281.26	−4092.39	−4245.60	−3765.05	−3582.84	−3729.93	−3404.46	−3394.35	−3366.71
Variable									
Constant	−2.8485	−3.169	−2.864	−1.1654	−1.2511	−1.1644	0.3093	0.2734	0.3207
	(0.173)	(0.184)	(0.174)	(0.089)	(0.091)	(0.089)	(0.072)	(0.076)	(0.073)
Estimate	0.2905	0.3024	0.2902	0.1235	0.1220	0.1250	0.0040	0.0038	0.0032
	(0.015)	(0.016)	(0.015)	(0.008)	(0.008)	(0.008)	(0.006)	(0.007)	(0.007)
Working Days	−0.3176	−0.3234	−0.3270	−0.1498	−0.1446	−0.1537	−0.0533	−0.0540	−0.0527
	(0.022)	(0.023)	(0.023)	(0.011)	(0.011)	(0.011)	(0.008)	(0.009)	(0.008)
Nbid-Fringe	−0.1835	−0.1913	−0.1885	−0.0882	−0.0875	−0.0905	−0.0613	−0.0599	−0.0624
	(0.027)	(0.027)	(0.027)	(0.013)	(0.013)	(0.013)	(0.007)	(0.008)	(0.008)
Distance	−0.5193	−0.4805	−0.5238	−0.2536	−0.2240	−0.2543	−0.1196	−0.0978	−0.1188
	(0.023)	(0.024)	(0.023)	(0.012)	(0.012)	(0.012)	(0.008)	(0.009)	(0.009)
# Plants within Region	0.1807	0.0513	0.1786	0.0638	0.0078	0.0632	−0.0051	−0.0193	−0.0052
	(0.051)	(0.054)	(0.052)	(0.025)	(0.024)	(0.024)	(0.014)	(0.015)	(0.015)
Backlog	−0.0835	−0.0856	−0.1079	−0.0383	−0.0372	−0.0528	−0.0127	−0.0127	−0.0162
	(0.015)	(0.015)	(0.053)	(0.007)	(0.007)	(0.025)	(0.004)	(0.005)	(0.017)
Firm_2		0.6784			0.2985			0.1204	
		(0.061)			(0.029)			(0.019)	
Firm_3		−0.0338			−0.0081			−0.0223	
		(0.073)			(0.034)			(0.024)	

TABLE III
(CONTINUED)

Estimation Method:	Probit[a]			Tobit[a]			Heckman[a]		
Dependent Variable:	Bid Submission			(R−Bid)/Estimate[b]			(R−Bid)/Estimate[b]		
Number of Observations:	22230	22230	22230	22230	22230	22230	22230	22230	22230
Chi-square:	1605.65	1984.17	1677.77	1518.99	1883.42	1589.23	420.41	444.64	420.44
Degrees of Freedom:	6	15	15	6	15	15	6	15	15
Log Likelihood:	−4281.26	−4092.39	−4245.60	−3765.05	−3582.84	−3729.93	−3404.46	−3394.35	−3366.71
Variable									
Firm_4		0.1499			0.0649			0.0011	
		(0.074)			(0.034)			(0.022)	
Firm_5		−0.0325			−0.0133			−0.0097	
		(0.073)			(0.033)			(0.023)	
Firm_6		−0.1885			−0.0976			−0.0458	
		(0.076)			(0.035)			(0.023)	
Firm_7		0.2011			0.0969			0.0054	
		(0.072)			(0.033)			(0.022)	
Firm_8		−0.0515			−0.0212			−0.0357	
		(0.073)			(0.034)			(0.023)	
Firm_9		−0.2070			−0.0893			−0.0372	
		(0.077)			(0.035)			(0.023)	
Firm_10		0.2277			0.1297			0.0742	
		(0.069)			(0.031)			(0.021)	
Backlog_2			0.0345			0.0214			0.0065
			(0.072)			(0.034)			(0.022)
Backlog_3			−0.0239			0.0033			0.0015
			(0.063)			(0.030)			(0.019)

TABLE III
(CONTINUED)

Estimation Method:	Probit[a]			Tobit[a]			Heckman[a]		
Dependent Variable:	Bid Submission			(R−Bid)/Estimate[b]			(R−Bid)/Estimate[b]		
Number of Observations:	22230	22230	22230	22230	22230	22230	22230	22230	22230
Chi-square:	1605.65	1984.17	1677.77	1518.99	1883.42	1589.23	420.41	444.64	420.44
Degrees of Freedom:	6	15	15	6	15	15	6	15	15
Log Likelihood:	−4281.26	−4092.39	−4245.60	−3765.05	−3582.84	−3729.93	−3404.46	−3394.35	−3366.71
Variable									
Backlog_4			−0.0637			−0.0338			−0.0258
			(0.078)			(0.037)			(0.024)
Backlog_5			0.3193			0.1530			0.0591
			(0.070)			(0.034)			(0.023)
Backlog_6			0.0495			0.0256			−0.0072
			(0.016)			(0.034)			(0.024)
Backlog_7			0.0970			0.0458			0.0128
			(0.074)			(0.035)			(0.023)
Backlog_8			−0.2357			−0.1117			−0.0344
			(0.075)			(0.036)			(0.024)
Backlog_9			0.0546			0.0261			0.0018
			(0.071)			(0.034)			(0.024)
Backlog_10			−0.0195			−0.0075			−0.0096
			(0.071)			(0.030)			(0.022)
Mills Ratio							0.2342	0.2233	0.2319
							(0.011)	(0.011)	(0.011)

[a] All variables except Backlog are in logarithm. The numbers in parenthesis are standard deviations.

[b] (R−Bid)/Estimate denotes the logarithm of the variable (Reserve price minus the Bid) over the engineers' Estimate plus one.

effects. As is evident in Table III, the coefficients do not change significantly. Thus, excluding unobserved firm heterogeneity does not affect the estimates of the backlog effect.

As described in Section 2, our model requires that bidders behave symmetrically conditional on observables. We can test this assumption by including a set of backlog and bidder identity interaction terms. We report these results in columns three, six, and nine of Table III. We find that the interaction terms are not significant for the majority of firms. An examination of individual coefficients reveals that 21 out of 27 coefficients are not significantly different from zero. Although we can reject the null of no significant firm specific backlog effects for all firms jointly in all three columns,[36] we cannot reject the null that firm specific backlog effects are zero for eight of the ten firms in all three specifications.[37] The backlog variable has the same effect for eight of ten bidders. The effect differs for two firms: One exception occurs for firm five, where the sign of the backlog effect is reversed. The other exception occurs for firm eight, where the effect of backlog is of larger magnitude. Overall, we interpret this evidence as partially supportive of our assumption of symmetric behavior conditional on observables.

The estimates in Table III provide preliminary support for our dynamic bidding model in at least three ways: First, capacity constraints appear important. Second, bidders' additional state variables, location, and size, are important. Third, a test of identical backlog effects cannot be rejected for the majority of bidders.

5. ESTIMATION RESULTS

This section discusses the estimates of the econometric model. Section 5.1 reports the estimates of the bid distribution functions. We discuss how well the estimates predict the data. We then illustrate the predicted effect of selected variables. Section 5.2 discusses the estimates of the value function. Section 5.3 discusses the estimates of the bidding function and inferred costs. The estimates suggest that the effect of the backlog variable is in accordance with the expected effect under the presence of capacity constraints. Moreover, the effect is substantial, suggesting that capacity constraints play an important role in highway bidding.

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[36]Under the null, the test statistic is a chi-squared random variable with nine degrees of freedom in the probit and it equals 70.99. In the Heckman model, the test statistic is a chi-squared random variable with 18 degrees of freedom and it equals 74.0. For the Tobit model, we constructed an F-test. The test statistic equals 7.42 with (9, 22215) degrees of freedom.

[37]Under the null, the test statistic is a chi-squared random variable with seven degrees of freedom in the probit and it equals 7.48. The test statistic is a chi-squared random variable with 14 degrees of freedom in the Heckman model and it equals 10.78. For the Tobit model, we construct an F-test. The test statistic equals 0.95 with (7, 22215) degrees of freedom.

5.1. *Estimates of the Bid Distribution Functions*

The parameter estimates are reported in Table IV. Columns one, two, and three report the parameter estimates for regular bids, for fringe bids, and for the lower bound respectively. Columns four, five, and six report the second set of parameter estimates, which include additionally a set of firm identity and backlog interaction variables. The variables entering Table IV are bidders' characteristics and contract characteristics. Bidders' characteristics include the standardized backlog from previously won uncompleted projects, the distance between the closest plant of the bidder to the contract location, and the number of plants within the region of the contract. The contract characteristics include the engineers' estimated cost of the project, the number of working days, the reserve price, and the number of fringe bids. Some fringe bids and a small number of regular bids are (substantially) below the engineer's estimate. We believe that these bids represent erroneous bids by inexperienced bidders, or coding errors and we omit bid observations that are more than 15% below the engineer's estimate.[38] Keeping the bid outliers in the data reduces the lower bound estimate, which amplifies the per period payoff estimates, the value function estimates, and also the effect of the backlog variable on the cost estimates. Thus, to the extent that our data selection rule may influence our estimates, our results provide a conservative assessment of the backlog effect.

As mentioned earlier, we also estimated the bid distribution functions using the consistent but less efficient two stage approach described in Smith (1985). The point estimates are qualitatively similar to the results reported here and are reported in Jofre-Bonet and Pesendorfer (1999).

Goodness of fit: As a measure of the goodness of fit of the model, we randomly draw bids from the estimated distribution of bids and compare them to the observed bids. To account for contract heterogeneity, we normalize bids by dividing them by the reserve price. The estimates predict well the observed distribution of fringe bids. We draw 10,000 fringe bids. On average, the predicted fringe bid equals 78.01% of the reserve price with a standard deviation of 11.71%. The observed fringe bid equals 77.72% of the reserve price with a standard deviation of 11.53%. The difference between the two means is not significant.

The predicted probability of observing a regular bid equals 5.97%. In the data, the probability of observing a regular bid equals 5.96%. The difference between the two numbers is not significant. Conditional on observing a regular bid, the mean predicted bid equals 82.36% of the reserve price with a standard deviation of 12.02%. The observed regular bid equals 79.03% with a standard deviation of 10.40%. We can reject the null hypotheses of equal mean and we reject the null of equal variance. Conditional on observing a regular bid,

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[38]In total we omit 25 regular bids and 273 fringe bids. Low bids occur mostly for small contracts and low regular bids do not arise on contracts with engineer's estimates exceeding $1,000,000.

TABLE IV
PARAMETER ESTIMATES OF THE BID DISTRIBUTIONS

Data:	Regular and Fringe Bids			Regular and Fringe Bids		
Number of Observations:	30,873			30,873		
Log Likelihood:	−1,467.46			−1,500.82		
Variables	Regular Bids	Fringe Bids	Lower Bound	Regular Bids	Fringe Bids	Lower Bound
θ_1	−13.1688			−13.1721		
	(22.689)			(14.779)		
θ_4		−1.3354			−1.3357	
		(0.155)			(0.154)	
Constant	5.6934	1.8821	−0.8228	5.6997	1.8820	−0.8228
	(0.193)	(0.170)	(0.019)	(0.193)	(0.151)	(0.022)
Ln Estimate	−0.3739	−0.0014	1.0541	−0.3767	−0.0014	1.0541
	(0.015)	(0.011)	(0.001)	(0.015)	(0.006)	(0.001)
Ln Working Days	0.2765	−0.0283	−0.0049	0.2813	−0.0283	−0.0049
	(0.021)	(0.018)	(0.002)	(0.021)	(0.019)	(0.002)
Estimate/Reserve_Price	−2.8037	−2.5396	−0.0188	−2.7803	−2.5395	−0.0188
	(0.200)	(0.158)	(0.013)	(0.200)	(0.158)	(0.013)
Nbid-Fringe	0.025	0.0049	0.0011	0.0258	0.0049	0.0011
	(0.006)	(0.008)	(0.001)	(0.007)	(0.006)	(0.001)
Distance	0.0559			0.0562		
	(0.003)			(0.003)		
# of Plants within the Region	−0.3144			−0.3159		
	(0.026)			(0.027)		
Backlog	0.0721			0.0803		
	(0.015)			(0.016)		
Sum_Distance	0.0015	0.0002	−0.0004	0.0015	0.0002	−0.0004
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Sum_# of Plants within the Region	0.1242	−0.0262	−0.0129	0.1271	−0.0262	−0.0129
	(0.012)	(0.013)	(0.002)	(0.012)	(0.014)	(0.002)
Sum_Backlog	0.0108	0.0278	−0.0022	0.0128	0.0277	−0.0022
	(0.006)	(0.007)	(0.001)	(0.006)	(0.007)	(0.001)

TABLE IV
(CONTINUED)

Data:	Regular and Fringe Bids			Regular and Fringe Bids		
Number of Observations:	30,873			30,873		
Log Likelihood:	−1,467.46			−1,500.82		
Variables	Regular Bids	Fringe Bids	Lower Bound	Regular Bids	Fringe Bids	Lower Bound
Backlog_Firm_1				−0.0497		
				(0.045)		
Backlog_Firm_2				0.0530		
				(0.035)		
Backlog_Firm_3				0.0274		
				(0.026)		
Backlog_Firm_4				0.0777		
				(0.059)		
Backlog_Firm_5				−0.2561		
				(0.045)		
Backlog_Firm_6				−0.0416		
				(0.056)		
Backlog_Firm_7				−0.0526		
				(0.046)		
Backlog_Firm_8				0.2264		
				(0.050)		
Backlog_Firm_9				−0.0375		
				(0.028)		

the predicted bid, on average, is higher and has a higher standard deviation. A closer inspection of the distribution of regular bids conditional on observing a regular bid reveals that the difference between the predicted and observed distribution is attributable to a small fall in the number of bid observations close to the reserve price. Indeed, a closer fit is obtained when we estimate the bids' distribution function as a product of a Weibull and a Beta distribution functions, instead of the Weibull distribution function that we use. When using the Weibull–Beta alternative specification, we obtain that, conditional on observing a regular bid, the resulting predicted mean bid equals 78.81% of the reserve price with a standard deviation of 9.81%. For this specification, the observed mean is not significantly different from the predicted mean. The reason for the improvement is that the estimated parameters for the Beta density permit a fall in density close to the reserve price, which improves the fit. But the Weibull–Beta specification violates the monotonicity assumption of the hazard function, which is required by the bidding model. Thus, we decided to impose the monotonicity condition in the estimation and report the estimates obtained using the Weibull distribution function.[39]

The effect of individual variables can be illustrated by evaluating their effect on the probability of submitting a bid at sample average values of explanatory variables. In general, the predicted effect confirms the intuition: The probability of submitting a bid decreases monotonically in backlog, which is consistent with the notion of capacity constraints. An increase in the number of competing fringe firms has a negative effect on the bid submission decision. Distance affects the probability of bid submission negatively, and the number of plants in the region has a positive effect.

Backlog: For regular bidders, the backlog variable enters in the scale parameter θ_2. Our assumption of symmetric bidding behavior conditional on observables requires that backlog have the same effect across bidders. We can test this assumption using our second set of estimates reported in columns four,

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[39]The fall in the number of bid observations close to the reserve price has at least two explanations: First, bidders do not fully understand the announced reserve price R^t. As described before, about 12% of all bid observations violate the reserve price requirement, which indicates that at least some bidders did not interpret the reserve price rule correctly. In general, the reserve price rule is complicated, varying with contract size and equaling a fixed dollar amount and a certain percentage (typically 10%) above the contract's budget. Additionally, a change in the reserve price rule occurred in the middle of the sample period. Second, as described before, Caltrans can use a secret reserve price in addition to the announced reserve price rule, by rejecting bids (below the reserve price) that are deemed too high. The data do not substantiate the importance of the secret reserve price rule, as there are no observed bids below the reserve price that were rejected. However, the lack of data may reflect that the support of the secret reserve price distribution is small and that most winning bids are substantially below the reserve price. Both explanations can account for the fall in observations close to the reserve price. Unfortunately, the data are not rich enough to explore these explanations further. For these reasons, we decided to impose the monotonicity requirement in the estimation and to assume that the support of the secret reserve price distribution is small, $\underline{R}$ equals R, ignoring the role of the secret reserve price.
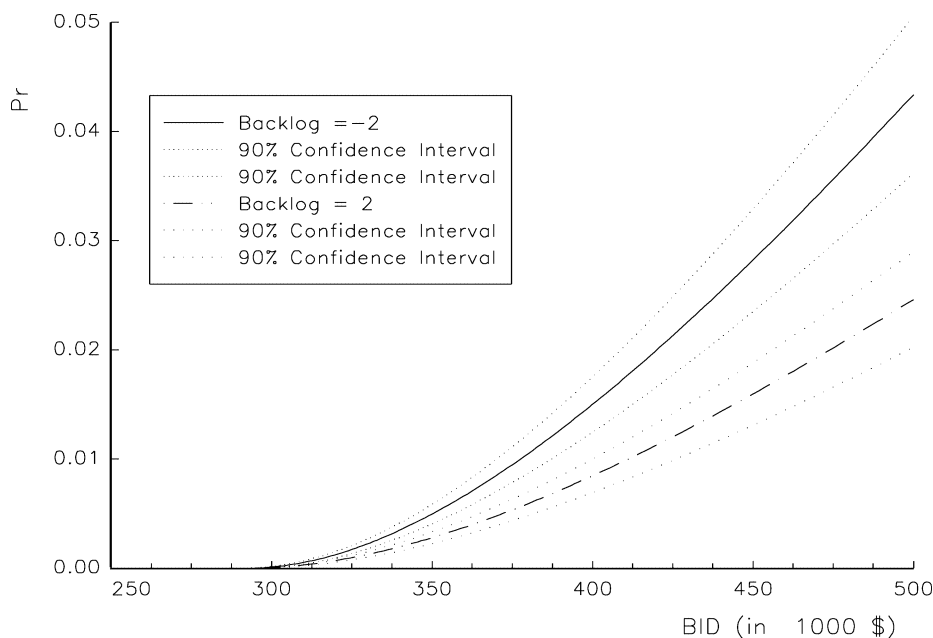
FIGURE 1.—Bid distribution function.

five, and six in Table IV. This second set of coefficients was estimated including additionally a set of bidder identity and backlog interaction variables. We find that differences in the backlog effect between bidders are not significant for the majority of bidders, reinforcing our earlier evidence in Table III. An examination of bidder identity and backlog interaction variables reveals that for seven cases out of nine, the bidder specific backlog coefficients are not significantly different from zero. The two significant effects are for firms five and eight. The null of jointly no bidder specific backlog effects is rejected. The test statistic is a chi-squared random variable with nine degrees of freedom and equals 66.72. However, taking only those seven firms with backlog effects not significantly different for zero, the null of jointly no significant backlog effects cannot be rejected. The test statistic is a chi-squared random variable with seven degrees of freedom and equals 7.88.

The effect of the backlog variable on the bid distribution of regular bidders is illustrated in Figure 1. It shows the bid distribution function between the lower bound of bids and the reserve price, evaluated at sample average values of state variables. Two bid distribution functions are reported: The solid function assumes a backlog equal to -2 (unconstrained bids) and the dashed function assumes a backlog equal to 2 (constrained bids). The dotted lines represent 90% confidence intervals. The confidence interval in Figure 1 (and all subsequent estimates' standard errors) are calculated using the delta method. The figure

illustrates that the distribution of constrained bids stochastically dominates, in the first order sense, the distribution of unconstrained bids. On average, unconstrained bidders are about twice as likely to submit a bid than constrained bidders. This finding confirms the importance of capacity constraints.

5.2. *Estimates of the Value Function*

The arguments of the value function are the bidders' state variables. Each bidder's state consists of a three dimensional vector consisting of the bidder's backlog, plant locations, and number of plants in all regions.

We approximate a value function for each bidder i separately. Doing so allows us to ignore bidder i's plant locations as an argument entering the value function. The reason is that bidder i's *plant locations do not change over time* and their level effect is accounted for in the per period payoff function. For the same reason we can ignore bidder i's size measure (number of plants in the region) as an argument in the value function. Thus, the only bidder i's state variable that remains in bidder i's value function approximation is *bidder i's backlog*. Similarly, competitors' plant locations and number of plants do not change over time and their direct effect on the value function is accounted for by estimating a value function for each bidder separately.

To simplify the dimensionality of the value function approximation further, we make use of the *symmetry* property. As explained in Section 3, symmetry implies that the coefficients associated with competitors' state variables in a polynomial approximation of the value function will be identical. However, imposing the symmetry restriction requires that we explicitly take into account the indirect effect manifested in the interaction terms involving backlog and the competitor's distances to the contract location and size within the region, which does change from period to period after the contract is realized. To account for this indirect effect, we include the average distance to contracts and the average number of plants per region interacted with backlog for every competitor as an additional variable in the value function. These averages are calculated by using the observed information. We assume that these variables adequately measure the distribution of distances and the distribution of the number of plants per region.

The variables entering the value function are bidder i's backlog and three variables for each competitor consisting of the competitor's backlog, the competitor's backlog interacted with average distance, and the competitor's backlog interacted with average number of plants per region.

The approximation of the value function for bidder 3, is depicted in Figure 2. We arbitrarily select bidder 3, who is the third largest bidder in dollar value won. We assume the projects are equally spaced over time and since our data contain on average 709 projects per year this assumption implies that a project
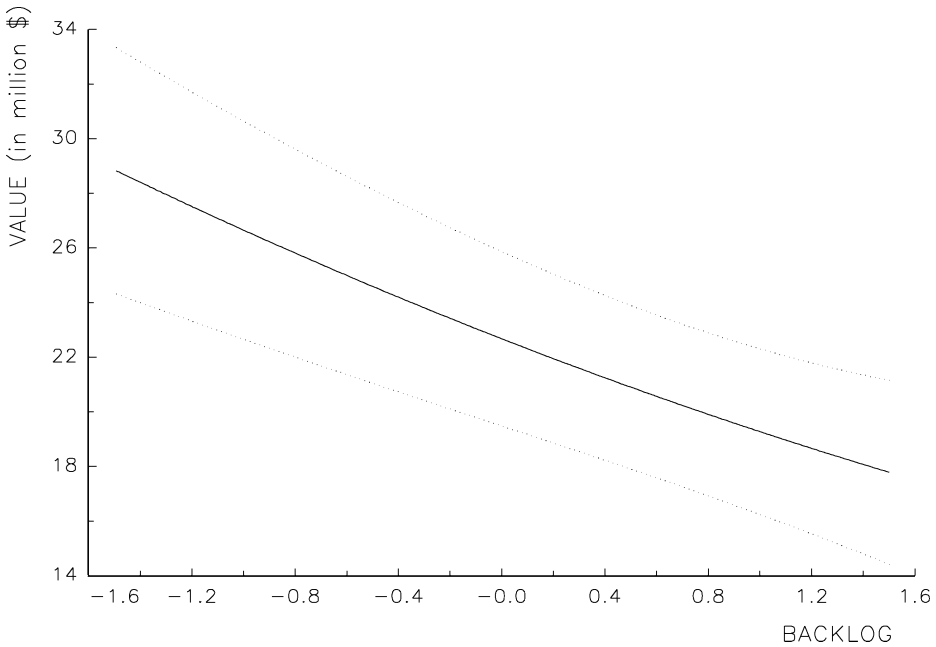
FIGURE 2.—Value function.

is offered for sale about every 12 hours.[40] We impose an annual discount factor of 0.80. We chose the 99th percentile of the distribution of winning bids as the lower endpoint of the support of the reserve price distribution. This percentile amounts to $0.85 \cdot R^t$. In the figure the competitors' state variables are fixed at their sample averages. The plot illustrates the discounted expected future profit of bidder 3 by varying the backlog variable of bidder 3 between -1.6 and 1.6. The dotted lines depict the 90% confidence interval calculated using the delta method.[41]

The average discounted sum of payoffs for bidder 3 equals $25 million. Other regular bidders' discounted sum of payoffs ranges between $7 million and $73 million. The assumed annual discount factor of 0.8 is a conservative measure for the effect of the future. Increasing the annual discount factor from 0.8 to 0.9 increases the discounted sum of payoffs by a factor of 2, approximately. The lower bound of the secret reserve price distribution affects the calculated value function as well. Increasing the lower bound from $0.85 \cdot R^t$ to

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[40]To permit randomness in the timing of projects, a shorter period length could be used and it could be assumed that with a certain probability no project is offered for sale.

[41]To calculate the derivative of the value function with respect to the parameter vector, we take the derivative in equation (3.3), which yields $V_i' = A_i' + \beta \sum B_{ij}' V_i + \beta \sum B_{ij} V_i'$. We numerically calculate the derivatives A_i' and B_{ij}' and the derivative of V_i' is then given by $V_i' = [A_i' + \beta \sum B_{ij}' V_i]/[1 - \beta \sum B_{ij}]$.

$0.9 \cdot R^t$ shifts the value function up by \$1.3 million on average. The shift ranges between \$1.1 million and \$1.45 million across backlog levels. The accuracy of the computations depends on the fineness of the selected grid of states. Doubling the number of state points, from 50 to 100, alters the value function of bidder 3 by a very small amount only. The difference between the value function based on 50 and based on 100 states equals at most 3.4% of its value, or \$840,000. Additionally, the accuracy of the computations depends on the selected random sample of contracts $\hat{S}_0$. To assess its importance we calculate a sample average value function obtained by drawing ten times the set $\hat{S}_0$ from the set of observed contracts instead of drawing it once only. By repeatedly drawing $\hat{S}_0$, the value function of bidder 3 shifts by \$1.64 million on average. The shift ranges between \$1.41 million and \$1.75 million across the backlog range.

The effect of backlog on the value function in Figure 2 is negative, which is in accordance with the expected effect under capacity constraints. In Figure 2, backlog reduces the value function in total by about 35%. Value function estimates for other bidders are of different magnitude, but in general of similar shape. An exception is bidder 4 for which the value function increases initially and then decreases as backlog increases. To illustrate the backlog effect for the average bidder, we regress the value functions of bidders evaluated at the 50 states on a set of bidder specific dummy variables and the state variables. To capture possible nonlinearities, we include both linear and quadratic coefficients of variables. We find that the predicted value function is about 39% higher if backlog equals -2 than if backlog equals 2, it is about 30% higher if backlog equals -1 than if backlog equals 2, and about 20% higher if backlog equals 0 than if backlog equals 2.

5.3. *Estimates of Costs*

First, we illustrate the equilibrium bid functions. Then, we describe markups and cost estimates.

The equilibrium bid function for bidder 3 is illustrated in Figure 3. The bid function is estimated using equation (3.1). The bid function is plotted by fixing the state variables at sample average values for bidder 3 and varying the cost. In addition to the bid function, the 45 degree line is reported. As is evident in the figure, the bid increases with the cost. In addition, the distance between the bid and the cost decreases as the cost increases. However, as the cost increases the bid does not approach the 45 degree line. The reason is the mark-up attributable to the negative effect on the future discounted profits if firm i wins the contract instead of another firm. The bid function is of similar shape for other contract characteristics and state variables. In some instances, the difference between bids and costs is large in magnitude at the lower end of the support of bids, which can result in a negative cost. We find negative costs implausible and set the cost to zero whenever the inferred cost would be negative.
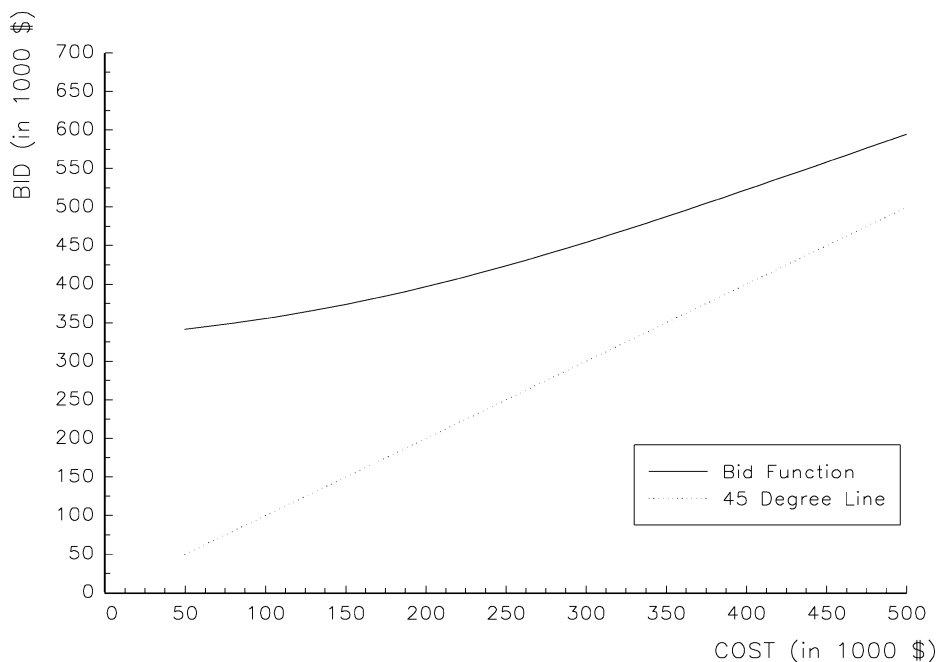
FIGURE 3.—Bid function.

The mark-up denotes the difference between the bid and the cost of a bidder. In the figure, the mark-up is the distance between the bid and the 45 degree line. An examination of all observed bids by bidder 3 reveals that the median estimated mark-up for this bidder equals 26.0% of the bid. The mean mark-up is higher and equals 35.6% of the bid. The estimated mark-up differs across bidders. The median mark-up across all observed regular bids equals 32.6% and the mean mark-up equals 40.3%. Although the magnitude of the mark-up may appear large, it appears in accordance with descriptive evidence in Table I and Table II. The difference between the lowest and second lowest bid is, on average, 9% of the value of the bid.

A substantial portion of the mark-up of regular bidders is attributable to the loss in future discounted value due to limited capacity. This loss reflects the cost of winning today versus winning later. We can measure this loss based on equation (3.1), which decomposes the mark-up into two parts: The first part reflects *contemporaneous competition*. The second part measures the loss in value of winning today versus winning later. For bidder 3, on average, across all observed bids, 64.2% of the mark-up is attributable to the second part, which is the *option value* of winning today versus winning later. The number varies across bidders. Across all regular bidders 55.9% of the mark-up is attributable to the second part.
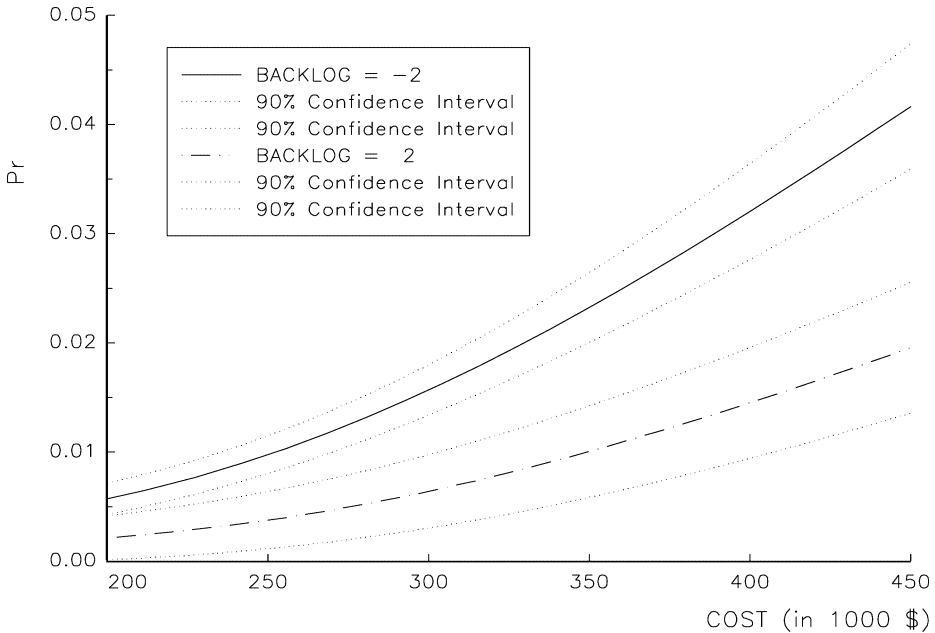
FIGURE 4.—Cost distribution function.

The mark-up estimates depend on the assumed discount factor. As we increase the annual discount factor, from 0.8 to 0.9, the following changes take place: For bidder 3 the median estimated mark-up becomes 44.5% of the bid and the mean mark-up increases to 51.1% of the bid. The median mark-up across all observed regular bids becomes 45.9% and the mean mark-up equals 50.6%. Finally, when the discount factor is 0.9, the option value for bidder i accounts for about 74.5% of the mark-up, and across all regular bidders for about 69.8%.

Cost estimates: Figure 4 depicts the distribution function of costs for bidder 3. Distribution functions are reported for two values of backlog and holding other state variables at sample average values. The backlog values are -2 and 2. The dotted lines represent 90% confidence intervals, which are calculated using the delta method. The estimated cost distribution functions are truncated at the point at which the equilibrium bid corresponding to the cost equals the reserve price and are reported for a common range of costs. Figure 4 documents that the cost distribution of the constrained bidder stochastically dominates in the first order sense the cost distribution of the unconstrained bidder. On average, the probability that the cost is below a certain threshold is more than twice when the bidder is unconstrained than when the bidder is constrained. Similar to our earlier evidence in Table IV, we find that cost realization depends not only on the bidder's own state variables, but also on other bidders' state vari-

ables. A possible explanation is that shortages of paving workers or shortages of paving equipment caused by high backlog levels of bidders can increase the contract costs for all bidders.

6. THE EFFECT OF BACKLOG AND INEFFICIENCIES

This section reports three applications of the estimates: First, we measure the benefits to the auctioneer of increasing capacity of all bidders simultaneously. Second, we measure the short run benefits of individual bidders of an increase in their capacity. Third, we determine to what extent the auction rule does not select the low cost bidder due to the presence of capacity constraints and bidder asymmetries. We quantify the magnitude of resulting inefficiencies.

6.1. *Price Effect*

We have shown that individual regular bids, costs, and future profits depend on the backlog level. Next, we report the hypothetical effect on the price paid for the service by the auctioneer as the backlog level of all regular bidders is increased simultaneously. We may interpret the effect as the benefit to the auctioneer of an increase in capacity of all bidders. We report the price effect for a hypothetical contract on which the fringe bidders are absent. Due to the absence of fringe bidders, the price equals the low regular bid, which is the low bid submitted from the set of regular bids. The reason why we select a contract on which the fringe bidders are absent is that their dependence on backlog was not modeled, as fringe bidders' backlog level is not observed in our data.

To compute the price effect we conduct the following exercise: We select a contract with an engineers' cost estimate equal to the sample average. For each regular firm, we randomly draw a bid from the bid distribution under the assumption that all firms' backlog equals -2. Then, we calculate the price that equals the low bid from this set of regular bids. Similarly, we randomly draw bids from the bid distributions of regular bidders under the assumption that their backlog equals 2. Then, we determine this draw's low regular bid. We repeat this sampling procedure to obtain 1,000 observations. Finally, we compare the distribution of prices between both cases.

The price effect is substantial. When backlog of all regular bidders equals -2, the average price equals \$539,845. When backlog equals 2, the average price increases to \$657,016. The difference in the mean is significant and equals about 18% of the average at the -2 backlog level. The shift in backlog from 2 to -2 is achievable with a 114% increase in regular bidders' capacity.

We repeat the above calculations for backlog levels equal to -1 and 1. The difference in the average of the price between backlogs of -1 and $+1$ is significant and equals about 12% of the average price at a backlog of -1. The shift in backlog from 1 to -1 is achievable with a 57% increase in regular bidders' capacity. We also conducted the experiment for other engineers' cost estimates with similar results.

We can conclude that the benefits to the auctioneer of an increase in available bidder capacity is substantial. Next we examine the effect of available capacity on bidders' returns.

6.2. *Period Returns*

We illustrate the effect of available capacity on the ex ante expected period return of an individual regular bidder. The benefits of an additional unit of capacity is an important part in the equation that determines the optimal capacity choice as a function of the returns to capacity and the cost of investing in capacity. The cost of additional capacity in the highway paving industry is difficult to measure and we do not attempt to quantify it. However, our estimates permit us to assess the short run bidder returns due to a reduction in backlog and holding total bidder capacity fixed. We calculate the ex ante expected period bidder return before contract characteristics and costs are observed. We emphasize that the exercise looks at one auction only. The long run effect of backlog on bidders' returns was illustrated earlier in the discussion of the value function estimates in Section 5.2 and exemplified in Figure 2.

The period returns are defined as the first expression on the right-hand side in equation (2.1). Due to our conditional symmetry assumption, variations in period returns are entirely determined by variations in the state variables. To assess bidder specific returns, we take a sample of 50 draws from the observed distribution of states for each regular bidder and we obtain the short run bidder return corresponding to each one of the 50 states, for each regular bidder. Our calculations indicate that regular bidders expect to receive $16,200 per contract on average, but there is substantial variation in ex ante expected returns for individual regular bidders ranging between $790 and $45,900 across bidders.

The effect of available capacity on ex ante expected returns can be illustrated by regressing these ex ante period returns on a set of bidder specific dummy variables and the state variables. To capture possible nonlinearities we include both linear and quadratic coefficients of the state variables. Using the estimated backlog coefficient, we conduct counterfactual predictions by varying the backlog level, while holding the other variables constant at sample average.

We find that the ex ante expected period payoffs are about 95% higher if backlog equals -2 than if the backlog equals 2, they are about 56% higher if backlog equals -1 than if the backlog equals 2, they are about 26% higher when backlog equals 0 than if the backlog equals 2, and they are about 8% higher when backlog equals 1 than if the backlog equals 2. If we compare the effects to our earlier results on the value function, we can conclude that the effect of capacity is of larger magnitude in the short run than in the long run.

The illustration shows that available capacity is important in the bidders calculus of bidding.

6.3. *Inefficiencies*

Next, we assess the magnitude of the inefficiencies. Notice, that due to the presence of intertemporal effects and due to bidder heterogeneity, a first-price auction need not select the efficient firm. The bidder with the lowest bid need not be the bidder with the lowest cost. The reason is that constrained (or smaller) bidders may bid more aggressively than unconstrained (or larger) bidders. The strategic bid shading can imply that a constrained firm wins although it did not have the lowest cost.

To assess the magnitude of inefficiencies at auction t, we conduct the following experiment: We select the firm that minimizes costs at auction t and take as given that the first-price auction is used at auction $t + 1$ and onwards. This selection rule gives us a lower bound on the efficiency loss.[42] To assess which firm to select, we take into account contemporary and future costs. Contemporary costs are those implied by the observed bids. The discounted sum of expected future costs, V^c, is approximated using the estimates reported in Section 5. Specifically, the discounted sum of expected future costs can be written as

$$
V^c(s) = E_{s_0} \left\{ \int \phi(., s_0, s) dG_{(1)}(.|s_0, s) \right.
$$

$$
\left. + \beta \sum_{j=0}^{n} \Pr(j \text{ wins}|s_0, s) V^c(\omega(s_0, s, j)) \right\},
$$

where $G_{(1)}(.|s_0, s)$ denotes the distribution function of the winning bid on contract s_0 with state s. We evaluate V^c numerically in the same way as the value function. The low cost firm is the firm j that minimizes $c_j + \beta V^c(\omega(s_0, s, j))$.

Table V reports the frequency and the inefficiencies' dollar amounts associated with the experiments. We perform the experiment for all observed contract characteristics, except contracts that we omitted in the estimation of the bid distribution functions due to suspiciously low bids, and using the observed bid data. Inefficiencies are reported as a fraction of the initial engineers' estimate. On 32% of all experiments, an inefficient bidder is selected. We decompose the efficiency loss into contemporary and future cost effects. We find that the inefficiency arises because a lower cost firm could have been chosen in auction t. The effect on future costs is negligible. The average efficiency loss across experiments amounts to 13% of the engineers' estimate. In dollar value, this amounts to $305,500 per experiment, on average. In addition to the overall results, Table V reports efficiency losses for a range of selected engineers'

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[42]The full cost minimizing problem is a dynamic decision problem involving ten state variables, one backlog variable for each regular bidder. This problem is too complex for the computing techniques available.

TABLE V
ESTIMATES OF EFFICIENCY LOSSES

Variable	Range of Engineers' Estimate (in $ 1,000)					Overall
	[0, 100]	(100, 400]	(400, 1000]	(1000, 5000]	(5000, .]	
All Contracts:						
Number of Contracts	49	893	525	545	184	2196
Prob of an Inefficiency	0.02	0.52	0.17	0.24	0.11	0.32
Average Efficiency Loss[a]	0.04	0.08	0.14	0.17	0.19	0.13
Contract Won by a Regular Bidder:						
Number of Contracts	1	90	94	147	42	374
Prob of an Inefficiency	0.00	0.20	0.55	0.38	0.27	0.36
Average Efficiency Loss[a]	0.36	0.29	0.29	0.22	0.24	0.25
Contract Won by a Fringe Bidder:						
Number of Contracts	48	803	431	398	142	1822
Prob of an Inefficiency	0.02	0.56	0.08	0.19	0.06	0.31
Average Efficiency Loss[a]	0.04	0.06	0.11	0.15	0.18	0.10

[a] Efficiency losses are reported as a fraction of the engineers' estimate.

estimate values. In general, inefficiencies arise for small and large contracts. Nevertheless, inefficiencies, measured in percent of engineers' estimate, are of larger magnitude for larger contracts. Table V also reports efficiency losses for two subsets of the data: contracts won by regular bidders and contracts won by fringe bidders. Table V illustrates that the probability of inefficiencies is about the same if a contract is won by a regular bidder than when it is won by a fringe bidder.

7. CONCLUSIONS

This paper proposes an estimation method for a repeated auction game under the presence of capacity constraints and bidder asymmetry. We apply the method to highway procurement auctions in California. We characterize costs as a function of state variables and illustrate the bidding equilibrium. The data suggests the presence of capacity constraints. Bidders that have a large fraction of their capacity committed have, on average, higher costs than bidders with little capacity committed. We find that when all bidders are capacity constrained, the resulting price paid by the auctioneer is about 18% higher than when all regular bidders are unconstrained. Moreover, an individual bidder expects to receive about twice as much from an auction in which the bidder is unconstrained than from an auction in which the bidder is constrained.

There are at least two policy implications from our analysis: First, scheduling and timing of contracts offered for sale influences the final price. Thus, optimal scheduling taking the endogeneity of capacity choices into account may save costs. Second, due to intertemporal constraints and bidder heterogeneity,

an inefficient firm may be chosen. Our experiments indicate that inefficiencies may arise on about 32% of all contracts and they may amount to 13% of the expected contract size. Our estimates suggest that auction rules that cope better with intertemporal effects and bidder asymmetry could be a cost saving alternative.

There are two shortcomings of our estimation method: First, our estimation method relies on the assumptions that bidders completely understand the auction environment and that our estimates of winning probabilities correctly capture bidders' perceived winning odds. While we feel that these assumptions are reasonable in our context, we emphasize that our data are too limited to test the adequacy of these assumptions beyond some regularity conditions that our data do satisfy. It may be fruitful to assess the accuracy of the models' predictions with data on independent and reliable information on the perceived winning odds and bidders' cost estimates. Second, our estimation method does not identify the discount factor. The nonidentification problem of the discount factor is similar to other dynamic estimation approaches; see Rust (1994).

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APPENDIX

The appendix contains the proofs to all propositions in the text as well as an auxiliary result.

SECTION 3.1—EQUATION (3.1): The probability that bidder i assigns to the event that bidder j wins the contract when bidder i bids b, can be written as

$$\int_{\underline{b}}^{b} g(x|s_0, s_j, s_{-j}) \prod_{l \neq i,j} [1 - G(x|s_0, s_l, s_{-l})] dx.$$

The first order condition of equilibrium bids by a regular bidder is given by

$$(A3.1) \quad [b-c] \cdot \sum_{j \neq i} \prod_{l \neq i,j} [1 - G(b|s_0, s_l, s_{-l})][-g(b|s_0, s_j, s_{-j})] + \prod_{j \neq i} [1 - G(b|s_0, s_j, s_{-j})]$$

$$+ \beta V_i(\omega(s_0, s, i)) \sum_{j \neq i} \prod_{l \neq i,j} [1 - G(b|s_0, s_l, s_{-l})][-g(b|s_0, s_j, s_{-j})]$$

$$+ \beta \sum_{j \neq i} \left[g(b|s_0, s_j, s_{-j}) \prod_{l \neq i,j} [1 - G(b|s_0, s_l, s_{-l})] \cdot V_i(\omega(s_0, s, j)) \right] = 0.$$

We can rearrange this expression by dividing by $\prod_{l \neq i}[1 - G(b|s_0, s_l, s_{-l})]$. This yields

$$(A3.2) \qquad [b - c] \cdot \sum_{j \neq i} \frac{-g(b|s_0, s_j, s_{-j})}{1 - G(b|s_0, s_j, s_{-j})} + 1 + \beta V_i(\omega(s_0, s, i)) \sum_{j \neq i} \frac{-g(b|s_0, s_j, s_{-j})}{1 - G(b|s_0, s_j, s_{-j})}$$

$$+ \beta \sum_{j \neq i} \frac{g(b|s_0, s_j, s_{-j})}{1 - G(b|s_0, s_j, s_{-j})} \cdot V_i(\omega(s_0, s, j)) = 0.$$

The hazard function,

$$h(.|s_0, s_j, s_{-j}) = \frac{g(.|s_0, s_j, s_{-j})}{1 - G(.|s_0, s_j, s_{-j})}$$

can be substituted into this equation. This substitution yields equation (3.1).

The first order condition for optimal bids by a fringe bidder is obtained in an analogous way. In particular, evaluating (A3.2) at $V_i = 0$ gives the corresponding first order condition of a fringe bidder.

PROOF OF PROPOSITION 1: To establish the claim in the proposition, we show that the value function, defined in equation (2.1), has a representation in terms of the distribution of bids. There are two steps involved: First, we make use of the first order condition of optimal bids. This condition provides us with an explicit expression of the cost in terms of bids and the value function. Second, we change the range of integration from the cost space to the bid space. The resulting equation (3.2) characterizes the value function as an infinite sum.

Note that the probability that bidder i assigns to the event that bidder j wins the contract when bidder i bids b, $\Pr(j \text{ wins}|b, s_0, s_i, s_{-i})$, can be written as: $\int_{\underline{b}}^{b} g(x|s_0, s_j, s_{-j}) \prod_{l \neq i, j}[1 - G(x|s_0, s_l, s_{-l})]dx$. In an abuse of notation, we denote by $b(c)$ the equilibrium bid by bidder i on contract s_0 if the state is s. We can substitute the first order condition for optimal bids, equation (3.1), into the value function, equation (2.1). Notice that we can use the first order condition only for costs such that the equilibrium bid is below the reserve price. If the equilibrium bid equals or exceeds the reserve price, then the bid is rejected and the current period payoff equals zero. We account for this distinction explicitly by including an indicator function $\mathbf{1}_A$ that equals one if event A is true and equals zero otherwise. Doing so yields

$$V_i(s) = E_0 \left\{ \int \left[\frac{1 - \beta \sum_{j \neq i} h(b(c)|s_0, s_j, s_{-j})[V_i(\omega(s_0, s, i)) - V_i(\omega(s_0, s, j))]}{\sum_{j \neq i} h(b(c)|s_0, s_j, s_{-j})} \right. \right.$$

$$\times \Pr(i \text{ wins}|b(c), s_0, s_i, s_{-i})\mathbf{1}_{\{b(c) \leq R(s_0)\}} + 0 \cdot \mathbf{1}_{\{b(c) > R(s_0)\}}$$

$$\left. \left. + \beta \sum_{j=0}^{n} \Pr(j \text{ wins}|b(c), s_0, s_i, s_{-i})V_i(\omega(s_0, s, j)) \right] \cdot f(c|s_0, s_i, s_{-i})dc \right\}.$$

Note that the expression $\int \beta \sum_{j=0}^{n} \Pr(j \text{ wins}|b(c), s_i, s_{-i})V_i(\omega(s_0, s, j))f(c|s_0, s_i, s_{-i})dc$ is the ex ante expected value of $\beta \sum_{j=0}^{n} \Pr(j \text{ wins}|s_0, s_i, s_{-i})V_i(\omega(s_0, s, j))$, and that the term involving bids above the reserve price, $0 \cdot \mathbf{1}_{\{b(c) > R(s_0)\}}$, vanishes. Furthermore, the expression

$$\frac{\sum_{j \neq i} h(b(c)|s_0, s_j, s_{-j})V_i(\omega(s_0, s, i))}{\sum_{j \neq i} h(b(c)|s_0, s_j, s_{-j})}$$

reduces to $V_i(\omega(s_0, s, i))$ and cancels with the second term involving $V_i(\omega(s_0, s, i))$. By making

these changes, we can write the value function as

$$V_i(s) = E_0 \left\{ \int \left[\frac{1 + \beta \sum_{j \neq i} h(b(c)|s_0, s_j, s_{-j}) V_i(\omega(s_0, s, j))}{\sum_{j \neq i} h(b(c)|s_0, s_j, s_{-j})} \right] \right.$$

$$\times \Pr(i \text{ wins}|b(c), s_0, s_i, s_{-i}) \cdot \mathbf{1}_{\{b(c) \leq R(s_0)\}} f(c|s_0, s_i, s_{-i}) dc$$

$$\left. + \beta \sum_{j \neq i} \Pr(j \text{ wins}|s_0, s_i, s_{-i}) V_i(\omega(s_0, s, j)) \right\}.$$

Next consider a change of variable of integration from c to b. Notice that

$$db = \frac{\partial b(c)}{\partial c} dc.$$

Let b^{-1} denote the inverse function of the equilibrium bid function. By assumption the inverse bidding function exists for bids below the reserve price. The inverse bidding function allows us to write the distribution function of cost in terms of the distribution functions of bids. Specifically, $F(b^{-1}(b)|s_0, s_i, s_{-i}) = G(b|s_0, s_i, s_{-i})$. Taking the partial derivative yields a relationship between the density of costs and bids:

$$f\left(b^{-1}(b)|s_0, s_i, s_{-i}\right) \cdot \frac{\partial b^{-1}(b)}{\partial b} = g(b|s_0, s_i, s_{-i}).$$

Also, notice that

$$\frac{\partial b^{-1}(b)}{\partial b} = \frac{1}{\partial b(c)/\partial c}.$$

Finally, the probability that bidder i wins can be written as $\Pr(i \text{ wins}|b, s_0, s_i, s_{-i}) = \prod_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})]$. Applying the change of variables in the above equation yields

$$V_i(s) = E_0 \left\{ \int_{\underline{b}}^R \frac{\prod_{k \neq i}[1 - G(b|s_0, s_k, s_{-k})]}{\sum_{j \neq i} h(b|s_0, s_j, s_{-j})} g(b|s_0, s_i, s_{-i}) db \right.$$

$$+ \beta \int_{\underline{b}}^R \left[\sum_{j \neq i} \frac{h(b|s_0, s_j, s_{-j}) \cdot V_i(\omega(s_0, s, j))}{\sum_{l \neq i} h(b|s_0, s_l, s_{-l})} \right] \prod_{k \neq i}[1 - G(b|s_0, s_k, s_{-k})]$$

$$\times g(b|s_0, s_i, s_{-i}) db$$

$$\left. + \beta \sum_{j \neq i} \Pr(j \text{ wins}|s_0, s_i, s_{-i}) V_i(\omega(s_0, s, j)) \right\}.$$

Observe that the expression in the second and third lines of the value function can be rewritten, by taking the sum $\sum_{j \neq i}$ outside, multiplying the expression by

$$\frac{1 - G(b|s_0, s_i, s_{-i})}{1 - G(b|s_0, s_i, s_{-i})} \quad \text{and} \quad \frac{g(b|s_0, s_j, s_{-j})}{g(b|s_0, s_j, s_{-j})},$$

and rearranging terms, as

$$\beta \sum_{j \neq i} \left[\int_{\underline{b}(s)}^R \frac{h(b|s_0, s_i, s_{-i})}{\sum_{l \neq i} h(b|s_0, s_l, s_{-l})} \prod_{l \neq j}[1 - G(b|s_0, s_l, s_{-l})] \cdot g(b|s_0, s_j, s_{-j}) db \right]$$

$$\times V_i(\omega(s_0, s, j)).$$

Finally, observe that $\prod_{l \neq j}[1 - G(b|s_0, s_l, s_{-l})]g(b|s_0, s_j, s_{-j})$ is the derivative of the probability that a bid of bidder j is the low bid, $dG^{(j)}(b|s_0, s)$, which yields the expression (3.2) in the proposition. *Q.E.D.*

PROOF OF PROPOSITION 2: (i) Fix β. Suppose there are two cost distribution functions $F_1(.|.)$ and $F_2(.|.)$ such that both distributions lead to the same bid distribution function $G(.|.)$ on the interval $[\underline{b}(s_0^t, s^t), R^t]$. By strict monotonicity of (3.1) we have

$$\underline{C}_i(s_0^t, s_i^t, s_{-i}^t) = \phi\big(\underline{b}(s_0^t, s^t)|s_0^t, s_i^t, s_{-i}^t, \beta, G\big).$$

Thus the lower end point of the support is the same for $F_1(.|.)$ and $F_2(.|.)$. Further monotonicity implies that

$$F_i(c|s_0^t, s_i^t, s_{-i}^t) = G\big(\phi^{-1}(c|s_0^t, s_i^t, s_{-i}^t, \beta, G)|s_0^t, s_i^t, s_{-i}^t\big).$$

It follows that $F_1 = F_2$ on the interval $[\underline{C}(.), \phi(R^t|.)]$.

(ii) Pick β_1, β_2, with $\beta_1 \neq \beta_2$. By the argument given in (i) there is a unique distribution function $F_1(.|.)$ for β_1 that leads to the bid distribution function $G(.|.)$ and a distribution function $F_2(.|.)$ for β_2 that leads to the bid distribution function $G(.|.)$. Therefore, (β_1, F_1) and (β_2, F_2) lead to the same bid distribution function and are observationally equivalent. *Q.E.D.*

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