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ESTIMATION OF ADDITIVE ERROR IN MIXED SPECTRA FOR STABLE PROCESSES

Rachid Sabre [1]

Laboratory LE2I UMR6306, CNRS, University of Bourgogne, Agrosup, Department DSIP, Dijon, France.

1. INTRODUCTION

In this paper, the class of symmetric alpha stable processes have been considered. It is a particular family of processes with infinite energy. Theory of these processes have been covered in numerous papers including Cambanis (1983), Cambanis and Maejima (1989), Marcus and Shen (1989), Masry and Cambanis (1984), Samorodnitsky and Taqqu (1994), Panki and Renming (2014), Zhen-Qing and Longmin (2016) to name a few. Symmetric alpha processes are considerably accurate models for many phenomenons in several fields such as: physics, biology, electronic and electric, hydrology, economies, communications and radar applications, see Sousa (1992), Shao and Nikias (1993), Nikias and Shao (1995), Kogon and Manolakis (1996), Azzaoui *et al.* (2002), Montillet and Kegen (2015), Pereyra and Batalia (2012), Zhong and Premkumar (2012), Ligang and Zidong (2015), Panki *et al.* (2017), Brice *et al.* (2017).

In this work, a stationary symmetric α stable harmonizable process is precisely discussed, $Z = \{Z_n : n \in \mathbb{Z}\}$. Alternatively Z has the integral representation

$$Z_n = \int_{-\pi}^{\pi} \exp[i(n\lambda)]\, d\xi(\lambda),$$

where $1 < \alpha < 2$ and ξ is a complex valued symmetric α-stable random measure on $\mathbb{R}$ with independent and isotropic increments. The measure defined by $m(A) = |\xi(A)|_\alpha^\alpha$ (see Masry and Cambanis, 1984) is called "control" measure or spectral measure. Suppose that this measure is absolutely continuous with respect to Lebegue measure: $md(x) = \phi(x)dx$. The function ϕ is called the spectral density. The spectral density function was already estimated by Masry and Cambanis (1984), when the time of the process is continuous, by Sabre (1995) when the time of the process is discrete and by Sabre (2012) when the time of the process is p-adic.
In this work, we consider the case where can not be observed this process without an unknown constant error. The process $X_n = a + Z_n$ is observed instead of the process Z alone.

[1] E-mail: r.sabre@agrosupdijon.fr

Our goal is to estimate this constant a, when the spectral measure is the sum of an absolutely continuous measure with respect to Lebesgue measure and a discrete measure

$$d\mu(\lambda) = \phi(x)dx + \sum_{i=1}^{q} c_i \delta_{w_i},$$

where δ is a Dirac measure, ϕ is nonnegative integrable and bounded function, c_i is unknown positive real number and w_i is unknown real number. Assume that $w_i \neq 0$. We study the case where the spectral density is zero at origin, particularly at $\phi(\lambda) = \sin^{2k\alpha}\left(\frac{\lambda}{2}\right) g(\lambda)$ and $\phi(\lambda) = |\lambda|^\beta g(\lambda)$. We show that the rate of convergence is improved in accordance with the value of β.

This paper is organized as follows. Section 2 gives some definitions and proprieties of symmetric stable processes and an estimator of the constant a is defined. We show that this estimate converges in probability to a. Then we show that the estimate converges to a in L_p $(p < \alpha)$ which will replace the convergence in mean square because the second moment of the processes is infinite. In Section 3, the spectral density of Z is assumed vanishing at origin precisely: $\phi(\lambda) = |\lambda|^\beta g(\lambda)$. We improve the rate of convergence in accordance with the values of β. Section 4 is reserved to numerical studies. Section 5, Appendix, is reserved to prove the elementary results.

2. THE ESTIMATE OF THE CONSTANT a

First, are introduced some basic notation and properties used throughout the paper. A random variable X is symmetric α-stable (SαS), $0 < \alpha < 2$, if its characteristic function is defined by

$$\phi_X(\theta) = e^{-\sigma^\alpha |\theta|^\alpha},$$

where α is the characteristic exponent and σ is the dispersion of the distribution. By letting α take the values 1 and 2, we obtain two important special cases of (SαS) distribution, namely Cauchy distribution and Gaussian distribution.

The random variables $X_1, \ldots, X_d$ are jointly (SαS) if there is a single positive measure $\Gamma_{X^{(d)}}$ on S_d, unit sphere of $\mathbb{R}^d$, where its characteristic function is of the form

$$\phi_X(\theta_1, \ldots, \theta_d) = \exp\left\{ -\int_{S_d} |\theta_1 s_1 + \ldots + \theta_d s_d|^\alpha d\Gamma_{X^{(d)}}(s_1, \ldots, s_d) \right\}.$$

When X_1 and X_2 are jointly (SαS), the covariation of (X_1, X_2) is defined in Cambanis (1983) by

$$[X_1, X_2]_\alpha = \int_{S_2} s_1.(s_2)^{<\alpha-1>} d\Gamma_{X_1,X_2}(s_1, s_2),$$

where $s^{<\beta>} = \text{sign}(s).|s|^\beta$. This covariation plays the same role as the covariance because the moment of second order is infinity.

From the definition of the covariation Schilder (1970) defined the following norm on the linear space of (SαS) random variables

$$|X|_\alpha = [X, X]_\alpha^{1/\alpha}.$$

The process $\xi = (\xi_t, t \in \mathbb{R})$ is symmetric α-stable if all linear combinations $\sum_{i=1}^{n} \lambda_i \xi_{t_i}$ are (SαS) variables.
This paper considers a (SαS) process where its spectral representation is

$$Z_N = \int_{-\pi}^{\pi} e^{iN\lambda} d\xi(\lambda),$$

where ξ is a isotropic symmetric α-stable with independent increments.

The measure defined by $\mu(]s,t]) = |\xi(t) - \xi(s)|_{\alpha}^{\alpha}$ is Lebesgue-Stiel measure called *the spectral measure* (see Cambanis, 1983; Masry and Cambanis, 1984). When μ is absolutely continuous $d\mu(x) = f(x)dx$, the function f is called *the spectral density* of the process Z.
As in Demesh (1988), Sabre (1994) and Sabre (1995), we give the definition of the Jackson polynomial kernel.

Let $Z_1, \dots, Z_N$ observations of the process Z: $\left(Z_{(n)}\right)_{0 \le n \le N-1}$, where N satisfies:
$N - 1 = 2k(n-1) \quad$ *with* $\quad n \in \mathbb{N} \quad k \in \mathbb{N} \cup \left\{\frac{1}{2}\right\}$
if $k = \dfrac{1}{2} \quad$ *then* $\quad n = 2n_1 - 1, n_1 \in \mathbb{N}$.
The Jackson's polynomial kernel is defined by: $|H_N(\lambda)|^{\alpha} = \left|A_N H^{(N)}(\lambda)\right|^{\alpha}$ *where*

$$H^{(N)}(\lambda) = \frac{1}{q_{k,n}} \left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})} \right)^{2k} \quad \text{with} \quad q_{k,n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right)^{2k} d\lambda.$$

In addition, we have $A_N = (B_{\alpha,N})^{\frac{-1}{\alpha}}$ *with* $B_{\alpha,N} = \displaystyle\int_{-\pi}^{\pi} \left|H^{(N)}(\lambda)\right|^{\alpha} d\lambda.$

We give the following lemmas which are used in the reminder of this paper. Their proof are given in Section 5.

LEMMA 1. *There is a non negative function h_k such as*

$$H^{(N)}(\lambda) = \sum_{m=-k(n-1)}^{k(n-1)} h_k\left(\frac{m}{n}\right) \cos(m\lambda).$$

LEMMA 2. *Let* $\quad B'_{\alpha,N} = \displaystyle\int_{-\pi}^{\pi} \left| \frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} d\lambda \quad$ *and* $\quad J_{N,\alpha} = \displaystyle\int_{-\pi}^{\pi} |u|^{\gamma} |H_N(\lambda)|^{\alpha} d\lambda,$
where $\gamma \in]0,2]$.

Then,
$$B'_{\alpha,N} \begin{cases} \ge 2\pi \left(\dfrac{2}{\pi}\right)^{2k\alpha} n^{2k\alpha - 1} & \text{if } 0 < \alpha < 2 \\ \le \dfrac{4\pi k\alpha}{2k\alpha - 1} n^{2k\alpha - 1} & \text{if } \dfrac{1}{2k} < \alpha < 2 \end{cases}$$

$$\text{and} \qquad J_{N,\alpha} \leq \begin{cases} \dfrac{\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma-2k\alpha+1)} \dfrac{1}{n^{2k\alpha-1}} & \text{if} \quad \dfrac{1}{2k} < \alpha < \dfrac{\gamma+1}{2k}, \\ \dfrac{2k\alpha\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma+1)(2k\alpha-\gamma-1)} \dfrac{1}{n^{\gamma}} & \text{if} \quad \dfrac{\gamma+1}{2k} < \alpha < 2. \end{cases}$$

In this paper, we propose an estimate of the constant error a defined by

$$\hat{a} = \frac{A_N}{H_N(0)} \sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(\frac{n'}{n}\right) X\big(n'+k(n-1)\big). \tag{1}$$

THEOREM 3. *Let p a real number such that $0 < p < \alpha$. Then*

$$|\hat{a}-a|^p = O\left(\frac{1}{n^{\frac{p}{\alpha}}}\right).$$

PROOF. From the spectral representation of the process, the estimator proposed becomes

$$\hat{a} = \frac{A_N}{H_N(0)} \sum_{n'=-k(n-1)}^{k((n-1)} \left[h_k\left(\frac{n'}{n}\right) \int_{-\pi}^{\pi} exp\left[i\big([n'+k(n-1)]\lambda\big)\right] d\xi(\lambda) + a \right].$$

Using Cambanis (1983), the characteristic function of $(\hat{a}-a)$ can be written as

$$\mathbf{E}\exp[i\Re e\overline{r}(\hat{a}-a)] = \exp\left[-C_\alpha |r|^\alpha \int_{-\pi}^{\pi} \left| \frac{A_N}{H_N(0)} \sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(\frac{n'}{n}\right) e^{in'\lambda} \right|^\alpha d\xi(\lambda) \right].$$

where $r = r_1 + i r_2$. It is easy to show that

$$\mathbf{E}\exp[i\Re e\overline{r}(\hat{a}-a)] = \exp(-C_\alpha |r|^\alpha \psi_N),$$

where $\psi_N = \psi_{N,1} + \psi_{N,2}$ with

$$\psi_{N,1} = \int_{-\pi}^{\pi} \frac{|H_N(\lambda)|^\alpha}{|H_N(0)|^\alpha} \phi(\lambda) d\lambda \text{ and } \psi_{n,2} = \sum_{i=1}^{q} c_i \frac{|H_N(w_i)|^\alpha}{|H_N(0)|^\alpha}.$$

The function ϕ being bounded on $[-\pi,\pi]$ and $|H_N(.)|^\alpha$ being a kernel, it can be shown that $\displaystyle\int_{-\pi}^{\pi} |H_N(\lambda)|^\alpha \phi(\lambda) d\lambda$ is converging to $\phi(0)$. On the other hand, from Lemma 2, we have

$$\frac{1}{|H_N(0)|^\alpha} = \frac{B'_{\alpha,N}}{n^{2k\alpha}} = O\left(\frac{1}{n}\right). \tag{2}$$

Therefore $\psi_{N,1}$ converges to 0.

$$\psi_{n,2} \leq \sum_{i=1}^{q} \frac{c_i}{B'_{\alpha,N}} \frac{1}{\left|\sin\left[\frac{1}{2}w_i\right]\right|^{2k\alpha}} \frac{B'_{\alpha,N}}{n^{2k\alpha}}.$$

Therefore

$$\psi_{N,2} = O\left(\frac{1}{n^{2k\alpha}}\right).$$

Thus

$$\psi_N = O\left(\frac{1}{n}\right).$$

Consequently, the characteristic function of $\hat{a} - a$ converges to 1 when N approaches infinity. Hence we have the convergence in probability of $\hat{a}$ to a.

We study now the convergence of $\hat{a}$ to a in L_p where $0 < p < \alpha$, which replaces the convergence in mean square, because the second order moment of X is infinity.

Let

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - e^{i r \cos\theta}}{|r|^{1+p}} d r d\theta.$$

Let $t = r' e^{i\theta'}$ and $x = \varrho e^{i\tau_0}$.
Assuming now $r = \varepsilon r'$, $\quad \tau = \tau' - \tau_0$

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\varepsilon r' \cos(\tau' - \tau_0)}}{|\varepsilon|^{1+p} |r'|^{1+p}} \varepsilon d r' d\theta'$$

$$D_p |x|^p = \Re e \int_0^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\Re e(\bar{t}x)}}{|t|^{1+p}} d|t| d\theta' - \Re e \int_{-\infty}^{0} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\Re e(\bar{t}x)}}{|t|^{1+p}} d|t| d\theta'.$$

Replacing in this formula x by $\hat{a} - a$, from the previous result we have

$$\begin{aligned} D_p \mathbf{E}|x|^p &= \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} d|t| d\theta' \\ &= \frac{\pi}{2} \int_{-\infty}^{\infty} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} dt. \end{aligned}$$

Let $u = t[\psi_N]^{\frac{1}{\alpha}}$ and using (2), we obtain

$$\frac{2}{\pi} C_{p,\alpha} \mathbf{E}|\hat{a} - a|^p = (\psi_N)^{\frac{p}{\alpha}} = O\left(\frac{1}{n^{p/\alpha}}\right), \tag{3}$$

where

$$C_{p,\alpha} = R_p F_{p,\alpha}^{-1} (C_\alpha)^{\frac{-p}{\alpha}}$$

with

$$R_p = \int \frac{1 - \cos(u)}{|u|^{1+p}} du \text{ and } F_{p,\alpha} = \int \frac{1 - e^{-|u|^\alpha}}{|u|^{\frac{1+p}{\alpha}}} du.$$

3. Improvement of the rate of convergence

In this section, we study the cases where the spectral density is zero at the origin. We prove that the convergence rate of the estimator $\hat{a}$ will be improved.

THEOREM 4. *Assume that the spectral density is satisfying*

$$\phi(\lambda) = |\lambda|^{\beta} g(\lambda),$$

where $\beta \in]0, 2k\alpha - 1[$, $\lambda \in [-\pi, \pi]$ *and* $g(\lambda)$ *is a bounded function on* $[-\pi, \pi]$, *continuous in neighborhood of* 0 *and* $g(0) \neq 0$. *Then*

$$2^{4kp} L \leq \lim_{N\to\infty} n^{\frac{p(\beta+1)}{\alpha}} \mathbf{E}|\hat{a} - a|^p \leq \pi^{4kp} L,$$

where L is the following constant

$$L = \frac{\pi}{2C_{p,\alpha}} \left[g(0) \int_{-\infty}^{\infty} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du \right]^{\frac{p}{\alpha}}.$$

PROOF. From (2), the function ψ_N can be written as

$$\psi_N = n^{-2k\alpha} \int_{-\pi}^{\pi} \left| \frac{\sin\frac{n\lambda}{2}}{\sin\frac{\lambda}{2}} \right|^{2k\alpha} |\lambda|^{\beta} g(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}.$$

Using the following inequality

$$\left|\sin\frac{x}{2}\right| \geq \frac{x}{\pi} \qquad 0 \leq x \leq \pi, \tag{4}$$

we maximize ψ_N as follows

$$\psi_N \leq \pi^{4k\alpha} n^{-2k\alpha} \int_{-\pi}^{\pi} \frac{\left|\sin\frac{n\lambda}{2}\right|^{2k\alpha}}{|\lambda|^{2k\alpha-\beta}} g(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{1}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}.$$

Putting $n\lambda = u$, we have

$$\psi_N \leq \pi^{4k\alpha} n^{-1-\beta} \left[\int_{-\infty}^{\infty} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} g\left(\frac{u}{n}\right) du + \frac{n^{-2k\alpha+1+\beta}}{\pi^{4k\alpha}} \sum_{i=1}^{q} c_i \left| \frac{1}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha} \right].$$

On the other hand, using Lebesgue's dominated convergence theorem, we show that

$$\lim_{N\to\infty} \int_{-\infty}^{\infty} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} g\left(\frac{u}{n}\right) du + \frac{n^{-2k\alpha+1+\beta}}{\pi^{4k\alpha}} \sum_{i=1}^{q} c_i \left| \frac{1}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha} = g(0) \int_{-\infty}^{\infty} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du. \tag{5}$$

Lemma 2 gives

$$\lim_{N\to\infty} n^{\frac{p(\beta+1)}{\alpha}} (\psi_N)^{\frac{p}{\alpha}} \le \pi^{4kp} \left(g(0) \int_{-\infty}^{+\infty} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du \right)^{\frac{p}{\alpha}}.$$

Thus ψ_N converges to zero. Using the following inequality

$$|\sin x| \le |x| \qquad \forall x \in [-\pi, \pi], \tag{6}$$

we obtain

$$\psi_N \ge 2^{4k\alpha} n^{-2k\alpha} \int_{-\pi}^{\pi} \frac{\left|\sin\frac{n\lambda}{2}\right|^{2k\alpha}}{|\lambda|^{2k\alpha-\beta}} g(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha},$$

$$\psi_N \ge 2^{4k\alpha} n^{-\beta-1} \left[\int_{-\pi}^{\pi} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} g(\frac{u}{n}) du + R_n \right],$$

where $R_n = \frac{n^{-2k\alpha+\beta+1}}{2^{4k\alpha}} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}$. Since R_n converges to zero, the equality (5) gives

$$\lim_{N\to\infty} n^{\frac{p(\beta+1)}{\alpha}} (\psi_N)^{\frac{p}{\alpha}} \ge 2^{4kp} \left(g(0) \int_{-\infty}^{+\infty} \frac{|\sin\frac{u}{2}|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du \right)^{\frac{p}{\alpha}}.$$

The first equality of (3) reaches the result of this theorem.

THEOREM 5. *Assuming that the spectral density satisfies*

$$\phi(\lambda) = \sin^{2k\alpha}\left(\frac{\lambda}{2}\right) g(\lambda),$$

where the function g *is integrable on* $[-\pi, \pi]$ *and* $g(0) \ne 0$*. Then*

$$cte \frac{\pi}{2C_{p,\alpha}} \left(\int_{-\pi}^{\pi} g(\lambda) d\lambda \right)^{\frac{p}{\alpha}} \le \lim_{N\to\infty} n^{2pk} \mathbf{E}|\hat{a} - a|^p \le$$

$$\frac{\pi}{2C_{p,\alpha}} \left(\int_{-\pi}^{\pi} g(\lambda) d\lambda + \sum_{i=1}^{q} c_i \left| \frac{1}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha} \right)^{\frac{p}{\alpha}}.$$

PROOF. From the definition of ψ_N and (2), we have

$$\psi_N = n^{-2k\alpha} \int_{-\pi}^{\pi} \left|\sin\frac{n\lambda}{2}\right|^{2k\alpha} g(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}.$$

As $1 \le k\alpha$ and the sinus function is smaller than 1, the next expression is connected to

$$\psi_N \le n^{-2k\alpha} \int_{-\pi}^{\pi} \left|\sin\frac{n\lambda}{2}\right|^{2} g(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}$$

$$\psi_N \le n^{-2k\alpha}\left[\int_{-\pi}^{\pi} g(\lambda)d\lambda + \sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]}\right|^{2k\alpha}\right].$$

So, from Lemma 2, we have

$$\lim_{N\to\infty} \psi_N n^{2k\alpha} \le \int_{-\pi}^{\pi} g(\lambda)d\lambda + \sum_{i=1}^{q} c_i \left|\frac{1}{\sin\left[\frac{w_i}{2}\right]}\right|^{2k\alpha}.$$

Using the fact that the sinus function is between -1 and 1 and that $k\alpha < [k\alpha]+1$ where $[k\alpha]$ represents the integer part of $k\alpha$, we obtain

$$\psi_N \ge n^{-2k\alpha}\int_{-\pi}^{\pi}\left[\left(\sin\frac{n\lambda}{2}\right)^2\right]^{[k\alpha]+1} g(\lambda)d\lambda + n^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]}\right|^{2k\alpha}$$

$$\psi_N \ge n^{-2k\alpha}\frac{n^{-1}}{2B'_{\alpha,N}}\int_{-\pi}^{\pi}\left[\frac{1-\cos n\lambda}{2}\right]^{[k\alpha]+1} g(\lambda)d\lambda + n^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]}\right|^{2k\alpha}.$$

The binomial formula gives

$$\begin{aligned} 2^{[k\alpha]+1}\psi_N &\ge n^{-2k\alpha}\int_{-\pi}^{\pi} g(\lambda)d\lambda \\ &+ n^{-2k\alpha}\sum_{r=1}^{[k\alpha]+1}(-1)^r C^r_{[k\alpha]+1}\int_{-\pi}^{\pi}\cos^r(n\lambda)g(\lambda)d\lambda \\ &+ n^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]}\right|^{2k\alpha}. \end{aligned}$$

We know that $\int_{-\pi}^{\pi}\cos^r(n\lambda)g(\lambda)d\lambda$ is converging to a constant. Indeed, using the binomial formula, we obtain

$$\cos^r(n\lambda_) = \left(\frac{e^{in\lambda}+e^{-in\lambda}}{2}\right)^r = \frac{1}{2^r}\sum_{j=0}^{r} C_r^j e^{ijn\lambda}e^{-i(r-j)n\lambda}.$$

Hence

$$\begin{aligned} \int_{-\pi}^{\pi}\cos^r(n\lambda)g(\lambda)d\lambda &= \frac{1}{2^r}\sum_{j=0}^{r} C_r^j \int_{-\pi}^{\pi} e^{i(2j-r)n\lambda} \\ &= \frac{1}{2^r}\sum_{j=0}^{r} C_r^j \int_{-\pi}^{\pi}\cos((2j-r)n\lambda)g(\lambda)d\lambda. \end{aligned}$$

The right side of the last equality converges to $\frac{1}{2^r}\int_{-\pi}^{\pi} g(\lambda)d\lambda$ when r is even, and converges to 0 when r is odd. Consequently,

$$\lim_{N\to\infty}\sum_{r=1}^{[k\alpha]+1}(-1)^r C^r_{[k\alpha]+1}\int_{-\pi}^{\pi}\cos^r(n\lambda)g(\lambda)d\lambda =$$

$$\begin{cases} \sum_{p=1}^{\frac{[k\alpha]+1}{2}} (-1)^{2p} C_{[k\alpha]+1}^{2p} \frac{1}{2^{2p}} \int_{-\pi}^{\pi} g(\lambda) d\lambda & \text{if } [k\alpha]+1 \text{ is even} \\ \sum_{p=1}^{\frac{[k\alpha]}{2}} (-1)^{2p} C_{[k\alpha]+1}^{2p} \frac{1}{2^{2p}} \int_{-\pi}^{\pi} g(\lambda) d\lambda & \text{else.} \end{cases}$$

Since $\lim_{N\to\infty} n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha} = 0$. The similar arguments are used for showing that

$$\lim_{N\to\infty} (\psi_N)^{\frac{p}{\alpha}} n^{2kp} \geq cte \left(\int_{-\pi}^{\pi} g(\lambda) d\lambda \right)^{\frac{p}{\alpha}}. \tag{7}$$

THEOREM 6. *Assume that spectral density satisfies*

$$\phi(\lambda) = |\lambda|^{\beta} g(\lambda),$$

where $\beta > 2k\alpha - 1$ *and* g *is measurable positive function bounded on* $[-\pi, \pi]$. *Then*

$$cte \times R \leq \lim_{N\to\infty} n^{2kp} \mathbf{E} |\hat{a} - a|^p \leq R,$$

where $R = \frac{\pi}{2C_{p,\alpha}} \left(\int_{-\pi}^{\pi} \frac{|\lambda|^{\beta}}{\left|\sin \frac{\lambda}{2}\right|^{2k\alpha}} g(\lambda) d\lambda \right)^{\frac{p}{\alpha}}.$

PROOF. Define the continuous function l as follows

$$l(\lambda) = \begin{cases} \pi^{2k\alpha} & \text{if } |\lambda| > \pi \\ \frac{|\lambda|^{2k\alpha}}{\left|\sin \frac{\lambda}{2}\right|^{2k\alpha}} & \text{if } 0 < |\lambda| \leq \pi \\ 2^{2k\alpha} & \text{if } \lambda = 0. \end{cases}$$

The function ψ_N can be written as

$$\psi_N = n^{-2k\alpha} \int_{-\pi}^{\pi} \left| \frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} \sin^{2k\alpha}\left(\frac{\lambda}{2}\right) h(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha},$$

where $h(\lambda) = l(\lambda)|\lambda|^{\beta-2k\alpha} g(\lambda)$. We know that the function h is integrable on $[-\pi, \pi]$. Indeed, since the function g is bounded, we get

$$\int_{-\pi}^{\pi} h(\lambda) d\lambda \leq \sup(g) \int_{-\pi}^{\pi} l(\lambda) |\lambda|^{\beta-2k\alpha} d\lambda.$$

Using the inequality (6), we obtain $l(\lambda) \leq \pi^{2k\alpha}$. Thus

$$\int_{-\pi}^{\pi} h(\lambda) d\lambda \leq 2\pi^{2k\alpha} \sup(g) \int_{0}^{\pi} (\lambda)^{\beta-2k\alpha} d\lambda.$$

Since $\beta > 2k\alpha - 1$, the function h is integrable. From (3) and Theorem 5, the result is obtained.

THEOREM 7. *Assume that the spectral density has the following form:* $\phi(\lambda) = |\lambda|^\beta g(\lambda)$ *where* $\beta < 2k\alpha - 1$ *and* g *is a continuous positive function on* $[-\pi, \pi]$. *Then*

$$Q \times 2^{2kp} \leq \lim_{N\to\infty} n^{\frac{(\beta+1)p}{\alpha}} |\mathbf{E}|\hat{a} - a|^p \leq \pi^{2kp} \times Q,$$

where $Q = \left(g(0) \int_{-\infty}^{\infty} \left| \sin \frac{u_1}{2} \right|^{2k\alpha} |u_1|^{\beta_1 - 2k\alpha} du_1 \right)^{\frac{p}{\alpha}}$.

PROOF. From the definition of the kernel $|H_N(\lambda)|^\alpha$ and the inequality (6), we obtain

$$\psi_N \leq \frac{\pi^{2k\alpha} n^{-1}}{B'_{\alpha,N}} \int_{-\pi}^{\pi} \left| \sin \frac{n\lambda}{2} \right|^{2k\alpha} |\lambda|^{\beta - 2k\alpha} g(\lambda) d\lambda + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}.$$

Putting $n\lambda = u$, we obtain

$$\psi_N \leq \frac{\pi^{2k\alpha} n^{2k\alpha - \beta_1 - 2}}{B'_{\alpha,N}} \int_{-\infty}^{\infty} \left| \sin \frac{u}{2} \right|^{2k\alpha} |u|^{\beta - 2k\alpha} g\left(\frac{u}{n}\right) du + n^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{1}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}.$$

when N approaches infinity, we obtain

$$\lim_{N\to\infty} n^{\beta+1} \psi_N \leq \pi^{2k\alpha} \int_{-\infty}^{\infty} \left| \sin \frac{u}{2} \right|^{2k\alpha} |u|^{\beta - 2k\alpha} g(0) du.$$

Similarly, from the inequality (6) we minimize ψ_N as follow

$$n\psi_N \geq \frac{2^{2k\alpha}}{B'_{\alpha,N}} \int_{-\pi}^{\pi} \left| \sin \frac{n\lambda}{2} \right|^{2k\alpha} |\lambda|^{\beta - 2k\alpha} d\lambda + Q_n,$$

where $Q_n = n^{-2k\alpha+1} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{nw_i}{2}\right]}{\sin\left[\frac{w_i}{2}\right]} \right|^{2k\alpha}$. Since Q_n converges to zero and the function $\lambda \longrightarrow g(\lambda)$ is continuous, Lemma 2 gives

$$\lim_{N\to\infty} n^{\beta+1} \psi_N \geq 2^{2k\alpha} \int_{-\infty}^{\infty} \left| \sin \frac{u_1}{2} \right|^{2k\alpha} |u_1|^{\beta - 2k\alpha} g(0) du.$$

Thus from (3) we obtain the result of this theorem.

4. NUMERICAL STUDIES

The estimator provided in this work can be used in some concrete applications. For example, in the models transmission rate in indoor environment for next generation of wireless communication systems. And also in mobile indoor and radio communication where multipath propagation causes severe degradation of the transmission quality. To solve this problem Clavier *et al.* (2001) proposed a model of arrival times based on Poisson distributions. Azzaoui *et al.* (2002) provided a model based on alpha stable distributions. It is assumed that we have a finite number of signals where

their arrival times are modeled by Poisson distributions independent to the direction (isotropic). From the series representation theory shown by Janicki and Weron (1993), the sum of the arrival times can be modeled by a stable harmonizable process Z_n as the process defined by (8). It is supposed that these signals are observed in an aquatic environment causing a constant disturbance. The signal arrival delay model in this case can be considered to be affected by a constant error ($X_n = Z_n + a$).
Throughout this section, we give the simulation of the studied process

$$Z_n = \int_{-\pi}^{\pi} e^{in\lambda} d\xi(\lambda), \tag{8}$$

where $1 < \alpha < 2$ and ξ is a complex symmetric α-stable measure on $\mathbb{R}$ with independent and isotropic increments and with control measure m such that $mdx = \phi(x)dx$. For that, we use the series representations given by Janick and Weron (1993). Therein the authors have shown that the process Z defined by (8) can be expressed as follows

$$Z_n = C_\alpha \left(\int \phi(x)dx \right)^{\frac{1}{\alpha}} \sum_{k=1}^{\infty} \varepsilon_k \Gamma_k^{-\frac{1}{\alpha}} e^{inV_k} e^{i\theta_k}, \tag{9}$$

where

- ε_k is a sequence of i.i.d. random variables such as $P[\varepsilon_k = 0] = P[\varepsilon_k = 1] = \frac{1}{2}$,
- Γ_k is a sequence of arrival times of Poisson process,
- V_k is a sequence of i.i.d. random variables independent of ε_k and of Γ_k having the same distribution of control measure m, which has probability density ϕ,
- θ_k is a sequence of i.i.d. random variables that have the uniform distribution on $[-\pi, \pi]$, independent of ε_k, Γ_k and V_k.

To generate N values ($N = 101, 501, 1001$) of the process Z_n, we use the following steps:

- generate 2000 values of ε_k,
- generate 2000 values of Γ_k,
- generate 2000 values of V_k,
- generate 2000 values of θ_k.

Then we calculate for all $0 \leq n \leq N$

$$Z_n = C_\alpha \left(\int \phi(x)dx \right)^{\frac{1}{\alpha}} \sum_{k=1}^{2000} \varepsilon_k \Gamma_k^{-\frac{1}{\alpha}} e^{inV_k} e^{i\theta_k}, \tag{10}$$

where the spectral density is chosen as $\phi(x) = |x|^\beta e^{-|x|}$ for $x \in [-\pi, \pi]$ and $\phi(x) = 0$ otherwise and $\alpha = 1,7$ and $k = 4$. The β value is chosen so as to have two cases: β greater than $2k\alpha - 1$ and beta less than $2k\alpha - 1$. Afterwards, we generate $X_n = a + Z_n$ where a is chosen equal to 30.

We calculate the estimator $\hat{a}$ given in (1) for different sizes of sample N =101, 501, 1001. The result is given in Table 1.

Comparing $\hat{a}$ to 30 (value of the constant a), we find that the estimator $\hat{a}$ increasingly approaching the constant error a when the sample size is large. And the approximation is better when the parameter β of the spectral density is greater than $2k\alpha-1$ compared to the case where β is smaller than $2k\alpha-1$.

TABLE 1
Values of the estimator for two values of β one is smaller than $2k\alpha-1$ other is bigger than $2k\alpha-1$.

$2k\alpha-1=12.6$	$\beta=0.21$	$\beta=38$
N=101	$\hat{a}=20.30$	$\hat{a}=36.45$
N=501	$\hat{a}=35.10$	$\hat{a}=31.03$
N=1001	$\hat{a}=28.89$	$\hat{a}=30.28$

5. Appendix

PROOF OF LEMMA 1. For all nonnegative integer, n, we have $\displaystyle\sum_{s=0}^{n-1} e^{is\lambda} = \frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})} e^{\frac{i(n-1)\lambda}{2}}$.

Therefore we can write $H^{(N)}(\lambda) = \dfrac{1}{q_{k,n}} \left(\displaystyle\sum_{s=0}^{n-1} e^{is\lambda} \right)^{2k} e^{-ik(n-1)\lambda}$

Hence $H^{(N)}(\lambda) = \dfrac{e^{-ik(n-1)\lambda}}{q_{k,n}} \displaystyle\sum_{t\in S_{2k}} \exp\Big[i(t_1+t_2+\cdots+t_{2k})\lambda \Big]$, where

$$S_{2k} = \Big\{ t=(t_1,t_2,\cdots,t_{2k}) \in \mathbb{Z}^{2k}; \quad \forall j \in \{1,2,\cdots,2k\} \quad 0 \le t_j \le n-1 \Big\}.$$

Let $\tau = t_1+t_2+\cdots+t_{2k}$, we have $H^{(N)}(\lambda) = \dfrac{e^{-ik(n-1)\lambda}}{q_{k,n}} \displaystyle\sum_{\tau=0}^{2k(n-1)} \exp\Big[i\tau\lambda \Big] Q_N^{(k)}(\tau)$,

where $Q_N^{(k)}(\tau) = \displaystyle\sum_{t\in S_{2k}} \chi_0(t_1+t_2+\cdots+t_{2k}-\tau)$.

$Q_N^{(k)}(\tau)$ is the number of the terms $(t_1,t_2,\cdots,t_{2k})$ which satisfy:
$\forall j \in \{1,2,\cdots,2k\} \quad t_j \in \mathbb{Z}, \quad 0 \le t_j \le n-1 \quad$ et $\quad t_1+t_2+\cdots+t_{2k}=\tau$.

If $k=\dfrac{1}{2}$ and $\tau \in \mathbb{Z}$ then $Q_N^{(k)}(\tau)=1$. On the other hand, putting $m=\tau-k(n-1)$, we get

$$H^{(N)}(\lambda) = \frac{1}{q_{k,n}} \sum_{m=-k(n-1)}^{k(n-1)} \exp[i\lambda m] Q_N^{(k)}(m+k[n-1]).$$

Since $H^{(N)}(.)$ is even function, by using $H^{(N)}(\lambda) = \dfrac{H^{(N)}(\lambda)+H^{(N)}(-\lambda)}{2}$
we obtain

$$H^{(N)}(\lambda) \quad = \quad \frac{1}{q_{k,n}} \sum_{m=-k(n-1)}^{k(n-1)} \frac{\exp\Big[i\lambda m\Big]+\exp\Big[-i\lambda m\Big]}{2} Q_N^{(k)}(m+k[n-1])$$

$$H^{(N)}(\lambda) \qquad \frac{1}{q_{k,n}} \sum_{m=-k(n-1)}^{k(n-1)} \cos(\lambda m) Q_N^{(k)}(m+k[n-1]).$$

It follows from the definition of $q_{k,n}$, we obtain

$$\begin{aligned}
q_{k,n} &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})}\right)^{2k} d\lambda \\
q_{k,n} &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\sum_{\tau=-k(n-1)}^{k(n-1)} \cos(\lambda\tau) Q_N^{(k)}(\tau+k[n-1])\right] d\lambda \\
q_{k,n} &= \frac{1}{2\pi}\sum_{\tau=-k(n-1)}^{k(n-1)}\left(\int_{-\pi}^{\pi}\cos(\lambda\tau)d\lambda\right) Q_N^{(k)}(\tau+k[n-1]).
\end{aligned}$$

Since $\tau \in \mathbb{Z}$, we have

$$\int_{-\pi}^{\pi}\cos(\lambda\tau)d\lambda = \begin{cases} 0 & \text{if} \quad \tau \neq 0 \\ 2\pi & \text{if} \quad \tau = 0 \end{cases}$$

We have $q_{k,n} = Q_N^{(k)}(k[n-1])$. By substituting this in the expression for $H^{(N)}(\lambda)$, we obtain $H^{(N)}(\lambda) = \sum_{m=-k(n-1)}^{k(n-1)} \cos(\lambda m)\left[\frac{Q_N^{(k)}(m+k[n-1])}{Q_N^{(k)}(k[n-1])}\right]$. Therefore the function h_k is defined by

$$h_k(u) = \begin{cases} \left[\frac{Q_N^{(k)}(nu+k[n-1])}{Q_N^{(k)}(k[n-1])}\right] & \text{if } nu \in \mathbb{Z} \\ 0 & \text{if } nu \notin \mathbb{Z} \end{cases}$$

We note that if $k = \frac{1}{2}$ then $q_{k,n} = h_k\left(\frac{m}{n}\right) = 1.$

PROOF OF LEMMA 2. We first state some inequalities which we are using in this proof

$$|\sin(\lambda)| \leq |\lambda| \qquad \forall \lambda \in \mathbb{R} \tag{11}$$

$$|\sin(\frac{\lambda}{2})| \geq \frac{|\lambda|}{\pi} \qquad \forall \lambda \in [-\pi,\pi] \tag{12}$$

$$|\sin(n\lambda)| \leq n|\sin(\lambda)| \qquad \forall \lambda \in \mathbb{R}, \tag{13}$$

It follows from (11) and (12) that

$$B'_{\alpha,N} = \int_{-\pi}^{\pi}\left|\frac{\sin\frac{n\lambda}{2}}{\sin\frac{\lambda}{2}}\right|^{2k\alpha} d\lambda \geq 2\int_0^{\frac{\pi}{n}}\left(\frac{n\lambda/\pi}{\lambda/2}\right)^{2k\alpha} d\lambda = 2\pi\left(\frac{2}{\pi}\right)^{2k\alpha} n^{2k\alpha-1}. \tag{14}$$

By splitting the integral out of two subregions, and using (12) and (11), we get

$$B'_{\alpha,N} = \int_{-\pi}^{\pi} \left| \frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} d\lambda \leq 2 \left(\int_0^{\frac{\pi}{n}} n^{2k\alpha} d\lambda + \int_{\frac{\pi}{n}}^{\pi} \frac{1}{\left(\frac{\lambda}{\pi}\right)^{2k\alpha}} d\lambda \right)$$

Thus we obtain

$$B'_{\alpha,N} \leq \left(\frac{4\pi k\alpha}{2k\alpha - 1} \right) n^{2k\alpha - 1}. \tag{15}$$

Using (12) and (11) we get

$$\begin{aligned} J_{N,\alpha} &= \frac{2}{B'_{\alpha,N}} \int_0^{\pi} (\lambda)^{\gamma} \left| \frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} d\lambda \leq \frac{2}{B'_{\alpha,N}} \left(\int_0^{\frac{\pi}{n}} \lambda^{\gamma} n^{2k\alpha} d\lambda + \int_{\frac{\pi}{n}}^{\pi} \lambda^{\gamma} \left(\frac{\pi}{\lambda}\right)^{2k\alpha} d\lambda \right) \\ &\leq \frac{2}{B'_{\alpha,N}} \left(\frac{\pi^{\gamma+1}}{\gamma+1} n^{2k\alpha-\gamma-1} + \frac{\pi^{2k\alpha}}{\gamma - 2k\alpha + 1} \left[\pi^{\gamma-2k\alpha+1} - \left(\frac{\pi}{n}\right)^{\gamma-2k\alpha+1} \right] \right) \\ &\leq \frac{2}{B'_{\alpha,N}} \left(\frac{\pi^{\gamma+1}}{\gamma+1} n^{2k\alpha-\gamma-1} + \frac{\pi^{\gamma+1}}{\gamma-2k\alpha+1} - \frac{\pi^{\gamma+1}}{\gamma-2k\alpha+1} \frac{1}{n^{\gamma-2k\alpha+1}} \right). \end{aligned}$$

We distinguish two cases according to the sign of $\gamma - 2k\alpha + 1$

$$J_{N,\alpha} \leq \frac{2}{B'_{\alpha,N}} \begin{cases} \dfrac{\pi^{\gamma+1}}{\gamma+1-2k\alpha} - \dfrac{2k\alpha\pi^{\gamma+1}}{(\gamma+1)(\gamma-2k\alpha+1)} \dfrac{1}{n^{\gamma-2k\alpha+1}} & \text{if} \quad \gamma - 2k\alpha + 1 > 0 \\ \dfrac{2k\alpha\pi^{\gamma+1}}{(\gamma+1)(2k\alpha-\gamma-1)} \dfrac{1}{n^{\gamma-2k\alpha+1}} - \dfrac{\pi^{\gamma+1}}{2k\alpha-\gamma-1} & \text{if} \quad \gamma - 2k\alpha + 1 < 0. \end{cases}$$

From (13) we get

$$J_{N,\alpha} \leq \begin{cases} \dfrac{2\pi^{\gamma+1}}{\gamma - 2k\alpha + 1} \dfrac{1}{2\pi\left(\frac{2}{\pi}\right)^{2k\alpha} n^{2k\alpha-1}} & \text{if} \quad \dfrac{1}{2k} < \alpha < \dfrac{\gamma+1}{2k} \\ \dfrac{4k\alpha\pi^{\gamma+1}}{(\gamma+1)(2k\alpha-\gamma-1)} \dfrac{n^{2k\alpha-\gamma-1}}{2\pi\left(\frac{2}{\pi}\right)^{2k\alpha} n^{2k\alpha-1}} & \text{if} \quad 2 > \alpha > \dfrac{\gamma+1}{2k}. \end{cases}$$

This inequality implies the results of the lemma.

Conclusion

In this paper, some results about the estimation of the constant additive error in spectral representation of (SαS) process are presented. This work could be applied to several cases when processes have an infinite variance and the observation of these processes are perturbed by a constant noise. For instance, the segmentation of a dynamic picture sequence, detecting weeds in a farm field, the detection of possible structural changes in the dynamics of an economic structural phenomenon depending on a constant parameter of sampling, and the study of the rate of occurrence of notes in melodic music in order to simulate the sensation of hearing from afar with an additive acoustic vibration. This work could be supplemented by the study of optimal smoothing parameters using cross validation methods that have been proven in the field.

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SUMMARY

Consider a symmetric α stable process having a spectral representation with an additive constant error. An estimator of that error and its rate of convergence are given. We study the rate of convergence when the spectral density have some behaviors at origin. Few long memory processes are taken here as example.

Keywords: Spectral density; Jackson kernel; Stable processes.