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Estimation of Correction of Ground State Energy of Hydrogen Atom in Presence of Quadratic GUP

Himangshu Barman[1], Palash Mandal[2]
Department of Physics, Department of Mathematics, Hooghly Mohsin College,West Bengal, 712101

Abstract: String Theory, Quantum Geometry, Loop Quantum Gravity and Black Hole physic all predict the existence of a observable minimal length at Planck scale. For example, in case of string theory it is conjectured that a particle described as a string does not interact at distances smaller than its size. As a consequence, the HUP has to be generalized to take into account this aspect. The models which are designed to implement the minimal length scale and/or the maximum momentum in different physical systems entered into the literature as the Generalized Uncertainty Principle (GUP). Here, quadratic GUP model has been used to estimate the quantum-gravitational correction of ground state energy of hydrogen atom (H-atom).
Keywords: quadratic GUP, Planck scale, GUP parameter, minimum energy, ground state energy.

I. INTRODUCTION

In recent years, the investigation of the effects of the Generalized Uncertainty Principle (GUP) on various physical systems has attracted much attention and many authors have found exact or approximate solutions in both classical and quantum mechanical domains [1–4]. Indeed, because of the universality of this gravitational effect, it couples to all forms of matter and modifies the corresponding Hamiltonians in both non-relativistic and relativistic limits. Moreover, the existence of a finite lower bound to the possible resolution of length proportional to the Planck length $l_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35}m$, where G is Newton's gravitational constant, naturally arises from various candidates of quantum gravity such as string theory [5–10], loop quantum gravity [11], and noncommutative spacetime [12–17].The problem of the hydrogen atom is studied in ordinary quantum mechanics and its well-known exact energy eigen values and eigen functions have already been obtained [18–22].

The quadratic GUP model [13] is taken here to estimate the ground state energy of hydrogen atom. In one dimension, the simplest generalized uncertainty relation which implies the appearance of a nonzero minimal uncertainty ΔX_0 in position has the form:

$$\Delta x\Delta p \geq \frac{\hbar}{2}(1 + \beta(\Delta p)^2 + \gamma) \quad (1)$$

Where β is the GUP parameter defined as $\beta = \frac{\beta_o}{M_{pl}C^2} = \frac{\beta_o {l_{pl}}^2}{\hbar^2}$ and $M_{pl}C^2 \approx 10^9$ GeV and l_{pl} is the Planck length. It is normally assumed that β_o is not far from unity. The second term on the RHS above is important at very high energies/small length scales (i.e., $\Delta x \sim l_{pl}$). The introduction of GUP automatically includes the Planck scale correction into the energy of a physical system that makes the GUP proposition more interesting. Moreover, β (GUP parameter) and γ are positive number and independent of Δx and Δp (but may in general depend on the expectation values of x and p). The curve of minimal uncertainty is illustrated in Fig. 1.

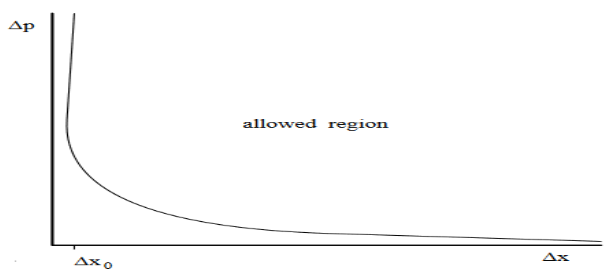

Fig. 1: Modified uncertainty relation, implying a 'minimal length' $\Delta X_0 > 0$

[1]Corresponding author (himu.phy@gmail.com)

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II. DESCRIPTION OF QUADRATIC GUP

To establish a concrete theory of quantum gravity is currently one of the main challenges in theoretical physics. Various approaches predict the existence of a minimum length scale [1, 23] that leads to the modification of the Heisenberg Uncertainty Principle-

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (2)$$

to the Generalized Uncertainty Principle (GUP) [24,25]

$$\Delta x \Delta p \geq \frac{\hbar}{2}(1 + \beta(\Delta p)^2 + \beta < p >^2) \quad (3)$$

Equation (3) comes from the general form of quadratic GUP [13,26] given below.

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}\ (1 + \beta((\Delta p)^2 + < p >^2) + 2\beta((\Delta p_i)^2 + < p_i >^2))) \quad (4)$$

Which follows from the modified commutation relation [13] given below.

$$[x_i, p_j] = i\hbar(\delta_{ij} + \beta(p^2\delta_{ij} + 2p_i p_j)) \quad (5)$$

The commutation relation (5) suggests the existence of minimal observable length as minimum uncertainty in position, $(\Delta x)_{min} = \hbar\sqrt{\beta}$ and admits the following representation in position space [27, 28] $x_i = x_{0i}$ and $p_i = p_{0i}(1 + \beta p_0^2)$.

Where x_{0i} and p_{0j} satisfy the canonical commutation relation $[x_{0i}, p_{0j}] = i\hbar\delta_{ij}$. Here x_{0i} and p_{0i} can be interpreted as position and momentum low energies (having standard representation in position space i.e., $x_{0i} = x_{0i}$ and $p_{0i} = -i\hbar\frac{\partial}{\partial x_{0i}}$) and x_i and p_i as that at high energies.

III. GROUND STATE ENERGY OF THE HYDROGEN ATOM

A. Ground State Energy of Hydrogen Atom without GUP

Bohr in 1913, combining the concepts of Rutherford's nuclear atom, Planck's quantum hypothesis and Einstein's photo electric effect, was able to explain the observed spectrum of atomic hydrogen. The total energy (E) of the electron in hydrogen atom is the sum of its kinetic energy (T) and its potential energy(V).

$$E = T + V = \frac{p^2}{2m} - \frac{e^2}{r} = \frac{p^2}{2m} - \frac{\alpha\hbar c}{r} \quad (6)$$

Where m and p are mass and momentum of the electron respectively and α is the fine structure constant and c is the velocity of light in free space.

Heisenberg Uncertainty Principle (HUP) gives

$$\Delta r \Delta p \sim \hbar \quad (7)$$

Assume the uncertainty in momentum, $\Delta p = a$, then uncertainty in position will be $\Delta r \sim \frac{\hbar}{a}$ as per eqn. (7). To calculate minimum energy we have to take $\Delta p = a$ and $\Delta r = \frac{\hbar}{a}$

Now, in terms of uncertainty in position Δr and in term of uncertainty in momentum Δp we have uncertainty in energy ΔE as

$$\Delta E = \frac{(\Delta p)^2}{2m} - \frac{\alpha\hbar c}{(\Delta r)} = \frac{a^2}{2m} - \alpha c a \quad (8)$$

Minimum energy of hydrogen atom for a particular value of a, is obtained by solving the equation $\frac{d(\Delta E)}{da} = 0$, and it gives

$$a = \alpha c m\ . \quad (9)$$

For $a = \alpha c m$, $\frac{d^2(\Delta E)}{da^2} > 0$, that implies that for that value of a, uncertainty in energy ΔE is minimum and equation (8) gives

$$E_{min} = -\frac{1}{2}\alpha^2 c^2 m \quad (10)$$

B. Ground State Energy of Hydrogen atom with quadratic GUP:

Heisenberg algebra generated by x and p obeying the commutation relation [13]

$[x, p] = i\hbar(1 + \beta p^2)$ gives the generalized uncertainty relation as

$$\Delta x \Delta p \geq \frac{\hbar}{2}[1 + \beta(\Delta p)^2] \quad (11)$$

Here the factor $\frac{1}{2}$ is dropped for our conveniences. If uncertainty in momentum is assumed $\Delta p = a$, then uncertainty in position will be, $\Delta r \geq \frac{\hbar}{a}(1 + \beta a^2)$ as per equation (11). To calculate minimum energy, following process will be observed.

Minimum energy of hydrogen atom for a particular value of a, is obtained by solving the equation $\frac{d(\Delta E)}{da} = 0$.

Now, in terms of uncertainty in position Δr and uncertainty in momentum Δp we have uncertainty in energy ΔE as

$$\Delta E = \frac{(\Delta p)^2}{2m} - \frac{\alpha\hbar c}{(\Delta r)} = \frac{a^2}{2m} - \frac{\alpha c a}{(1+\beta a^2)},$$

which can be written as

$$\Delta E = \frac{a^2}{2m} - \alpha c a(1-\beta a^2) \quad (12)$$

Differentiating equation (12) we get

$$\frac{d(\Delta E)}{da} = \frac{a}{m} + 3\alpha c\beta a^2 - \alpha c = 0.$$

That leads to

$3\alpha c\beta a^2 + \frac{a}{m} - \alpha c = 0$, which gives

$$a = \alpha cm \quad \text{Or} \quad a = -(\frac{3\alpha c\beta}{m} + \alpha cm). \quad (13)$$

See that for $a = \alpha cm$, $\frac{d^2E}{da^2} > 0$. This implies that at that value of a, uncertainty in energy ΔE is minimum and equation (12) gives

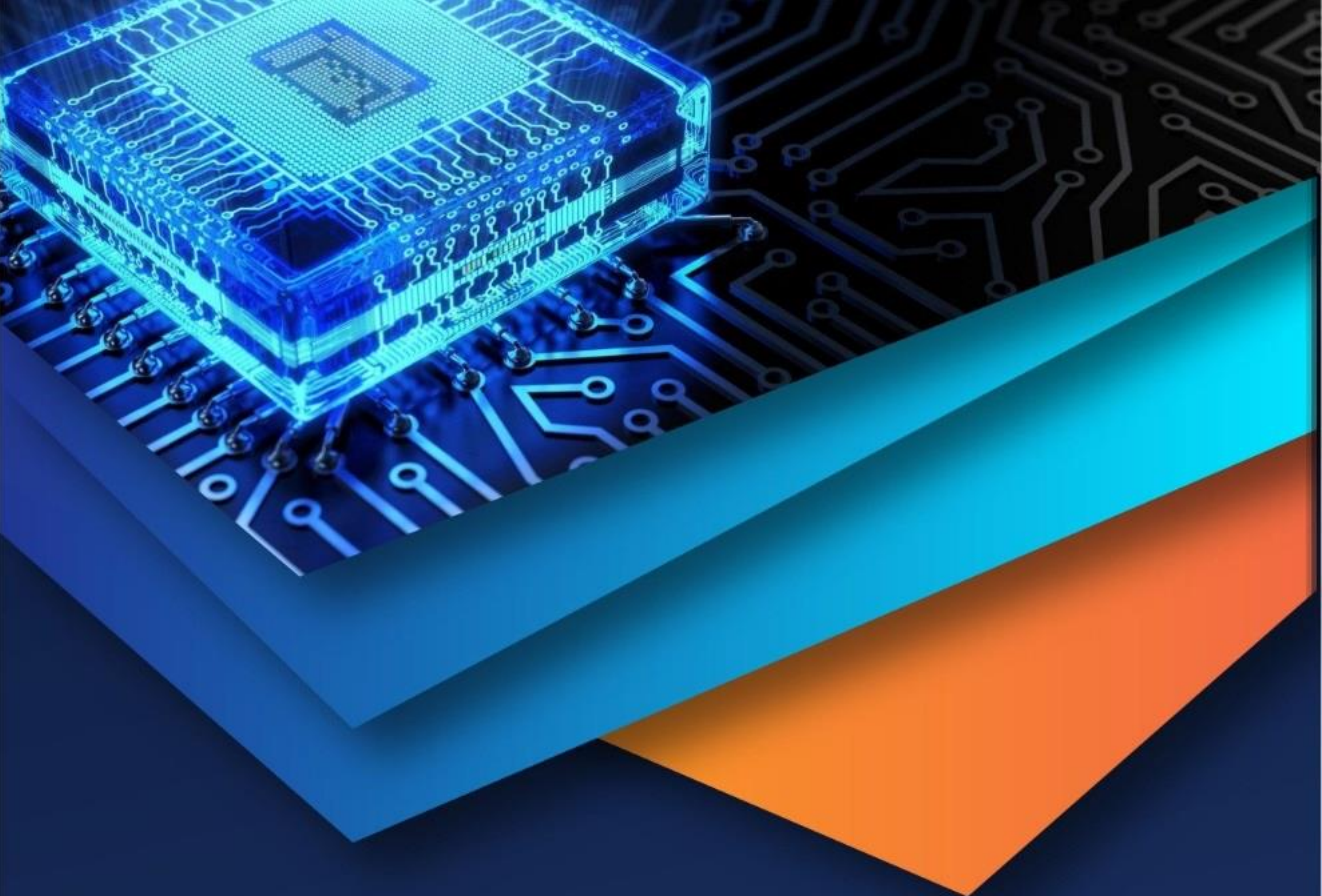
$$E_{min} = -\frac{1}{2}\alpha^2c^2m + \alpha^4c^4m^3\beta \quad (14)$$

IV. DISCUSSION

In this article, we have used quadratic GUP to calculate quantum-gravitational correction of ground state energy of hydrogen atom. The final expression gives the correction in terms of β. The correction up to the 1st order of β has been considered here. The correction gives positive contribution since β is positive and less than one. If $\beta = 0$ in eqn.(14), we have usual expression of ground state energy without GUP. The ground state appeared here is expected to be the correction need in the vicinity of Plank scale. So to build up any theory in the vicinity of Plank scale if the zero point energy of hydrogen atom becomes important, this investigation will be helpful in that case.

REFERENCES

[1] S. Hossenfelder, "Minimal length scale scenarios for quantum gravity," Living Reviews in Relativity 16, (2013), 2
[2] P. Pedram,"On the modification of the Hamiltonians' spectrum in gravitational quantum mechanics" Europhys. Lett. 89 (2010) 50008.
[3] P. Pedram and K. Nozari K, "Minimal length and bouncing-particle spectrum"Europhys. Lett. 92 (2010) 50013.
[4] P. Pedram, K. Nozari, and S.H. Taheri, "The effects of minimal length and maximal momentum on the transition rate of ultra cold neutrons in gravitational field" JHEP 1103 (2011) 093.
[5] G. Veneziano, "A Stringy Nature Needs Just Two Constants" Europhys. Lett. 2 (1986) 199.
[6] E. Witten, "Reflections on the fate of spacetime" Phys. Today 49 (1996) 24.
[7] D. Amati, M. Ciafaloni, G. Veneziano, "Can spacetime be probed below the string size? Phys. Lett. B 216 (1989) 41.
[8] D. Amati, M. Ciafaloni, G. Veneziano, "Higher-order gravitational deflection and soft bremsstrahlung in planckian energy superstring collisions" Nucl. Phys. B 347 (1990) 550.
[9] D. Amati, M. Ciafaloni, G. Veneziano, "Effective action and all-order gravitational eikonal at planckian energies"Nucl. Phys. B 403 (1993) 707.
[10] K. Konishi, G. Paffuti, P. Provero, "Minimum physical length and the generalized uncertainty principle in string theory"Phys. Lett. B 234 (1990) 276.
[11] L.J. Garay, Int. J. Mod. "Quantum gravity and minimum length" Phys. A 10 (1995) 145.
[12] M. Maggiore, "The algebraic structure of the generalized uncertainty principle" Phys. Lett. B 319 (1993) 83
[13] A. Kempf, G. Mangano, R.B. Mann, "Hilbert space representation of the minimal length uncertainty relation" Phys. Rev. D 52 (1995) 1108.
[14] A. Kempf, G. Mangano, "Minimal length uncertainty relation and ultraviolet regularization" Phys. Rev. D 55 (1997) 7909.
[15] J. Magueijo and L. Smolin, "Lorentz invariance with an invariant energy scale" Phys. Rev. Lett. 88 (2002) 190403.
[16] J. Magueijo and L. Smolin, "String theories with deformed energy momentum relations, and a possible non-tachyonic bosonic string"Phys. Rev. D 71 (2005)
[17] J.L. Cortes and J. Gamboa, "Quantum Uncertainty in Doubly Special Relativity" Phys. Rev. D 71 (2005) 065015.
[18] J.A. Reyes and M. del Castillo-Mussot, "1D Schrödinger equations with Coulomb-type potentials" J. Phys. A 32 (1999) 2017.
[19]] Y. Ran, L. Xue, S. Hu and R-K. Su, "On the Coulomb-type potential of the one-dimensional Schrödinger equation" J. Phys. A 33 (2000) 9265.
[20] A.N. Gordeyev and S.C. Chhajlany, "One-dimensional hydrogen atom: a singular potential in quantum mechanics"J. Phys. A 30 (1997) 6893.
[21] I. Tsutsui, T. Fulop and T. Cheon, "Connection conditions and the spectral family under singular potentials"J. Phys. A 36 (2003) 275.
[22] H.N. Nunez Yepez, C.A. Vargas and A.L.S. Brito, "The one-dimensional hydrogen atom in momentum representation"Eur. J. Phys. 8 (1987) 189.
[23] Ng Y.J., Modern Physics Letters A, 2003; 18(16):1073–1097.
[24] Maggiore M., Physics Letters B, 1993; 304(1-2):65–69.
[25] Garay L.J., International Journal of Modern Physics A, 1995; 10(02):145–165.
[26] Das S, Vagenas E.C. Canadian Journal of Physics, 2009; 87(3): 233–240.
[27] Das S, Vagenas E.C., Physical Review Letters, 2008; 101(22): Article ID 221301.
[28] Kempf A, Journal of Physics A, 1997; 30(6): 2093–2101.



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