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# ESTIMATION OF A DISEQUILIBRIUM AGGREGATE LABOR MARKET

by

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## I. Introduction

An important question in contemporary economics is whether or not the real wage clears the labor market. Its answer has bearing on issues as diverse as the nature of unemployment, the efficacy of fiscal and monetary policies, and the incidence of income taxes. Unfortunately, consensus as to the correct answer seems to be lacking. While much of modern macroeconomic theory allows for the possibility that the real wage fails to equate the supply and demand of labor [Barro and Grossman, 1971], much analysis is based on the assumption of equilibrium in the labor market [Patinkin, 1965]. The purpose of the present paper is to carry out an econometric test for which view of the labor market is more appropriate.

Although the model we build is very aggregative and much too crude to be used as a basis for policy, we believe that it provides a first step in making operational the theoretical literature on disequilibrium macro models. Our tentative conclusion is that the hypothesis of a labor market in continuous equilibrium must be rejected.

In Section II we describe briefly some earlier work on modelling the aggregate supply and demand for labor. It is shown that prior studies either assume equilibrium in the labor market, or deal with disequilibrium inadequately. In Section III we specify the disequilibrium model. Section IV

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contains a discussion of estimation problems, an interpretation of the results, and a comparison with an equilibrium version of the model. A concluding section has a summary and an agenda for future research.

## II. Antecedents

In this section we review several attempts to estimate the parameters of an aggregative model of the labor market.<sup>1</sup> In the discussion 'disequilibrium' describes a situation in which price fails to equate notional supply and demand; hence a 'disequilibrium price' is a 'non-market-clearing' price. This usage is widespread and in conformity with some of the recent literature ([Barro and Grossman, 1971], [Fair and Jaffee, 1972]) but differs from that of Nadiri and S. Rosen [1974]. They characterize a model as disequilibrium if the economic actors fail to reach an optimum in a given period, even though supply and demand are always equal.

An early attempt to estimate the supply and demand of labor is that of Mosbaek [1959]. His model consists of a labor force participation equation for the entire economy, and a demand equation for labor in the bituminous coal industry. The model is estimated by ordinary least squares. The use of OLS in such a context is inappropriate and it seems curious to estimate the supply curve for the entire economy jointly with the demand curve for the coal industry. Nevertheless, it is interesting that the justification offered for ignoring simultaneous equations problems is that "... the labor market is not in equilibrium" [Mosbaek, 1959, p. 140].

In the Black-Kelejian (B-K) model [1970], an aggregate constant

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<sup>1</sup>Lucas and Rapping [1970] provide a concise summary of attempts to estimate labor supply functions prior to the late 1960's. Summaries of some of the best examples of work on labor supply since then can be found in Hall [1973]. The literature on the demand for labor is discussed by Nadiri and S. Rosen [1974] and Hamermesh [1976].

elasticity of substitution (CES) production function is assumed, and the demand for labor is derived from the marginal productivity condition for profit maximization.<sup>2</sup> The supply side of the model is represented by two labor force participation rate equations, one for primary and one for secondary workers. The real wage does not appear as an argument in either of these equations.<sup>3</sup> The B-K model explicitly assumes the existence of disequilibrium in the labor market and hence includes a wage adjustment equation in which percentage changes in money wages are related to the unemployment rate and to percentage changes in both product prices and the marginal productivity of labor. The simultaneous system is estimated by two-stage least squares. A problem \_\_\_\_\_ in B-K's analysis is that they fail to note that in a disequilibrium model a given observation is on the demand curve or on the supply curve (or possible on neither) but certainly not on both. In general, it cannot be known a priori which schedule an observation lies upon.

The last model we shall consider is Lucas and Rapping's (L-R) [1970]. Although we cannot do justice to the rich theoretical detail of the L-R model, we describe its main components. The aggregate supply of labor depends upon current and anticipated wages and prices, the interest rate, and the market value of household assets. The 'demand'<sup>4</sup> side of the model is derived from

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<sup>2</sup>They also include an equation to explain the division of total manhours between number of workers and hours per worker.

<sup>3</sup>The variables included are a time trend, total employment divided by total labor force, ratio of manufacturing employment to total private employment, a dummy to correct for changes in census definitions, and the ratio of labor compensation less taxes plus transfers to household wealth.

<sup>4</sup>L-R property stress that since output, which is endogenous, appears as an explanatory variable, a marginal productivity condition for labor is different from a demand function for labor.

the marginal productivity condition for a CES production function. There is no disequilibrium in the model: "... The current wage is assumed to equate quantity demanded and quantity supplied each period" [Lucas and Rapping, 1970, p. 272]. Nevertheless, L-R do allow for unemployment, and posit that it is due to job search. Using this theory of unemployment and making certain simplifying assumptions, L-R complete their model with an equation which relates the unemployment rate to current and lagged wages and prices and to the lagged unemployment rate. The model is estimated with annual data from 1929 to 1965.

In both the supply and demand equation, L-R exercise great care to account for slow adjustment in behavior. In spite of this, they take as a maintained hypothesis that the labor market itself is always in equilibrium and that there is no lag in the response of the real wage to changes in supply and demand. L-R thus avoid the econometric problems associated with disequilibrium, but at the cost of imposing a possibly unrealistic constraint. The model developed in the next section is similar in some respects to that of L-R, but it allows for the possibility that the labor market may fail to equilibrate.

### III. The Model

In this section we construct a simple model of the labor market based upon microeconomic foundations. The model consists of four equations: one each for the marginal productivity of labor, the supply of labor, the observed quantity of labor, and the real wage adjustment. Each of these is discussed in turn.

Marginal Productivity of Labor. A necessary condition for profit maximization requires that the marginal product of labor equal the real wage:

$$w_t = f_L(L_t, K_t, t) \quad (1)$$

where  $f_L$  is the partial derivative of the production function

$$Q_t = f(L_t, K_t, t) \quad (2)$$

and where  $Q_t$  is output,  $L_t$  is manhours of labor,  $K_t$  is the flow of services of capital, and  $t$  is a time trend representing the state of technical progress in period  $t$ . Eliminating  $K_t$  between (1) and (2) and assuming that the resulting equation can be solved for  $L_t$ ,

$$L_t^D = \delta(w_t, Q_t, t) \quad (3)$$

The superscript  $D$  indicates that this is the quantity demanded given the level of output. Although (3) is a proper structural relationship, it is not a reduced form equation and hence not in the usual form for the demand equation because of the appearance of the endogenous variable  $Q_t$  on the right hand side.<sup>5</sup>

For purposes of estimation a log linear formulation (except for  $t$ ), is employed for (3)<sup>6</sup>

$$\ln L_t^D = \alpha_0 + \alpha_1 \ln w_t + \alpha_2 \ln Q_t + \alpha_3 t \quad (4)$$

Ideally, one would want to study a multi-market model in which output was

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<sup>5</sup>If some fraction of the labor force is hired on long-term contracts, the first-order condition for a profit maximum must be modified. However, an equation similar to (3) will continue to hold.

<sup>6</sup>If the underlying production function is assumed to be CES, the coefficients in (4) can be used to solve the following set of interdependent equations for the CES parameters:  $\alpha_1 = -\sigma$ ,  $\alpha_2 = (\sigma h + 1 - \sigma)/h$ , and  $\alpha_3 = -\lambda \sigma (1 - \sigma)/h$ , where  $\sigma$  is the elasticity of substitution,  $h$  measures returns to scale, and  $\lambda$  is the rate of Hicks-neutral technological change. Although this is an interesting interpretation, the usefulness of (4) does not rest upon the CES specification.

treated econometrically as an endogenous variable. This task is beyond the scope of the current study since the econometric problems of estimating multiple markets in disequilibrium are as yet unsolved. For tractability it will be assumed that output is exogenous. This assumption is common to most earlier studies in this and related areas.<sup>7</sup>

Supply of Labor. The labor supply function is based upon the theory of leisure-income choice. For an individual with a net wage  $w_{nt}$  and net unearned income  $A_{nt}$ , the indirect utility function can be written

$$V = V(w_{nt}, A_{nt}) \quad (5)$$

where  $V(\cdot)$  is the indirect utility function. Since, by Roy's Identity, the consumer's demand for work can be written in terms of partial derivatives of the indirect utility function,

$$L_t^S = h(w_{nt}, A_{nt}). \quad (6)$$

This model is atemporal in the sense that the leisure-income choice in period  $t$  depends only upon the wage and unearned income in that period. This is considerably simpler than the model of Lucas-Rapping in that it contains no assumptions with respect to expectations mechanisms. Since our observations are annual, this does not seem too severe a restriction.

Since population changes over time, (6) must be augmented by a scale variable which captures changes in the size of the potential labor force. Calling the potential number of manhours  $P_t$ , and using a log-linear form, (6) becomes

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<sup>7</sup>For example, it is a common assumption in studies of investment demand. See [Jorgenson, 1971]. It is also assumed in the L-R model [Lucas and Rapping, 1970].

$$\ln L_t^S = \beta_0 + \beta_1 \ln w_{nt} + \beta_2 \ln A_{nt} + \beta_3 \ln P_t. \quad (7)$$

$A_{nt}$  is assumed to be exogenous but in reality it depends upon past hours of work. Therefore, its estimated coefficient is likely to be biased. Similarly,  $P_t$ , which depends on population growth, would be endogenous in a more detailed model. Since there are different 'types' of labor in the economy,  $\beta_1$  is a weighted average of different groups' responses to changes in the net wage.

Observed Quantity of Labor. In an equilibrium model, the observed quantity of labor is given by the intersection of the supply and demand curves. In a disequilibrium model, this is not the case. We will assume, as does much of the recent work in disequilibrium theory [Barro and Grossman, 1971], that the quantity observed is the minimum of quantity supplied and quantity demanded at the current wage:<sup>8</sup>

$$\ln L_t = \min(\ln L_t^S, \ln L_t^D). \quad (8)$$

Clearly, (8) is not the complete story. One problem is that it does not explain how rationing takes place. Moreover, if aggregation is over sub-markets some of which are characterized by excess demand and some by excess supply, the observed quantity of labor might be some combination of  $L_t^S$  and  $L_t^D$ . Averaging over markets may produce an observed  $L_t$  less than both  $L_t^S$  and  $L_t^D$ , since the expected value of the minimum of two normal variables is smaller than the expected value of either. If both sides of the market possess some monopoly power, the realized outcome may be between  $L_t^D$  and  $L_t^S$  in spite of the oft repeated assumption in the macro disequilibrium literature that no participant can be forced to take (sell) more than his notional demand

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<sup>8</sup>For some econometric uses of min conditions see [Fair and Jaffee, 1972] and [Goldfeld and Quandt, 1975].



(supply). Some of these qualifications could be captured by specifying (8) to have an error term on the right hand side. Although the likelihood function can be deduced in this case, it appears at present unmanageable in practice. For this reason we shall employ (8) as a fairly reasonable approximation to how the observed  $L_t$  is determined.

Real Wage Adjustment. Standard Walrasian analysis suggests that if the wage fails to clear the labor market during a given period, the forces of supply and demand will tend to move it toward equilibrium. This construct has played an important role in theoretical and empirical work [McCallum, 1974]. Unfortunately, economic theory says little about why the wage is 'sticky,' or the determinants of the speed at which it moves toward equilibrium. Search costs, uncertainty, and the time needed to negotiate wage contracts are probably key parts of the answer. Although the standard view lacks a choice-theoretic foundation, we adopt the following simple version of it:

$$\ln w_t - \ln w_{t-1} = \gamma_1 (\ln L_t^D - \ln L_t^S), \quad (9)$$

where  $\gamma_1 > 0$ .

It is possible that noncompetitive elements in the economy can induce movements in the real wage independent of changes in supply and demand. For example, the presence of unions may influence real wages, although the direction of this effect can be argued to be indeterminate.<sup>9</sup> It has been argued by Hines [1964] that wage changes are influenced by the rate of change in the degree of unionization of the labor force. To account for these

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<sup>9</sup>If unions manage to increase the real wage of their members, those who lose jobs in the unionized sector may enter the nonunion sector, driving down the real wage there. The net effect depends upon the magnitudes of the relevant behavioral and technological elasticities. See [Lucas and Rapping, 1970, p. 262].

possibilities, we augment (9) with a variable  $V_t$ , which will be defined in alternative ways:

$$\ln w_t - \ln w_{t-1} = \gamma_1 (\ln L_t^D - \ln L_t^S) + \gamma_2 V_t. \quad (10)$$

Letting  $U_t$  be the percent of the labor force which is unionized in period  $t$ , the first alternative is to define  $V_t = U_t$ . The interpretation is that unions endow workers with monopoly power which is exercised at a uniform rate. A second alternative is to define  $V_t = U_t - U_{t-1}$  in which case unions have only a current impact after which the monopoly power dissipates. Finally,  $V_t = \text{constant}$  (1.0) yields an autonomous drift in real wages and suggests a response to a change in market-clearing wages reflecting anticipated improvements in technology.

#### IV. Estimation and Results

In this section we discuss the data, outline the estimation procedure, present parameter estimates and analyze their implications. We test a number of variants of the basic model, including one in which equilibrium is taken as a maintained hypothesis. For purposes of reference we restate the model:

$$\ln L_t^D = \alpha_0 + \alpha_1 \ln w_t + \alpha_2 \ln Q_t + \alpha_3 t + \epsilon_{1t} \quad (11)$$

$$\ln L_t^S = \beta_0 + \beta_1 \ln w_{nt} + \beta_2 \ln A_{nt} + \beta_3 \ln P_t + \epsilon_{2t} \quad (12)$$

$$\ln L_t = \min(\ln L_t^S, \ln L_t^D) \quad (13)$$

$$\ln w_t - \ln w_{t-1} = \gamma_1 (\ln L_t^D - \ln L_t^S) + \gamma_2 V_t + \epsilon_{3t}. \quad (14)$$

Equations (11), (12) and (14) differ from their counterparts above only by

the addition of the error terms  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_3$ , whose joint distribution is specified below. As indicated earlier, we specify (13) without an error term in order to avoid serious computational problems.

Data. The data are annual observations on the U.S. economy for the years 1930 through 1973.  $L_t$  is total private hours worked per year expressed in billions. It is the product of private domestic hours per person and number of persons engaged in private production.<sup>10</sup>  $w_t$  is total wages and salaries in the private sector expressed in 1958 dollars, divided by the number of private hours worked.<sup>11</sup>  $Q_t$  is gross national product expressed in billions of 1958 dollars.

The net wage  $w_{nt}$  is the product of the gross wage  $w_t$  and a factor  $(1-\theta_t)$ , where  $\theta_t$  is the ratio of personal taxes to personal income in period  $t$ . This adjustment for taxes is similar to that used by Abbott and Ashenfelter [forthcoming].  $A_{nt}$  is the sum of rent, dividends, interest, and profits in 1958 dollars divided by the number of workers, adjusted for taxes by multiplying by  $(1-\theta_t)$ .  $A_{nt}$  does not include retained earnings, unrealized capital gains, or the imputed income from durables. Although it would have been desirable to include these categories in principle, conceptual ambiguities in measurement prevented us from doing so.

$P_t$  is the potential number of hours of work available in year  $t$  expressed in billions. It is calculated by multiplying the number of civilians between the ages of 16 and 64 by the average number of hours worked per year.

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<sup>10</sup>The numbers used are from an updated and revised version of the series found in Table 4 of [Christensen and Jorgenson, 1970]. We are grateful to L. Christensen and D. Jorgenson for making these unpublished figures available to us.

<sup>11</sup>The consumer price index is used to convert to 1958 dollars. All data are from [BLS Handbook of Labor Statistics, 1972] and [Office of Business Economics]. The wage series  $w_t$  could be defined as the BLS index of adjusted hourly earnings in the private nonagricultural sector. This series corrects for overtime and interindustry employment shifts. Unfortunately, these data were available only for the latter part of the sample period.

The implicit assumption here is that in any given year, those absent from the labor force can potentially contribute an annual number of hours equal to the average of those in the labor force. An alternative specification is to set  $P_t$  equal to the number of civilian individuals between 16 and 64. Finally,  $U_t$  is the percentage of the work force unionized in year  $t$ .

Stochastic Specification and Estimation. We assume that the error terms  $\epsilon_{it}$  ( $i=1,2,3$ ) are distributed normally with mean zero and variance-covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$$

Given that the error terms are normally and independently distributed, Eqs. (11), (12) and (14) define the joint density function of the endogenous variables  $\ln L_t^D$ ,  $\ln L_t^S$ , and  $\ln w_t$ . Denoting this density function by  $g(\ln L_t^D, \ln L_t^S, \ln w_t)$ , it is straightforward to show (see [Quandt, forthcoming]) that the joint density of the observable random variables  $\ln L_t$ ,  $\ln w_t$  is

$$h(\ln L_t, \ln w_t) = \int_{\ln L_t}^{\infty} g(\ln L_t, \ln L_t^S, \ln w_t) d\ln L_t^S + \int_{\ln L_t}^{\infty} g(\ln L_t^D, \ln L_t, \ln w_t) d\ln L_t^D \quad (15)$$

The likelihood function is obtained from (15) as

$$L = \prod_{t=1}^T h(\ln L_t, \ln w_t) \quad (16)$$

Maximum likelihood is the estimation technique.<sup>12</sup> The asymptotic standard

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<sup>12</sup>The numerical optimizations were performed using the Davidon-Fletcher-Powell and the quadratic hill-climbing algorithms [Goldfeld and Quandt, 1972].

errors of the estimates are computed by taking the square roots of the diagonal elements of the negative inverse Hessian matrix of the loglikelihood function.

Results. The parameter estimates are shown in Table 1. The four versions of the disequilibrium model (Columns 1 through 4) yield qualitatively and numerically very similar results. The version with  $P_t$  = potential hours and  $V_t = U_t$  yields the highest value for the loglikelihood and is discussed in detail below. We note that  $\alpha_1$  and  $\alpha_2$ , the elasticities of quantity demanded of labor with respect to the real wage and output, are close to unity in absolute value. These values are within the range of other estimates of the labor marginal productivity condition.<sup>13</sup> The coefficient of the time trend has the expected negative sign, but it is statistically insignificant from zero.<sup>14</sup>

The elasticity of supply with respect to the net wage ( $\beta_1$ ) is small in absolute value and insignificantly different from zero. This result is common in virtually all time series and many cross section studies,<sup>15</sup> and suggests that the income and substitution effects of real wage changes are approximately offsetting. An apparently counterintuitive result is the sign of  $\beta_2$ , which suggests a positive elasticity with respect to unearned income. This

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<sup>13</sup>A summary of these results is presented by Hamermesh [1976, pp. 512, 516].

<sup>14</sup>If the marginal productivity relation is assumed to be derived from a CES production function, the implied parameters are  $\sigma = .984$  (.105),  $h = .114$  (.826), and  $\lambda = .027$  (.047). See footnote 6. Observe that if  $\alpha_2$  were 1.0, the point estimate of  $h$  would also be 1.0. Due to the extreme nonlinearity of the relation between  $h$  and  $\alpha_2$  around the point  $h = 1$ , even small deviations away from  $\alpha_2 = 1$  (such as our value of 1.095) lead to drastic changes in the value of the returns to scale parameter. This phenomenon is reflected in the large standard error associated with  $h$  and in the fact that the implied values of  $h$  and  $\lambda$  for column 3 (the  $\alpha_1$  and  $\alpha_2$  estimates of which are within 10 percent of those in column 1) are  $h = 1.250$ ,  $\lambda = .041$ .

<sup>15</sup>See, for example, [Lucas and Rapping, 1970] or [Hall, 1973]. Of course, various subgroups in the population may exhibit labor force behavior quite responsive to changes in the real net wage.

Table 1

Maximum Likelihood Estimates\*

	Disequilibrium Model $P_t = \text{Potential } V_t = U_t$ Hours	Disequilibrium Model $P_t = \text{Population } V_t = U_t$ Hours	Disequilibrium Model $P_t = \text{Potential } V_t = U_t - U_{t-1}$ Hours	Disequilibrium Model $P_t = \text{Potential } V_t = 1.0$ Hours	Equilibrium Model $P_t = \text{Potential } V_t = 1.0$ Hours
$\alpha_0$	-1.330 (.199)	-1.413 (.357)	-1.641 (.404)	-1.460 (.261)	-2.442 (.985)
$\alpha_1$	-.984 (.105)	-.982 (.097)	-1.085 (.120)	-.974 (.094)	-1.480 (.452)
$\alpha_2$	1.095 (.038)	1.111 (.069)	1.153 (.074)	1.122 (.051)	1.241 (.196)
$\alpha_3$	-.003 (.003)	-.003 (.003)	-.003 (.002)	-.004 (.003)	.012 (.019)
$\beta_0$	.209 (.496)	3.422 (.313)	1.132 (1.199)	.059 (1.196)	3.616 (.904)
$\beta_1$	.008 (.040)	-.019 (.057)	.031 (.051)	-.0001 (.051)	.015 (.075)
$\beta_2$	.490 (.046)	.610 (.062)	.482 (.049)	.492 (.048)	.526 (.075)
$\beta_3$	.871 (.091)	.347 (.070)	.700 (.219)	.899 (.219)	.216 (.168)
$\gamma_1$	.182 (.058)	.228 (.073)	.207 (.103)	.125 (.056)	-
$\gamma_2$	.002 (.0003)	.001 (.0003)	.006 (.007)	.030 (.006)	-
logL	202.64	197.22	187.86	199.91	178.30

\* Variables are defined in the text. Numbers in parentheses are standard errors.

phenomenon has been noted in a number of previous studies: when Lucas and Rapping [1970] include real non-human wealth per household in their model, the coefficient is insignificant, and in one of their equations it is positive. A similar result based on cross section data is in [DaVanzo, DeTray and Greenberg, 1976]. This may be due to the fact that, in a life-cycle context, asset income is determined simultaneously with work effort [Smith, 1977]. In an experiment not reported here, we substitute a net wealth for  $A_{nt}$ , and this result was essentially unchanged.

$\beta_3$ , the elasticity of number of hours worked with respect to the potential number of hours of work, is about .87. The estimated standard error indicates that it does not differ significantly from one, which accords with prior expectations. Thus, failure to correct for demographic changes in the labor force may not be too serious a problem.

The coefficient on excess demand,  $\gamma_1$ , is positive and differs significantly from zero, precisely as theory suggests. Finally,  $\gamma_2$  is positive and statistically significant, and implies that an increase of ten percentage points in the labor force that is unionized would lead to a two percent annual increase in real wages, ceteris paribus.

In summary, the parameter estimates of the disequilibrium model generally accord with a priori expectations. The supply and demand parameters are quantitatively in line with the results of earlier studies. In particular, they do not differ greatly from Lucas and Rapping's long run elasticities, despite the fact that our underlying equations contain no lagged variables.<sup>16</sup> Since there are no wage-adjustment equations such as (14) in the earlier

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<sup>16</sup>In terms of our notation, L-R find  $\alpha_1 = -1.09$ ,  $\alpha_2 = 1.00$ . Compare these with the results from Table 1:  $\alpha_1 = -.984$ ,  $\alpha_2 = 1.095$  for column 1 or  $\alpha_1 = -1.085$  and  $\alpha_2 = 1.153$  for column 3.

literature, comparisons are impossible, but the results do not seem unreasonable.

Further Discussion. A number of variations on the basic model were investigated. One experiment was to substitute the number of civilian individuals of age 19 through 64 for potential number of hours in the supply equation. The results, which are shown in column 2 are somewhat less satisfactory than those for the basic model. The coefficient on the population variable falls to .347, and the elasticity of labor supply with respect to unearned income becomes even more positive. The usual likelihood ratio test cannot be used to discriminate among the models of columns 1 through 4 because they express alternative hypotheses which are not nested with respect to one another. Heuristically it may be noted, however, that some of the less reasonable parameter estimates are accompanied by a lower likelihood.

Another question is whether workers correctly perceive the effect of taxes when making their work decision, i.e. whether it is the gross or net wage that enters the supply of labor function. As (12) stands, it is assumed that the net wage is the appropriate variable. However, several investigators have suggested that individuals fail to take taxes into account. (See [Barlow, et.al., 1966] or [Break, 1957]). In order to examine this issue,  $w_t$  and  $(1-\theta_t)$  were entered as separate variables in (12) and the system re-estimated. The coefficients of  $w_t$  and  $(1-\theta_t)$  differed negligibly and -2 times the loglikelihood ratio for testing the null hypothesis that these coefficients were equal yielded a test statistic of 2.56. Thus, the hypothesis that the coefficients of these variables were equal could not be rejected, confirming that it is the net wage that matters.

Next we examined  $V_t$  in the wage-adjustment equation. The results from letting  $V_t$  be alternately  $U_t - U_{t-1}$  or 1.0 are shown in columns 3 and 4.



The coefficient  $\gamma_2$  indicates a 6 percent annual increase in the real wage for a 10 percentage point increase in the unionization rate for column 3 and a steady 3 percent annual increase in the real wage for column 4. The loglikelihoods are lower than in column 1 and on this basis the latter model must be preferred although all three are acceptable on the basis of coefficient estimates.<sup>17</sup>

Perhaps the most interesting question associated with the model is whether or not it is 'better' than its equilibrium counterpart. This is not a straightforward problem, because the hypothesis of equilibrium is not strictly a nested hypothesis.<sup>18</sup> Nevertheless, an approximate test can be made by estimating the equilibrium version and applying the likelihood ratio test. It is also of interest to ascertain the extent to which parameter estimates change when equilibrium is assumed.

The equilibrium system consists of the marginal productivity condition for labor (11), the supply equation (12), and the market clearing relation

$$\ln L_t = \ln L_t^D = \ln L_t^S . \quad (17)$$

In contrasting this system with its disequilibrium counterpart we stress that the equilibrium system is somewhat artificial in that it was specifically designed to facilitate easy comparison with the disequilibrium system that we found to be estimable if not ideal. Thus, in particular, the comparison does not

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<sup>17</sup>Several other variations on the basic model were investigated. In one, a binary variable was added to the supply equation to shift the intercept during the World War II years. In another, nonlabor income was omitted from the supply equation. These results did not have a major impact on the basic qualitative conclusions. Finally,  $V_t$  was defined as  $V_t = t$ . This slightly improves the loglikelihood but the coefficient estimates are barely changed.

<sup>18</sup>It is nested only asymptotically. See [Quandt, forthcoming] for a discussion.

affect the standing of a sophisticated equilibrium model such as the L-R model.

Maximum likelihood estimation of system (11), (12), (17) is routine. The estimates corresponding to the disequilibrium model in columns 1 or 3 are in column 5. The comparison of the results with those in column 1 points toward disequilibrium as the favored hypothesis: (a) The parameter estimates in column 5 are less reasonable ( $\alpha_1$  is numerically too large,  $\alpha_3$  has the wrong sign and  $\beta_3$  is much too small); (b)  $-2 \ln$  (likelihood ratio) is 48.7 for column 1 and 19.0 for column 3, rejecting equilibrium strongly. Finally, (c) it has been suggested [Quandt, forthcoming] that  $1/\gamma_1$  be examined for significant departures from zero: if such were found, the equilibrium model would tend to be rejected. In the present case  $1/\hat{\gamma}_1$  divided by an estimate of its standard error is 1.75 for column 1 and 2.41 for column 3; casting doubt on the equilibrium model mildly in one case and rather more strongly in the other.

Two aspects of the model remain to be explored. The first is the speed with which the system moves toward equilibrium after it is shocked. Assume that all the exogenous variables in the system are held constant and that error terms are zero. Then it can be shown that the steady state value of the logarithm of the gross wage is

$$\ln w_t = \psi k_1^t + \frac{k_2^t}{c} + \frac{k_3}{c} - \frac{k_2}{c} \quad (18)$$

where  $k_1 = 1/(1-(\alpha_1-\beta_1)\gamma_1)$ ,  $k_2 = \gamma_1\alpha_3/(1-(\alpha_1-\beta_1)\gamma_1)$ ,  $k_3 = (\gamma_1(\alpha_2-\beta_2)+\gamma_2)/(1-(\alpha_1-\beta_1)\gamma_1)$ ,  $c = 1 - k_1$  and  $\psi$  is a constant which depends upon initial conditions. Substituting the estimates from column 1 yields  $k_1 = .847$ ,  $k_2 = -.005$ ,  $k_3 = .095$ . Equ. (18) thus is  $\ln w_t = \psi(.847)^t - .003t + .619$ . Disregarding the autonomous part of the solution, one observes from the transient part that an initial discrepancy

between  $\ln w_t$  and its equilibrium path will be reduced by 50 percent in about 4 periods. This conclusion, however, is somewhat sensitive to which version of the model is employed: similar calculations for column 2 indicate a speed of adjustment about twice as rapid although the result for column 3 is nearly the same as for column 1. In any event, the value of  $k_1$  appears to be sufficiently high to suggest that adjustment is sluggish and that the estimated disequilibrium model is not, in fact, mimicking the behavior of an equilibrium model.

The second issue concerns the implications for estimates of 'true' unemployment rates. As Lucas and Rapping [1974, p. 274] emphasize, the question used in unemployment surveys does not ask individuals whether or not they are seeking employment at the current market wage. It is therefore possible that individuals are counted as unemployed even though they are unwilling to work at the going wage. In short, measured unemployment rates probably do not yield a measure of involuntary unemployment as the concept is commonly used in economic theory.

However, one can develop such a measure by using the information reported in Table 1. For each year substitute the values of the right hand side variables into (11) and (12), thus generating predictions  $\hat{L}_t^D$  and  $\hat{L}_t^S$ , respectively. The expression  $\hat{R}_t = (\hat{L}_t^S - \hat{L}_t^D) / \hat{L}_t^S$  provides an estimate of the rate of involuntary unemployment in year  $t$ . By employing a Taylor series expansion of  $\hat{R}_t$  about the parameter estimates, it is possible to obtain approximate expressions for the expected value of  $\hat{R}_t$ ,  $E(\hat{R}_t)$ , and for its standard deviation,  $\sigma(\hat{R}_t)$ .

The point estimates for  $R_t$  were disappointing, since positive estimated excess demand appears in all years up to 1946. The remaining years show results that are not counterintuitive: 1946, 1947 and 1949 exhibit small excess supplies; excess demand reigns in 1948 and 1950-53; finally, the period 1954-73

is characterized by excess supply. The point estimates may not be sufficiently accurate, however, to distinguish periods characterized by common measures as periods of excess demand from those of excess supply. For this reason one may examine quasi-confidence bounds given by  $(E(\hat{R}_t) - 3\sigma(\hat{R}_t), E(\hat{R}_t + 3\sigma(\hat{R}_t)))$ . For any period there are three possible outcomes: (a) the lower limit is positive, in which case the period is classified as one of excess demand; (b) the upper limit is negative, in which case the period is one of excess supply; (c) the lower limit is negative and the upper limit positive in which case the classification is ambiguous. According to this criterion 1931, 1933-35, 1947-50 and 1953-55 are ambiguous, 1930, 1932, 1936-45, 1951-52 are excess demand periods, and the remaining years, 1946 and 1956-73 excess supply periods. It is disappointing that the years of the Great Depression do not show up more unambiguously on the excess supply side, although the excess demand during World War II and the Korean War as well as the excess supply in the last decade do make sense. Clearly further study is needed to investigate the ability of disequilibrium models to predict the values of unobservable endogenous variables.

#### V. Concluding Remarks

We have specified and estimated a simple aggregative disequilibrium model of the labor market. The supply and demand equations are based upon choice theoretic considerations, and the wage adjustment mechanism follows from standard Walrasian analysis. The supply and demand elasticities are generally in accord with those of earlier studies, and the evidence suggests that movement of the system toward equilibrium is quite sluggish.

As suggested throughout this paper, the labor market is only one part of a general disequilibrium system. The other markets must be included in the model

in order to obtain more reliable estimates. The theoretical and statistical problems involved are formidable, but not insurmountable. It is hoped that the current paper will provide an impetus for the required research.

References

- Abbott, Michael and Orley C. Ashenfelter, "Labor Supply, Commodity Demand, and the Allocation of Time," Review of Economic Studies, forthcoming.
- Barlow, Robin, et.al., Economic Behavior of the Affluent. Washington, D.C.: The Brookings Institution, 1966.
- Barro, R.J. and H.I. Grossman, "A General Disequilibrium Model of Income and Employment," American Economic Review, 61(1971), 82-93.
- Black, S. and Harry Kelejian, "A Macro Model of the U.S. Labor Market," Econometrica, 38(1970), 712-741.
- BLS Handbook of Labor Statistics. Washington, D.C.: U.S. Government Printing Office, 1972.
- Break, G., "Income Taxes and Incentives to Work," American Economic Review, 47(1957), 529-549.
- Christensen, Laurits and Dale W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929-1967," The Review of Income and Wealth, 16(1970), 19-50.
- DaVanzo, Julie, Dennis N. DeTray and David H. Greenberg, "The Sensitivity of Male Labor Supply Estimates to Choice of Assumptions," The Review of Economics and Statistics, 58(1976), 313-325.
- Economic Report of the President. Washington, D.C.: U.S. Government Printing Office, various editions.
- Fair, Ray and Dwight Jaffee, "Methods of Estimation for Markets in Disequilibrium," Econometrica, 49(1972), 497-514.
- Goldfeld, Stephen M. and Richard E. Quandt, Nonlinear Methods in Econometrics. Amsterdam: North-Holland Publishing Co., 1972.
- \_\_\_\_\_, "Estimation in a Disequilibrium Model and the Value of Information," Journal of Econometrics, 3(1975), 325-348.
- Hall, Robert E., "Wages, Income and Hours of Work in the U.S. Labor Force," in Income Maintenance and Labor Supply, E. Cain and H. Watts (eds.). Chicago: Rand McNally, 1973.
- Hamermesh, D.S., "Econometric Studies of Labor Demand and Their Application to Policy Analyses," Journal of Human Resources, 11(1976), 507-525.
- Hines, A.G., "Trade Unions and Wage Inflation in the United Kingdom 1893-1961," Review of Economic Studies, XXXI(1964), 221-252.
- Jorgenson, Dale W., "Econometric Studies of Investment Behavior: A Survey," The Journal of Economic Literature, 9(1971), 1111-1147.

- Korliras, D.G., "A Disequilibrium Macroeconomic Model," Quarterly Journal of Economics, 89(1975), 56-80.
- Lucas, Robert and Leonard Rapping, "Real Wages, Employment and Inflation," in Microeconomic Foundations of Employment and Inflation Theory, Edmund Phelps (ed.). New York: W.W. Norton and Co., 1970.
- McCallum, B.T., "Competitive Price Adjustments: An Empirical Study," American Economic Review, LXIV(1974), 56-65.
- Mosbaek, Ernest, "Fitting a Static Supply and Demand Function for Labor," Weltwirtschaftliches Archiv, 82(1959), 133-146.
- Nadiri, Ishaq and Sherwin Rosen, A Disequilibrium Model of Production. New York: National Bureau of Economic Research, 1974.
- Office of Business Economics, Survey of Current Business, various editions.
- Patinkin, Don, Money, Interest and Prices, 2nd edition. New York: Harper and Row, 1965.
- Quandt, Richard E., "Tests of the Equilibrium vs. Disequilibrium Hypotheses," International Economic Review, forthcoming.
- Smith, James, "Assets and Labor Supply" in James P. Smith (ed.), Female Labor Supply: Theory and Estimation. The Rand Corporation, 1977.