ESTIMATION OF FINITE POPULATION MEAN WITH KNOWN COEFFICIENT OF VARIATION OF AN AUXILIARY CHARACTER

H.P. Singh, R. Tailor

1. INTRODUCTION AND THE SUGGESTED ESTIMATOR

It is well known that the use of auxiliary information at the estimation stage provides efficient estimators of the parameter (s) of the study character y. When the population mean $\bar{X}$ of the auxiliary character x is known, a large number of estimators such as ratio, product and regression estimators and their modifications, have been suggested by various authors. Das and Tripathi (1980) have advocated that the coefficient of variation C_x of the auxiliary character x is also available in many practical situations. Keeping this in view, Sisodia and Dwivedi (1981), Singh and Upadhyaya (1986) and Pandey and Dubey (1988) have made the use of coefficient of variation C_x alongwith the population mean $\bar{X}$ in estimating the population mean $\bar{Y}$ of y.

Suppose *n* pairs (x_i, y_i) (i =1,2,...,*n*) observations are taken on n units sampled from *N* population units using simple random sampling without replacement (SRSWOR) scheme. The classical ratio and product estimators for $\bar{Y}$ are respectively defined by

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1}$$

and

$$\hat{\bar{Y}}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \tag{2}$$

where $\bar{x} = \sum_{i=1}^{n} x_i / n$, $\bar{y} = \sum_{i=1}^{n} y_i / n$ and $\bar{X} = \sum_{i=1}^{N} x_i / N$ is the known population mean of the auxiliary character x.

When the population coefficient of variation C_x alongwith the population mean $\bar{X}$ of x is also known, Sisodia and Dwivedi (1981) suggested a ratio-type estimator for $\bar{Y}$ as

$$\hat{\bar{Y}}_{MR} = \bar{y}\frac{(\bar{X}+C_x)}{(\bar{x}+C_x)} \tag{3}$$

and Pandey and Dubey (1988) proposed a product – type estimator for $\bar{Y}$ as

$$\hat{\bar{Y}}_{MP} = \bar{y}\frac{(\bar{x}+C_x)}{(\bar{X}+C_x)} \tag{4}$$

Motivated by Rao and Mudholkar (1967) and Singh and Ruiz Espejo (2003), we suggest a ratio - product estimator for $\bar{Y}$ as

$$\hat{\bar{Y}}_{MRP} = \bar{y}\left[\alpha\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)+(1-\alpha)\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right], \tag{5}$$

where α is a suitably chosen scalar. We note that for α =1, $\hat{\bar{Y}}_{MRP}$ reduces to the estimator $\hat{\bar{Y}}_{MR}$ suggested by Sisodia and Dwivedi (1981) while for α = 0 it reduces to the estimator $\hat{\bar{Y}}_{MP}$ reported by Pandey and Dubey (1988).

2. BIAS OF $\hat{\bar{Y}}_{MRP}$

To obtain the bias of $\hat{\bar{Y}}_{MRP}$, we write

$$\bar{y} = \bar{Y}(1+e_0)$$
$$\bar{x} = \bar{X}(1+e_1)$$

such that

$$\left.\begin{aligned} &E(e_0)=E(e_1)=0\\ &E(e_0^2)=\frac{(1-f)}{n}C_y^2\\ &E(e_1^2)=\frac{(1-f)}{n}C_x^2\\ &E(e_0e_1)=\frac{(1-f)}{n}\rho\, C C_x^2 \end{aligned}\right\} \tag{6}$$

where $C_y = S_y / \bar{Y}$, $C_x = S_x / \bar{X}$, $\rho = S_{yx} / S_x S_y$, $C = \rho C_y / C_x$,

$$S_x^2 = \sum_{i=1}^{N} (x_i - \bar{X})^2 / (N-1), \quad S_y^2 = \sum_{i=1}^{N} (y_i - \bar{Y})^2 / (N-1),$$

$$S_{xy} = \sum_{i=1}^{N} (x_i - \bar{X})(y_i - \bar{Y}) / (N-1).$$

Expressing (5) in terms of e's we have

$$\hat{\bar{Y}}_{MRP} = \bar{Y}(1+e_0)[\alpha(1+\theta e_1)^{-1} + (1-\alpha)(1+\theta e_1)], \tag{7}$$

where $\theta = \bar{X} \big/ (\bar{X} + C_x)$

We now assume that $|\theta e_1| < 1$ so that we may expand $(1+\theta e_1)^{-1}$ as a series in powers of θe_1. Expanding, multiplying out and retaining terms of e's to the second degree, we obtain

$$\hat{\bar{Y}}_{MRP} \cong \bar{Y}[\alpha(1+e_0 - \theta e_1 + \theta^2 e_1^2 - \theta e_0 e_1) + (1-\alpha)(1+e_0 + \theta e_1 + \theta e_0 e_1)]$$

or

$$(\hat{\bar{Y}}_{MRP} - \bar{Y}) \cong \bar{Y}\left[e_0 + \theta e_1 + \theta e_0 e_1 + \alpha(\theta^2 e_1^2 - 2\theta e_0 e_1 - 2\theta e_1)\right] \tag{8}$$

Taking expectations of both sides of (8), we obtain the bias of $\hat{\bar{Y}}_{MRP}$ to order $0(n^{-1})$ as

$$B(\hat{\bar{Y}}_{MRP}) = \frac{(1-f)}{n} \bar{Y} \theta C_x^2 [C + \alpha(\theta - 2C)] \tag{9}$$

which vanishes if

$$\alpha = C / (2C - \theta) \tag{10}$$

Thus the estimator $\hat{\bar{Y}}_{MRP}$ with $\alpha = C/(2C-\theta)$ is almost unbiased. We also note from (9) that the bias of $\hat{\bar{Y}}_{MRP}$ is negligible if the sample size n is sufficiently large.

To the first degree of approximation, the biases of $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$ are respectively given by

$$B(\hat{\bar{Y}}_R) = \frac{(1-f)}{n} \bar{Y} C_x^2 (1-C) \tag{11}$$

$$B(\hat{\bar{Y}}_P) = \frac{(1-f)}{n}\bar{Y} C C_x^2 \tag{12}$$

$$B(\hat{\bar{Y}}_{MR}) = \frac{(1-f)}{n}\bar{Y}\theta C_x^2(\theta - C) \tag{13}$$

$$B(\hat{\bar{Y}}_{MP}) = \frac{(1-f)}{n}\bar{Y}\theta\, C C_x^2 \tag{14}$$

From (9) and (11) we note that

$$\left|B(\hat{\bar{Y}}_{MRP})\right| < \left|B(\hat{\bar{Y}}_{R})\right| \quad \text{if}$$

$$\left|\theta\{C + \alpha(\theta - 2C)\}\right| < \left|(1-C)\right|$$

i.e. if

$$\left.\begin{array}{ll} \textit{either} & \dfrac{-\{1-C(1-\theta)\}}{\theta(\theta-2C)} < \alpha < \dfrac{\{1-C(1+\theta)\}}{\theta(\theta-2C)} \\ \textit{or} & \dfrac{\{1-C(1+\theta)\}}{\theta(\theta-2C)} < \alpha < \dfrac{-\{1-C(1-\theta)\}}{\theta(\theta-2C)} \end{array}\right\} \tag{15}$$

From (9) and (12) we note that

$$\left|B(\hat{\bar{Y}}_{MRP})\right| < \left|B(\hat{\bar{Y}}_{P})\right| \quad \text{if}$$

$$\left|\theta\{C + \alpha(\theta - 2C)\}\right| < \left|C\right|$$

i.e. if

$$\left.\begin{array}{ll} \textit{either} & -\dfrac{C(1+\theta)}{\theta(\theta-2C)} < \alpha < \dfrac{C(1-\theta)}{\theta(\theta-2C)} \\ \textit{or} & \dfrac{C(1-\theta)}{\theta(\theta-2C)} < \alpha < -\dfrac{C(1+\theta)}{\theta(\theta-2C)} \end{array}\right\}\theta \tag{16}$$

From (9) and (13) it follows that

$$\left|B(\hat{\bar{Y}}_{MRP})\right| < \left|B(\hat{\bar{Y}}_{MR})\right| \quad \text{if}$$

$$\left|\{C + \alpha(\theta - 2C)\}\right| < \left|(\theta - C)\right|$$

i.e. if $\quad -1 < \alpha < 1$ (17)

Further from (9) and (14) we see that

$$\left|B(\hat{\bar{Y}}_{MRP})\right| < \left|B(\hat{\bar{Y}}_{MP})\right| \quad \text{if}$$

$$\left|\{C + \alpha(\theta - 2C)\}\right| < |C|$$

i.e. if

$$\left.\begin{array}{ll} \textit{either} & -\dfrac{2C}{(\theta - 2C)} < \alpha < 0 \\ \textit{or} & 0 < \alpha < -\dfrac{2C}{(\theta - 2C)} \end{array}\right\} \tag{18}$$

3. VARIANCE OF $\hat{\bar{Y}}_{MRP}$

Squaring both sides of (8) and neglecting terms of e's having power greater than two we have

$$(\hat{\bar{Y}}_{MRP} - \bar{Y})^2 = \bar{Y}^2[e_0^2 + (1-2\alpha)\theta\{(1-2\alpha)\theta e_1^2 + 2e_0e_1\}] \tag{19}$$

Taking expectations both sides in (19), we get the variance of $\hat{\bar{Y}}_{MRP}$ to the first degree of approximation as

$$V(\hat{\bar{Y}}_{MRP}) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + (1-2\alpha)\theta C_x^2\{(1-2\alpha)\theta + 2C\}] \tag{20}$$

which is minimized for

$$\alpha = \frac{(\theta + C)}{2\theta} = \alpha_0 \text{ (say)} \tag{21}$$

Substitution of (21) in (5) yields the asymptotically optimum estimator (AOE) for $\bar{Y}$ as

$$\hat{\bar{Y}}_{MRP}^{(0)} = \frac{\bar{y}}{2\theta}\left[(\theta + C)\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right) + (\theta - C)\left(\frac{\bar{x} + C_x}{\bar{X} + C_x}\right)\right] \tag{22}$$

Putting (21) in (9) and (20) we get the bias and variance of $\hat{\bar{Y}}_{MRP}^{(0)}$ respectively as

$$B(\hat{\bar{Y}}_{MRP}^{(0)}) = \frac{(1-f)}{2n}\bar{Y}C_x^2(\theta - C)(\theta + C) \tag{23}$$

and

$$V(\hat{\bar{Y}}_{MRP}^{(0)}) = \frac{(1-f)}{n}S_y^2(1-\rho^2) \tag{24}$$

It is to be mentioned here that the variance of $\hat{\bar{Y}}_{MRP}^{(0)}$ at (24) is same as that of the approximate variance of the usual linear regression estimator $\bar{y}_{lr} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$, where $\hat{\beta}$ is the sample regression coefficient of y on x.

Remark 3.1. In practice, with a good guess of 'C' obtained through pilot surveys, past data or experience gathered in due course of time, an optimum value of α fairely close to its true value α_0 can be obtained. This problem has been also discussed among others by Murthy (1967, pp. 96-99), Reddy (1978) and Srivenkataramana and Tracy (1980). Further if a good guess of the interval containing 'C' (i.e. $C_1 \le C \le C_2$) which is more realistic than a specific guess about C, can be made on theory, accumulated experience and/or a scatter diagram for at least a part of current data then it is also advisable to use the suggested estimator in practice.

4. EFFICIENCY COMPARISONS

It is well known under SRSWOR that

$$V(\bar{y}) = \frac{(1-f)}{n}\bar{Y}^2C_y^2 \tag{25}$$

and the variance of $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$ to the first degree of approximation are respectively given by

$$V(\hat{\bar{Y}}_R) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + C_x^2(1-2C)] \tag{26}$$

$$V(\hat{\bar{Y}}_P) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + C_x^2(1+2C)] \tag{27}$$

$$V(\hat{\bar{Y}}_{MR}) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + \theta C_x^2(\theta - 2C)] \tag{28}$$

$$V(\hat{\bar{Y}}_{MP}) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + \theta C_x^2(\theta + 2C)] \tag{29}$$

From (20) and (25) we have

$$V(\hat{\bar{Y}}_{MRP}) - V(\bar{y}) = \frac{(1-f)}{n}\bar{Y}^2 C_x^2 \theta(1-2\alpha)\{\theta(1-2\alpha) + 2C\}$$

which is negative if

$$\left.\begin{array}{ll} \textit{either} & \dfrac{1}{2} < \alpha < \left(\dfrac{1}{2} + \dfrac{C}{\theta}\right) \\ \textit{or} & \left(\dfrac{1}{2} + \dfrac{C}{\theta}\right) < \alpha < \dfrac{1}{2} \end{array}\right\} \tag{30}$$

From (20) and (26) we have

$$V(\hat{\bar{Y}}_{MRP}) - V(\hat{\bar{Y}}_{\mathrm{R}}) = \frac{(1-f)}{n}\bar{Y}^2 C_x^2 \{(1-2\alpha)\theta + 1\}\{(1-2\alpha)\theta + 2C - 1\}$$

which is negative if

$$\left.\begin{array}{ll} \textit{either} & \dfrac{(1+\theta)}{2\theta} < \alpha < \left(\dfrac{\theta + 2C - 1}{2\theta}\right) \\ \textit{or} & \left(\dfrac{\theta + 2C - 1}{2\theta}\right) < \alpha < \dfrac{(1+\theta)}{2\theta} \end{array}\right\} \tag{31}$$

From (20) and (27) we have

$$V(\hat{\bar{Y}}_{MRP}) - V(\hat{\bar{Y}}_{P}) = \frac{(1-f)}{n}\bar{Y}^2 C_x^2 \{(1-2\alpha)\theta - 1\}\{(1-2\alpha)\theta + 2C + 1\}$$

which is negative if

$$\left.\begin{array}{ll} \textit{either} & \left(\dfrac{\theta + 2C + 1}{2\theta}\right) < \alpha < \dfrac{(\theta - 1)}{2\theta} \\ \textit{or} & \dfrac{(\theta - 1)}{2\theta} < \alpha < \dfrac{(\theta + 2C + 1)}{2\theta} \end{array}\right\} \tag{32}$$

From (20) and (28) we have

$$V(\hat{\bar{Y}}_{MRP}) - V(\hat{\bar{Y}}_{MR}) = 4\frac{(1-f)}{n}\bar{Y}^2 C_x^2 \theta(\alpha - 1)(\alpha\theta - C)$$

which is negative if

$$\left.\begin{array}{ll} \textit{either} & 1<\alpha<\dfrac{C}{\theta} \\ \textit{or} & \dfrac{C}{\theta}<\alpha<1 \end{array}\right\} \tag{33}$$

Further from (20) and (29) we have

$$V(\hat{\hat{Y}}_{MRP})-V(\hat{\hat{Y}}_{MP})=4\frac{(1-f)}{n}\bar{Y}^2C_x^2\alpha\theta\{\theta(\alpha-1)-C\}$$

which is negative if

$$\left.\begin{array}{ll} \textit{either} & 0<\alpha<\left(1+\dfrac{C}{\theta}\right) \\ \textit{or} & \left(1+\dfrac{C}{\theta}\right)<\alpha<0 \end{array}\right\} \tag{34}$$

5. ESTIMATOR BASED ON ESTIMATED 'OPTIMUM'

If exact or good guess of 'C' is not available, we can replace 'C' by the sample estimate $\hat{C}$ in (22) and get the estimator (based on estimated optimum) as

$$\hat{\hat{Y}}^{(0)}_{MRP}=\frac{\bar{y}}{2\theta}\left[(\theta+\hat{C})\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)+(\theta-\hat{C})\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right] \tag{35}$$

where $\hat{C}=(s_{xy}/\bar{y})\{1/\bar{X}C_x^2\}$ where, we recall, $\bar{X}$ and C_x are known, and $s_{xy}=\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})/(n-1)$.

To obtain the variance of $\hat{\hat{Y}}^{(0)}_{MRP}$ we write

$$\hat{C}=C(1+e_2)$$

with $E(\hat{C})=C+o(n^{-1})$.

Expressing $\hat{\hat{Y}}^{(0)}_{MRP}$ in terms of $e's$ we have

$$\hat{\hat{Y}}^{(0)}_{MRP}=\frac{\bar{Y}}{2\theta}(1+e_0)[\{\theta+C(1+e_2)\}(1+\theta e_1)^{-1}+\{\theta-C(1+e_2)\}(1+\theta e_1)]$$

where e_0 and e_1 are same as defined in section 2. The variance of $\hat{\hat{Y}}^{(0)}_{MRP}$ is

$$V(\hat{\hat{\bar{Y}}}_{MRP}^{(0)}) = E(\hat{\hat{\bar{Y}}}_{MRP}^{(0)} - \bar{Y})^2$$
$$= \bar{Y}^2 E\left(\frac{(1+e_0)}{2\theta}[\{\theta + C(1+e_2)\}(1+\theta e_1)^{-1} + \{\theta - C(1+e_2)\}(1+\theta e_1)] - 1\right)^2 \quad (36)$$

Expanding the terms on the right hand side of (36) and neglecting power of $e's$ that are greater than two we have

$$V(\hat{\hat{\bar{Y}}}_{MRP}^{(0)}) = \bar{Y}^2 E(e_0 - C e_1)^2 = \bar{Y}^2 E(e_0^2 - 2C e_0 e_1 + C^2 e_1^2)$$
$$= \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + C^2 C_x^2 - 2C \rho C_y C_x)] = \frac{(1-f)}{n} S_y^2 (1-\rho^2)$$

which is same as that of $\hat{\bar{Y}}_{MRP}^{(0)}$ *i.e.* $V(\hat{\bar{Y}}_{MRP}^{(0)}) = V(\hat{\hat{\bar{Y}}}_{MRP}^{(0)})$. Thus it is established that the variance of the estimator $\hat{\hat{\bar{Y}}}_{MRP}^{(0)}$ in (35) based on the estimated optimum, to terms of order n^{-1}, is the same as that of $\hat{\bar{Y}}_{MRP}^{(0)}$ in (22).

6. EMPIRICAL STUDY

To examine the merits of the suggested estimator we have considered five natural population data sets. The description of the population are given below.

Population – I: Murthy (1967, p. 228)
N= 80, y: Output
n = 20, x: Fixed Capital
$\bar{Y} = 51.8264$, $\bar{X} = 11.2646$, $C_y = 0.3542$, $C_x = 0.7507$,
$\rho = 0.9413$, $C = 0.4441$ $f = 0.25$, $\theta = 0.9375.$,

Population – II: Murthy (1967, p. 228)
N= 80, y: Output
n = 20, x: Number of Workers
$\bar{Y} = 51.8264$, $\bar{X} = 2.8513$, $C_y = 0.3542$, $C_x = 0.9484$
$\rho = 0.9150$, $C = 0.3417$ $f = 0.25$, $\theta = 0.7504.$,

Population – III: Das (1988)
N= 278, y: Number of agricultural labourers for 1971
n = 30, x: Number of agricultural labourers for 1961
$\bar{Y} = 39.0680$, $\bar{X} = 25.1110$, $C_y = 1.4451$, $C_x = 1.6198$,
$\rho = 0.7213$, $C = 0.6435$ $f = 0.1079$, $\theta = 0.9394.$,

<u>Population – IV</u>: Steel and Torrie (1960, p. 282)
N= 30, y: Log of leaf burn in secs
n = 6, x: Clorine percentage

$\bar{Y} = 0.6860$, $\bar{X} = 0.8077$, $C_y = 0.700123$, $C_x = 0.7493$,

$\rho = -0.4996$, $C = -0.3202$ $\theta = 0.5188$ $f = 0.20$,

<u>Population – V</u>: Maddala (1977)
N= 16, y: Consumption per capita
n = 4, x: Deflated prices of veal

$\bar{Y} = 7.6375$, $\bar{X} = 75.4313$, $C_y = 0.2278$, $C_x = 0.0986$,

$\rho = -0.6823$ $C = -1.5761$ $\theta = 0.9987$ $f = 0.25$

We have computed the ranges of α for which the proposed estimator $\hat{\bar{Y}}_{MRP}$ is better than $\bar{y}$, $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$, optimum value of α and common range of α and displayed in Table 1. Table 2 shows the percent relative efficiencies of $\hat{\bar{Y}}_{MRP}$ with respect to $\bar{y}$, $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$.

TABLE 1

Range of α in which $\hat{\bar{Y}}_{MRP}$ is better than $\bar{y}$, $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$

Population	Range of α in which $\hat{\bar{Y}}_{MRP}$ is better than					Optimum value of α	Common range of α in which $\hat{\bar{Y}}_{MRP}$ is better than $\bar{y}$, $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$.
	$\bar{y}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_P$	$\hat{\bar{Y}}_{MR}$	$\hat{\bar{Y}}_{MP}$	α_0	
I	(0.50,0.9737)	(0.4404,1.0333)	(-0.033, 1.5070)	(0.4737, 1.00)	(0.00, 1.4737)	0.73685	(0.50, 0.9737)
II	(0.50,0.9554)	(0.2891,1.1663)	(-0.1663,1.6217)	(0.4554, 1.00)	(0.00, 1.4554)	0.72768	(0.50, 0.9554)
III	(0.50,1.1850)	(0.6528,1.0323)	(-0.0323,1.7173)	(0.6850, 1.00)	(0.00, 1.6850)	0.84251	(0.6850, 1.00)
IV	(-0.1172,0.50)	(-1.0810,1.4638)	(-0.4638,0.8466)	(-0.6172, 1.00)	(0.00, 0.3828)	0.19140	(0.00, 0.3828)
V	(-1.0782,0.50)	(-1.5788,1.0007)	(-0.577,-0.0007)	(-1.5782, 1.00)	(-0.5782, 0.00)	-0.28908	(-0.5775,0.0007)

TABLE 2

Percent relative efficiencies of $\hat{\bar{Y}}^{(0)}_{MRP}$ or $\hat{\bar{Y}}^{(0)}_{MRP}$ with respect to $\bar{y}$, $\hat{\bar{Y}}_R$, $\hat{\bar{Y}}_P$, $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$

Population	Percent relative efficiencies of $\hat{\bar{Y}}_{MRP}$ with respect to:				
	$\bar{y}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_P$	$\hat{\bar{Y}}_{MR}$	$\hat{\bar{Y}}_{MP}$
I	877.62	1318.18	*	1059.44	*
II	614.40	2008.96	*	835.86	*
III	208.46	133.29	*	122.94	*
IV	133.26	*	249.90	*	112.80
V	187.37	*	111.91	*	111.88

* Data not applicable.

Table 1 exhibits that there is enough scope of selecting the scalar 'α' in $\hat{\bar{Y}}_{MRP}$ to get better estimators. It is observed that even if α slides away from its true optimum value, the efficiency of the suggested estimator $\hat{\bar{Y}}_{MRP}$ can be increased considerably. Table 2 clearly indicates that the suggested estimator $\hat{\bar{Y}}^{(0)}_{MRP}$ *or* $\hat{\hat{\bar{Y}}}^{(0)}_{MRP}$ is more efficient (with substantial gain) than the usual unbiased estimator $\bar{y}$, classical ratio estimator $\hat{\bar{Y}}_R$ and product estimator $\hat{\bar{Y}}_P$, and the modified estimators $\hat{\bar{Y}}_{MR}$ and $\hat{\bar{Y}}_{MP}$ suggested by Sisodia and Dwivedi (1981) and Pandey and Dubey (1988) respectively. Thus the proposed estimator $\hat{\hat{\bar{Y}}}^{(0)}_{MRP}$ is to be preferred in practice.

7. CONCLUSION

This article is concerned with estimating the population mean $\bar{Y}$ of the study variate y using auxiliary information at the estimation stage. When the population mean $\bar{X}$ and coefficient of variation C_x of an auxiliary variable x is known, a class of estimators for estimating $\bar{Y}$ is suggested. 'Optimum' estimator in the class is identified with its approximate variance formula. Estimator based on estimated optimum values is also proposed with its approximate variance formula. It is interesting to note that the estimators based on 'optimum value' and 'estimated optimum value' have the same approximate variance formula. Thus we conclude that the studies carried out in the present article can be used fruitfully even if the optimum values are not known. An empirical study is carried out to throw light on the performance of the suggested estimator over already existing estimators. Further empirical studies carried out in this article clearly reflect the usefulness of the proposed estimators in practice.

School of Studies in Statistics H.P. SINGH
Vikram University, Ujjain, India RITESH TAILOR

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RIASSUNTO

Stima della media di un popolazione finita con coefficiente di variazione di un carattere ausiliario noto

Il contributo si occupa del problema della stima di una media di popolazione $\overline{Y}$ di una variabile oggetto di studio *y* utilizzando l'informazione sulla media di popolazione $\overline{X}$ e sul coefficiente di variazione C_x di un carattere ausiliario *x*. Viene suggerito uno stimatore per il parametro $\overline{Y}$ e ne vengono studiate le proprietà nel contesto di singoli campioni. Si dimostra che lo stimatore proposto, sotto alcune condizioni realistiche, è più efficiente degli stimatori proposti da Sisodia e Dwivedi (1981) e da Pandey e Dubey (1988). Tramite una analisi empirica vengono esaminati i meriti dello stimatore costruito rispetto agli antagonisti.

SUMMARY

Estimation of finite population mean with known coefficient of variation of an auxiliary character

This paper deals with the problem of estimating population mean $\overline{Y}$ of the study variate y using information on population mean $\overline{X}$ and coefficient of variation C_x of an

auxiliary character x. We have suggested an estimator for $\overline{Y}$ and its properties are studied in the context of single sampling. It is shown that the proposed estimator is more efficient than Sisodia and Dwivedi (1981) estimator and Pandey and Dubey (1988) estimator under some realistic conditions. An empirical study is carried out to examine the merits of the constructed estimator over others.