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Estimation of formation temperature from borehole measurements

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Summary. A new numerical method is described for modelling the thermal disturbance around a borehole and a comparison is made with the commonly used line source model. The numerical model generally gives an estimate of the equilibrium formation temperature which is up to about 20 per cent higher than the 'Horner plot' method.

The relaxation of the borehole temperature is sensitive to the contrast between the thermal properties of the drilling fluid and of the surrounding rock as well as to the disturbance time. The importance of vertical temperature gradients and fluid motion (both free convection in the borehole and flow in the formation) is also examined. It is found that vertical temperature gradients are unlikely to be important provided temperatures are measured more than 10 borehole radii above the bottom. Convective heat transfer can be important under some drilling conditions.

Introduction

Knowledge of the undisturbed temperature in a formation is required for geothermal calculations, for the interpretation of electrical resistivity logs and for various well completion operations.

Ideally the steady state temperature would be obtained by direct measurement. However, the temperatures recorded during normal wireline logging are critically dependent on the drilling history of the borehole. It has been known for some time (see Bullard 1947) that it is necessary to shut the well for periods which vary from a few days to a few months before thermal equilibrium is re-established. Clearly in most commercial drilling operations such long shut-in times are impractical.

Thus there is a need for a model of the thermal disturbance caused by drilling and of the subsequent thermal relaxation during the shut-in period. The aim of such a model would be to predict a theoretical temperature build-up curve for the time period covered by the wireline logging operations. The matching of such a curve to the logged temperatures can then be used to estimate a value for the equilibrium formation temperature.

As the drilling process is variable with depth and time it makes considerable sense to localize the model to the bottom 10–20 m of the borehole. Hopefully one can then assume that parameters such as mud circulation rate, weight and inlet temperature are almost constant and the variation in other parameters (such as geothermal gradient and lithology) is small. At the very least the cumulative errors due to such assumptions can be minimized, because integrations are for a more limited time.

The current published models fall into two classes. One class attempts to describe the evolution of the temperature of the complete mud column. The other set of models concentrate on the region of the borehole where the bottom hole temperatures (BHTs) are actually measured.

Models of the complete fluid column in the hole have been published by Jaeger (1961), Holmes & Swift (1970), Edwardson *et al.* (1962). Tregasser, Crawford & Crawford (1967), Keller, Couch & Berry (1973) and Wooley (1980). The main difficulty with all these models is that they need a value for the equilibrium geothermal gradient which for many applications is the property we are trying to estimate. Hence to use these models to fit measured BHTs would require an iterative technique with the vertical equilibrium temperature gradient as the control variable. An iterative procedure based on the model of Jaeger (1961) has been used by Burch & Langseth (1981).

The sensitivity of these complete mud column models to possible variations in parameters such as mud composition, inlet temperature, etc. is unknown. Usually parameters such as fluid circulation rate, inlet temperature and physical properties are taken as constant. Making such assumptions over an extended depth of hole and period of time could lead to the danger of accumulation of errors of an unknown magnitude.

The second set of models concentrate on describing the section of the borehole near the bottom. The drilling process is assumed to introduce a temperature disturbance of an unknown magnitude at the time the drill bit cuts through the depth at which the BHTs were measured. The amplitude of this temperature disturbance is left as a free parameter, which is to be fitted by the model.

Basically these models have two free parameters: (a) the temperature of the drilling mud during circulation [or in the case of the line source model (see Bullard 1947) the temperature at surface of the borehole at the end of circulation] and (b) the equilibrium formation temperature.

The line source/sink model proposed by Bullard (1947) forms the basis of the popular 'Horner plot' method (see Dowdle & Cobb 1975) and because of its use in this guise will be described in more detail below.

Middleton (1979) proposed a model in which the borehole was created instantaneously and is of rectangular cross-section. This model ignores the finite time taken to drill the bottom section of the borehole and the subsequent mud circulation period while the hole is being conditioned. The physical properties of the mud and rock are taken to be identical. Leblanc, Pascoe & Jones (1981) pointed out a mathematical inconsistency in Middleton's model and gave a corrected version. Leblanc *et al.* also gave the solution for a similar model with circular hole cross-section.

The main difficulties with the Middleton (1979) and Leblanc *et al.* (1981) models are the assumptions concerning zero disturbance/circulation time and identical mud and rock thermal properties. As will be shown below the thermal disturbance and subsequent relaxation are sensitive to the finite circulation time and to the contrast in the mud/rock properties.

Leblanc *et al.* (1982) have applied the results of their 1981 paper to some real data. Their model has three free parameters, namely T_∞ (the equilibrium temperature), T_m (the initial mud temperature) and κ (a composite thermal diffusivity which describes the thermal inertia of the system as a whole, i.e. mud plus rock). They fitted this model to data from a group of wells with T_∞ and T_m as free parameters with various values of κ. They selected the value of κ which gave the closest agreement with the T_∞ calculated using the 'Horner plot' method for the data from many wells. The authors suggest that this value of κ is a good estimate of the thermal diffusivity of the well contents. There are some difficulties with drawing this conclusion:

(i) It assumes that the 'Horner plot' method is generally valid for the extrapolation of BHTs to ambient temperature.

(ii) The diffusivity in the Leblanc *et al.* (1982) model is for the whole composite system, i.e. it is the value that would be obtained if the borehole rock system *could* be replaced with some homogeneous material which extended from $r = 0$ to ∞.

(iii) There are likely to be marked differences between the drilling muds used in different boreholes, the largest being between water-based and oil-based muds.

As will be argued below the validity of the 'Horner plot' approximation is limited for both mathematical and physical reasons and the detailed model which forms the basis of this paper shows that the size of the contrast in the mud and rock thermal properties is also an important parameter.

The paper is organized as follows: Section 1 discusses the Bullard line source model and the 'Horner plot' approximation. Section 2 describes the new 'EFT' (for Equilibrium Formation Temperature) model. Examples of the use of this model and a comparison with the line source models are given in Section 3. A general discussion and conclusions are given in Section 4 and 5, respectively. The numerical technique used to solve the 'EFT' model is described in Appendix A.

Temperature build-up models in the past have generally ignored the effect of fluid flow in the formation, convection in the borehole and of vertical temperature gradients on the temperature build-up. These are considered in Appendices B, C and D.

1 Line source model

The Bullard (1947) model was described by the conductive heat flow equation and assumed that whilst mud was being circulated heat was extracted by an infinitely thin and long axial source at the rate $-Q$. The end of circulation was simulated by switching on an additional source $+Q$. The solution to this problem was given by Bullard,

$$T(r, t) = T_\infty + \left(\frac{Q}{4\pi K}\right) \left\{E_i\left(\frac{-r^2}{4\kappa t}\right) - E_i\left(\frac{-r^2}{4\kappa t_2}\right)\right\} \qquad (1)$$

where,

$T(r, t)$ = the temperature at radius r,

K = thermal conductivity,

κ = thermal diffusivity of the system,

t_1 = total mud circulation time at the depth of measurement,

$t_2 = t - t_1$ = shut-in time after the end of circulation.

The function $E_i(-z)$ is the exponential integral. In practice r is taken as the borehole radius.

This model has been used in practice by applying the following asymptotic expansion of equation (1) given by Bullard,

$$T(a, t) \simeq T_\infty + \left(\frac{Q}{4\pi K}\right) \ln\left\{\frac{t_2}{t_1 + t_2}\right\} \qquad \text{for} \quad t_2 > 0 \qquad (2)$$

which is valid provided that

$$\frac{a^2}{4\kappa t_2} \ll 1 \tag{3}$$

where a is the borehole radius. Note that this condition depends on the square of the borehole radius.

The temperature extrapolation method given by equation (2) has been commonly referred to as the 'Horner plot' method, due to its similarity to the pressure build-up model given by Horner (1951). In the temperature build-up case shut-in times of between 20 and 50 hr are needed in an 8.5 in. diameter hole before equation (2) becomes a good approximation to equation (1) (assuming a diffusivity of about $5 \times 10^{-7} \mathrm{m}^2 \mathrm{s}^{-1}$). Also the shut-in time needed for (3) to be satisfied varies as the square of the borehole radius and hence for larger diameters the 'Horner plot' approximation rapidly becomes untenable on mathematical grounds alone.

There is a more serious physical problem with the line source model which is lost in the 'Horner plot' approximation. Equation (2) implies that the borehole temperature increases monotonically during the shut-in period. A careful evaluation of the full line source solution (equation 1) shows that during shut-in the temperature at the hole surface decreases initially before starting to build up (see, e.g. Fig. 7). This means that the temperature build-up phase is delayed beyond the end of the circulation phase. Under certain circumstances this could lead to the line source model predicting a value for T_∞ which is *lower* than the measured BHTs!

The length of time for which the build-up is retarded is a strong function of hole diameter, circulation time and diffusivity. This leads to some ambiguity due to the somewhat *ad hoc* value of diffusivity which must be used (since the mud and rock properties are taken to be the same).

The physical reason for this delay in the build-up can be traced to the manner in which the mud circulation and shut-in phase are modelled. The line source model assumes that heat is continuously extracted by an axial sink for all time. The end of circulation is simulated by switching on a heat source at the axis which is equal in magnitude to the original sink. This heat source takes some time to have an appreciable effect at the hole boundary due to the finite diameter of the borehole. Thus initially during the shut-in period the temperature at the horehole surface continues to decrease because of the cooling effect of the original line sink. The boundary temperature only starts to build up once the source begins to have a large enough effect at the hole surface.

This effect is built into equation (1) and if this is used for temperature inversion then if the cutoff threshold is not exceeded the predicted T_∞ will clearly be unreasonable. The danger lies in using the 'Horner plot' technique which will give what are apparently plausible values for T_∞ when its physical basis is clearly no longer sound.

The above arguments imply that the indiscriminate use of the 'Horner plot' method for inversion of BHTs should be discouraged.

2 The 'EFT' (Equilibrium Formation Temperature) model

The 'EFT' model has been designed to relax some of the assumptions made by previous models. The basic assumptions are:

heat flow is purely radial;
the borehole contents and surrounding rock have different properties in general;

the temperature of the fluid in the borehole is constant and uniform during circulation; and no convective heat flow during shut-in (see Appendices B and C).

The model is described by the following equation

$$\rho c \frac{\partial T}{\partial t} = K \left\{ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right\} . \tag{4}$$

The initial conditions are

$$T(r, 0) = T_m \quad \text{for} \quad 0 \leqslant r \leqslant a \tag{5}$$

$$T(r, t < t_1) = T_m \quad \text{for} \quad 0 \leqslant r \leqslant a$$

$$T(r, 0) = T_\infty \quad \text{for} \quad r > a \tag{6}$$

the boundary conditions are

$$T(r, t) = T_\infty \quad r \to \infty, \quad t > 0 \tag{7}$$

$$T(a, t)|_1 = T(a, t)|_2 \; \begin{matrix} t > t_1 \\ r = a \end{matrix} \tag{8}$$

$$K_1 \left.\frac{\partial T}{\partial r}\right|_1 = K_2 \left.\frac{\partial T}{\partial r}\right|_2 \; \begin{matrix} t > t_1 \\ r = a \end{matrix} \tag{9}$$

where

$(\rho c)_1$ = heat capacity of the borehole contents,
$(\rho c)_2$ = heat capacity of the rock,
K_1 = thermal conductivity of the borehole contents,
K_2 = thermal conductivity of the rock.

The subscripts 1 and 2 refer to the inside and outside of the borehole/rock boundary.

The initial conditions (5) and (6) state that at time $t = 0$ (i.e. when the drill bit cuts through the depth of BHT measurement) the temperature of the hole becomes T_m and in the rock it is T_∞ (i.e. there is a temperature step at the boundary). Condition (5) also describes the mud circulation phase. Conditions (8) and (9) describe the shut-in phase (i.e. the temperature and heat flux are continuous at the borehole boundary).

Note that equation (4) assumes that the thermal properties are independent of temperature and this is not necessarily a safe assumption especially in the case of the specific heat and density of the mud. Although allowing for this non-linearity is simple in principle (given a relation for the temperature dependence of the properties) it would make the inversion problem very complicated.

Solutions in integral form have been found for two special cases of the model described by equations (4) to (9):

(i) Carlslaw & Jaeger give a solution for the case when the borehole is created instantaneously (i.e. $t_1 = 0$; see Carslaw & Jaeger 1959, p. 346, equations 7 and 8),

(ii) Jaeger (1956) gave the solution for the case when the borehole and rock properties are the same but the circulation time is non-zero [i.e. $t_1 > 0, K_1 = K_2$ and $(\rho c)_1 = (\rho c)_2$].

The model used by Leblanc *et al.* (1982) is a special case of either of the above two analytical solutions. The integrals describing these two solutions need to be evaluated numerically. They do not describe the general case of different borehole and rock properties with non-zero circulation time, but they are usful as a benchmark for the full solution described below.

Equation (4) in conjunction with conditions (5) to (9) is easily solved by finite difference means. The particular algorithm used here is described in Appendix A. The numerical accuracy of the scheme has been checked against the analytical solutions given by Carslaw & Jaeger (1959) and Jaeger (1956) and agreement to better than 1 per cent was found.

Three examples of the evolution of the radial temperature profile calculated using this scheme are shown in Fig. 1(a, b, c). The physical properties used and circulation time assumed are shown in the captions. The temperature scale has been normalized and is equivalent to $(T-T_m)/(T_\infty-T_m)$. The normalized temperature is plotted as a function of r/a.

Fig. 1(a) shows the temperature history assuming a zero circulation time. Note the discontinuity in the temperature gradient at the borehole boundary which is due to the continuity of heat flux condition (see equation 9). The effect of a finite disturbance time of 5 hr is shown in Fig. 1(b). The circulation period gives a smaller temperature gradient at the start of the shut-in period thus leading to a smaller initial heat flux into the hole and giving a lower temperature build-up rate in the fluid (*cf.* Fig. 1a). Assuming that the borehole properties are equal to those of the rock gives the result shown in Fig. 1(c) which shows a higher build-up rate than Fig. 1(b) due to the greater diffusivity

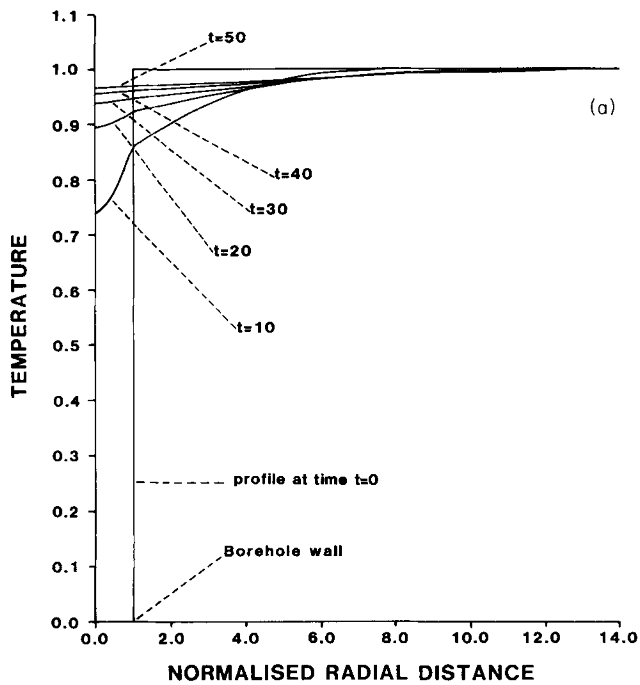
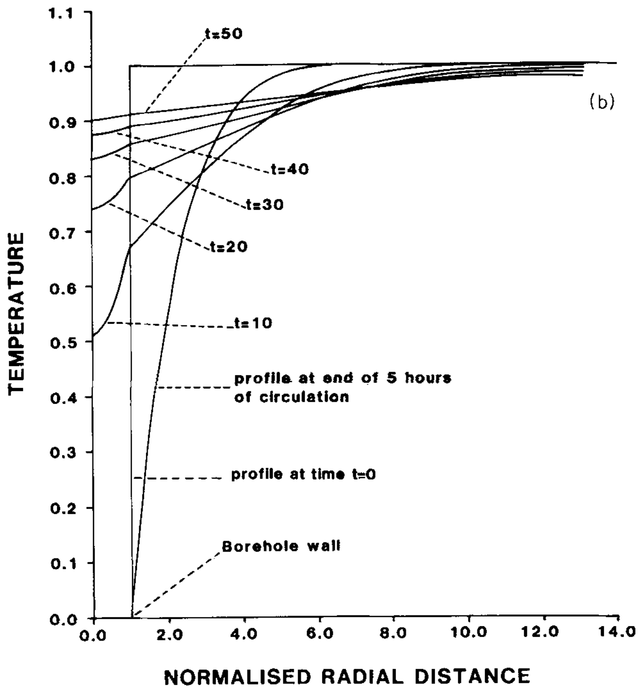


Figure 1. (a) Temperature profiles after intervals of 10, 20, 30, 40 and 50 hr, assuming no mud circulation prior to shut-in. t = shut-in time. D1 = 1.8E−7 m^2 s^{-1}. D2 = 1.1 E−6 m^2 s^{-1}, K1 = 0.8 W m^{-1} K^{-1}, K2 = 2.8 W m^{-1} K^{-1} and hole diameter = 8.5 in. (b) Temperature profiles after intervals of 10, 20, 30, 40 and 50 hr, assuming the hole was circulated for 5 hr prior to shut-in. t = shut-in time. D1 = 1.8E−7 m^2 s^{-1}, D2 = 1.1E−6 m^2 s^{-1}, K1 = 0.8 W m^{-1} K^{-1}, K2 = 2.8 W m^{-1} K^{-1} and hole diameter = 8.5 in. (c) Temperature profiles after intervals of 10, 20, 30, 40 and 50 hr, assuming the hole was circulated for 5 hr prior to shut-in. t = shut-in time. D1 = 1.1E−6 m^2 s^{-1}. D2 = 1.1E−6 m^2 s^{-1}, K1 = 2.8 W m^{-1} K^{-1}, K2 = 2.8 W m^{-1} K^{-1} and hole diameter = 8.5 in.



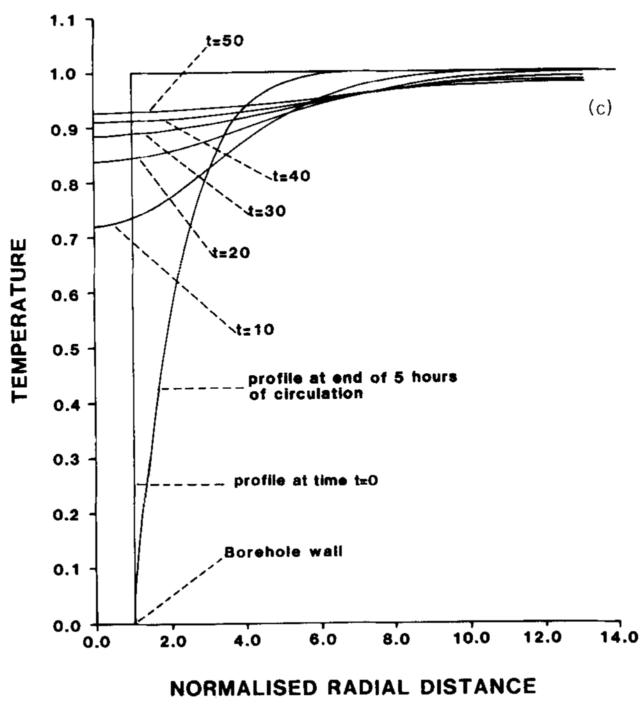


Figure 1 – *continued*

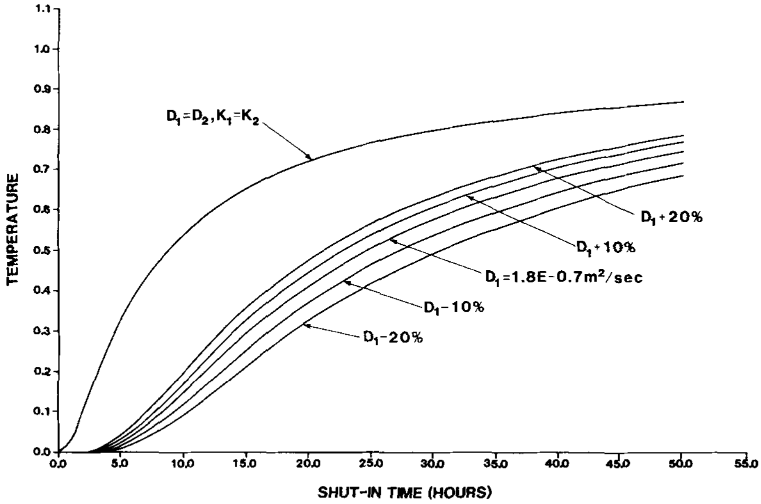


Figure 2. The sensitivity of the axial temperature build-up to changes in the mud diffusivity, D1. D2 = 1.1E–6 $m^2 s^{-1}$, circulation time = 5 hr, K1 = 0.8 $W m^{-1} K^{-1}$, K2 = 2.8 $W m^{-1} K^{-1}$ and hole diameter = 15 in.

in the hole. As the thermal conductivity is continuous there is no temperature discontinuity at the hole boundary.

In order to establish the importance of the various parameters it is necessary to perform a sensitivity study. The results of such a study using ranges of parameters which are likely in oil exploration are given in Figs 2–5, which show the axial temperature build-up during shut-in when only one parameter is allowed to vary.

Fig. 2 shows the sensitivity to the mud diffusivity (note that the thermal conductivity is held constant). The curves labelled as $D_1 \pm x$ per cent etc. are obtained by changing the

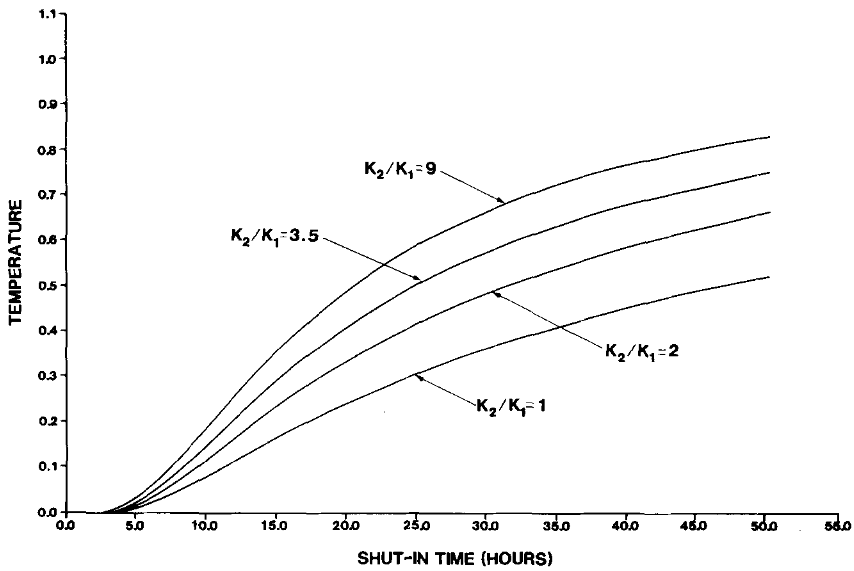


Figure 3. The sensitivity of the axial temperature build-up to changes in the ratio K2/K1. D1 = 1.8E–7 $m^2 s^{-1}$, D2 = 1.1E–6 $m^2 s^{-1}$, circulation time = 5 hr and hole diameter = 15 in.

mud diffusivity by x per cent of the 'median' value of $1.8 \times 10^{-7} m^2 s^{-1}$. As expected increasing the mud diffusivity leads to a faster build-up rate (since the heat capacity of the mud is reduced). For comparison Fig. 2 also shows the build-up curve for the case when the mud properties are equal to those of the rock clearly showing the effect of ignoring the additional thermal inertia of the mud.

It is found that in general the temperature build-up curves are only slightly sensitive to the thermal diffusivity of the rock.

The build-up curves are very sensitive to the conductivity ratio K_2/K_1. This is shown in Fig. 3 where curves for K_2/K_1 values of between 1 and 9 are shown (this range contains the likely extreme values of this parameter).

The effect of varying the mud circulation period is shown in Fig. 4. This shows build-up curves for cases from $t_1 = 0$ (i.e. instant hole creation) to $t_1 = 20$ hr. As would be expected increasing the circulation time leads to lower build-up rates.

Changing the borehole diameter has a very significant effect as shown in Fig. 5. Hence it is important that this be determined accurately in practical application. The scale of this sensitivity is not surprising as characteristic diffusion times usually vary as the square of the length scale.

The importance of these variations in practice will depend on the distribution of the measured data in shut-in time. The larger the shut-in time the less the sensitivity of the build-up curves to variations in the model parameters.

One point that holds in general is that fitting data to a curve with a faster build-up rate over the data window will lead to a lower estimate for T_∞ than using a curve with a lower build-up rate. This implies, for example that if one fitted a given set of data to a curve with a particular value of mud thermal diffusivity then this will predict a T_∞ which is lower than if a smaller diffusivity was used. The variation in the estimated T_∞ will depend on how the data points are distributed in time and on the change in the diffusivity (or whatever parameter was being altered).

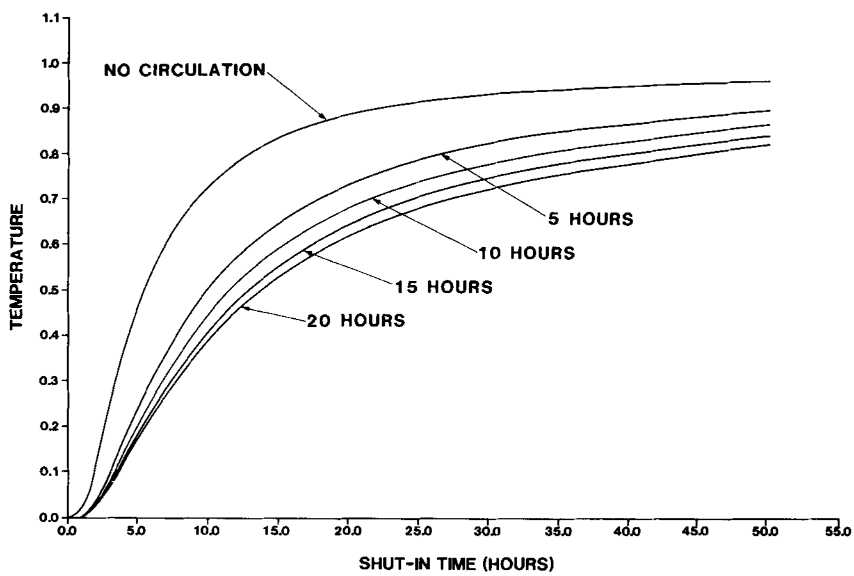


Figure 4. The sensitivity of the axial temperature build-up to changes in the circulation time. D1 = 1.8E–7 $m^2 s^{-1}$, D2 = 1.1E–6 $m^2 s^{-1}$, K1 = 0.8 $W m^{-1} K^{-1}$, K2 = 2.8 $W m^{-1} K^{-1}$ and hole diameter = 8.5 in.

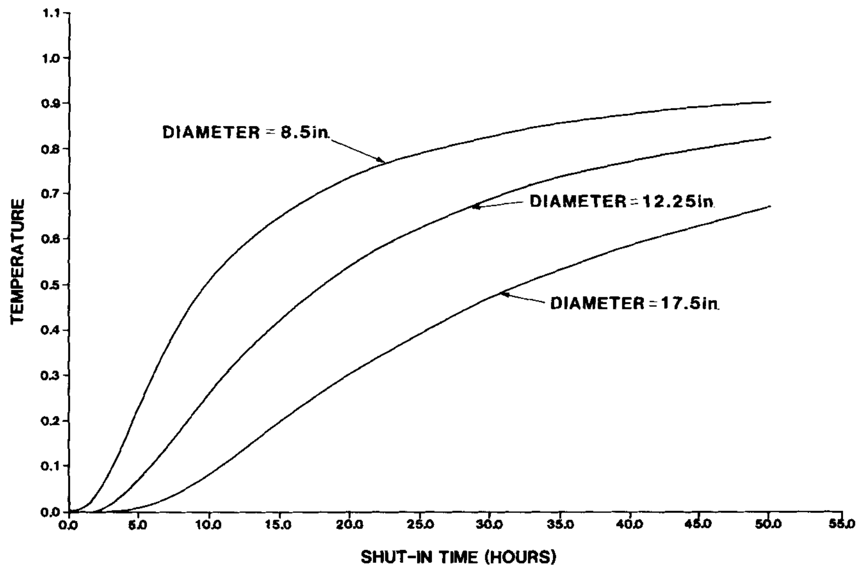


Figure 5. The effect of borehole diameter on temperature build-up. D1 = 1.8E−7 $m^2 s^{-1}$, D2 = 1.1E−6 $m^2 s^{-1}$, K1 = 0.8 $W m^{-1} K^{-1}$, K2 = 2.8 $W m^{-1} K^{-1}$, circulation time = 5 hr.

Figs 2, 3, 4 and 5 only considered temperature build-up at the axis. From Fig. 1 it can be seen that the build-up curve will depend, in general, on the radial position in the borehole surface. The build-up rate at the axis is faster than that at the borehole surface. Hence even for a fixed set of physical parameters it is possible to obtain a range of estimates for T_∞. The predicted value of T_∞ increases with increasing radius in the borehole.

One way of removing this ambiguity (as to where exactly in the hole the temperature is measured) is to assume that fluid is stirred up by the action of raising and lowering the logging tool. It is common logging practice to 'yo–yo' the tool when it is near hole bottom to prevent sticking. Hence it is reasonable to assume that this will effectively stir up the mud thus leading to an equalization of the temperature within the borehole.

It is possible to model this fluid mixing quite simply. The 'EFT' model assumes that at the shut-in time when the temperature is measured the fluid in the borehole is mixed instantly. This mixing redistributes the energy in the hole leading to a uniform fluid temperature T_{av} which can be found from a simple energy balance thus

$$T_{av} = \frac{2}{a^2} \int_0^a T(r, t)\, r\, dr \tag{10}$$

where the time t corresponds to the shut-in time when the temperature value was measured. Thus for each measurement the temperature in the hole is averaged by using equation (10) and the borehole temperature then continues relaxing from this new base position. An example of this temperature averaging is shown in Fig. 6, where three measurements have been made.

This facility for equalizing the mud temperature is optional in the programme and hence it is possible to evaluate its effect by comparison with cases where mud mixing was excluded.

For purposes of comparison the temperature build-up curves at the borehole surface predicted by the 'EFT', Bullard (equation 1) and 'Horner plot' (equation 2) methods are

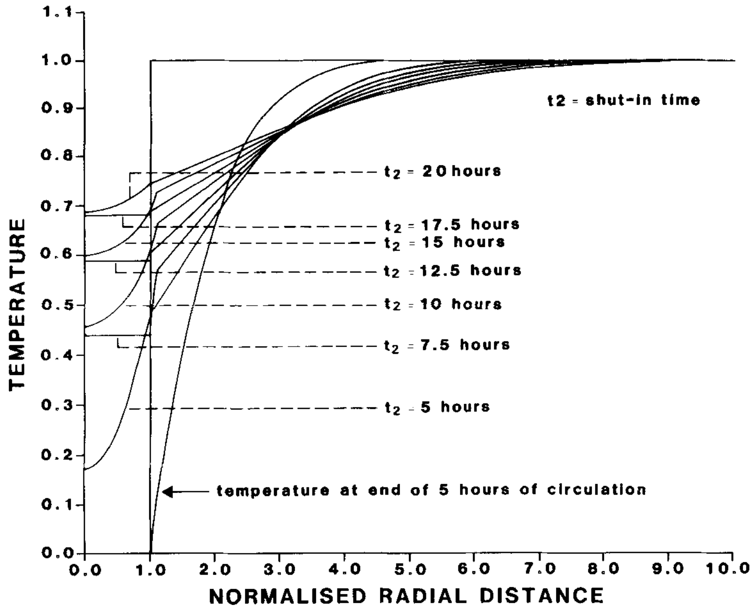


Figure 6. Example of a temperature profile when the mud is mixed at the BHT measurement time. D1 = 1.4E–7 $m^2 s^{-1}$, D2 = 5.7E–7 $m^2 s^{-1}$, K1 = 0.7 $W m^{-1} K^{-1}$, K2 = 1.9 $W m^{-1} K^{-1}$, diameter = 8.5 in, circulation time = 5 hr.

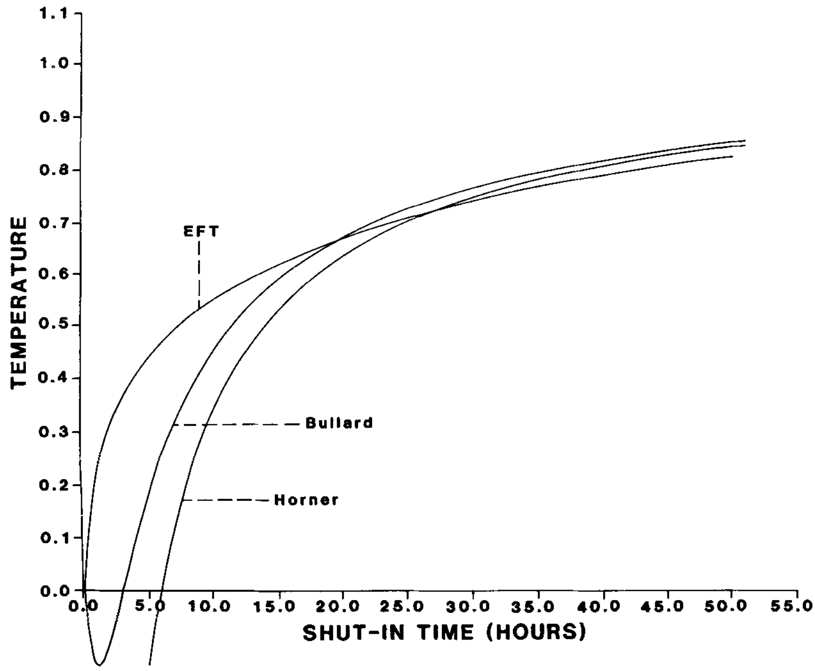


Figure 7. Comparison of the temperature build-up predicted by the 'EFT', Bullard and Horner models. D1 = 1.8E–7 $m^2 s^{-1}$, D2 = 1.1E–6 $m^2 s^{-1}$, K1 = 0.8 $W m^{-1} K^{-1}$, K2 = 2.8 $W m^{-1} K^{-1}$, circulation time = 5 hr and hole diameter = 15 in. A diffusivity of 1.1E–6 $m^2 s^{-1}$ was used in the line source curves.

shown in Fig. 7. The line source curves have been calculated assuming a diffusivity of $1.1 \times 10^{-6} m^2 s^{-1}$. The retarded build-up effect predicted by the Bullard model is evident in Fig. 7. If a diffusivity of $5 \times 10^{-7} m^2 s^{-1}$ is used then the temperature does not increase above its value at zero shut-in time until 15 hr have elapsed!

The equilibrium temperature predicted using any of the three models depends on the curvature of the theoretical curve over the time window covered by the measured data. The curve with the higher build-up rate over this time window will predict a lower value for T_∞ than a curve with a smaller build-up rate.

In practice this almost always means that 'EFT' predicts the highest value for T_∞ followed by the full line source solution, with the 'Horner plot' method giving the lowest value. The spread in values predicted by the three curves can be up to about 20 per cent depending on the data, the physical properties and the circulation time.

Generally for oil exploration data it has been found that the 'EFT', the Bullard and the Horner approximation are in good agreement for borehole diameters less than 8.5 in. For larger diameter holes the differences tend to get progressively bigger.

3 Examples

Ideally for this type of study one needs to specify all the input parameters needed by each model and then compare the predicted values of T_∞ with a measured value.

A certain amount of additional information is also needed to decide whether the assumptions underlying each model are satisfied. This sort of information is essential in order to ascertain that the physics behind any given model is representative of the real world. The sort of information required is described in Section 4.2. Without such information it would be difficulty to assess the reliability of the T_∞ predicted by any method. Unfortunately such information has not been routinely available in the past which makes it difficult to validate any of the models. The most serious lack is in the acquisition of the BHT data and their reliability (see Section 4).

In practice it has been necessary to make assumptions about both input parameters (e.g. circulation time and mud and rock properties) and about the reliability of the measurements. Hence the philosophy of the present study was to apply the three models described filling in any unknown parameters by guesswork. This will at least illustrate the behaviour of the models, while a validation of the models awaits further experimental work. All that can be done here is to offer physical reasons as to why the new 'EFT' is more reasonable than existing models.

3.1 EXAMPLE 1

The data for this example are from a well in the North Sea. Many of the input data needed by the 'EFT' model are available from the well logs allowing the specification of the following parameters:

borehole diameter = 8.5 in,
circulation time at measurement depth = 12.25 hr,
rock thermal conductivity = $1.9 W m^{-1} K^{-1}$,
heat capacity = $\rho c = 3.3 \times 10^6 J m^{-3} K^{-1}$.

Little is known about the properties of the drilling mud used, hence the following values were assumed:

borehole thermal conductivity = $0.7 W m^{-1} K^{-1}$,
borehole heat capacity = $5 \times 10^6 J m^{-3} K^{-1}$.

The values assumed for the physical properties are reasonable at surface ambient atmospheric conditions. The effect of high temperature and pressure at depth are not very important as far as the rock parameters are concerned. However, the properties of the drilling muds may vary considerably with increases in temperature and pressure and these factors need further study.

For comparison the BHTs were also analysed using the Bullard model and the Horner approximation. Temperature inversion using the Horner approximation is independent of all the above parameters with the exception of the circulation time. The Bullard model needs as input the borehole diameter and a thermal diffusivity, as well as the circulation time. The composite value of diffusivity used was $5 \times 10^{-7} m^2 s^{-1}$ which is in between the value of water and that for a typical sediment.

The BHT data were fitted using the three models under discussion and the results are shown in Fig. 8. In the case of the fit using 'EFT' the mud mixing option was not used here and the BHTs were fitted to the axial build-up curve. Fig. 8 shows that the 'EFT' model predicts the highest value for T_∞ (248.8°F), the Bullard models gives T_∞ = 245.3°F and the Horner plot gives T_∞ = 240.7°F. In the case of the 'EFT' model if the build-up curve at the hole surface is used the estimated T_∞ goes up to 258.7°F.

If we assume that the mud is thoroughly stirred by the logging tool then the axial build-up curve is shown in Fig. 9, which gives a T_∞ of 251.5°F. The discontinuous shape of the curve in Fig. 9 is due to the fact that at the measurement time the mud temperature is equalized (using equation 10), which means that there is a sudden change in the borehole temperature profile. Clearly as the mud temperature is equalized in this way the same value of T_∞ will be found regardless of the radial position in the hole.

An indication of the sensitivity of the T_∞ predicted using the 'EFT' model is given in Table 1 (D_1 is the mud diffusivity).

The only independent check on the formation temperature comes from a production drill stem test (DST) performed 50 m above the hole bottom. This recorded a fluid

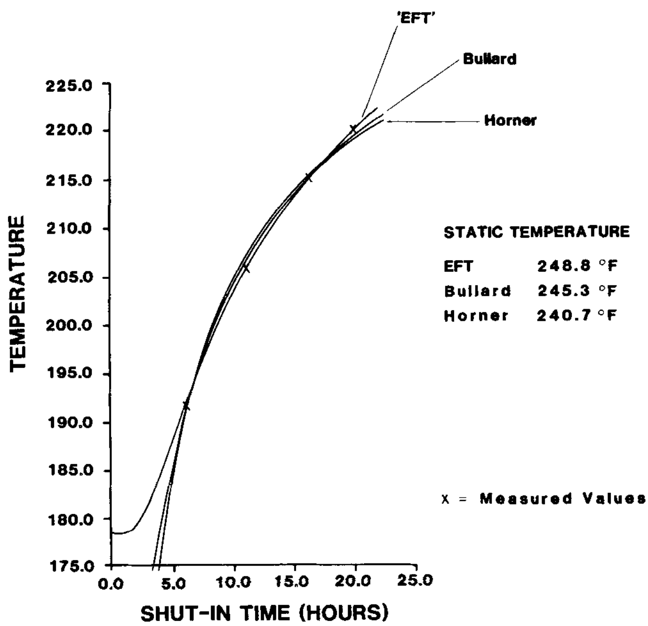


Figure 8. Fit of data for example 1 using 'EFT', Bullard and 'Horner plot'.

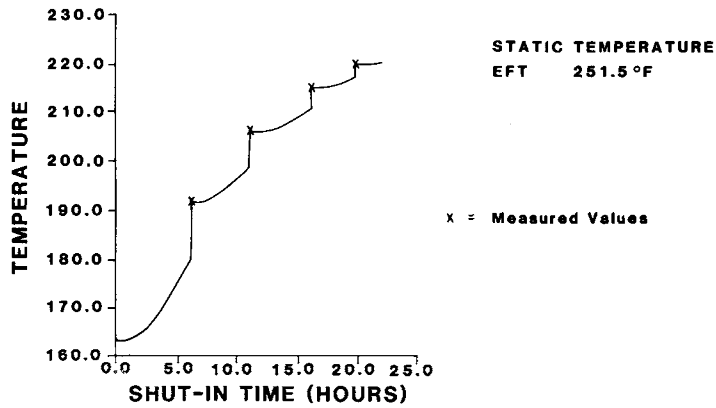


Figure 9. Fit of data for example 1 using the 'EFT' model assuming mud mixing at the BHT measurement times.

temperature of 241°F. Assuming that this DST temperature represents the true formation temperature then 'Horner plot' estimate is the closest to the true T_∞. It is possible to lower the estimates given by the Bullard and 'EFT' methods by changing the physical properties. For example an increase of the rock conductivity to 3.0 W m^{-1} K^{-1} reduces the T_∞ predicted by 'EFT' to 243°F. There is no justification in doing this as the uncertainties in the measurements are such that differences of about 10°F are not significant.

Table 1. Sensitivity of the T_∞ predicted using 'EFT'.

Parameters in Fig. 8	K2/K1 = 1	K2/K1 = 9	Circ. time = 25 hr	Circ. time = 0	$D_1 - 20$ per cent	$D_1 + 20$ per cent
248.5	270.3	240.4	257.0	229.0	253.4	246.4

All three models predict plausible values for T_∞ in this case. For this particular data set the uncertainties are in the BHT measurements themselves and in the lack of information on the physical properties of the borehole contents near the hole bottom.

3.2 EXAMPLE 2

The only data available for this example are the logged temperatures, the circulation time (9 hr) and the borehole diameter (12.25 in.). The following values were assumed for the remaining parameters:

rock conductivity = 1.9 W m^{-1} K^{-1},
rock heat capacity = 3.3×10^6 J m^{-3} K^{-1},
mud conductivity = 0.7 W m^{-1} K^{-1},
mud heat capacity = 5×10^6 J m^{-3} K^{-1}.

The least squares fit is shown in Fig. 10. No measured value for T_∞ is available and hence it is not possible to comment on the accuracy of any of the three predicted values shown in Fig. 10. However, it is clear that EFT predicts a significantly higher value of T_∞ than either the full Bullard model or the 'Horner plot'. The EFT model predicts a T_∞ which is 16 per cent larger than that given by the Horner method.

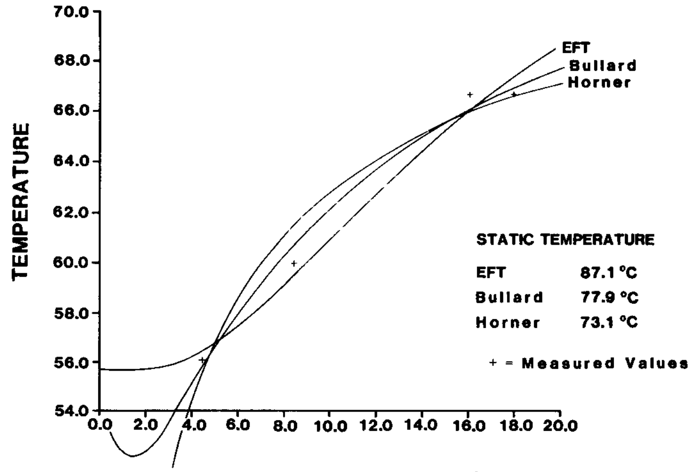


Figure 10. Fit of data for example 2 using 'EFT', Bullard and 'Horner plot'.

The above two examples were chosen for purposes of illustration. As more data become available where T_∞ has been measured it should become possible to perform a rigorous analysis of which of the current models is the most reliable.

One incidental point worth mentioning here is that both the 'EFT' and Bullard models produce an estimate of the borehole temperature at the beginning of the shut-in period, with 'EFT' generally giving a higher value of T_m. The 'Horner plot' completely breaks down for zero shut-in time and gives a value for T_m of $-\infty$.

4 Discussion

The study discussed in this paper has raised several questions regarding the theory of BHT modelling. It has been difficult to judge the significance of some of the issues as far as prediction of equilibrium temperatures is concerned, because of uncertainty in the data. There are two related points that need to be considered in order to specify a technique for obtaining reliable bottom hole equilibrium temperatures: (a) the theoretical and (b) the practical aspects.

4.1 MODELLING CONSIDERATIONS

As pointed out in Section 1 the 'Horner plot' is probably the most popularly used method for BHT extrapolation. Since this is only an approximation to the Bullard line source model its use is only justifiable if: (a) the approximation is valid mathematically and (b) the parent model is itself physically sound.

There are two main objections to the line source model, namely (a) it only describes a single component system and (b) the mechanism used to model the drilling disturbance can lead to some curious results.

Clearly what is required is a model that describes the temperature build-up in the borehole fluid. The drilling fluid has thermal properties which are quite distinct from those of the formation. There is no *a priori* reason for assuming that such a two-component system could be adequately modelled by using some average properties in a one-component model (at least not for the relatively short periods of between 20–30 hr after shut-in). The contrast in the mud/rock thermal properties does have a significant effect. Hence on a theoretical basis a single homogeneous system cannot be expected to be reliable under all conditions.

The second problem with the Bullard model is with the constant line source/sink assumption. This has two consequences: (a) during circulation the temperature in the hole is continuously reduced (physically one would expect a near steady state to be established leading to a near constant mud temperature in the hole during circulation) and (b) the temperature in the hole does not start to build up immediately circulation ceases (see, e.g. Fig. 7). The latter effect can lead to the result that over the shut-in period covered by the data the model is predicting that the borehole temperature is *falling*. This initial drop in temperature is controlled by the circulation time, borehole diameter and the diffusivity assumed in the model.

This means that the Horner approximation can be invalid both because the shut-in time is not long enough and because the underlying line source model itself breaks down.

This situation is clearly unsatisfactory since the regime in which the line source model is reasonable (and hence the Horner approximation) cannot be precisely defined in general because of the somewhat arbitrary nature of the diffusivity used in the model.

The particular difficulty with the composite diffusivity also applies to the models given by Middleton (1979) and Leblanc *et al.* (1982). In these models the temperature builds up monotonically for all shut-in times but the thermal inertia of the system is defined by a single parameter.

The additional difficulty with the Middleton/Leblanc *et al.* models relates to their assumption that the borehole can be considered to be created instantly with zero circulation time. As shown in Fig. 4 temperature build-up is very sensitive to the total disturbance time.

The numerical model described in this paper tries to overcome the above difficulties by using the assumptions described in Section 2, which allow a constant mud temperature during circulation and different mud and rock properties. Numerical experiments have shown that the temperature build-up is sensitive to the additional parameters implied by this model.

The 'EFT' model does require the specification of a greater number of input parameters than previous models. It should be possible to get good estimates of these parameters from the well logs and thus be able to predict an unambiguous theoretical build-up curve which can be used for fitting to BHT data from a particular depth in a given well.

There are several further assumptions which are common to most models which need to be considered. One assumption which is specific to the 'EFT' model is that of constant borehole temperature during circulation. This was the simplest most reasonable assumption to make. However, if a detailed history of the circulating bottom hole temperature is available (either from measurements or from a model) then it would be a simple matter to include this in the model. Similarly for the assumption of a two component system: if the need for including transition zone(s) (e.g. invaded zone or mud cake) between the mud and surrounding rock becomes apparent then such zones could easily be added to the finite difference model.

The three other main assumptions which have been commonly made are:

(1) no convective heat transfer due to fluid motion in the formation,
(2) no free convection in the mud during shut-in, and
(3) vertical temperature gradients are negligible.

These points have been considered and the methods used to estimate their importance are described in the appendices. It is found that free convection in the borehole is feasible but that the vertical heat transfer due to this mechanism is negligible. However, if free convection is present then it may be necessary to modify the boundary conditions used in the 'EFT' model and instead of using equation (9) it may be more appropriate to assume that the borehole is a perfect conductor (as presumably the convective instability will stir up the fluid in the hole). This would not be unreasonable provided that the mixing time was much shorter than the time taken for heat to diffuse across the borehole.

The vertical temperature gradient due to the proximity of the hole bottom to the measurement depth is found to have an important effect only for positions which are less than 10 hole radii above the hole bottom.

The effect of fluid loss to or gain from the formation, however, can be extremely important. Drilling fluid can get lost to the formation at rates up to about 25 barrel hr^{-1} (e.g. when drilling into vuggy limestone) and fluid gain rates of about 20–25 barrel day^{-1} are also found in practice. If one assumes that this loss or gain occurs over a depth interval of some tens of metres then Peclet numbers (vL/κ, where v is the fluid velocity, L is a scale length and κ is a diffusivity) $|\mathrm{Pe}| \sim 1-10$ are feasible.

The results described in the Appendices show that for $\mathrm{Pe} \sim 10$ the extent of the thermal disturbance into the rock is very much larger than for a similar case where purely conductive heat flow is assumed. This implies that the subsequent thermal relaxation (once the fluid flow is stopped and the well is shut-in) will take a much longer time (since diffusion times vary roughly as the square of the disturbance diameter).

Fluid loss or gain is not unusual during drilling operations and if it is established that this flow is occurring at a depth where BHTs have been measured, then it will be necessary to assess the thermal effect. The extension to the 'EFT' algorithm described in Appendix B could be used to calculate the temperature profile at the end of the flow period. This profile could then be used as the initial condition in the 'EFT' model which assumed purely conductive heat transfer to describe the well shut-in conditions.

4.2 PRACTICAL ASPECTS

There is no doubt that regardless of which model is used for inversion more control needs to be exerted over the data acquisition. The quality of the data must be controlled and the necessary ancillary information must be logged. Without such measures it is not possible to give reliable estimates of T_∞ and one cannot establish a level of confidence for such predictions. The following points need to be considered:

(i) BHTs are routinely measured using maximum reading mercury in glass thermometers. These are not ideal instruments for the job in hand as it is not possible to be sure that the maximum reading recorder refers to any given depth (mud temperature does not necessarily increase monotonically with depth).

Even more serious is the susceptibility of these thermometers to vibration which can alter their readings. The meniscus has also been found to drop if the thermometers are allowed to stand in ambient temperature (a drop from 390 to 380°F was found when stood at room temperature, for example).

The uncertainties relating to using such instruments would suggest the use of a continuous reading electronic thermometer.

Regardless of which type of thermometer is used the response time when the instrument is placed in the logging tool needs to be known.

(ii) The fluid circulation time at the depth of measurement should be carefully measured. This should include the period taken to drill from the relevant depth to the bottom of the hole.

(iii) Little work has been published on the thermal properties of drilling muds (see Tanaka & Miyazawa 1976). More work needs to be done on the measurement of these properties and their variation with temperature, pressure and chemical composition.

(iv) The rock lithology and porosity at the measurement depth should be noted in order to estimate the thermal conductivity (see Sass, Lachenbruch & Jessop 1971).

(v) The borehole radius should be read from a caliper log if possible and if this shows large variations in diameter then the rms value would probably be the best value to use.

(vi) Careful records should be kept of mud circulation (especially any additional circulation periods which occurred after logging had started) and of any loss or gain of fluid.

5 Conclusions

Several new features have been included in the 'EFT' model of BHT build-up described in this paper. The model has allowed for different borehole and formation thermal properties and also for non-zero circulation time. Using modifications to the basic algorithm it was also possible to estimate the effect of vertical temperature gradients and the flow of formation fluids.

The general conclusions are that: (a) the contrast in mud/rock thermal properties is significant, (b) the temperature build-up curves are sensitive to the total disturbance time and (c) fluid losses or gains during drilling (assuming they are due to flow of formation fluids) can be high enough to give Peclet numbers in the range about 1–10, thus leading to a greatly extended thermal disturbance diameter (for fluid loss to the formation) with consequently greatly increased build-up times.

It has been found that (excluding cases where formation fluid flow has occurred) the values of T_∞ predicted using the numerical model are not much different from those calculated using the Bullard (1947) model provided the borehole diameter is less than 10 in. and a diffusivity of about $5 \times 10^{-7}\,m^2\,s^{-1}$ is used in the latter. For larger diameters the physics of the line source model can lead to a value of T_∞ which is lower than the measured BHTs (obviously in such cases using the Horner approximation would not make sense).

The new model almost produces values of T_∞ which are higher than the commonly used 'Horner plot' method. There is no scope for varying the estimates given by the 'Horner plot' model since it is only sensitive to the circulation and shut-in times. The parent line source model (Bullard 1947) is sensitive to circulation time, borehole diameter and thermal diffusivity; hence given the circulation time and hole diameter it would be possible to juggle the diffusivity used to give different estimate for T_∞. The 'EFT' model gives a unique unambiguous build-up curve provided all the input parameters are known.

On theoretical physical grounds the new 'EFT' model is more reasonable than those previously published as it allows the specification of a more realistic system. The previously published models may be applicable under more restricted circumstances but because of the underlying assumptions of such models there is some ambiguity in the results they predict. This model may need further modification in the light of more detailed data. The priority is for further case-studies using data acquired under carefully controlled conditions (or at the very least under known conditions) in order to establish the validity of the model.

Lee (1982) has recently published a paper which covers some of the work reported above. Independently Lee has produced a model which is similar to the finite difference analysis discussed in this paper. He presented a model which describes temperature build-up assuming instantaneous hole creation and a more general model which includes a finite mud circulation time (the former is merely a special case of the latter).

The model that assumes no mud circulation prior to shut-in is the analytical solution given by Carslaw & Jaeger (1959, p. 346) which has been used by the present author to check the validity of the finite difference model in this regime. Lee, however, has suggested using this solution to describe borehole temperature build-up in general. In the author's opinion this is unlikely to be of practical use under normal exploration conditions since even if one assumes a very rapid drilling rate there will still be a significant period of mud circulation time that will need to be accounted for. This will be true even if one assumes the temperatures are measured actually at hole bottom, because mud circulation is usually continued after the end of drilling for hole conditioning purposes.

Apart from the no circulation model Lee has also described a model which is identical to the 'EFT' algorithm described above and includes both different mud and rock properties and a non-zero circulation time. There is one relatively minor technical difference between Lee's approach and that of the present author's in that Lee uses a finite element technique to solve the equations whereas the 'EFT' programme uses a finite difference method.

However, Lee does not use this more realistic model to analyse the case-studies presented in his paper. Instead he uses a graphical application of the Carslaw & Jaeger (1959) solution to estimate parameters such as the equilibrium temperature and initial mud temperature. As shown in Sections 2 and 3 of this report the temperature build-up is sensitive to even quite short mud circulation periods (e.g. 5 hr) and it is likely that allowing for this mud circulation will give a more reliable estimate of the equilibrium temperature.

There is one further difference between the model presented here and that given by Lee (1982). This originates in the method used to remove the uncertainty of the radial position in the hole where the temperature is measured. The 'EFT' model assumes that the fluid is mixed at the time of measurement (see equation 9) thus leading to discontinuities in the temperature build-up in the hole (see Figs 6 and 9). Lee suggests calculating the temperature disturbance using the Carslaw & Jaeger (1959) solution and then calculating the radial average of the borehole temperature to represent the measured values. Lee's method gives a continuous build-up curve. It is not clear whether a similar averaging process would be applied to cases where a non-zero mud circulation time is used in the finite element model.

This paper presents a quantitative discussion of the effects of fluid convection (both in the borehole and through fluid loss or gain) and of vertical heat flow due to the proximity of the hole bottom (see Appendices B, C and D). These aspects of the problem are not discussed in Lee (1982).

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Appendix A: finite difference scheme

Equation (4) is solved numerically using the 'method of lines'. The partial differential equation is replaced by a finite set of coupled ordinary differential equations thus:

$$\left.\frac{dT}{dt}\right|_{r=(j-1)\delta r} = \frac{K}{\rho c}\left\{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right\}_{r=(j-1)\delta r} \qquad j = 1, 2 \ldots j_{\max} \tag{A1}$$

where δr is the node spacing.

The right-hand side of (A1) is approximated using central differences

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \approx \left\{\frac{T(j+1) - 2T(j) + T(j-1)}{\delta r^2} + \frac{1}{(j-1)\delta r}\left(\frac{T(j+1) - T(j-1)}{2\delta r}\right)\right\}. \tag{A2}$$

Equation (A2) is substituted into (A1) and the resulting set of ordinary differential equations are solved numerically.

The one main difficulty found in solving the current problem originates with the continuity conditions given by equations (8) and (9). It is necessary to devise an approximation to the spatial derivatives in (A1) which honours these conditions at the borehole boundary. This is done as follows:

Assume that the borehole boundary coincides with node $j = n$ and postulate two fictitious temperatures at nodes $n+1$ and $n-1$, namely $T'(n+1)$ and $T'(n-1)$ respectively [note

$T'(n+1) \neq T(n+1)$ and $T'(n-1) \neq T(n-1)$]. Then the slope just inside the hole is given by

$$\left.\frac{\partial T}{\partial r}\right|_1 \approx \frac{T'(n+1) - T(n-1)}{2\delta r} \tag{A3}$$

and similarly for the slope just outside the hole

$$\left.\frac{\partial T}{\partial r}\right|_2 \approx \frac{T(n+1) - T'(n-1)}{2\delta r}\,. \tag{A4}$$

The boundary condition (9) can be approximated by

$$K_1\,\{T'(n+1) - T(n-1)\} \approx K_2\,\{T(n+1) - T'(n-1)\}\,, \tag{A5}$$

while equation (8) implies that

$$\left.\frac{dT}{dt}\right|_1 = \left.\frac{dT}{dt}\right|_2 \tag{A6}$$

and using the fictitious temperatures $T'(n+1)$ and $T'(n-1)$ this gives

$$\frac{K_1}{(\rho c)_1}\left\{\frac{T'(n+1) - 2T(n) + T(n-1)}{\delta r^2} + \frac{T'(n+1) - T(n-1)}{2(n-1)\,\delta r^2}\right\} \tag{A7}$$

$$= \frac{K_2}{(\rho c)_2}\left\{\frac{T(n+1) - 2T(n) + T'(n-1)}{\delta r^2} + \frac{T(n+1) - T'(n-1)}{2(n-1)\,\delta r^2}\right\}\,.$$

We can solve for $T'(n+1)$ and $T'(n-1)$ using (A5) and (A7). After some simple algebra $T'(n+1)$ is found to be

$$\begin{aligned} T'(n+1) = R\,\Bigg\{&\frac{D_2}{D_1}\left[\frac{T(n+1) - 2T(n)}{\delta r^2} + \frac{T(n+1)}{2r\delta r}\right] \\ &+ \frac{2T(n) - T(n-1)}{\delta r^2} + \frac{T(n-1)}{2r\,\delta r} \\ &- \left(\frac{1}{\delta r^2} + \frac{1}{2r\delta r}\right)\left[\frac{K_2}{K_1}\,T(n+1) + T(n-1)\right]\Bigg\} \end{aligned} \tag{A8}$$

where

$$D_1 = \frac{K_1}{(\rho c)_1}\,; \quad D_2 = \frac{K_2}{(\rho c)_2} \tag{A9}$$

and

$$R = -\left\{\frac{K_2}{K_1}\left(\frac{1}{\delta r^2} + \frac{1}{2r\delta r}\right) + \frac{D_2}{D_1}\left(\frac{1}{\delta r^2} - \frac{1}{2r\delta r}\right)\right\} \tag{A10}$$

$T'(n-1)$ is easily found from equation (A5).

The values of $T'(n+1)$ and $T'(n-1)$ can then be used in conjunction with the left-hand side or the right-hand side of equation (A7), respectively, to calculate the rate of change of temperature at the borehole boundary.

The scheme described by equations (A3) to (A10) is necessary to deal with the discontinuity of the temperature gradient implied by the continuity condition given by equation (9).

Comparison of the numerical results obtained using this scheme with the analytical solution given by Carslaw & Jaeger (1959, p. 346, equations 7 and 8) and by Jaeger (1956) gave excellent agreement. This implies that the scheme can handle both a discontinuous change in physical properties and a non-zero circulation time.

Appendix B: formation fluid flow

Flow of formation fluids during circulation can become apparent through the loss or gain of drilling fluid. This is not an uncommon occurrence in the field and hence it is necessary to evaluate the thermal effects of such flows assuming that they occur at or near the depth of measurement.

The model described below deals with the time period starting when the drill bit cuts through a given depth until mud circulation finally ceases. However, we assume that the flow of formation fluid continues when the mud circulation is stopped, i.e. the model does not extend into the time period when the well is finally stabilized. Assuming that the well is eventually stabilized then the radial temperature profile at this time can be fed into the conduction only thermal model described by equations (4)–(9) to calculate the subsequent build-up history.

The model makes the following assumptions:

(1) heat transfer is by conduction and convection;

(2) the temperature of the borehole fluid is constant during drilling and circulation;

(3) the properties of the fluid in the borehole are taken to be identical to the formation fluid;

(4) fluid motion and heat transfer are purely radial;

(5) the fluid in the formation is in local themodynamic equilibrium with the rock;

(6) fluid flow is incompressible with a constant rate of gain or loss;

(7) fluid gain or loss is taken to be continuous both during circulation and the subsequent 'shut-in' time.

The fluid flow assumptions imply that there is a constant line source or sink of fluid at the axis of the hole, with a temperature equal to that of the borehole fluid.

The heat flow in the formation is described by the following equation

$$\rho c \, \frac{\partial T}{\partial t} = K \left\{ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \, \frac{\partial T}{\partial r} \right\} - \frac{(\rho c)_\omega a U_0 \phi}{r} \, \frac{\partial T}{\partial r} \tag{B1}$$

where a = borehole radius, ϕ = porosity, $(\rho c)_\omega$ = fluid heat capacity, (ρc) = formation heat capacity = $\phi(\rho c)_\omega + (1-\phi)(\rho c)_r$, $(\rho c)_r$ = rock heat capacity, K = formation conductivity = $K_\omega^\phi K_r^{(1-\phi)}$, K_ω = fluid conductivity, K_r = rock conductivity, U_0 = radial fluid velocity at the hole surface = $Q/(2\pi a \phi)$, Q = constant rate of fluid injection or extraction at the hole axis in $m^3\, s^{-1}\, m^{-1}$ of borehole.

The temperature of the fluid in the borehole when drilling and circulation is stopped is calculated assuming perfect conductivity in the hole. This is not unreasonable in view of the radial motion which would be expected to stir up the fluid in the hole over a relatively short time-scale.

Thus after the end of the mud circulation the borehole temperature is given by

$$\frac{(\rho c)_\omega a}{2} \, \frac{dT}{dt}\bigg|_{\text{borehole}} = K \, \frac{\partial T}{\partial r}\bigg|_{r=a} \tag{B2}$$

where the right-hand side is the gradient just outside the hole. Clearly below some flow rate equation (B2) will become a poor approximation to the heat transfer within the hole. For out present purposes however it should provide an adequate approximation.

Taking the ratio of the convective to conductive term on the right-hand side of (B1) gives

$$\mathrm{Pe} \approx \frac{a(\rho c)_\omega U_0 \phi}{K} \tag{B3}$$

which shall be referred to as the Peclet number. Convective heat transfer dominates when $\mathrm{Pe} > 1$.

Equation (B1) is solved numerically assuming a constant borehole temperature during circulation and using equation (B2) as a boundary condition when circulation stopped. The algorithm used is a simple modification of the one described in Appendix A.

As an example we consider a borehole/formation system with the following parameters,

diameter = 8.5 in.
$(\rho c)_\omega = 4.2 \times 10^6 \, \mathrm{J\,K^{-1}\,m^{-3}}$,
$K_\omega = 0.6 \, \mathrm{W\,m^{-1}\,K^{-1}}$,
$(\rho c)_r = 2.4 \times 10^6 \, \mathrm{J\,K^{-1}\,m^{-3}}$,
$K_r = 2.8 \, \mathrm{W\,m^{-1}\,K^{-1}}$,
porosity = 20 per cent.

The fluid properties used are those for water.

Figs B1 to B4 show the evolution of the temperature assuming conduction and convection are present. Fig. B5 shows the case of conductive heat flow only calculated using the 'EFT' model. In all cases mud circulation is assumed to take place for 5 hr at the depth of interest. This is then followed by a period of 10 hr of 'shut-in' (i.e. no pumping). It is unlikely that the well will be left in this unstable state for such a long period; this figure

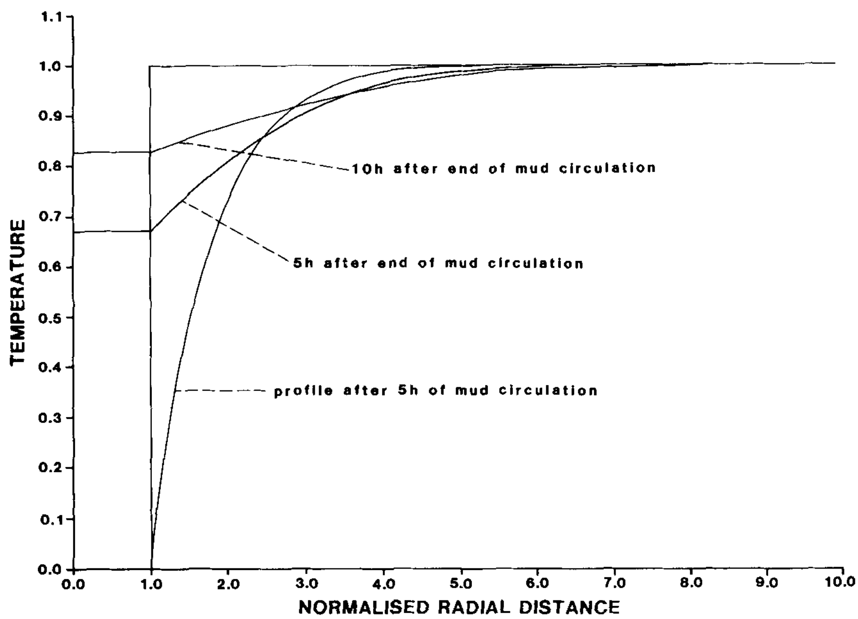

Figure B1. Temperature profile assuming a Peclet number = −1.

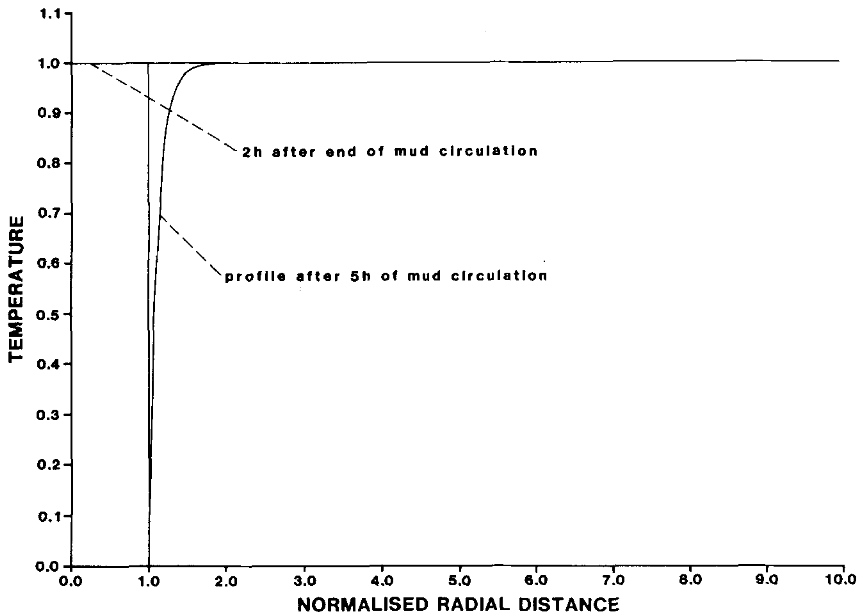

Figure B2. Temperature profile assuming a Peclet number = −10.

was chosen for illustration only. The actual temperature profiles are plotted at intervals of 5 hr after pumping has ceased.

Figs B1 to B4 correspond to different values of Peclet number as defined in equation (B3). For the above parameters a Peclet number of +1 corresponds to a loss of fluid to the formation at the rate of 1.68 barrel day^{-1} m^{-1} of hole. Coversely Pe = −1 corresponds

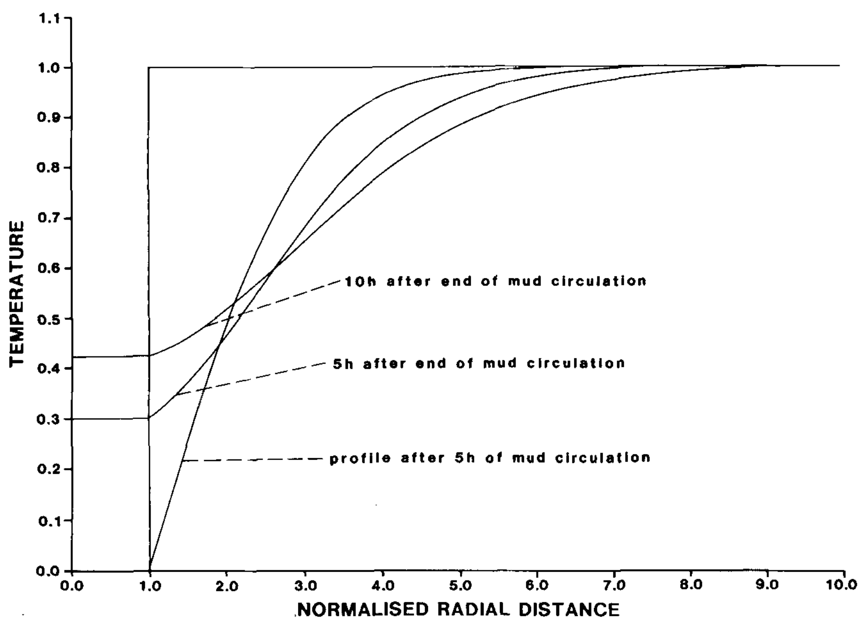

Figure B3. Temperature profile assuming a Peclet number = 1.

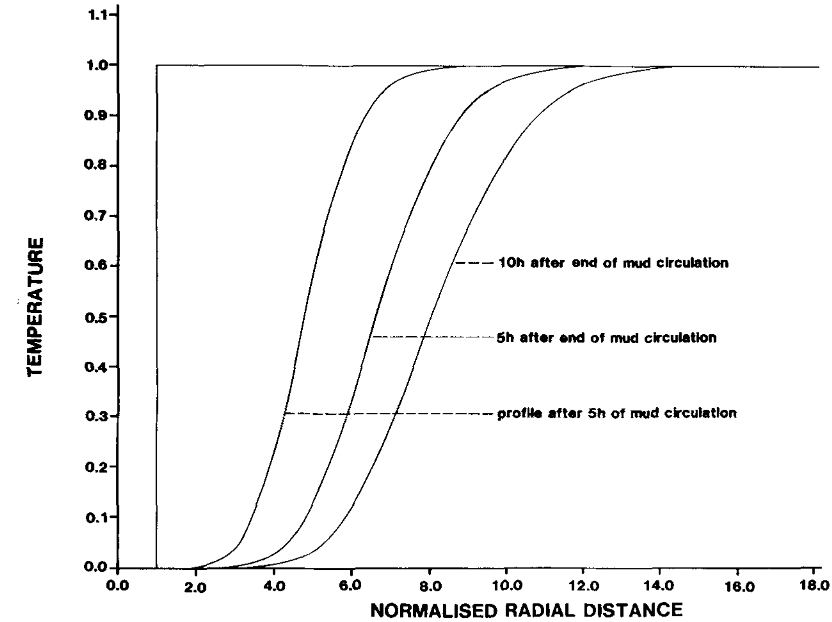

Figure B4. Temperature profile assuming a Peclet number = 10.

to a gain of fluid at the rate of 1.68 barrel day^{-1} m^{-1} of hole. The rate/unit length of loss or gain is directly proportional to the Peclet number.

In practice a fluid gain of about 20–25 barrel day^{-1} and losses of up to 5 barrel hr^{-1} can be seen during drilling. Hence the likely Peclet number range is $-10 \lesssim Pe \lesssim 10$.

Figs B1 and B2 show the effect of fluid gain with Pe = −1 and −10. The physical effect is to reinforce the conductive heat flow thus leading to faster build-up rates (*cf.* Fig. B5).

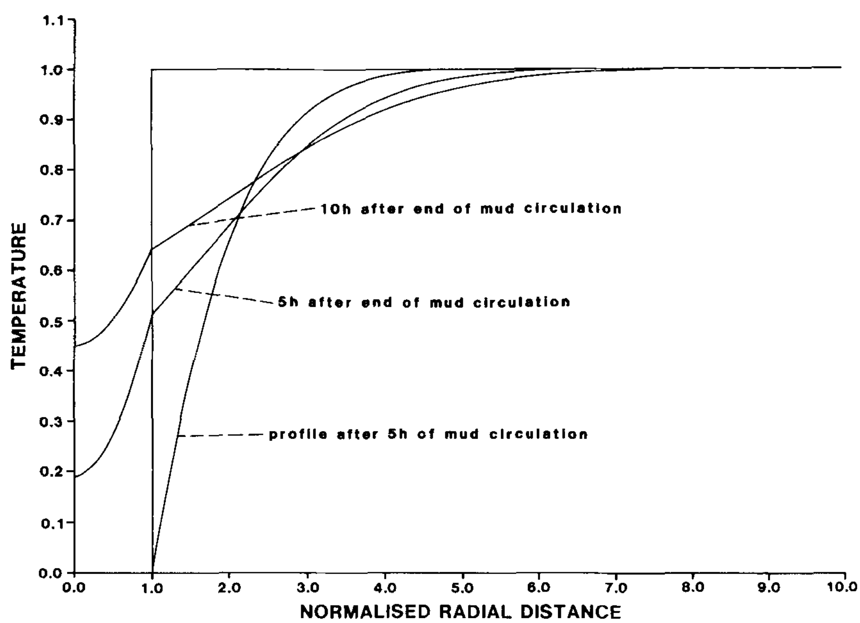

Figure B5. Temperature profile assuming purely conductive heat flow.

Table B1. Typical flow rates (bpd).

Pe	Thicknesses 10 m	50 m	100 m
1	16.8	84	168
10	168	840	1680
100	1680	8400	16 800

In the case of Fig. B2 (Pe = −10) the effect is quite dramatic and after about 2 hr of 'shut-in' the temperature in the hole has reached equilibrium.

The situation where fluid is lost to the formation is shown in Figs B3 and B4. Here the convective heat flow opposes the conductive term leading to lower build-up rates. In fact in the case of Fig. B4 (with Pe = 10) this effect is so high that the temperature disturbance continues to propagate outwards through the formation as long as the flow persists.

The important point to note is that the radius of the disturbance can be greatly increased for Pe > 1. This means that when the well is finally stabilized the build-up rate in the borehole will be much reduced.

The conclusion is that if one uses a build-up model which *ignores* the fluid flow (i.e. one which assumes purely conductive heat flow throughout) then for a *gain* (*loss*) of fluid the equilibrium temperature will be *overestimated* (*underestimated*). This is because, for example, in the case of fluid gain the disturbance in the formation is less than if conduction was the only heat transfer mechanism (and conversely for the case of fluid loss).

It should be emphasized that such fluid flows are only of interest in the BHT build-up problem if they occur through a section of hole where the temperature is being measured. Unless such flows occur down the whole length of the well then clearly they will introduce a significant vertical temperature gradient at the surface of the layer through which the flow is occurring. Hence for the present approach to be reasonable (i.e. assuming purely radial heat and fluid flow) it is necessary that the point of interest in the borehole be far away from these edge effects.

Some typical flow rates for various Peclet numbers and a range of rock thicknesses (corresponding to the section through which the fluid is being lost or gained) are given in Table B1. These rates are independent of borehole diameter and are calculated using the physical parameters applied in Figs B1 to B4.

Appendix C: free convection in the borehole

BHT models assume that the mud in the borehole is convectively stable. It is interesting to consider whether such an assumption is reasonable, because if convection cells can form then it may not be safe to assume that heat transfer inside the borehole is via radial conduction only. Convection cells can introduce a vertical heat flow which may be significant and can also lead to an enhanced radial heat flow which may be high enough to allow the borehole to be considered a perfect conductor. Hence we need to examine two points, namely (a) whether the mud is unstable and (b) if so is the vertical heat flux significant.

Convection is usually characterized by the dimensionless Rayleigh number R, defined by

$$R = \frac{\alpha g \nabla T}{\nu \kappa} \left(\frac{r_0}{\gamma}\right)^4 \tag{C1}$$

where α = coefficient of thermal expansion of the fluid, ∇T = vertical temperature gradient, g = gravitational acceleration, γ = aspect ratio = r/L, L = height of convective cell, ν = kinematic viscosity, κ = thermal diffusivity.

The fluid is unstable if $R > R_c$ where R_c is a critical Rayleigh number. Thus we need to find the smallest vertical temperature gradient which will support free convection and this will correspond to the smallest critical Rayleigh number for the system.

Charlson & Sani (1970, 1971) have evaluated R_c for the case of a fluid enclosed in a cylindrical container. This is not strictly equivalent to the case of mud in the borehole but their results should be valid here to at least order of magnitude, especially for aspect ratios $\ll 1$ (i.e. cells which are very high compared with their width).

The critical gradient ∇T_c is given by

$$\nabla T_c = \frac{\nu \kappa R_c \gamma^4}{\alpha g r_0^4} \tag{C2}$$

and its smallest value will correspond to the minimum value of $R_c\gamma^4$. The minimum value of $R_c\gamma^4$ is given by Charlson & Sani (1971, fig. 1) to be about 230. Taking the following values as typical for the borehole fluid

$\kappa = 1.8 \times 10^{-7}\,\mathrm{m^2\,s^{-1}}$,
$\nu = 0.001\,\mathrm{m^2\,s^{-1}}$ (i.e. viscosity = 1000 centipoise),
$\alpha = 5 \times 10^{-4}\,\mathrm{K^{-1}}$

then for an 8.5 in. diameter hole $\nabla T_c = 0.06\,\mathrm{K\,m^{-1}} = 60\,\mathrm{K\,m^{-1}}$ and for a 17 in. hole $\nabla T_c = 0.004\,\mathrm{K\,m^{-1}} = 4\,\mathrm{K\,m^{-1}}$.

The ambient geothermal gradient is typically 20–30 $\mathrm{K\,m^{-1}}$ and presumably this limits the gradient in the mud. Hence from the above estimates it would appear that free convection is feasible especially in the large diameter sections of a borehole.

Given that free convection is feasible we need to determine whether the vertical heat flux carried by this motion is comparable to the radial conductive flux.

Heat transfer due to free convection is characterized by the dimensionless Nusselt number Nu given by

$$\mathrm{Nu} = \frac{\text{total vertical heat flux}}{\text{vertical conductive heat flux}}. \tag{C3}$$

From the definition Nu = 1 for $R < R_c$ (i.e. no convection) and Nu > 1 for $R > R_c$.

Jones, Moore & Weiss (1976) have studied axysymmetric convection in a cylinder. The model they used is not strictly applicable to our particular problem (they assume slippery boundaries and zero heat flux across the walls of the hole, whereas in the borehole one would expect friction at the boundary and a non-zero heat flow). Nevertheless their results should be applicable to at least order of magnitude. Numerical experiments performed by these authors indicate that for cases when the Prandtl number $\nu/\kappa \gg 1$ (which it is in our case as $\nu/\kappa \sim 10\,000$) then Nu obeys a power law of the form

$$\mathrm{Nu} \simeq 2\left(\frac{R}{R_c}\right)^{1/3}. \tag{C4}$$

This form for the relation of Nu to Rayleigh number is typical. Alterations in boundary conditions and geometry tend to change the exponent within the range 0.3–0.5 and also changes the numerical constant by factors less than 10. Thus we can use this result to estimate the order of magnitude of the ratio R/R_c which would make convective heat transfer comparable with the radial conductive flux.

The conductive heat flux is proportional to the temperature gradient. Typically the 'EFT' build-up model produces gradients of the order of $50\,\mathrm{K\,m^{-1}}$. Vertical gradients are unlikely to exceed about $0.05\,\mathrm{K\,m^{-1}}$. Thus for the total vertical flux to be comparable with the radial conductive flux would require a Nusselt number Nu ≈ 50/0.05 ≈ 1000. This in turn would imply a ratio $R/R_c \sim 10^8$ (using equation C4).

If we consider convective cells of height about 10 m (which is the height above hole bottom where BHTs are typically measured) then for an 8.5 in. hole the aspect ratio is about 0.01. From fig. 1 in Charlson & Sani (1971) we find that for the most unstable mode $R_c \sim 2 \times 10^{10}$. The vertical temperature gradients used above (i.e. $0.05\,\mathrm{K\,m^{-1}}$) give a Rayleigh number $\mathscr{R} \sim 1.4 \times 10^{10}$, which to order of magnitude gives a ratio $R/R_c \sim 1$. This value of R/R_c is 8 orders of magnitude lower than required to give a significant vertical heat flux.

Obviously the numbers quoted above are model- and parameter-dependent but changing the boundary conditions and physical properties within reasonable ranges is unlikely to make sufficient difference to give a significant vertical heat flux with a feasible vertical temperature gradient.

Thus the conclusion is that although free convection is feasible in the mud, the consequent vertical heat flux is almost certainly negligible. It may, however, be necessary to take account of the enhancement of the radial heat flux via the mixing effect of the fluid motion.

Appendix D: vertical temperature gradients

Most BHT models (including the one described in this paper) ignore vertical temperature gradients. In the real borehole there will undoubtedly be a non-zero vertical as well as a radial temperature gradient. The vertical slope will be due to the following sources

(1) there is a general geothermal gradient,

(2) drilling rates are finite so that the disturbance in the formation will vary with depth, and

(3) there will be a vertical heat flux into the hole due to the proximity of the hole bottom.

In general point (1) is negligible since the radial gradients are typically 100–1000 times larger than the equilibrium geothermal gradient.

The question of the finite drilling rate is more complex: it can only be sensibly ignored if the time taken to drill the section from the depth of measurement to the hole bottom is small compared with the post-drilling circulation time. The 'EFT' model implies that the borehole is created instantly and then the disturbing mechanism (i.e. the mud circulation) is switched on, thus ensuring that all depths are disturbed for the same length of time.

The soundness of this assumption can be checked *a posteriori* by comparing the radial temperature profile assuming a disturbance time equal to: (a) the circulation time after drilling has ceased and (b) the circulation time plus the time taken to drill from the depth of interest to the hole bottom. If the two profiles calculated in (a) and (b) are significantly different then it is likely that the variation of disturbance with depth will be an important consideration and a more complicated two-dimensional model may be necessary.

Point (3) is concerned with how far above the bottom the BHTs must be measured in order to be able to neglect vertical heat flow due to proximity of the hole bottom. This effect can be estimated by modifying the 'EFT' model to include a vertical gradient in an approximate way in order to avoid the need for a full two-dimensional model.

The heat flow is described by

$$\rho c \frac{\partial T}{\partial t} = K \left\{ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right\} . \tag{D1}$$

This is approximated by

$$\rho c \frac{\partial T}{\partial t} \simeq K \left\{ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{T_\infty - T}{h^2/2} \right\} \tag{D2}$$

where here T is a function of r and t only and the vertical gradient has been approximated by assuming that the horizontal plane through the hole bottom is held fixed at the equilibrium temperature. A simple finite difference approximation is used for the $\partial^2 T/\partial z^2$ term. Since the temperature at the bottom is fixed at T_∞ this effectively exaggerates the vertical heat flux and hence the model describes a worst case situation.

Equation (D2) describes the radial temperature profile at a distance h above the bottom and is solved numerically by modifying the 'EFT' algorithm described in Appendix A.

Fig. D1 shows a typical example calculated using the following parameters,

mud heat capacity = 4.6×10^6 J m^{-3} K^{-1},
mud conductivity = 0.8 W m^{-1} K^{-1},
rock heat capacity = 2.4×10^6 J m^{-3} K^{-1},
rock conductivity = 2.8 W m^{-1} K^{-1},
borehole diameter = 8.5 in.,
circulation time = 5 hr.

Fig. D1 shows the axial temperature build-up for various heights above hole bottom. The curve labelled $h/a = \infty$ was calculated using the purely radial 'EFT' model.

The results shown in Fig. D1 (and similar ones for larger diameter holes) show that for practical purposes the effect of the hole bottom can be neglected for distances $h/a > 10$. Thus for an 8.5 in. diameter hole if the temperatures are measured more than 1 m above the bottom then it would be reasonable to ignore the proximity of the bottom.

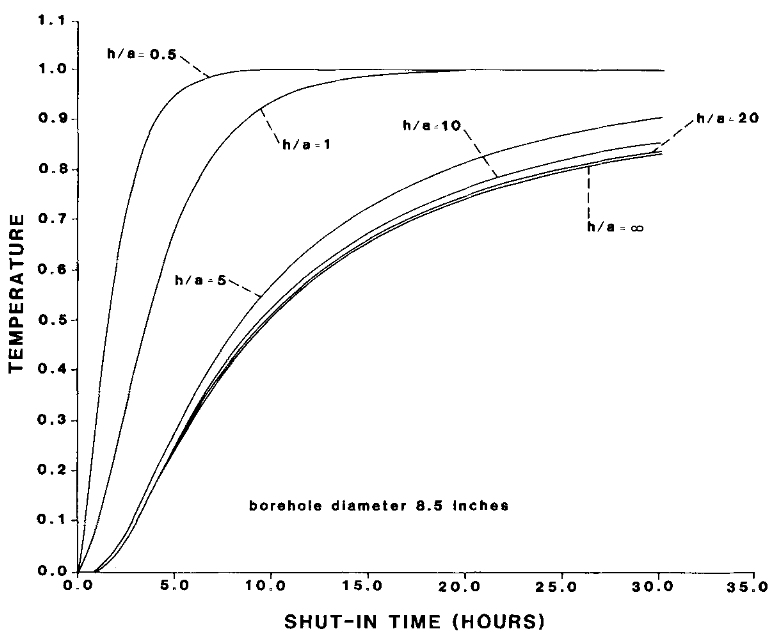

Figure D1. Axial temperature build-up at various heights above hole bottom.

Normally temperature sensors are placed on top of the logging tools, which are typically about 10 m long and hence safely far away from the bottom. However, if sensors are placed on the bottom of the tool then it may be necessary to apply corrections to the build-up curve to account for this effect (unless, of course, the logging tool is not allowed to get too close to the bottom).