



## Estimation of Growth Rate of Electromagnetic Plasma Wave through Vlasov-Maxwell Mathematical Frame in Ionospheric Plasma

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### Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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## ABSTRACT

Unique forms of nonlinear wave energy exchange phenomena are observed in the Earth's ionosphere region. Energy upconversion of nonresonant plasma waves in the top ionospheric and auroral zone are noticed. Origin of these phenomena are tried to explained by linear and nonlinear theoretical approach. Wave-wave and wave-particle-wave interaction processes may be possible role takes place here. In this theoretical investigation we wish to derive probable growth rate expression of high frequency electromagnetic O-mode wave in the presence of low frequency electrostatic ion sound wave through wave-particle interaction process known as plasma maser instability and estimate its value by using observational data.

*Keywords: Ionospheric plasma; O-mode; ion sound wave; plasma maser instability.*

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## 1 INTRODUCTION

Over the several decades theoretical and experimental investigations on the Earth's ionosphere have continued to escalate in different aspects. Ionospheric region has weak particle collision rate and weak magnetic field strength gradient. Solar wind is the main source of energy and momenta to this open plasma system. Both electromagnetic plasma waves and electrostatic plasma waves are interacting by means of wave-wave interaction and wave-particle-wave interaction process within this near-Earth space region[1,2,3,4,5,6]. This region has many linear and nonlinear properties for the energy exchange among waves and particles. Complex radiation emission phenomena are observed at different altitudes by ground based and satellite based observatories. Amplification of plasma radiation emission may take place through nonlinear energy exchange process in the presence of discrete types of ionospheric instabilities. Several theoretical investigations are carried to analysis such phenomenon by means of either fluid or kinetic theory approach.

From spectra of reflected waves it has been revealed that nonlinear responses in the ionospheric plasma may appear for the mode conversion and the wave energy upconversion of plasma waves. Stubbe et al.[7] established that secondary electromagnetic wave was originated through the mode conversion process from a primary electromagnetic wave in the ionospheric plasma. In another observation [ 8] it was derived that a high frequency electromagnetic wave could stimulate electromagnetic radiation in the ionosphere. After investigation on amplified radiations in top ionospheric regions of earth Mellot et al.[9] proclaimed that auroral kilometric radiation (AKR) was composed of electromagnetic plasma radiations. Khound et al.[10] reported that in presence of low frequency electrostatic plasma wave, plasma maser effect might be the possible mechanism of origin of Jupiter's kilometric radiation. From last several years a lot of investigations had been carried on wave energy upconversion of high frequency plasma waves in both space and laboratory plasma system. In this regard Nambu[11] and Tsyvovich et al.[12] suggested

plasma maser instability was one of the possible nonlinear wave-particle interaction for wave energy upconversion of plasma wave. By using PIC code, theoretically it was found that through plasma maser effect, nonresonant Langmuir wave could be generated from resonant whistler wave[13]. Gyobu et al.[14] suggested possible conversion of an electromagnetic wave from an electrostatic plasma through the plasma maser process. Deka et al.[15] investigated on amplification of electromagnetic plasma wave through plasma maser instability approach in inhomogeneous magnetosphere near the auroral region.

Generation of high frequency electromagnetic O-mode wave and low frequency ion sound wave in the upper ionosphere region were mentioned in several studies [9,10,16]. Interaction between plasma particles and ion sound plasma wave in the ionospheric zone was discussed by Kantor et al.[17]. In inhomogeneous plasma system most of the available turbulent wave energy is in the form of resonant wave mode. A low frequency plasma wave may play a crucial role in nonlinear energy exchange process with high frequency plasma wave. In this study we wish to investigate probable energy upconversion of electromagnetic O-mode wave in presence of electrostatic ion sound wave through plasma maser instability in inhomogeneous ionospheric plasma. According to weak turbulence theory plasma maser effect is an effective nonlinear mode-mode coupling interaction. In this instability the high frequency mode gains energy from low frequency mode in presence of plasma particles. For this theoretical investigation we use Vlasov-Maxwell system of equations in addition with Fourier transform and method of characteristics [18] to evaluate growth rate expression of nonresonant electromagnetic O-mode wave.

## 2 FORMULATION

In this problem we consider a magnetised inhomogeneous open plasma system in where direction of magnetic field is along z-axis in a cartesian coordinate frame. We assume in the presence of magnetic field high frequency electromagnetic O-mode wave has propagation

vector  $\vec{K} = (K_{\perp}, 0, 0)$  and low frequency electrostatic ion sound wave has propagation vector  $\vec{k} = (0, 0, k_{\parallel})$  where  $\parallel$  and  $\perp$  sign represent along parallel and perpendicular to the direction of external magnetic field. It is assumed ion sound wave has wave fields  $\vec{B}_l = (0, 0, 0)$  and  $\vec{E}_l = (0, 0, E_l)$ .

We consider the zero order particle distribution

function of non uniform plasma [18] for  $j^{th}$  species is

$$f_{oj} \simeq f_{oj} \left\{ 1 + \epsilon' \left( y - \frac{v_x}{\Omega_j} \right) \right\}. \quad (1)$$

where  $\Omega_j = \frac{e_j B_o}{m_j c}$  is cyclotron frequency,  $\epsilon' = -\frac{1}{f_{oj}} \left( \frac{\partial f_{oj}}{\partial x} \right)_{x=x_o}$  is density gradient.

### 3 MATHEMATICAL ANALYSIS

The interaction of high frequency electromagnetic O-mode plasma wave with low frequency electrostatic ion sound wave are governed by Vlasov-Maxwell system of equations which are

$$\left[ \frac{\partial}{\partial t} + \vec{v}_j \cdot \frac{\partial}{\partial \vec{r}} - \frac{e_j}{m_j} \left( \vec{E} + \frac{\vec{v}_j \times \vec{B}}{c} \right) \cdot \frac{\partial}{\partial \vec{v}_j} \right] F_j(\vec{r}, \vec{v}_j, t) = 0. \quad (2)$$

$$\vec{\nabla} \times \vec{E} = \frac{1}{c} \frac{\partial \vec{B}}{\partial t}. \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{J}. \quad (4)$$

$$\vec{J} = e_j n_j \int \vec{v} f_{oj}(\vec{r}, \vec{v}, t) d\vec{v}. \quad (5)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi e_j n_j \int f_{oj}(\vec{r}, \vec{v}, t) d\vec{v}. \quad (6)$$

In this problem we consider the unperturbed distribution functions for charged particles, the unperturbed electric field and the unperturbed magnetic field are taken as

$$F_{oj} = f_{oj} + \epsilon f_{1j} + \epsilon^2 f_{2j}. \quad (7)$$

$$\vec{E}_{ol} = \epsilon \vec{E}_l. \quad (8)$$

$$\vec{B}_{ol} = \vec{B}_o. \quad (9)$$

where  $f_{oj}$  is space and time averaged part of the distribution function,  $f_{1j}$  and  $f_{2j}$  are fluctuating parts due to low frequency ion sound wave turbulence and  $\epsilon$  represents order of ion sound wave turbulence. Now equation (2) to the order of  $\epsilon$  can be written as

$$\left[ \frac{\partial}{\partial t} + \vec{v}_j \cdot \frac{\partial}{\partial \vec{r}} - \frac{e_j}{m_j} \left( \vec{E} + \frac{\vec{v}_j \times \vec{B}_o}{c} \right) \cdot \frac{\partial}{\partial \vec{v}_j} \right] f_{1j}(\vec{r}, \vec{v}_j, t) = \frac{e_j}{m_j} \left( \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} f_{oj} \right). \quad (10)$$

Under the boundary conditions  $x'(0) = x, \vec{v}'(\tau = 0) = \vec{v}$ , the unperturbed particle orbits are

$$x' - x = -\frac{v_{\perp}}{\Omega_j} \sin(\theta - \Omega_j \tau) + \frac{v_{\perp}}{\Omega_j} \sin \theta$$

$$y' - y = \frac{v_{\perp}}{\Omega_j} \cos(\theta - \Omega_j \tau) - \frac{v_{\perp}}{\Omega_j} \cos \theta$$

$$z' - z = v_{\parallel} \tau.$$

where

$$\tau = t' - t$$

Applying Fourier transforms and the method of characteristics [18] from equation(10) we have

$$\begin{aligned} f_{ij} &= \frac{e_j}{m_j} \int_{-\infty}^0 \left( \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} f_{oj} \right) e^{i\{\vec{k} \cdot (\vec{r}^l - \vec{r}) - \omega t\}} d\tau. \\ &= -i \frac{e_j}{m_j} E_{l\parallel} \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{(k_{\parallel} v_{\parallel} - \omega)}. \end{aligned} \quad (11)$$

Consider this quasisteady system is perturbed by a nonresonant O-mode wave. This electromagnetic plasma wave has wave fields  $\mu \delta \vec{E}_h$  with fields vector  $\delta \vec{E}_h = (0, 0, \delta E_h)$ ,  $\delta \vec{B}_h = (0, \delta B_h, 0)$  and a frequency  $\Omega$ .

For this perturbation the total perturbed electric and magnetic fields and charged particle distribution functions are

$$\delta \vec{E} = \mu \delta \vec{E}_h + \mu \varepsilon \delta \vec{E}_{lh}. \quad (12)$$

$$\delta \vec{B} = \mu \delta \vec{B}_h + \mu \varepsilon \delta \vec{B}_{lh}. \quad (13)$$

$$\delta f_j = \mu \delta f_{hj} + \mu \varepsilon \delta f_{lhj} + \mu \varepsilon^2 \Delta f_j. \quad (14)$$

where  $\delta \vec{E}_h$  and  $\delta \vec{B}_h$  are fluctuating parts of modulating fields,  $\delta \vec{B}_{lh}$  and  $\delta \vec{E}_{lh}$  are modulated fields,  $\delta f_{hj}$  is fluctuating part due to nonresonant wave and  $\delta f_{lhj}$  and  $\Delta f_j$  are fluctuating parts of particle distribution functions corresponds to modulated fields. Under perturbed state equation (2) become

$$\left[ \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e_j}{m_j} \left\{ (\vec{E} + \delta \vec{E}) + \frac{\vec{v} \times (\vec{B}_o + \delta \vec{B})}{c} \right\} \cdot \frac{\partial}{\partial \vec{v}} \right] (f_{oj} + \delta f_j) = 0. \quad (15)$$

Using

$$\delta \vec{B}_h = \frac{K_{\perp} c}{\Omega} \delta E_h \hat{j}. \quad (16)$$

and

$$\left( \delta \vec{E}_h + \frac{\vec{v} \times \delta \vec{B}_h}{c} \right) \cdot \frac{\partial f_{oj}}{\partial \vec{v}} = \delta E_h \frac{\partial f_{oj}}{\partial v_{\parallel}}. \quad (17)$$

From equation (15) we have

$$\begin{aligned} \delta f_{hj} &= \frac{e_j}{m_j} \int_{-\infty}^0 \left( \delta \vec{E}_h + \frac{\vec{v} \times \delta \vec{B}_h}{c} \right) \cdot \frac{\partial f_{oj}}{\partial \vec{v}} e^{i\{\vec{k} \cdot (\vec{r}^l - \vec{r}) - \omega t\}} d\tau \\ &= -\frac{i e_j}{m_j} \delta E_h \frac{\partial f_{oj}}{\partial v_{\parallel}} \Gamma_{a,b}. \end{aligned} \quad (18)$$

where

$$\Gamma_{a,b} = \sum_{a,b} \frac{J_a(\alpha_j) J_b(\alpha_j) e^{i(b-a)\theta}}{(a\Omega_j - \Omega) + i\sigma^+}; \alpha_j = \frac{K_{\perp} v_{\perp}}{\Omega_j}. \quad (19)$$

and

$$\delta f_{lhj} = \frac{e_j}{m_j} \int_{-\infty}^0 \left[ \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} \delta f_{hj} + \left( \delta \vec{E}_h + \frac{\vec{v} \times \delta \vec{B}_h}{c} \right) \cdot \frac{\partial f_{1j}}{\partial \vec{v}} + \left( \delta \vec{E}_{lh} + \frac{\vec{v} \times \delta \vec{B}_{lh}}{c} \right) \cdot \frac{\partial f_{oj}}{\partial \vec{v}} \right] \times e^{[i\{(\vec{K}-\vec{k}) \cdot (\vec{r}^l - \vec{r}) - (\Omega - \omega)\tau\}]} d\tau \quad (20)$$

$$= \left( \frac{e_j}{m_j} \right)^2 E_{l\parallel} \delta E_h \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) \Gamma_{a,b} \Theta_{s,t} - \left( \frac{e_j}{m_j} \right)^2 E_{l\parallel} \delta E_h \frac{\partial f_{oj}}{\partial v_{\parallel}} \frac{\partial}{\partial v_{\parallel}} \left[ \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{(k_{\parallel} v_{\parallel} - \omega)} \right] \Theta_{s,t} + \frac{i e_j k_{\parallel}}{m_j K_{\parallel}} \delta E_{lh} \frac{\partial f_{oj}}{\partial v_{\parallel}} \Theta_{s,t} \quad (21)$$

where

$$\Theta_{s,t} = \sum_{s,t} \frac{J_s(\alpha_j) J_t(\alpha_j) e^{[i(t-s)\theta]}}{(s\Omega_j - k_{\parallel} v_{\parallel} + \Omega - \omega) + i0^+}; \alpha_j = \frac{K_{\perp} v_{\perp}}{\Omega_j} \quad (22)$$

and

$$\begin{aligned} \Delta f(\vec{K}, \Omega) &= \frac{e_j}{m_j} \int_{-\infty}^0 \left[ \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} \delta f_{lhj} + \left( \delta \vec{E}_{lh} + \frac{\vec{v} \times \delta \vec{B}_{lh}}{c} \right) \cdot \frac{\partial f_{1j}}{\partial \vec{v}} \right] \cdot e^{[i\{\vec{K} \cdot (\vec{r}^l - \vec{r}) - \Omega\tau\}]} d\tau \\ &= i \left( \frac{e_j}{m_j} \right)^3 E_{l\parallel}^2 \delta E_h \frac{\partial}{\partial v_{\parallel}} \left[ \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) \right] \Gamma_{a,b} \Theta_{s,t} \Gamma_{a,b} + i \left( \frac{e_j}{m_j} \right)^3 \delta E_{l\parallel}^2 \delta E_h \times \\ &\frac{\partial}{\partial v_{\parallel}} \left[ \frac{\partial f_{oj}}{\partial v_{\parallel}} \frac{\partial}{\partial v_{\parallel}} \left\{ \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{k_{\parallel} v_{\parallel} - \omega} \right\} \right] \Theta_{s,t} \Gamma_{a,b} + \left( \frac{e_j}{m_j} \right)^2 E_{l\parallel} \delta E_{lh} \frac{k_{\parallel}}{K_{\perp}} \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) \Theta_{s,t} \Gamma_{a,b} + \\ &\left( \frac{e_j}{m_j} \right)^2 E_{l\parallel} \delta E_{lh} \frac{k_{\perp}}{K_{\parallel}} \frac{\partial}{\partial v_{\parallel}} \left\{ \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{k_{\parallel} v_{\parallel} - \omega} \right\} \Gamma_{a,b} \end{aligned} \quad (23)$$

For the modulated fields we are using Maxwell's equations

$$\begin{aligned} \vec{\nabla} \times \vec{B}_{lh} &= \frac{1}{c} \frac{\partial}{\partial t} \delta \vec{E}_{lh} + \frac{4\pi}{c} \vec{J} \\ \vec{J} &= -e_j n_j \int \vec{v} f_{lhj} d\vec{v} \end{aligned} \quad (24)$$

after calculations we obtain the mixed modulation field as

$$\begin{aligned} \delta \vec{E}_{lh}(\vec{K} - \vec{k}, \Omega - \omega) &= \frac{4\pi i e_j n_j (\Omega - \omega)}{(\Omega - \omega)^2 - k_{\parallel}^2 c^2} \int v_{\parallel} \delta f_{lhj} d\vec{v} \\ &= \frac{4\pi i e_j n_j (\Omega - \omega)}{M [(\Omega - \omega)^2 - k_{\parallel}^2 c^2]} \int v_{\parallel} \left[ - \left( \frac{e_j}{m_j} \right)^2 E_{l\parallel} \delta E_h \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) \Gamma_{a,b} \Theta_{s,t} - \right. \\ &\left. \left( \frac{e_j}{m_j} \right)^2 E_{l\parallel} \delta E_h \frac{\partial f_{oj}}{\partial v_{\parallel}} \left( \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{k_{\parallel} v_{\parallel} - \omega} \right) \Theta_{s,t} \right] dv \end{aligned} \quad (25)$$

where

$$M = 1 + \sum_{j=i,e} \frac{4\pi i e_j n_j (\Omega - \omega)}{m_j [(\Omega - \omega)^2 - k_{\parallel}^2 c^2]} \int v_{\parallel} \frac{k_{\parallel}}{K_{\perp}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \Theta_{s,t} \right) dv. \quad (26)$$

To obtain the nonlinear dispersion relation we are using Maxwell's equation as

$$\vec{\nabla} \times \vec{B}_h = \frac{1}{c} \frac{\partial}{\partial t} \delta \vec{E}_h + \frac{4\pi}{c} \vec{J}.$$

and

$$\vec{J} = -e_j n_j \int v_{\parallel} (\delta f_h + \Delta f) d\vec{v}. \quad (27)$$

After calculation we got

$$\delta \vec{E}_h(\vec{K}, \Omega) = \frac{4\pi i e_j n_j (\Omega - \omega)}{\Omega^2 - K_{\perp}^2} \int v_{\parallel} (\delta f_h + \Delta f) d\vec{v}. \quad (28)$$

After simplification we obtain the nonlinear dispersion relation as

$$\delta E_h \left( \varepsilon_o(\vec{K}, \Omega) + \varepsilon_d(\vec{K}, \Omega) + \varepsilon_p(\vec{K}, \Omega) \right) = 0. \quad (29)$$

where  $\varepsilon_o(\vec{K}, \Omega)$  is linear part,  $\varepsilon_d(\vec{K}, \Omega)$  is direct coupling part and  $\varepsilon_p(\vec{K}, \Omega)$  is polarization coupling part.

Here

$$\varepsilon_o(\vec{K}, \Omega) = 1 - \sum_j \frac{4\pi e_j n_j}{\Omega^2 - c^2 K_{\perp}^2} \frac{e_j}{m_j} \int \Gamma_{a,b} \frac{\partial f_{oj}}{\partial v_{\parallel}} d\vec{v}. \quad (30)$$

and

$$\begin{aligned} \varepsilon_d(\vec{K}, \Omega) = & - \sum \frac{4\pi e_j n_j}{\Omega^2 - c^2 K_{\perp}^2} i \left( \frac{e_j}{m_j} \right)^3 E_{l\parallel}^2 \Theta_{s,t} \Gamma_{a,b} \int \left[ \frac{\partial}{\partial v_{\parallel}} \left\{ \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) \right\} \right] \Gamma_{a,b} + \\ & \frac{\partial}{\partial v_{\parallel}} \left\{ \frac{\partial}{\partial v_{\parallel}} \left( \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{k_{\parallel} v_{\parallel} - \omega} \right) \frac{\partial f_{oj}}{\partial v_{\parallel}} \right\} + \frac{m_j k_{\parallel}}{i e_j K_{\parallel}} \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) \right] d\vec{v}. \end{aligned} \quad (31)$$

and

$$\varepsilon_p(K, \Omega) = \frac{16\pi_2 e_j^4 n_j^2 \Omega (\Omega - \omega)}{m_j^2 (\Omega^2 - c^2 K_{\perp}^2) M [(\Omega - \omega)^2 - c^2 K_{\perp}^2]} \left( \frac{e_j}{m_j} \right)^2 [(A + B) \times (C + D)]. \quad (32)$$

where

$$\begin{aligned} A &= \int E_{l\parallel} \frac{k_{\parallel}}{K_{\perp}} \Theta_{s,t} \Gamma_{a,b} \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) v_{\parallel} dv_{\parallel} \\ B &= \int E_{l\parallel} \frac{k_{\parallel}}{K_{\perp}} \Gamma_{a,b} \frac{\partial}{\partial v_{\parallel}} \left( \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{k_{\parallel} v_{\parallel} - \omega} \right) v_{\parallel} dv_{\parallel} \end{aligned}$$

$$C = - \int E_{l\parallel} \Theta_{s,t} \Gamma_{a,b} \frac{\partial}{\partial v_{\parallel}} \left( \frac{\partial f_{oj}}{\partial v_{\parallel}} \right) v_{\parallel} dv_{\parallel}$$

$$D = - \int E_{l\parallel} \Theta_{s,t} \frac{\partial f_{oj}}{\partial v_{\parallel}} \frac{\partial}{\partial v_{\parallel}} \left( \frac{\frac{\partial f_{oj}}{\partial v_{\parallel}}}{k_{\parallel} v_{\parallel} - \omega} \right) v_{\parallel} dv_{\parallel}.$$

After neglecting other sub-dominant processes,we consider the wave energy upconversion of O-mode plasma wave occurs only for the plasma maser instability in the presence of low frequency ion sound wave.

The growth rate expression of O-mode is

$$\frac{\gamma}{\Omega} = - \left[ \frac{Im\varepsilon_d + Im\varepsilon_p + \frac{1}{2} \frac{\partial^2 \varepsilon_o}{\partial \Omega \partial t}}{\Omega \left( \frac{\partial \varepsilon_o}{\partial \Omega} \right)} \right]. \quad (33)$$

In this problem we consider only the polarisation coupling term.From previous studies it is found that imaginary part of polarization coupling term take play a major role to the plasma maser effect.In such case the external magnetic field act as momentum source and the plasma maser effect is effective for this.

Based on casuality principle,here expression of  $Im\varepsilon_p$  is

$$Im\varepsilon_p = - \frac{16\pi^2 e_j^2 n_j^2 \Omega (\Omega - \omega)}{\{(\Omega^2 - \omega^2) - c^2 k_{\parallel}^2\} M(\Omega^2 - c^2 K_{\perp}^2)} \left( \frac{e_j}{m_j} \right)^2 (A \times ImD + C \times ImB). \quad (34)$$

where

$$A = \frac{E_{l\parallel}}{\Omega^2 - \Omega_j^2} \times \frac{k_{\parallel}^2}{K_{\perp}^2}.$$

$$ImB = \sqrt{\pi} \frac{E_{l\parallel} k_{\parallel}^2}{(\Omega_j - \Omega)^2 K_{\perp}} \frac{1}{k_{\parallel} |k_{\parallel}|} \frac{\omega}{v_j^3} e^{-\left(\frac{\omega}{k_{\parallel} v_j}\right)}.$$

$$C = -\sqrt{\pi} \frac{E_{l\parallel}}{|k_{\parallel}|} \left( \frac{K_{\perp} v_j}{\Omega_j} \right)^2 \frac{1}{(\Omega_j - \Omega)} \frac{K_{\perp} \Omega_j}{\Omega}.$$

$$ImD = - \frac{E_{l\parallel} K_{\perp}}{\Omega(\Omega_j - \Omega)(\Omega - \omega - \Omega_j)} \left( \frac{K_{\perp} v_j}{\Omega_j} \right).$$

Here in evaluation process we consider only the most dominant terms to Bessels function comes from after taking  $a=b=0$  and  $s=t=1$ .Also,

$$Im \int_{-\infty}^{\infty} \frac{\frac{\partial}{\partial v_{\parallel}} f_{oj}(v_{\parallel})}{k_{\parallel} v_{\parallel} - \omega} = \frac{2\sqrt{\pi}}{v_j^3} \frac{\omega}{k_{\parallel} |k_{\parallel}|} e^{-\left(\frac{\omega}{k_{\parallel} v_j}\right)^2}.$$

From equation (26) taking lowest order argument  $\vec{k}$  and  $\omega$  we have as lowest order approximation

$$\frac{1}{M[(\Omega - \omega)^2 - k_{\parallel} c^2]} \simeq \frac{1}{k_{\parallel} c^2}. \quad (35)$$

Neglecting direct coupling part and the reverse absorption effect, after taking only the most dominant terms we obtain from equation (34) the approximate expression of growth rate for electromagnetic O-mode as

$$\frac{\gamma_p}{\Omega} \simeq \sqrt{\pi} \left( \frac{\omega_{pj}}{\Omega_j} \right) \left( \frac{e_j}{m_j} \right)^2 \frac{1}{v_j^2} \frac{k_{\parallel}^2}{K_{\perp}^2} E_{l\parallel}^2 \frac{1}{(\Omega_j - \Omega)^2} e^{-\left(\frac{\omega}{k_{\parallel} v_j}\right)^2}. \quad (36)$$

where  $\omega_{pj}$  is charged particle plasma frequency and  $\Omega_e$  is charged particle gyrofrequency.

## 4 RESULTS

To estimate the growth rate of O-mode, we are using the following observational data:

1) The Plasma parameters and O-mode plasma wave parameters in space [19]:

$$\Omega \sim \Omega_{j=e} \sim 10^6 \text{ Hz}, \frac{\omega_{pj(=e)}}{\Omega_{j(=e)}} \sim 0.3, v_{j=e} \sim 4.19 \times 10^6 \text{ m/sec}$$

$$K_{\perp} \sim 10^{-3} \text{ m}^{-1}, \frac{e_{j=e}}{m_{j=e}} \sim 1.75 \times 10^{11} \text{ CKg}^{-1}$$

2) The ion sound wave parameters and ionospheric parameters from ROSE satellite observations and other sources [20,21,22 ]:

$$k_{\parallel} \sim 10^{-5} \text{ m}^{-1}, E_{t\parallel} \sim 125 \times 10^{-6} \text{ V/m}, T_e \sim 400 \text{ K}, \frac{\omega}{k_{\parallel}} \sim 4.9 \times 10^3 \text{ m/sec}$$

The growth rate for O-mode from polarisation coupling term is found as

$$\frac{\gamma_p}{\Omega} \sim 10^{-8}$$

We have plotted growth rate [equation (36)] variation of  $X = \frac{\omega_{pe}}{\Omega_e}$  and it has been observed that the growth rate is enhanced (Fig.1).

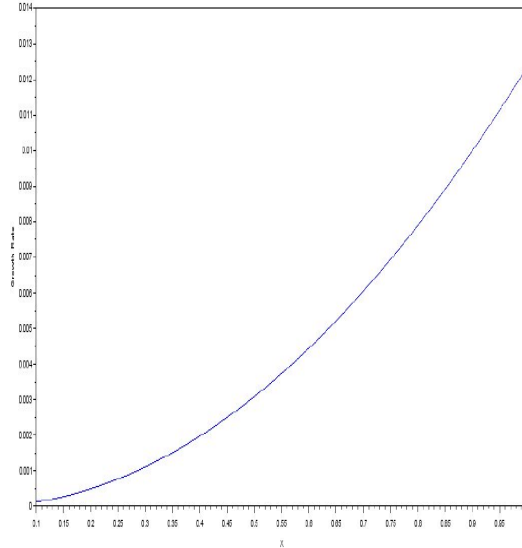


Fig. 1. Variation of Growth Rate with  $X = \frac{\omega_{pe}}{\Omega_e}$



## 5 CONCLUSION

In this investigation we have found that electromagnetic O-mode gets enhanced in the presence of ion sound wave in ionospheric plasma. Though density gradient parameter ( $\epsilon'$ ) is neutralized here for the propagation vector of ion sound wave, but the external magnetic field acts as momentum source for energy upconversion of the nonresonant O-mode. This study may suggest that plasma maser effect may be one of the sources of secondary electromagnetic radiation phenomenon in top ionospheric and magnetospheric region.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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