Estimation of origin-destination matrix from traffic counts: the state of the art

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Abstract

The estimation of up-to-date origin-destination matrix (ODM) from an obsolete trip data, using current available information is essential in transportation planning, traffic management and operations. Researchers from last 2 decades have explored various methods of estimating ODM using traffic count data. There are two categories of ODM; static and dynamic ODM. This paper presents studies on both the issues of static and dynamic ODM estimation, the reliability measures of the estimated matrix and also the issue of determining the set of traffic link count stations required to acquire maximum information to estimate a reliable matrix.

Keywords: Origin-Destination matrix; Static origin-destination matrix; Dynamic origin-destination matrix; Traffic count stations.

1. Introduction

In developing countries, changes in the land-use and economic state of affairs require momentous transportation planning. One of the most crucial requirements for the transportation planning is on arriving at the traffic pattern between various zones through Origin-Destination matrix (ODM) estimation. Traditional methods of estimating ODM are through large scale sampled surveys like home interview survey, roadside interview and license plate method conducted once in every 1-2 decades. But in situations of financial constraints these surveys become impossible to conduct. And by the time the survey data are collected and processed, the O-D data obtained become obsolete. In consequence, from 1970s many models have been proposed and widely applied for updating/estimating an old/sampled matrix using the current data of traffic counts collected on a set of links. The accuracy of these estimated matrix depends on

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the estimation model used, the input data errors, and on the set of links with collected traffic counts.

From the past studies, the ODM estimation models can be categorized as static and dynamic based on its application. In static methods the traffic flows are considered as time-independent and an average O-D demand is determined for long-time transportation planning and design purpose. Whereas from last two decades different dynamic approaches are proposed which are meant for short-term strategies like route guidance, traffic control on freeways, intersections etc. The assignment matrix which provides an approximate trip proportions based on the route choice behavior of the trip makers is the complicated part in the estimation problem. Another aspect, on which the reliability of the estimated ODM largely depends, is the optimum traffic counting locations. The traffic counts collected should provide as much traffic information as possible saving subsequent manpower requirement in data collection. Various rules are proposed in literature to select the traffic counting location points. This paper gives a vast study on the ODM estimation and the various related issues. It gives ideas about the various methodologies developed till date, the optimization algorithms used to solve the problem, convergence problems of those algorithms and most vital issue the reliability or the accuracy of the estimated ODM.

Though there are some softwares which can estimate ODM. But it is always wise to have coded algorithms which can be flexible with respect to the type of data available according to the study area. Still research is going on to estimate a reliable ODM efficiently. A comprehensive state-of-the-art review has been conveyed through this paper. Also the studies based on the optimum number of traffic counting locations required and their influences on estimated ODM are covered. This can help to acquire a good knowledge regarding the various developed models and also gives a direction for future research in this area.

The paper is organized as follows. Section 2 discusses about the static ODM estimation, its problem formulation and reviews the various methodologies developed till date. Discussions on dynamic and time-dependent ODM estimation are presented in Section 3. It encloses the models developed from past two decades. Section 4 gives a brief description of how the reliability of the estimated ODM is measured. The studies based on determining the traffic counting location points and its optimum number are presented in Section 5. Lastly Section 6 gives the final conclusions and the directions for future research.

2. Static ODM estimation from traffic counts

A static ODM estimation problem does not consider the time-dependent traffic flows and is assumed to represent a steady-state situation over a time period. The average traffic counts are collected for a longer duration to determine the average O-D trips. Let a transportation network is defined by W O-D pairs and a set of L links with $A \subseteq L$ as the subset of links with the observed traffic counts. Considering following notations:

t_w is the number of trips of O-D pair $w, w \in W$

p_w^a is the proportion of trips O-D pair $w \in W$ traversing link $a \in A$

v_a is the expected link flow for the link $a \in A$

The link flows, the trips between O-D pairs and the proportional matrix is related by the formulation given by eqn. (1).

$$\sum_{w \in W} p_w^a t_w = v_a \qquad\qquad a \in A \tag{1}$$

This estimation problem is underspecified as *A* is less than *W* ODs and there is no unique solution. So, additional information (a prior or a target or a sampled matrix) is needed to determine a unique trip matrix.

2.1 Travel Demand Model Based Methods

Initially the researchers tried to relate the trip matrix as a function of models (like the gravity models) with related parameters. Some of the researchers like Robillard (1975), Hogberg (1976) used Gravity (GR) model based approaches and some (Tamin and Willumsen, 1989; Tamin et al, 2003) used Gravity-Opportunity (GO) based models for estimating ODM. These techniques require zonal data for calibrating the parameters of the demand models. The main drawback of the gravity model is that it cannot handle with accuracy external-external trips (refer Willumsen, 1981). The fundamental model for estimating the ODM based on traffic counts in this approach is given by combining eqn. (1) for trip purpose *m* with the standard gravity model,

$$v_a^m = \sum_m \sum_i \sum_j .O_i^m D_j^m .A_i^m .B_j^m .f_{ij}^m .p_{ij}^{am} \tag{2}$$

where v_a^m gives the flows in particular link *a* for trip purpose *m*, O_i^m and D_j^m are the trips produced from zone *i* and attracted by zone *j* respectively for trip purpose *m*, f_{ij}^m here is the deterrence function, A_i^m and B_j^m are the balancing factors and p_{ij}^{am} is the proportion of trips traveling from zone *i* to zone *j* using link *a* for trip purpose *m*. Tamin and Willumsen (1989), for the calibration of unknown parameters, considered three estimation methods viz. non-linear-least-squares (NLLS), weighted-non-linear-least-squares (WNLLS) and maximum likelihood (ML). The GO models found to consume more time than GR model and does not guarantee the reliability of the estimated matrix.

2.2 Information Minimization (IM) and Entropy Maximization (EM) Approach

EM and IM techniques are used as model building tools in transportation, urban and regional planning context, after Wilson (1970) introduced the concept of Entropy in modeling. The entropy-maximizing procedure analyzes the available information to obtain a unique probability distribution. The number of micro-states $W\{t_w\}$ giving rise to meso-state t_w is given by,

$$W\{t_w\} = \frac{TN!}{\Pi_w t_w !} \tag{3}$$

where *TN* is the total number of trips and t_w is the trips of O-D pair $w \in W$. With the information contained in the observed flows and with other available information, EM is used by Willumsen (1978) and IM approach by Van Zuylen (1978) (refer Van Zuylen and Willumsen, 1980) to estimate trip matrix. Van Zuylen and Willumsen (1980) have shown that both the methods are same except the unit of observation. Van Zuylen's model uses a counted vehicle and Willumsen's a trip. Both the models results in multi-proportional problem. But the convergence of the algorithm has not been proved. Van Zuylen and Branston (1982) further extended the study of Van Zuylen and Willumsen (1980) considering the inconsistency in traffic counts for the case when there is more than one count available on some or all links of the network. Unfortunately, IM and EM based models have the disadvantage of not considering the uncertainties in traffic counts and prior matrix which can be erroneous and can influence the output.

2.3 Statistical Approaches

Several models have been presented in order to estimate or to update ODM from traffic counts for the networks without congestion and with congestion effects via parametric estimation techniques like; Maximum Likelihood (ML), Generalized Least Squares (GLS) and Bayesian Inference (BI).

The method of ***ML*** is one of the oldest and most important in estimation theory. The ML estimates are the set of parameters which will generate the observed sample most often. In ODM estimation maximum likelihood approach maximizes the likelihood of observing the target ODM and the observed traffic counts conditional on the true trip matrix. The data consist of $\hat{\mathbf{V}} = [\hat{v}_1, \hat{v}_2 .... \hat{v}_a]'$ representing a vector of a set of observed traffic counts, and a set of sampled O-D flows $\mathbf{N} = [n_1, n_2, ... n_w]'$. The likelihood of observing two sets of statistically independent data is expressed as

$$L(\mathbf{N}, \hat{\mathbf{V}} \mid \mathbf{T}) = L(\mathbf{N} \mid \mathbf{T}).L(\hat{\mathbf{V}} \mid \mathbf{T}) \qquad (4)$$

The sampled O-D flows may be assumed to follow Multivariate normal distribution (MVN) or Poisson probability distribution. This is dependent on small sampling fractions *α*. Consider observed link counts on a set of *A* links and vector $\mathbf{T} = [t_1, t_2, ... t_w]'$ representing the populated trip matrix with elements t_w.

For the logarithm of the probability $L(\mathbf{N}/\mathbf{T})$ we have:

$$\ln L(\mathbf{N}/\mathbf{T}) = -\frac{1}{2}(\mathbf{N} - \alpha\mathbf{T})'\mathbf{Z}^{-1}(\mathbf{N} - \alpha\mathbf{T}) + const. \qquad \text{(MVN)} \qquad (5)$$

$$\ln L(\mathbf{N}/\mathbf{T}) = \sum_{w \in W}(-\alpha t_w + n_w \ln(\alpha t_w)) + const. \qquad \text{(Poisson)} \qquad (6)$$

where $\mathbf{Z}$ is the covariance matrix of $\mathbf{N}$. If the observed traffic counts are also assumed to be generated either by a MVN distribution or Poisson probability distribution, then the similar expression for the probability $L(\hat{\mathbf{V}}/\mathbf{V}(\mathbf{T}))$ is obtained:

$$\ln L(\hat{\mathbf{V}} / \mathbf{V}(\mathbf{T})) = -\frac{1}{2}(\hat{\mathbf{V}} - \mathbf{V}(\mathbf{T}))'\mathbf{W}^{-1}(\hat{\mathbf{V}} - \mathbf{V}(\mathbf{T})) + const. \qquad \text{(MVN)} \qquad (7)$$

$$\ln L(\hat{\mathbf{V}} / \mathbf{V}(\mathbf{T})) = \sum_{a \in \hat{A}} (\hat{v}_a \ln(V_a(\mathbf{T})) - V_a(\mathbf{T})) + const. \qquad \text{(Poisson)} \qquad (8)$$

where $V_a(\mathbf{T})$ denotes the flow volume on link $a \in A$, resulting from an assignment of $\mathbf{T}$ and $\mathbf{W}$ as the variance-covariance matrix. Hence ML function is the log-likelihood of the sum of eqns. (5) or (6) with (7) or (8).

Among a number of branches of regression analysis, the method of **GLS** estimation based on the well-known Gauss-Markov theory plays an essential role in many theoretical and practical aspects of statistical inference based model. The advantage of this approach is that no distributional assumptions are made on the sets of data and it allows the combination of the survey data relating directly to O-D movements with traffic count data, while considering the relative accuracy of these data (Bell, 1991a). Let vector $\hat{\mathbf{T}}$ denote the survey estimate of $\mathbf{T}$ obtained from the grossing up the sample counts, independently of the sampling technique used. In GLS the following stochastic system of the equations in $\mathbf{T}$ is considered:

$$\hat{\mathbf{T}} = \mathbf{T} + \eta \qquad (9)$$
$$\hat{\mathbf{V}} = \mathbf{V}(\mathbf{T}) + \varepsilon \qquad (10)$$

where η is the sampling error with a variance-covariance matrix $\mathbf{Z}$, and ε is the traffic count error with dispersion matrix $\mathbf{W}$. Thus the GLS estimator T_{GLS} of $\mathbf{T}$ is obtained by solving:

$$T_{GLS} = \arg \min_{T \in S}(\hat{\mathbf{T}} - \mathbf{T})'\mathbf{Z}^{-1}(\hat{\mathbf{T}} - \mathbf{T}) + (\hat{\mathbf{V}} - \mathbf{V}(\mathbf{T}))'\mathbf{W}^{-1}(\hat{\mathbf{V}} - \mathbf{V}(\mathbf{T})) \qquad (11)$$

S is the feasible set of $\mathbf{T}$. There the dispersion matrix $\mathbf{Z}$ depends on the sampling estimator adopted.

BI method has also been applied in various transportation planning problems where prior beliefs are combined with the observations to produce the posterior beliefs. For further application and details of BI, refer Dey and Fricker (1994) paper where BI has been used for updating trip generation data. The BI approach considers the target ODM as a prior probability function $\Pr(\mathbf{T})$ of the estimated ODM $\mathbf{T}$. If the observed traffic counts are considered as another source of information with a probability $L(\hat{\mathbf{V}} / \mathbf{T})$, then Bayes theorem provides a method for combining the two sources of information. The posterior probability $f(\mathbf{T} / \hat{\mathbf{V}})$ of observing $\mathbf{T}$ conditional on the observed traffic counts is obtained as:

$$f(\mathbf{T} / \hat{\mathbf{V}}) \approx L(\hat{\mathbf{V}} / \mathbf{T}).\Pr(\mathbf{T}) \qquad (12)$$

2.3.1 Models without Considering Congestion Effects

The ODM estimation methods developed for networks with no congestion effects basically assume the route choice proportions and are independently determined outside the estimation process. For such networks Bayesians inference based approach has been first introduced by Maher (1983) for the ODM estimation. GLS estimator based approach has been studied by Cascetta (1984), Bell (1991a) etc. Bell (1991a) solved the GLS problem subject to inequality constraints and presented a simple algorithm but its application on real network has not been found in literature. Bierlaire and Toint (1995) proposed an ODM estimation method, called the Matrix Estimation Using Structure Explicitly (MEUSE), considering the information obtained from the parking surveys. Maximum likelihood based model is studied by Spiess (1987), Cascetta and Nguyen (1988), Hazelton (2000) etc. Two classical inference approaches; the ML and the GLS methods are derived and contrasted to the Bayesian method by Cascetta and Nguyen (1988). In all these studies, the link choice proportions used are estimated from the traffic assignment (TA) model and are assumed to be constant which may not estimate a dependable matrix. In consequence, Lo et al (1996) incorporated the randomness of the link choice proportions and discussed both Maximum likelihood and Bayesian approach for the estimation of the ODM by testing with a small network. Liu and Fricker (1996) introduced a stochastic logit model for calculating driver's route choice behaviour but it has certain drawback like all the link counts are considered to be known (further refer Yang et al, 2001). Lo et al (1999) extended the approach of Lo et al (1996) and developed a coordinate descent method using the partial linearization algorithm (PLA) for obtaining the optimum estimates and solved the new approach for large networks. Hazelton (2000) tested the performance of both multivariate normal (MVN) likelihood approximation and GLS techniques and found that the MVN method performed better. Hazelton (2001) studied the fundamental theoretical aspects of the ODM problem based on BI, defining the estimation, prediction and reconstruction problems as; a 'reconstruction' problem estimates the actual ODM occurring during the observation period, an expected number of O-D trips is obtained in 'estimation' problem and future O-D trips are obtained in 'prediction' problem. It has been shown that the estimation and reconstruction problems are different. There are some more studies (Hazelton, 2003; Van Aerde et al, 2003; Li, 2005, etc.) based on statistical approaches.

2.3.2 Models Considering Congestion Effects

Some authors included congestion effects in the estimation problem in which the dependence of the link costs, path choices and assignment fractions on link flows is considered. Equilibrium assignment approaches are particularly adopted for such cases. Nguyen (1977) first introduced the equilibrium based approach to estimate ODM through a mathematical programme (refer Leblanc and Farhangian, 1982). Also Yang et al (1994), Cascetta and Posterino (2001) and Yang et al (2001) solved the trip matrix estimation problem including congestion effects by considering different TA models. Cascetta and Posterino (2001) considered SUE assignment as a fixed-point problem whereas Yang et al (2001) considered the same problem of Liu and Fricker (1996) (developed without considering congestion effects) and proposed a non-linear optimization model (considering a weighted least square estimate) for the simultaneous

estimation of the ODM and travel-cost coefficient based on the logit-based SUE model. For solving this non-convex optimization problem a successive quadratic programming (SQP) method (which is a descent-feasible direction algorithm solving KKT solution) has been used. Further, Lo and Chan (2003) with SUE principle (multinomial logit model) estimated both the disperion parameter θ in multinomial logit model and the trip matrix simultaneously using Quasi-Newton method. The objective function considered is a likelihood function. Most of the above developed models are found to be tested using small networks. And the models do not assure their applicability for large real size congested networks.

Combined Distribution and Assignment (CDA) Based Problem

CDA is a network equilibrium based approach combining distribution and assignment problem. The model by Erlander et al (1979) for the simultaneous prediction of the flows and the demands is modified and used for matrix estimation by some researchers. Fisk and Boyce (1983) introduced the link count data in calibrating the dispersion parameter in Erlander et al (1979) model. The observed link flows are assumed to be available for all the network links. Fisk (1989) examined that the problem of Fisk and Boyce (1983) is simplest to solve and recommended to use for the problems where link cost functions are separable. Kawakami and Hirobata (1992) extended the Fisk's model by including the mode choice behaviour and proposed an optimization problem in the form of a combined distribution, modal split, and TA method. An entropy constraint condition with respect to the traffic mode choice behaviour has been considered. The objective function is an entropy function for the traffic distribution. The constraints include a Beckmann-type user equilibrium for all traffic modes and the entropy constraint with respect to the traffic modes. An iterative convergent algorithm has been proposed and is applied for a road network in Nagoya (Japan) considering two categories of automobiles; large-sized trucks and buses, and small trucks and cars. The network consists of 16 zones, 154 nodes and 240 links. But while applying it to the road network, the estimated values found to be deviated from the observed value, might be because of generalized cost function.

Bi-level Programming Approach

Bi-level programming approach has been used for the problem of ODM estimation in case of congested network. In this approach the upper-level problem is the trip matrix estimation problem and the lower-level problem represents a network equilibrium assignment problem. Genetic Algorithm (GA, a probabilistic global searching method) can also be seen introduced to solve the bi-level programming models.

Spiess in 1990 formulated a convex minimization, gradient based model (method of steepest descent) which can be applicable for large real networks. This ODM adjustment problem, allows adjustments between the traffic flows from the assignment algorithm and counts. It has been implemented using EMME/2 transportation planning software. The convex minimization problem is the distance between the observed and the assigned volumes shown below,

$$\text{Min } F(T) = \sum_{a \in A} (v_a - \hat{v}_a)^2 \tag{13}$$

subject to

$$\mathbf{V} \ = \ \mathbf{P(T)} \tag{14}$$

where v_a and $\hat{v}_a$ are the estimated and the counted flows for the link a ($a \in A$) respectively. For the convexity of the problem a set of non-decreasing link cost functions on all links of the network are assumed to be obtained from equilibrium assignment externally. The approach estimates an approximate gradient of the objective function with respect to the O-D demands. The Spiess model can estimate non-zero values for O-D pairs with zero trips considered initially, which can be a matter of concern for the planners who wants to preserve the structure of the target ODM. Thus, Doblas and Benitez (2005) further extended the above Spiess (1990) study for cases when one need to preserve the structure of the target ODM. An efficient approach has been proposed to update trip matrix for large study areas with minimum stored information. A new formulation in addition to the above formulation (eqns. 13 and 14) has been proposed which incorporates constraints based on trip generation, trip attraction, total trips in the network and trips between the O-D pairs with their upper and the lower bounds decided according to the planner. The problem has been transformed to an augmented lagrangian function and optimized using Frank-Wolfe algorithm. The approach has been tested on a real large size network.

Yang et al (1992) through a heuristic technique solved the integrated problem of GLS technique/entropy maximization with equilibrium TA model in the form of a convex bi-level optimization problem. In this study, the GLS estimate needs a matrix inversion of the size equal to the number of O-D pairs. Such an inversion requirement can have computational problem for large networks. Yang (1995) extended the bi-level programming problem by including link flow interaction and developed a model with heuristic algorithms. Kim et al (2001) discussed the problem of the bi-level models developed by Yang et al (1992). The author discussed the problem of dependency on the target ODM of the bi-level model and proposed an alternative model using GA. The upper level is solved by a combined solution method with GA for ODM estimation. The mathematical proof of the solution being optimal has not been discussed. To circumvent the difficulties with the non-differentiabilities of upper level problem Codina et al (2006) presented two alternative algorithms for solving bi-level problems and tested on small networks. First is a hybrid scheme proximal point-steepest descent method (for upper level sub-problem) and considering elastic demand TA problem and second a simple bi-level program considering fixed point mapping using linear TA problem. An iterative column generation algorithm which converges into a local minimum under the concept of continuity in path cost function is formulated by Garcia and Verastegui (2008) and applied on small networks. Recently, Lundgren and Peterson (2008) developed a heuristic bi-level problem solving it by a descent algorithm and demonstrated the algorithm using a large size network for the city of Stockholm, with 964 links and 1642 O-D pairs.

Path Flow Estimation (PFE) based Models

Recently, models based on path flow estimator which determines ODM according to the solutions of path flows have been adopted. It is a single level mathematical program in which the interdependency between O-D trip table and route choice proportion

(congestion effect) is taken into account. The core component of PFE is a logit based path choice model, which interacts with link cost functions to produce a stochastic user equilibrium traffic pattern. Sherali et al (1994) proposed a linear path flow model employing user equilibrium based solution for reproducing the observed link flows (known for all links). The procedure utilizes shortest path network flow programming sub-problem and a column generation technique is applied to generate the paths out of alternate paths that will determine the optimal solution to the linear programming model. To avoid the path enumeration required in the model proposed by Sherali et al (1994), Nie and Lee (2002) solved the linear programming model considering an exogenous K-shortest-path for determining the equilibrium path flow pattern. Nie et al (2005) further extended the decoupled path flow estimator by Nie and Lee (2002) considering the generalized least squares framework in aspect of the limitations of the linear programming structure. Sherali et al (2003) enhanced the linear programming model of Sherali et al (1994) for situations where only a partial set of link volumes are available. This introduces nonlinear cost function because of the dependence of the link travel cost on link volumes and a fixed point solution is tried. Further tests using larger and real-size networks are required with these PFE based models for better assessment and efficiency checking of these models.

2.4 Multi-Vehicle ODM Estimation

In spite of single-vehicle information (aggregated information) some authors included additional information obtained from the survey of the individual vehicles types. The multiclass O-D estimation leads to eliminate the internal inconsistency of traffic flows among different vehicle classes. Baek et al (2004) used multiple-vehicle information for ODM estimation from traffic counts. The multi-vehicle ODM estimation method is given as:

$$\text{Min } F(t_w) = \frac{1}{2}\sum_c \sum_{a \in A}(v_a^c - \hat{v}_a^c)^2 + \gamma\frac{1}{2}\sum_c \sum_{w \in W}(t_w^c - \hat{t}_w^c)^2 \tag{15}$$

subject to

$$\mathbf{T} \geq 0 \tag{16a}$$
$$\mathbf{V} = \mathbf{P(T)} \tag{16b}$$

where $\mathbf{T}$ is the ODM with elements t_w^c as the trips for the vehicle type c between O-D pairs w, $\hat{t}_w^c$ is the target O-D trips of vehicle type c between O-D pairs w, $\mathbf{V}$ as the vector with elements v_a^c as traffic volume of the vehicle type c on the link a, $\hat{v}_a^c$ is the observed volumes of vehicle type c on the link a, γ is the parameter reflecting the reliability of the target ODM and $\mathbf{P(T)}$ is the multi-vehicle traffic assignment map. For solving the non-convex problem, genetic algorithm has been used. The algorithm has been demonstrated only using a small network of 16 links with 8 O-D pairs and 9 nodes. It required more computing time due to involvement of genetic algorithm. An entropy maximization based ODM estimation of all vehicle classes with multiclass traffic counts observed directly from the network has been proposed by Wong et al (2005). A set of

multiplication factors for adjusting the demand matrix is used which may change because of changes in land use pattern or network configuration. The estimated matrix largely depends on the reliability of the input data.

2.5 Fuzzy Based Approach

Many non-linear real-life problems in the field of transportation planning and traffic control have been solved by fuzzy logic systems. Its application can be found in ODM estimation problem too. Fuzzy logic is used to model situations in which user making decisions are so complex that it is very hard to develop a mathematical model. Xu and Chan (1993a, b) estimated ODM with fuzzy weights (refer Teodorovic, 1999). Reddy and Chakroborty (1998) proposed a bi-level optimization approach on a multipath, fuzzy inference based flow dependent assignment algorithm for generating the route choice proportions which along with the observed link flows are then used in the ODM estimation. The proposed technique has been observed to give good O-D estimates irrespective of the flow pattern. Still not much study has been carried with fuzzy logic applications. The real-size network application of fuzzy logic systems should be further studied to ensure its efficiency.

2.6 Neural Network Based Approach

Considering the highly dynamic, large scale, complex and uncertain nature of many transportation systems, neural networks are recently considered as an efficient tool in solving numerous transportation problems. Gong (1998) developed the Hopefield Neural Network (HNN) model to estimate the urban ODM from link volumes, to promote the solving speed and the precision. Though there are quite a few studies carried out by researchers using Artificial Neural Networks (ANNs), but their application still need to be studied for real networks.

3. Dynamic/Time-Dependent ODM Estimation from Traffic Counts

Dynamic/time-dependent ODM estimation is crucial estimation problem for online/offline applications such as route guidance, dynamic traffic assignment (DTA) and freeway corridor control. Also used in various microscopic simulation based studies. Dynamic/time-dependent ODM estimation got much attention due to the development of Intelligent Transportation Systems (ITS). For dynamic (online) ODM estimation traffic counts are observed for short time say 15 min interval. Time-dependent (offline) ODM estimation considers time-series of traffic counts. Mostly the developed algorithms for both dynamic and time-dependent ODM estimation are applied for "closed" networks like intersections/interchanges and small freeways. The dynamic formulation of the matrix estimation process can be expressed as

$$f_{sh} = \sum_k \sum_w p_{sh}^{wk} t_{wk} \tag{17}$$

where f_{sh} is the flow crossing sensor s in time interval h, t_{wk} is the flow between O-D pair w that departed its origin during time interval k and p_{sh}^{wk} is an assignment parameter reflecting the proportion of the demand t_{wk} crossing sensor s in time interval h. The difference of dynamic formulation from static estimations lies in parameters k and h. And very few of them are developed considering large size network application. Classical techniques of dealing these systems are State-Space Modeling and Kalman filter which are discussed below.

To develop a **State-Space model**, a state is defined first. Once a state is defined, transition and measurement equations are specified. In dynamic systems, transition equations describe the evaluation of the state over time. Measurement equations on the other hand relate the unknown state to their observed indicators.

Measurement Equation: $y_h = C_h x_h + e_h$ (18)

Transition Equation: $x_{h+1} = D_h x_h + z_h$ (19)

Where x_h is the vector that represents the latent "true state" of the system during interval h, y_h is a vector of observations made in interval h, C_h and D_h are known matrices and, e_h and z_h are vectors of random errors.

Kalman filtering is an optimal state estimation process applied to a dynamic system that gives a linear unbiased and minimum error variance recursive algorithm. ODM estimation problem is formulated with a linear mapping from the state variable to the measurement variable. The formulation is stochastic which allows for measurement and state errors, and the initial state could be described stochastically. The transition equation for Kalman Filter is of the form

$$x_h = \phi_{h-1} x_{h-1} + \Gamma_{h-1} u_{h-1} + \Lambda_{h-1} q_{h-1}$$ (20)

where x_h is the state vector to be determined at the time h, ϕ_h is the matrix of autoregressive coefficients, u_h is a deterministic input, Γ_h the control gain, Λ_h the disturbance gain, and q_h is the disturbance input. The measurement equation is given by

$$x_h = H_{h-1} x_{h-1} + n_h$$ (21)

where n_h is the error (noise) term with zero mean and positive definite covariance matrix and the matrix H_h relates the state to the measurement of x_h. The filter computes the estimate of the state while minimizing the spread of the estimate error probability distribution function.

3.1 Dynamic or On-line ODM Estimation

The dynamic ODM estimation also stated as real-time ODM estimation, characterizes the time-variant trips between each origin and destinations. It is an essential input for DTA models and used for on-line identification of travel pattern for traffic controls systems. The dynamic models can be characterized as non-DTA based models and DTA

based models. The non-DTA based models like Kalman filter based models and Parameter optimization based models are basically applied for small networks like intersections, freeways etc where entry and exit flow information are available.

A state-space model with unknown O-D flows as the state vector has been first introduced by Okutani in 1987 (refer Kachroo et al, 1997). Ashok and Ben-Akiva (1993) (refer Sherali and Park, 2001) proposed a Kalman filtering approach to dynamically update an ODM. The O-D flow deviations from the prior estimates based on historical data are considered (for capturing the structural information) as the state-vectors in order to overcome inadequacy of autoregressive specification for O-D flows in Okutani's approach. Kachroo et al (1997) studied the applicability of Kalman filtering approaches for network ODM estimation from link traffic counts to explore the characteristics of the error terms in the underlying dynamic process of the O-D departures. The inconsistencies between the observed O-D flow patterns and Kalman Filter modeling assumptions is analyzed. It has been concluded that the noise is not a white Gaussian sequence. Ashok and Ben-Akiva (2000) further extended Ashok and Ben-Akiva's (1993) approach and presented a new formulation based on deviations of departure rates from origin and destination shares over time instead of destination flows.

Cremer and Keller (1987), Nihan and Davis (1987), Bell (1991b), Wu and Chang (1996), determined split parameters (averaged values) for input-output network relationships that is applicable for traffic flows at intersections or small freeway segments. Sherali et al (1997) developed a constrained optimization algorithm but with high computational cost. Li and Moor (1999) also proposed a recursive-based algorithm. The above approaches need all entry and exit information which is somewhat unrealistic. For the situations with incomplete traffic counts at some entrances and exits, Li and Moor (2002) formulated an optimization problem with linear equality constraints and non-negative inequality constraints. Van der Zijpp and Lindveld (2001) formulated a dynamic user optimal departure time and route choice (DUO-D&R) assignment problem which is used to estimate dynamic ODM with preferred departure times. There are some more studies (Van der Zijpp, 1997; Suzuki et al, 2000 etc.) on dynamic ODM estimation for intersections and freeways.

Due to disadvantages of Kalman filtering formulation (needs sufficient data and intensive matrix operations) Wu (1997) developed a real-time ODM updating algorithm based on a balancing method called multiplicative algebraic reconstruction technique (MART of Lamond and Stewart, 1981) considering entropy-maximization model. MART has been revised (RMART) by incorporating a normalization scheme, without giving any theoretical explanation of doing so. A diagonal searching technique is considered for improving the convergence speed. A numerical test has been carried out for checking the efficiency of the RMART with artificially generated database from some computer simulated problems. Zhou et al (2003) included the historical static information and ITS real-time link-level information to determine the dynamic O-D demand. The variation in day-to-day demand is studied by using multiday traffic counts. Nie and Zhang (2008) gives a brief review on dynamic ODM estimation algorithms and formulated a variational inequality problem determining the dynamic traffic assignment endogenously considering the user response to traffic congestion through a dispersion parameter θ. The problem finds out path flows denoted here as $\mathbf{f}$ which satisfy the conditions given in eqn. (22) which are transformed into a variational inequality formulation.

$$\begin{cases} f_w^{jh}(c_w^{jh} - \theta d_w^{jh}) = 0 \\ c_w^{jh} \ge \theta d_w^{jh}, f_w^{jh} \ge 0 \end{cases} \tag{22}$$

Here d_w^{jh} denotes the path derivatives between O-D pairs w for assignment interval h and c_w^{jh}, the path travel time of path j during assignment interval h. A solution for the variational inequality formulation has been proposed using a space-time expanded network (STEN, refer the journal paper for further details) to generate paths. A column generation algorithm has been proposed to solve the dynamic matrix estimation problem iterating the two sub-problems; generating paths to construct a restricted VI problem and to find an optimum solution of the restricted problem. The results depend on the initial path flows and the convergence issues still need to be studied.

3.2 Time-Dependent or Off-Line ODM Estimation

This estimation is carried out off-line, given a time-series of link counts, travel times and prior O-D information. Chang and Tao (1999) proposed a model integrating the link constraints and intersection turning flows from available DTA model, for determining the time-varying O-D trips for intersections applying Kalman filtering approach. A parametric optimization approach for off-line processing purpose is developed by Sherali and Park (2001). The proposed model seeks path flows that compromise between the least cost O-D paths and those that provide a match for the observed link flows. But with the increase in O-D paths, this model seems to be difficult to solve. Ashok and Ben-Akiva (2002) introduced the stochasticity of the assignment matrix in estimating the time-dependent O-D flows from link volumes. A GLS based solution has been studied for minimizing the error criteria for each interval and is evaluated for a case study. Tsekeris and Stathopoulos (2003) coupled multi-proportional procedure (MPP) of Murchland (1977) and multiplicative algebraic reconstruction technique (MART) of Gordon et al (1970) respectively, with a quasi-DTA model and estimated dynamic trip departure rates and ODM over a series of successive time interval. Combining the algorithms a doubly iterative matrix adjustment procedure (DIMAP) has been proposed to obtain a consistency between the trip departure rates from each origin zone and the observed link flows. The simulation based quasi-dynamic model used is based on the instantaneous link travel cost definition. The effect of congestion is incorporated and the performance of the algorithms is studied using a real network. Estimation is carried out separately using simulated link flows and real traffic flows. The DIMAP has been compared with MART and RMART (Wu, 1997). While considering simulated link flows the DIMAP algorithm found to perform better than the case using real traffic flows. Recently, BI based parsimonious parameterized model for estimating time varying ODM with traffic counts (collected on daily basis) has been recommended by Hazelton (2008).

4. The Measure of reliability of the estimated ODM

Statistical measures

The outcome of a situation which is difficult to predict is generally measured through some statistical measures. Likewise in ODM estimation problem some statistical measures are used by the authors to verify the performance of their proposed algorithms. The statistical measures only can measure the closeness of the estimated values (trips and link flows) and their true values, if known. Following are the statistical measures mostly adopted:

Relative error (RE) %:

$$RE\ (\%) = \sqrt{\frac{1}{2}\sum_{w \in W}\left(\frac{t_w^* - t_w}{t_w}\right)^2} \times 100\% \tag{23}$$

Total Demand Deviation (TDD) %:

$$TDD\ (\%) = \frac{\left|\sum_{w \in W} t_w^* - \sum_{w \in W} t_w\right|}{\sum_{w \in W} t_w} \times 100\% \tag{24}$$

Mean absolute error (MAE) %:

$$MAE\ (\%) = \frac{\sum_{w \in W}\left|t_w^* - t_w\right|}{N} \times 100\% \tag{25}$$

Root Mean Square Error (RMSE) %:

$$RMSE\ (\%) = \frac{\sqrt{\frac{1}{N}\sum_{w \in W}\left(t_w^* - t_w\right)^2}}{\frac{1}{N}\sum_{w \in W} t_w} \times 100\% \tag{26}$$

where t_w^* is the true ODM and N is the number of O-D pairs. The TDD gives the quality of the estimated ODM. The RMSE percent error quantifies the total percentage error of the estimate. The mean percent error indicates the existence of consistent under- or-over-prediction in the estimate. Smaller values of these measures will indicate the high quality of the estimated ODM. But in situations when the true values are not known these statistical measures cannot be used.

Maximum Possible Relative Error (MPRE)

Yang et al (1991) through a simple quadratic programming problem introduced a concept of maximum possible relative error (MPRE) which represents the maximum possible relative deviation of the estimated ODM from the true one and can be used only when the route choice proportions are correctly specified and the traffic counts are error free. The reliability of the estimated ODM from traffic counts is measured as,

$$\text{Re}(t) = \frac{1}{1+E}, \quad E \geq 0 \tag{27}$$

where E is the measure of the error (average relative deviation) in the estimated ODM depending upon the relative deviation of the estimated O-D flows from the true ones for the O-D pairs.

Travel Demand Scale (TDS)

Based on statistical analysis the quality measure of both static and dynamic ODM models is proposed by Bierlaire (2002) by means of TDS which is independent of the estimation method and a priori matrix (say obtained from a previous study), but depends upon the network topology and route choice assumptions. The Travel Demand Scale is computed as,

$$TDS = \varphi_{\max} - \varphi_{\min} \tag{28}$$

where

$$\varphi_{\min} = \min_{t} \mathbf{T}^{'} \mathbf{e} \tag{29}$$

and

$$\varphi_{\max} = \max_{t} \mathbf{T}^{'} \mathbf{e} \tag{30}$$

subject to

$$\mathbf{PT} = \hat{v}_{a} \tag{31a}$$

$$\mathbf{T} \geq 0 \tag{31b}$$

where $\varphi_{\min}$ and $\varphi_{\max}$ are minimum and maximum total level, $\mathbf{e}$ is the vector only of ones and $\mathbf{P}$ is the vector notation of the assignment matrix corresponding to the links where flow observations are available. The TDA value (for values refer the journal paper) helps to optimize the resources allocated during the surveys by identifying the nature of the additional information required. It finds out the unbounded O-D pairs (O-D pairs not captured by link flow data) so that surveys can be conducted to increase the quality of the corresponding entries in the a priori ODM. Thus it helps to assess the

level of investment necessary to collect data and build the a priori matrix. It is recommended to use in addition to the statistical measures.

5. Traffic Counting Location

   For the ODM estimation, traffic counting or sampling survey data collection are carried out where a road or rail route crosses a cordon line and screen lines. The accuracy of the ODM estimated increases with the number of traffic counting stations adopted. But due to resource limitations, it may not be possible. Again, the traffic count at each location has different degree of influence to the ODM estimation. Hence it is necessary to determine the optimum number of counting stations and their locations on the network to intercept maximum O-D pairs. Yang and Zhou (1998) introduced four basic rules of locating traffic counting points based on the maximal possible relative error (MPRE) concept proposed by Yang et al (1991). Based on the *O-D covering rule* stated by Yang et al (1991), following are the rules proposed by Yang and Zhou (1998);
(1) *O-D covering rule*: At least one traffic counting point on the network must be located for observing trips between any O-D pair.
(2) *Maximal flow fraction rule*: The traffic counting points on a network must be located at the links between a particular O-D pair such that flow fraction ϕ_{aw} is as large as possible.

$$\phi_{aw} = \frac{p_{aw}t_w}{v_a} \tag{32}$$

where v_a is the link flow, $a \in A$ between the O-D pair $w \in W$ and p_{aw} is the proportion of trips used by link $a \in A$ for each O-D pair w.
(3) *Maximal flow-intercepting rule*: From the links to be observed, the chosen links should intercept as many flows as possible.
(4) *Link independence rule*: The traffic counts on all chosen links should be linearly independent.
   For determining traffic count locations, the O-D covering rule and the link independence rule are treated as constraints and maximal flow fraction rule and maximal flow-intercepting rule are incorporated in objective function. Yim and Lam (1998) presented rules of maximum net O-D captured and maximum total O-D captured, and formulated in a linear programming model to determine the locations for ODM estimation. Bianco et al (2001) through proposed heuristic method solved the sensor location problem by proposing a two-stage procedure; determining the minimum number and location of counting points with known turning probabilities (assumed) and estimating the ODM with the resulting traffic flows. Yang et al (2003) studied the scheduling installation of traffic counting stations for long duration planning purpose and a Genetic Algorithm based sequential greedy algorithm has been proposed. Kim et al (2003) formulated two models, link-based and road-based model, to determine the location of the counting points on the link which minimizes the total cost. Three solution algorithm: Greedy Adding (GA) algorithm, Greedy Adding and Substituting

(GAS) algorithm and Branch and Bound (BB) algorithm are proposed and tested for a simple artificial network. Ehlert et al (2006) extended the problem of Chung (2001) (refer Ehlert et al, 2006) to optimize the additional counting locations assuming that some detectors are already installed increasing the O-D coverage. A software tool is developed based on mixed integer problem (MIP). With the budget restrictions for practical problems it is stated that the OD covering rule cannot always be satisfied i.e. the ODM estimation error measure MPRE cannot be applied. Gan et al (2005) studied both the traffic counting location and the error estimation measure in ODM estimation problem taking into consideration the route choice assumptions made in the TA models and the levels of traffic congestion on the networks. It has been noted that both MPRE and TDS measures are closely related to traffic counting locations. When the O-D covering rule is not satisfied by the link counting locations both the measures become infinite. Yang et al (2006) formulated an integer linear programming (ILP) problem using shortest path-based column generation procedure and branch-and-bound technique in determining screen line based traffic counting locations.

6. Conclusions

The basic goal of this paper is to explore the studies on one of the most promising topics for research which is the estimation of ODM using traffic counts on a set of links. The review shows the intricacy of the ODM estimation problem using traffic counts, the reason being the under-specification of the trip matrix estimation problem with less link count information than the number of unknowns. Till date both static and dynamic ODM estimation problems have been investigated by many researchers and models have been developed with different problem formulations, using different route choice decisions process and various solution algorithms.

The statistical approaches (ML, GLS and BI) have been mostly adopted by the researchers to solve the static problem considering congestion and without considering congestion effects. Both bi-level programming approach and simultaneous (single-level) optimization approaches have been studied in literature. Also Path flow estimation based algorithms are proposed assuming that all link costs are available, which may not be available in practical situations. Very few authors used fuzzy logic and neural network based approaches and their applicability need to be analyzed further. The review shows that most of the algorithms developed for static ODM have its own advantages and disadvantages and are implemented on small networks. However the most important consideration required is the applicability of the algorithms for real world networks which are large in size and highly congested. It is surprising to see a few realistic approaches in the literatures focused on large size network applications. Thus the developed algorithms need to be checked regarding their practical applicability for large size real networks.

The static ODM determined for long-time transportation planning and design purpose is easier to estimate as compared to dynamic ODM used for traffic management and operations for large networks because availability of real-time traffic information for all the O-D pairs required for dynamic trip matrix estimation is not possible. Compared to static ODM estimation, dynamic estimation based studies are few and mostly for intersections, freeways and small networks; as it is convenient to study the dynamic

state of these networks. Some authors tried to extend the study for large networks. But for practical applications dynamic ODM still is not much in use except for performing DTA on small scale networks.

To identify the reliability of the estimated ODM the statistical measures and the MPRE needs real trip values which are not always available in practical cases. The TDS though measures quality for both static and dynamic matrices but it does not serve alone the purpose of measuring the reliability. Some authors gave emphasis on finding the optimum number and location of the traffic counting points. Quite some rules have been proposed in the literature which can help in obtaining the optimum traffic counting locations and in receiving more information of travel pattern between O-D pairs.

As indicated earlier more studies and checking (mainly regarding computational difficulties) of the developed algorithms still need to be carried out especially for the case of its application for planning and designing purpose done for large size networks.

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