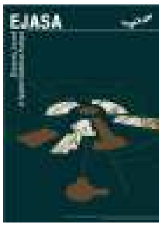

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ESTIMATION OF RELIABILITY IN MULTICOMPONENT STRESS-STRENGTH MODEL: LOG-LOGISTIC DISTRIBUTION

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Abstract: *A multicomponent system of k components having strengths following k-independently and identically distributed random variables x_1, x_2, ...x_k and each component experiencing a random stress Y is considered. The system is regarded as alive only if at least s out of k (s<k) strengths exceed the stress. The reliability of such a system is obtained when strength, stress variates are given by log-logistic distribution with different scale parameters. The reliability is estimated using Moment Method, ML method, Modified ML method and Best Linear Unbiased method of estimation in samples drawn from strength and stress distributions. The reliability estimators are compared asymptotically. The small sample comparison of the reliability estimates is made through Monte- Carlo simulation.*

Keywords: *Log-logistic distribution, reliability estimation, stress- strength, ML estimation, moment method, BLUE.*

1. Introduction

Let X, Y be two independent random variables following log – logistic distribution suggested by Balakrishnan *et. al.* [2] with scale parameters σ_1, σ_2 respectively and common shape parameter β. Then the p.d.f.'s and c.d.f.'s of *X* and *Y* are given by:

$$f(x;\sigma_1) = \frac{\beta}{\sigma_1}\frac{(x/\sigma_1)^{\beta-1}}{\left[1+(x/\sigma_1)^{\beta}\right]^2};\quad x \geq 0, \sigma_1 > 0, \beta > 1 \tag{1}$$

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$$F(x;\sigma_1) = \frac{(x/\sigma_1)^{\beta}}{[1+(x/\sigma_1)^{\beta}]};\ x \ge 0, \sigma_1 > 0, \beta > 1 \tag{2}$$

$$g(y;\sigma_2) = \frac{\beta}{\sigma_2}\frac{(y/\sigma_2)^{\beta-1}}{\left[1+(y/\sigma_2)^{\beta}\right]^2};\ y \ge 0, \sigma_2 > 0, \beta > 1 \tag{3}$$

$$G(y;\sigma_2) = \frac{(y/\sigma_2)^{\beta}}{[1+(y/\sigma_2)^{\beta}]};\ y \ge 0, \sigma_2 > 0, \beta > 1 \tag{4}$$

Let the random $y, x_1, x_2, ...x_k$ be independent, G(y) be the continuous cdf of Y and F(x) be the common continuous cdf of $x_1, x_2, ...x_k$. The reliability in a multicomponent stress- strength model developed by Bhattacharyya and Johnson [3] is given by:

$$R_{s,k} = \mathrm{P}\left[\text{at least } s \text{ of the } (x_1, x_2, ...x_k) \text{ exceed } Y\right]$$
$$= \sum_{i=s}^{k}\binom{k}{i}\int_{-\infty}^{\infty}[1-G(y)]^i[G(y)]^{k-i}\,dF(y), \tag{5}$$

Where $x_1, x_2, ...x_k$ are iid with common cdf F(x) this system is subjected common random stress Y. Assuming that F(.) and G (.) are log – logistic distributions with unknown scale parameters σ_1, σ_2 and common shape parameter β and that independent random samples $x_1 < x_2 < .... < x_n$ and $y_1 < y_2 < .... < y_m$ are available from F(.) and G (.), respectively. The reliability in multicomponent stress- strength for log- logistic distribution using (5) we get:

$$R_{s,k} = \sum_{i=s}^{k}\binom{k}{i}\int_{0}^{\infty}\frac{Z^{k-i}\lambda^{\beta(k-i)}dZ}{(1+Z\lambda^{\beta})^k(1+Z)^2}, \tag{6}$$

where $Z = (y/\sigma_1)^{\beta}$ and $\lambda = \sigma_1/\sigma_2$. The probability in (6) is called reliability in a multicomponent stress- strength model (Bhattacharyya and Johnson [3]). The survival probability of a single component stress – strength versions have been considered by Enis and Geisser [6], Downtown [5], Awad and Gharraf [1], McCool [12], Nandi and Aich [13], Surles and Padgett [22], Raqab and Kundu [16], Kundu and Gupta [9 &10], Raqab et.al. [17], Kundu and Raqab [11]. The reliability in multicomponent stress- strength was developed by Bhattacharyya and Johnson [3], Pandey and Borhan Uddin [14].

Suppose a system, with k identical components, functions if $s\,(1 \le s \le k)$ or more of the components simultaneously operate. In its operating environment, the system is subjected to a stress Y which is a random variable with cdf G(.). The strengths of the components, that are the minimum stresses to cause failure, are independent and identically distributed random variables with cdf F(.). Then the system reliability, which is the probability that the system does not fail, is the function $R_{s,k}$ given in (5). The estimation of survival probability in a multicomponent stress – strength system when the stress, strength variates are following log- logistic distribution is not paid much attention. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress–strength model with reference to log- logistic probability distribution and the findings are presented in Sections 2 and 3. Finally conclusions in Section 4.

2. Different Methods of Estimation of Parameters in $R_{s,k}$

If σ_1, σ_2 are not known, it is necessary to estimate σ_1, σ_2 to estimate $R_{s,k}$. In this paper we estimate σ_1, σ_2 by ML method and modification to ML method, Method of moment, BLUEs thus giving rise to four estimates. The estimates are substituted in λ to get an estimate of $R_{s,k}$ using equation (6). The theory of methods of estimation is explained below.
It is well known that the method of Maximum Likelihood Estimation (MLE) has invariance property. When the method of estimation of parameter is changed from ML to any other traditional method, this invariance principle does not hold good to estimate the parametric function. However, such an adoption of invariance property for other optimal estimators of the parameters to estimate a parametric function is attempted in different situations by different authors. Travadi and Ratani [24], Kantam and Srinivasa Rao [7] and the references therein are a few such instances. Srinivasa Rao and Kantam [20] studied point estimation of system reliability exemplified for the log- logistic distribution, in this article they have studied various methods of estimation of scale parameter involved in system reliability $R(x) = 1 - F(x) = [1 + (x/\sigma)^{\beta}]^{-1}$. In this direction, we have proposed some estimators for the reliability of multicomponent stress – strength model by considering the estimators of the parameters of stress, strength distributions by standard methods of estimation in log- logistic distribution.

2.1 Method of Maximum Likelihood Estimation (MLE)

Let $x_1 < x_2 < .... < x_n$; $y_1 < y_2 < .... < y_m$ be two ordered random samples of size n, m respectively on strength, stress variates each following log – logistic distribution with scale parameters σ_1, σ_2 and shape parameter β. The log- likelihood function of the observed sample is:

$$Log\, L(\beta, \sigma_1, \sigma_2) = (m+n)\log\beta - n\log\sigma_1 - m\log\sigma_2 + (\beta - 1)\left[\sum_{i=1}^{n}\log(x_i/\sigma_1) + \sum_{j=1}^{m}\log(y_j/\sigma_2)\right]$$

$$-2\sum_{i=1}^{n}\log[1+(x_i/\sigma_1)^{\beta}] - 2\sum_{j=1}^{m}\log[1+(y_j/\sigma_2)^{\beta} \qquad (7)$$

The MLEs of β, σ_1 and σ_2, say $\hat{\beta}, \sigma_1^{(1)}$ and $\sigma_2^{(1)}$ respectively can be obtained as the iterative solution of

$$\frac{\partial \log \mathrm{L}}{\partial \beta}=0 \Rightarrow \frac{m+n}{\beta}+\sum_{i=1}^{n}\log u_i+\sum_{j=1}^{m}\log v_j-2\sum_{i=1}^{n}\frac{u_i^{\beta}\log u_i}{1+u_i^{\beta}}-2\sum_{j=1}^{m}\frac{v_j^{\beta}\log v_j}{1+v_j^{\beta}}=0, \quad (8)$$

$$\frac{\partial \log \mathrm{L}}{\partial \sigma_1}=0 \Rightarrow \sum_{i=1}^{n}F(u_i)-\frac{n}{2}=0, \quad (9)$$

$$\frac{\partial \log \mathrm{L}}{\partial \sigma_2}=0 \Rightarrow \sum_{j=1}^{m}F(v_j)-\frac{m}{2}=0, \quad (10)$$

Where $u_i = x_i / \sigma_1$ and $v_j = y_j / \sigma_2$.

To avoid computational difficulty, here we consider β is a known value and on solving (9) and (10) we get MLEs of σ_1 and σ_2 are denoted as $\sigma_1^{(1)}$ and $\sigma_2^{(1)}$. The asymptotic variance of the MLE is given by:

$$\left[-E(\partial^2 \log L/\partial\sigma_i^2)\right]^{-1}=(3/\beta^2)(\sigma_i^2/n)\ ;i=1,2 \qquad \text{when m=n.} \quad (11)$$

The MLE of survival probability of multicomponent stress – strength model is given by $R_{s,k}^{(1)}$ with λ is replaced by $\lambda^{(1)}=\sigma_1^{(1)}/\sigma_2^{(1)}$ in (6).

It can be seen that equations (9) and (10) can not be solved analytically for σ_1, σ_2. Therefore Srinivasa Rao [19] approximated F (.) by a linear function say $F(u_i) \cong \gamma_i + \delta_i u_i$ and $G(v_j) \cong \mu_j + \vartheta_j v_j$ where $u_i = x_i/\sigma_1$ and $v_j = y_j/\sigma_2$ and $\gamma_i, \delta_i, \mu_j, \vartheta_j$ are to be suitably found. Here we present Tikku [23] method of finding $\gamma_i, \delta_i, \mu_j, \vartheta_j$ after using approximation in equations (9) and (10), solutions for σ_1, σ_2 are given by:

$$\sigma_1^{(2)}=\frac{2\sum_{i=1}^{n}x_i\delta_i}{n-2\sum_{i=1}^{n}\gamma_i} \quad \text{and} \quad \sigma_2^{(2)}=\frac{2\sum_{j=1}^{m}y_j\vartheta_j}{m-2\sum_{j=1}^{m}\mu_j}.$$

The above estimators are named as MMLE of σ_1, σ_2, which are linear estimators in x_i's and y_j's. Hence its variance can be computed using the variances and covariances of standard order statistics provided we have the values of $\gamma_i, \delta_i, \mu_j, \vartheta_j$. Tikku [23] is based on linearization of certain portion of log likelihood equation. This method would result in linear estimators requiring certain constants such as $\gamma_i, \delta_i, \mu_j, \vartheta_j$ tabulated by Srinivasa Rao [19] for n=1(1) 10,

$\beta = 3(1)6$. Borrowing these tabulated values of MMLE we can find estimates of σ_1, σ_2 and have estimate of λ and hence estimate of $R_{s,k}$. Thus for a given pair of samples on stress, strength variates we get an estimate of $R_{s,k}$ by the above method is $R_{s,k}^{(2)}$. Thus the MMLE of survival probability of multicomponent stress – strength model is given by $R_{s,k}^{(2)}$ where λ is replaced $\lambda^{(2)} = \sigma_1^{(2)} / \sigma_2^{(2)}$ in (6).

2.2 Method of Moment Estimation (MOM)

We know that, if $\bar{x}, \bar{y}$ are the sample mean of samples on strength, stress variates then moment estimators of σ_1, σ_2 are $\sigma_1^{(3)} = \bar{x} / \Gamma(1+1/\beta)\Gamma(1-1/\beta)$ and $\sigma_2^{(3)} = \bar{y} / \Gamma(1+1/\beta)\Gamma(1-1/\beta)$ respectively. The moments are obtained from Balakrishnan *et. al.* [2]. The third estimator, we propose here is $R_{s,k}^{(3)}$ with λ is replaced by $\lambda^{(3)} = \sigma_1^{(3)} / \sigma_2^{(3)}$ in (6).

2.3 Best Linear Unbiased Estimation (BLUE)

Srinivasa Rao [19] has developed the coefficients to get the BLUEs of σ in a scaled log–logistic distribution. Hence the BLUEs of σ_1, σ_2 are $\sigma_1^{(4)} = \sum_{i=1}^{n} l_i x_i$ and $\sigma_2^{(4)} = \sum_{j=1}^{m} c_j y_j$ where $(l_1, l_2, ..., l_m)$ and $(c_1, c_2, ..., c_n)$ are to be borrowed from Srinivasa Rao [19]. The fourth estimator that we propose be $R_{s,k}^{(4)}$ with λ is replaced by $\lambda^{(4)} = \sigma_1^{(4)} / \sigma_2^{(4)}$ in (6).

Thus for a given pair of samples on stress, strength variates we get 4 estimates of $R_{s,k}$ by the above 4 different methods. The asymptotic variance (AV) of an estimate of $R_{s,k}$ which is a function of two independent statistics (say) t_1, t_2 is given by Rao [15]:

$$\text{AV}(\hat{\text{R}}_{s,k}) = \text{AV}(\text{t}_1)\left(\frac{\partial \text{R}_{s,k}}{\partial \sigma_1}\right)^2 + \text{AV}(\text{t}_2)\left(\frac{\partial \text{R}_{s,k}}{\partial \sigma_2}\right)^2 \tag{13}$$

Where t_1, t_2 are to be taken in four different ways namely, exact MLE, TMMLE on lines of Tiku [23], MOM moment estimator and BLUEs. In the present case using (6) we get:

$$\frac{\partial R_{s,k}}{\partial \sigma_1} = \sum_{i=s}^{k} \binom{k}{i} \int_0^{\infty} \frac{2\beta Z^{k-i+1} \lambda^{\beta(k-i)} dZ}{\sigma_1 (1 + Z\lambda^{\beta})^k (1+Z)^3},$$

$$\frac{\partial R_{s,k}}{\partial \sigma_2} = \sum_{i=s}^{k} \binom{k}{i} \int_0^{\infty} \frac{\beta Z^{k-i} \lambda^{\beta(k-i)} (iZ\lambda^{\beta} - k + i) dZ}{\sigma_2 (1 + Z\lambda^{\beta})^{k+1} (1+Z)^2},$$

which can be used to get $\text{AV}(\hat{\text{R}}_{s,k})$.

Because we are using linear estimator as MMLE, which is obtained through admissible approximation to log likelihood function, this estimator is asymptotically as efficient as exact MLE. From the asymptotic optimum properties of MLEs (Kendall and Stuart [8]) and of linear unbiased estimators (David [4]), we know that MLEs and BLUEs are asymptotically equally efficient having the Cramer – Rao lower bound as their asymptotic variance as given in (11). Thus from Equation (13), the asymptotic variance of $\hat{R}_{s,k}$ when (t_1, t_2) are replaced by MLE, TMMLE and BLUE in succession, we get same result. In the log – logistic distribution the moment estimator of the scale parameter is sample mean divided by $\Gamma(1+1/\beta)\Gamma(1-1/\beta)$. Under central limit property for *iid* variates the asymptotic distribution of the moment estimator is normal with the asymptotic variance is given by:

$$\text{Asymptotic variance of moment estimator} = \frac{\left\{\Gamma(1+2/\beta)\Gamma(1-2/\beta) - \left[\Gamma(1+1/\beta)\Gamma(1-1/\beta)\right]^2\right\}}{\left[\Gamma(1+1/\beta)\Gamma(1-1/\beta)\right]^2}\sigma^2/n.$$

As exact variances of our estimates of $R_{s,k}$ are not analytically tractable, the small sample comparisons are studied through simulation in Section 3.

3. Small Sample Comparison

3000 random sample of size 3(1) 10 each from stress population, strength population are generated for λ= 1, 2, 3 on lines of Bhattacharyya and Johnson [3]. The scale parameters σ_1, σ_2 of the variates are estimated by MLE, MMLE, Moment, and BLUE and are used in estimating λ. These four estimators of λ are used to get the multicomponent reliability for (*s*, *k*) = (1, 3), (2, 4). The sampling bias, mean square error (MSE) of the reliability estimates over the 3000 such sample is given in Table 1 and Table 2 for $\beta = 3 \& 5$. The other values of β are also available with authors. Also, true value of reliability in multicomponent stress- strength increases with the increase of shape parameter β and λ values. The true value of reliability is close to 0.99 when $\beta = 6$ and λ=4 for (*s*, *k*) = (1, 3). With respect to bias TMMLE shows very close to exact MLE than to other two methods regarding minimum bias in most of the parametric and sample combinations, in some combinations TMMLE shows minimum bias than exact MLE. With respect to MSE the choice is BLUE and the nearest estimator is exact MLE regarding minimum MSE over the other suggested methods of estimation.

Table 1: Results of the Simulation Study of Bias and MSE for Estimates of Reliability ($\beta = 3$)

(s,k)	(m,n)	λ	True $R_{s,k}$	Bias				MSE			
				mle	tmmle	mom	blue	mle	tmmle	mom	blue
(1,3)	3	1	0.511	-0.00101	0.02823	-0.03031	-0.03287	0.02084	0.02617	0.02108	0.01913
	4	1	0.511	-0.00126	0.01101	-0.02183	-0.02581	0.01598	0.01779	0.01719	0.01466
	5	1	0.511	-0.00011	0.00355	-0.01206	-0.02049	0.01317	0.01389	0.01497	0.01238
	6	1	0.511	0.00919	0.01004	-0.00384	-0.00810	0.01271	0.01283	0.01472	0.01157
	7	1	0.511	-0.00222	-0.00423	-0.01170	-0.01762	0.00968	0.00964	0.01167	0.00918
	8	1	0.511	0.00063	-0.00218	-0.00712	-0.01238	0.00895	0.00886	0.01059	0.00849
	9	1	0.511	0.00146	-0.00200	0.00056	-0.01014	0.00789	0.00795	0.01139	0.00766
	10	1	0.511	0.00174	-0.00226	-0.00151	-0.00920	0.00715	0.00716	0.00918	0.00691
	3	2	0.635	0.00178	0.03781	-0.03335	-0.03996	0.04174	0.04907	0.04592	0.03902
	4	2	0.635	0.00158	0.01706	-0.02491	-0.02890	0.03227	0.03510	0.03608	0.03089
	5	2	0.635	0.00368	0.00784	-0.00777	-0.02135	0.02751	0.02856	0.03482	0.02664
	6	2	0.635	0.01279	0.01428	-0.00084	-0.00818	0.02428	0.02446	0.02956	0.02287
	7	2	0.635	0.00028	-0.00234	-0.00853	-0.01902	0.01970	0.01967	0.02642	0.01908
	8	2	0.635	-0.00099	-0.00455	-0.00916	-0.01718	0.01832	0.01820	0.02273	0.01795
	9	2	0.635	0.00551	0.00116	0.00730	-0.00865	0.01652	0.01668	0.02444	0.01632
	10	2	0.635	0.00162	-0.00320	-0.00183	-0.01140	0.01476	0.01490	0.01988	0.01464
	3	3	0.656	0.01509	0.05318	-0.01795	-0.02795	0.05346	0.06330	0.06036	0.04813
	4	3	0.656	0.01195	0.02850	-0.01363	-0.01912	0.04053	0.04439	0.04624	0.03801
	5	3	0.656	0.01210	0.01650	0.00427	-0.01342	0.03441	0.03571	0.04665	0.03280
	6	3	0.656	0.01964	0.02134	0.00834	-0.00166	0.02920	0.02947	0.03688	0.02728
	7	3	0.656	0.00577	0.00316	0.00007	-0.01383	0.02365	0.02373	0.03463	0.02277
	8	3	0.656	0.00304	-0.00060	-0.00332	-0.01351	0.02185	0.02164	0.02836	0.02124
	9	3	0.656	0.01048	0.00610	0.01553	-0.00383	0.01988	0.02005	0.03140	0.01951
	10	3	0.656	0.00491	0.00007	0.00293	-0.00823	0.01758	0.01771	0.02444	0.01736
(2,4)	3	1	0.465	0.00957	0.03614	-0.01561	-0.01855	0.01772	0.02307	0.01715	0.01508
	4	1	0.465	0.00774	0.01903	-0.00995	-0.01375	0.01325	0.01513	0.01384	0.01140
	5	1	0.465	0.00746	0.01087	-0.00176	-0.01021	0.01114	0.01186	0.01266	0.00991
	6	1	0.465	0.01496	0.01580	0.00483	-0.00030	0.01080	0.01093	0.01242	0.00942
	7	1	0.465	0.00407	0.00227	-0.00297	-0.00948	0.00797	0.00789	0.00970	0.00718
	8	1	0.465	0.00551	0.00304	-0.00026	-0.00590	0.00733	0.00719	0.00864	0.00668
	9	1	0.465	0.00643	0.00345	0.00724	-0.00367	0.00658	0.00659	0.00985	0.00618
	10	1	0.465	0.00567	0.00225	0.00387	-0.00380	0.00591	0.00585	0.00774	0.00552
	3	2	0.554	0.01888	0.05230	-0.01094	-0.01814	0.03685	0.04470	0.03921	0.03211
	4	2	0.554	0.01652	0.03120	-0.00649	-0.01052	0.02857	0.03175	0.03076	0.02577
	5	2	0.554	0.01599	0.02001	0.00765	-0.00630	0.02469	0.02582	0.03147	0.02282
	6	2	0.554	0.02268	0.02414	0.01227	0.00364	0.02201	0.02223	0.02656	0.01994
	7	2	0.554	0.00989	0.00748	0.00406	-0.00762	0.01748	0.01738	0.02380	0.01622
	8	2	0.554	0.00787	0.00461	0.00231	-0.00683	0.01619	0.01595	0.02025	0.01529
	9	2	0.554	0.01337	0.00943	0.01722	0.00053	0.01505	0.01506	0.02269	0.01439
	10	2	0.554	0.00862	0.00435	0.00732	-0.00313	0.01325	0.01321	0.01806	0.01273
	3	3	0.568	0.02937	0.06443	0.00139	-0.00872	0.04618	0.05588	0.05107	0.03944
	4	3	0.568	0.02472	0.04023	0.00257	-0.00276	0.03523	0.03919	0.03908	0.03160
	5	3	0.568	0.02253	0.02673	0.01698	-0.00016	0.03003	0.03135	0.04079	0.02765
	6	3	0.568	0.02790	0.02953	0.01925	0.00859	0.02575	0.02605	0.03227	0.02334
	7	3	0.568	0.01405	0.01166	0.01070	-0.00372	0.02049	0.02048	0.03033	0.01906
	8	3	0.568	0.01093	0.00760	0.00677	-0.00407	0.01889	0.01858	0.02466	0.01782
	9	3	0.568	0.01719	0.01321	0.02351	0.00419	0.01766	0.01768	0.02810	0.01689
	10	3	0.568	0.01116	0.00686	0.01090	-0.00072	0.01543	0.01539	0.02155	0.01484

Table 2: Results of the Simulation Study of Bias and MSE for Estimates of Reliability ($\beta = 5$)

(s,k)	(m,n)	λ	True $R_{s,k}$	Bias				MSE			
				mle	tmmle	mom	blue	mle	tmmle	mom	Blue
(1,3)	3	1	0.679	-0.03729	0.06399	-0.06043	-0.06222	0.02985	0.03675	0.03203	0.03191
	4	1	0.679	-0.03075	0.02073	-0.04660	-0.04923	0.02390	0.02568	0.02569	0.02496
	5	1	0.679	-0.02406	0.00506	-0.03301	-0.03955	0.01921	0.01971	0.02086	0.02001
	6	1	0.679	-0.01085	0.00913	-0.02158	-0.02339	0.01754	0.01785	0.01932	0.01779
	7	1	0.679	-0.02090	-0.00839	-0.02920	-0.03285	0.01466	0.01456	0.01621	0.01514
	8	1	0.679	-0.01616	-0.00730	-0.02300	-0.02600	0.01363	0.01366	0.01485	0.01395
	9	1	0.679	-0.01294	-0.00683	-0.01461	-0.02135	0.01156	0.01171	0.01344	0.01193
	10	1	0.679	-0.01121	-0.00717	-0.01504	-0.01961	0.01059	0.01079	0.01196	0.01092
	3	2	0.822	-0.03092	0.06004	-0.05206	-0.05484	0.04099	0.04299	0.04500	0.04393
	4	2	0.822	-0.02435	0.02197	-0.04043	-0.04170	0.03363	0.03312	0.03631	0.03562
	5	2	0.822	-0.01856	0.00700	-0.02564	-0.03267	0.02773	0.02707	0.03037	0.02914
	6	2	0.822	-0.00686	0.01148	-0.01558	-0.01789	0.02297	0.02252	0.02559	0.02351
	7	2	0.822	-0.01545	-0.00441	-0.02227	-0.02664	0.02033	0.01986	0.02329	0.02109
	8	2	0.822	-0.01658	-0.00858	-0.02290	-0.02588	0.01908	0.01865	0.02141	0.01968
	9	2	0.822	-0.00729	-0.00207	-0.00740	-0.01488	0.01636	0.01631	0.01878	0.01685
	10	2	0.822	-0.01074	-0.00720	-0.01480	-0.01824	0.01490	0.01506	0.01698	0.01551
	3	3	0.827	-0.02646	0.06449	-0.04678	-0.05027	0.04365	0.04696	0.04789	0.04601
	4	3	0.827	-0.02153	0.02471	-0.03736	-0.03874	0.03512	0.03497	0.03793	0.03697
	5	3	0.827	-0.01658	0.00880	-0.02310	-0.03056	0.02880	0.02822	0.03174	0.03015
	6	3	0.827	-0.00532	0.01285	-0.01374	-0.01623	0.02369	0.02327	0.02646	0.02422
	7	3	0.827	-0.01419	-0.00325	-0.02043	-0.02525	0.02091	0.02048	0.02437	0.02170
	8	3	0.827	-0.01565	-0.00772	-0.02176	-0.02487	0.01962	0.01919	0.02207	0.02021
	9	3	0.827	-0.00624	-0.00106	-0.00600	-0.01373	0.01687	0.01682	0.01944	0.01736
	10	3	0.827	-0.01006	-0.00654	-0.01399	-0.01747	0.01534	0.01551	0.01750	0.01595
(2,4)	3	1	0.624	-0.02406	0.07671	-0.04594	-0.04758	0.02799	0.03952	0.02919	0.02861
	4	1	0.624	-0.01996	0.03096	-0.03513	-0.03760	0.02258	0.02645	0.02366	0.02277
	5	1	0.624	-0.01534	0.01322	-0.02336	-0.03019	0.01860	0.02014	0.02009	0.01886
	6	1	0.624	-0.00333	0.01642	-0.01307	-0.01554	0.01730	0.01826	0.01880	0.01712
	7	1	0.624	-0.01405	-0.00184	-0.02156	-0.02572	0.01423	0.01448	0.01565	0.01433
	8	1	0.624	-0.01079	-0.00209	-0.01706	-0.02041	0.01334	0.01357	0.01445	0.01339
	9	1	0.624	-0.00748	-0.00146	-0.00825	-0.01565	0.01159	0.01190	0.01356	0.01176
	10	1	0.624	-0.00697	-0.00292	-0.01031	-0.01510	0.01060	0.01089	0.01193	0.01075
	3	2	0.741	-0.01617	0.07984	-0.03741	-0.04005	0.04266	0.04986	0.04585	0.04388
	4	2	0.741	-0.01203	0.03707	-0.02841	-0.02969	0.03600	0.03819	0.03784	0.03698
	5	2	0.741	-0.00850	0.01859	-0.01538	-0.02306	0.03028	0.03095	0.03309	0.03110
	6	2	0.741	0.00213	0.02169	-0.00628	-0.00952	0.02589	0.02629	0.02857	0.02596
	7	2	0.741	-0.00825	0.00347	-0.01468	-0.02003	0.02249	0.02241	0.02561	0.02280
	8	2	0.741	-0.01011	-0.00163	-0.01604	-0.01994	0.02117	0.02101	0.02369	0.02145
	9	2	0.741	-0.00116	0.00446	-0.00042	-0.00920	0.01888	0.01902	0.02183	0.01916
	10	2	0.741	-0.00560	-0.00168	-0.00924	-0.01348	0.01700	0.01728	0.01933	0.01741
	3	3	0.746	-0.01288	0.08277	-0.03360	-0.03666	0.04494	0.05284	0.04826	0.04571
	4	3	0.746	-0.00996	0.03892	-0.02618	-0.02748	0.03737	0.03969	0.03932	0.03830
	5	3	0.746	-0.00707	0.01984	-0.01365	-0.02152	0.03127	0.03195	0.03425	0.03207
	6	3	0.746	0.00331	0.02272	-0.00492	-0.00826	0.02660	0.02701	0.02941	0.02668
	7	3	0.746	-0.00730	0.00434	-0.01335	-0.01898	0.02308	0.02301	0.02655	0.02340
	8	3	0.746	-0.00942	-0.00099	-0.01522	-0.01920	0.02171	0.02154	0.02434	0.02199
	9	3	0.746	-0.00033	0.00525	0.00062	-0.00830	0.01940	0.01954	0.02247	0.01968
	10	3	0.746	-0.00508	-0.00118	-0.00866	-0.01290	0.01746	0.01773	0.01984	0.01787

4. Conclusions

We conclude that in order to estimate the multicomponent stress- strength reliability the TMML method of estimation is very close to exact MLE method with respect to the least value for bias than to the other methods of estimation like BLUE and Method of Moments and BLUE method of estimation shows least MSE than to exact MLE, TMMLE and Method of Moments. Hence the suggested TMML methods of estimation with respect to bias and BLUE with respect to MSE are preferable than the exact MLE method and Method of Moments show poor performance with respect to bias and MSE as compared with exact MLE, TMMLE and BLUE.

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