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ESTIMATION OF RESIDUAL STRESSES IN METAL SURFACE LAYERS AFTER THE ROLLER BURNISHING PROCESS

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The roller burnishing process is reduced to the classical problem of rolling of a rigid cylinder on a deformable half-space. Considerations are focussed on the case of high loads when a plastic wave appears ahead of the roller. The half-space is assumed to be rigid-perfectly plastic during loading and elastic during unloading. It enables one to estimate residual stresses in the rolled material. The estimations take a form of simple analytical expressions. The results are compared with the predictions made by Merwin and Johnson (1963), Pomeroy and Johnson (1969).

Key words: residual stress, surface layer, burnishing

1. Introduction

The roller burnishing process is one of the most popular metal surface technologies, which assure high durability and reliability of machine elements. From the mechanical point of view, the process may be considered as a steady rolling of rigid cylinder on a deformable half-space. The length of the cylinder is assumed to be large compared with its radius R, so that the deformation can be assumed to be plane strain. During the process, the arc of contact between the cylinder and the half-space is small, and one can assume that no slipping of the roller appears. The motion of the cylinder is due to a vertical load W and a horizontal driving force F only. The forces are applied to the cylinder axis and they are referred to its unit length. When the maximum contact pressure reaches the value of $3.1k$ (where k is the yield shear stress), the plastic flow appears below the rolled surface (cf Johnson (1985)).

A complete but approximate numerical analysis of the elastic-plastic behaviour of metal surface layers under repeated rolling was made by Merwin and

Johnson (1963). They have used the Reuss stress-strain relations assuming that strains in elastic-plastic zone are equal to the elastic ones. The above assumption is reasonable when the maximum contact pressure does not greatly exceed the shakedown limit that is equal to $4k$ (cf Johnson (1962)). The calculated residual stresses are in qualitative agreement wiyh the measurement results obtained by Pomeroy and Johnson (1969).

During the rolling burnishing process the maximum contact pressure very often exceeds the value of $4k$. Sometimes, it leads to checking and peeling of rolled surface layers. To control the process, estimation of residual stresses in the material after rolling with high loads is necessary. In the present paper, a certain simplified approach, which leads to analytical expressions for residual stress components, is proposed. It is assumed that the deformed material is rigid-perfectly plastic during loading and elastic during unloading. This strong assumption involves a certain overestimating of the results. Due to the lack of elastic response of the material during loading, calculated residual stresses are higher than the real ones. Nevertheless, the proposed approach enables one to draw interesting and important conclusions for the engineering practice.

A simplified solution for the rolling of rigid cylinder on rigid-perfectly plastic half-space was given by Collins (1972). The case of cylinder loaded by forces W and F, as well as, a torque Q applied to the cylinder axis was considered. The arc of contact was approximated by the straight-line chord. It enabled the Author to find a simple slip-line field composed of two triangles and a centred fan (Fig.1). During a steady motion of the cylinder, the above slip-line field has determined a plastically deformed layer of the thickness h. The exact solutions of the problem given later by Collins (1978) and Petryk (1983) are not so simple and they are not used in the present analysis.

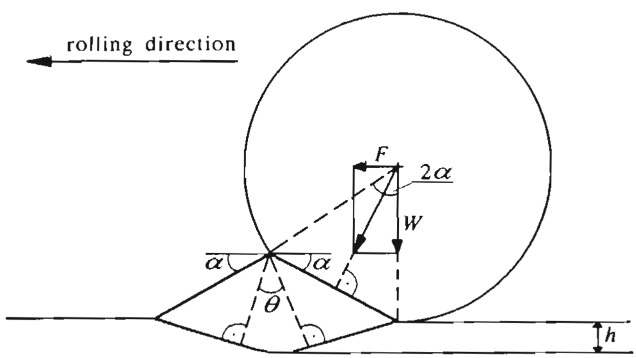

Fig. 1. Slip-line field in Collins' solution for force-driven frictionless rolling

According to Collins' solution for the torque Q equal zero, the force F may be expressed by: the force W, the yield shear stress k and the radius R

$$F = \frac{W^2}{2(2+\pi)kR} \tag{1.1}$$

Denote by α the angle of slope of the contact chord to the contact surface. Because lack of friction forces on that surface, the following relation holds

$$F = W \tan\alpha \tag{1.2}$$

For a given roller radius R, the problem is described by four variables: W, F, α and k. Only two of them are independent. From Eqs (1.1) and (1.2) yields, that α and k are uniquely determined by the driving force F. Then, our problem may be described by W and K, or equivalently, by α and k. The above fact will enable us to estimate the current values of residual stresses in the material, loaded by a fixed vertical force W, by measuring the changes of driving force F during the rolling process.

2. Stress state during loading

Let us consider the slip-line field $\{\alpha, \beta\}$ proposed by Collins for the case of the force-driven cylinder (Fig.2).

Assume that the situation shown in Fig.2 takes place at the time t. At the moment $t + \Delta t$, the material particles lying on the line BDC will be unloaded. Then, we should look for the stress distribution along this line. In the triangle ABD, the stress field is uniform (cf Hill (1950))

$$\begin{aligned} \sigma_{xx}^p &= \sigma - k\sin 2\psi \\ \sigma_{yy}^p &= \sigma + k\sin 2\psi \\ \sigma_{xy}^p &= k\cos 2\psi \end{aligned} \tag{2.1}$$

where

$$\psi = \alpha - \frac{\pi}{4} \qquad \sigma = -k(1+2\theta) \qquad \theta = \frac{\pi}{2} - 2\alpha \tag{2.2}$$

In the centred fan ACD, the stress field depends on the angle χ

$$\begin{aligned} \sigma_{xx}^p &= -k(1+\theta+2\chi) + k\sin 2\chi \\ \sigma_{yy}^p &= -k(1+\theta+2\chi) - k\sin 2\chi \\ \sigma_{xy}^p &= k\cos 2\chi \end{aligned} \tag{2.3}$$

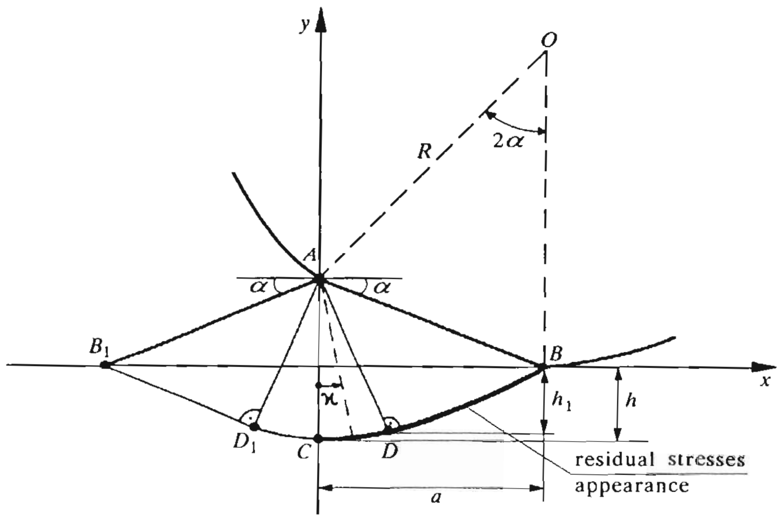

Fig. 2. Analysis of plastic zone assumed in Collins' solution

The angle *chi* may be expressed by a depth of plastically deformed layer determined by the co-ordinate y. Using the equations of arc CD

$$
\begin{aligned}
x &= \sqrt{2}R\sin\alpha\sin\chi \\
y &= -\sqrt{2}R\sin\alpha\cos\chi + 2R\sin^2\alpha
\end{aligned}
\tag{2.4}
$$

one can find, that

$$
\chi = \arccos\frac{2\sin^2\alpha - \frac{y}{R}}{\sqrt{2}\sin\alpha}
\tag{2.5}
$$

The depth $|y|$ for particles lying on the arc CD changes from the value $h_1 = R\sin\alpha(\cos\alpha - \sin\alpha)$ to the value $h = 2R\sin\alpha(\sqrt{2}/2 - \sin\alpha)$, see Fig.2.

To complete the stress field analysis, the transverse stress component σ^p_{zz} should be given. Under the plane strain conditions it is

$$
\sigma^p_{zz} = \frac{1}{2}(\sigma^p_{xx} + \sigma^p_{yy})
\tag{2.6}
$$

As it was mentioned earlier, the rolling process is determined by two parameters: W and F, or equivalently, by α and k. For prescribed α and k, Eqs (2.1) ÷ (2.6) represent stresses on the line BDC as functions of a depth $|y|$ in the plastically deformed layer.

3. Equivalent formulation of the loading process

The slip-line field shown in Fig.2 describes a stationary process of force-driven frictionless rolling. On the other hand, the same slip-line field may describe an initial plastic yield process in which a rigid cylinder is applied to a rigid-perfectly plastic half-space. In the last case, the residual stresses on the line BDC may be found as a sum of the stresses given by Eqs (2.1) $\div$ (2.6) and those for the elastic unloading. The question arises: is it possible to use the same procedure for the considered rolling process? The slip-line field running away from the line BDC will interact with the residual stresses formed on this line. Yes, but this interaction has elastic character only. When the slip-line field is going away, its influence on the residual stresses on the line BDC wanes, and for the plastic zone far enough it may be neglected. Then, to find the residual stresses due to the rolling process, the solution (2.1) $\div$ (2.6) may be added to that for the elastic unloading.

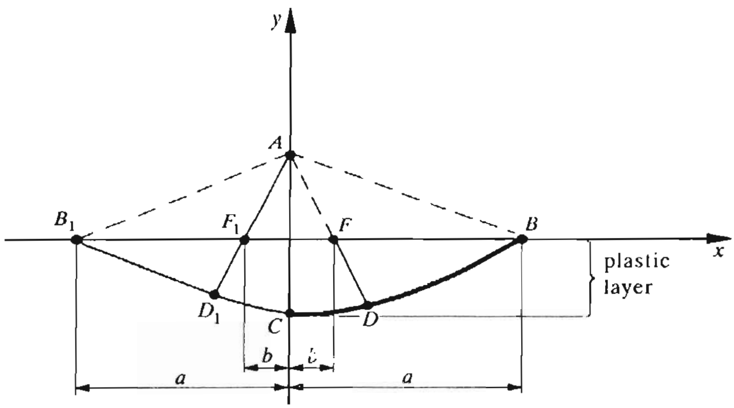

Fig. 3. Levelled plastic zone

Consider the unloading process. One can unload the half-space simply by removing the rolling cylinder at the time t. Then, the residual stresses in the wave formed in front of the cylinder will interact with those generated on the line BDC. In reality, after passing off the roller, the plastically formed wave will disappear and the unloaded surface will remain flat. To avoid this undesired effect, using the solution (2.1) $\div$ (2.6), an equivalent description of the loading process will be proposed.

In the previous formulation of the problem, the load is applied to the region AB that is a part of plastic wave formed ahead of the cylinder. The stress field will not change if we remove the plastic wave ABB_1 and apply the equivalent normal and tangential loads: $p(x) = \sigma^p_{yy}(x, y \equiv 0)$ and $t(x) = \sigma^p_{xy}(x, y \equiv 0)$, to the flat region BB_1 (Fig.3).

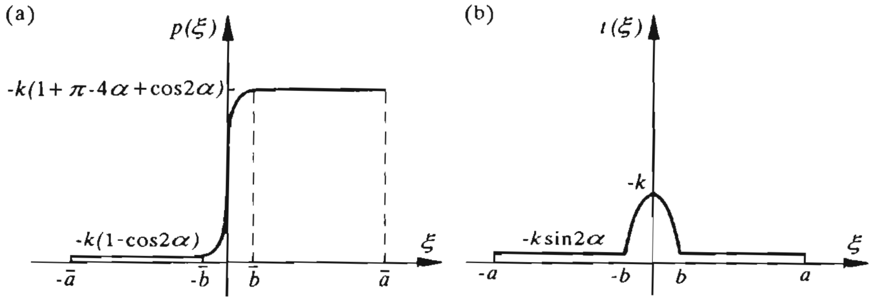

Fig. 4. Loading of the levelled plastic zone; (a) – normal, (b) – tangential

Let us introduce the following co-ordinates

$$\xi = \frac{x}{R} \qquad \eta = \frac{y}{R} \tag{3.1}$$

For the points B and F, these co-ordinates will be equal to $\bar{a} \equiv \sin 2\alpha$ and $\bar{b} \equiv (1 - \cos 2\alpha)\tan(\pi/4 - \alpha)$, respectively. The sought formulae for $p(\xi)$ and $t(\xi)$ may be derived from Eqs (2.1) ÷ (2.5). They take the following forms (Fig.4a,b)

$$p(\xi) = \begin{cases} -k(1 - \cos 2\alpha) & \text{for} \quad -\bar{a} \le \xi \le -\bar{b} \\ -k(1 + \pi/2 - 2\alpha + 2\chi) & \text{for} \quad |\xi| \le \bar{b} \\ -k(1 + \pi - 4\alpha + \cos 2\alpha) \equiv p_1 & \text{for} \quad \bar{b} \le \xi \le \bar{a} \end{cases} \tag{3.2}$$

$$t(\xi) = \begin{cases} -k \sin 2\alpha & \text{for} \quad -\bar{a} \le \xi \le -\bar{b} \\ -k \cos \chi) & \text{for} \quad |\xi| \le \bar{b} \\ -k \sin 2\alpha \equiv t_1 & \text{for} \quad \bar{b} \le \xi \le \bar{a} \end{cases} \tag{3.3}$$

It remains only to consider the problem of half-space unloading with a flat surface loaded by the normal and tangential loads (3.2) and (3.3).

4. Stress state during unloading

Now, let us consider the elastic half-space loaded by normal and tangential forces $p(\xi)$ and $t(\xi)$, according to Eqs (3.2) and (3.3). We will look for an elastic stress field along the line BDC. To simplify the problem, the normal load will be assumed to be uniformly distributed over the interval $\{0, \bar{a}\}$ with the amplitude p_1 (see Eq (3.3) and Fig.4a). Outside the interval the normal load is equal zero. The tangential load is taken as a sum of constant load with *the amplitude* t_1 *(see the rule (11)* and Fig.4b) and the concentrated force

$$T = -2Rk\Big[\bar{a}(\bar{a} - \bar{b}) + \bar{b}\Big(1 - \frac{2}{3}\bar{b}^2\Big)\Big] \tag{4.1}$$

acting at the point $\{0, 0\}$.

Using a standard procedure based on the Green function (cf Johnson (1985)), one can calculate the elastic stresses on the line CDB (see Fig.3)

$$\begin{aligned}\sigma^p_{xx} &= \frac{p_1}{\pi}\Big[\frac{(\xi - \bar{a})\eta}{(\xi - \bar{a})^2 + \eta^2} - \frac{\xi\eta}{\xi^2 + \eta^2} + \arctan\frac{\xi}{\eta} - \arctan\frac{\xi - \bar{a}}{\eta}\Big] + \\ &+ \frac{2t_1}{\pi}\Big[\frac{(\xi + \bar{a})\eta^2}{(\xi + \bar{a})^2 + \eta^2} - \frac{(\xi - \bar{a})\eta}{(\xi - \bar{a})^2 + \eta^2} + \ln\frac{(\xi + \bar{a})^2 + \eta^2}{(\xi - \bar{a})^2 + \eta^2}\Big] + \frac{4t_2}{\pi}\frac{\xi^3}{(\xi^2 + \eta^2)^2}\end{aligned} \tag{4.2}$$

$$\begin{aligned}\sigma^p_{yy} &= \frac{p_1}{\pi}\Big[\frac{\xi\eta}{\xi^2 + \eta^2} - \frac{(\xi - \bar{a})\eta}{(\xi - \bar{a})^2 + \eta^2} + \arctan\frac{\xi}{\eta} - \arctan\frac{\xi - \bar{a}}{\eta}\Big] + \\ &+ \frac{2t_1}{\pi}\Big[\frac{\eta^2}{(\xi - \bar{a})^2 + \eta^2} - \frac{\eta^2}{(\xi + \bar{a})^2 + \eta^2}\Big] + \frac{4t_2}{\pi}\frac{\xi\eta^2}{(\xi^2 + \eta^2)^2}\end{aligned} \tag{4.3}$$

$$\begin{aligned}\sigma^p_{xy} &= \frac{p_m}{\pi}\Big[\frac{\eta^2}{(\xi - \bar{a})^2 + \eta^2} - \frac{\eta^2}{\xi^2 + \eta^2}\Big] + \frac{4t_1}{\pi}\frac{\xi^3}{(\xi^2 + \eta^2)^2} + \\ &+ \frac{2t_m}{\pi}\Big[\frac{(\xi - \bar{a})\eta}{(\xi - \bar{a})^2 + \eta^2} - \frac{(\xi + \bar{a})\eta}{(\xi + \bar{a})^2 + \eta^2} + \arctan\frac{\xi + \bar{a}}{\eta} - \arctan\frac{\xi - \bar{a}}{\eta}\Big]\end{aligned} \tag{4.4}$$

where $t_2 \equiv T/2R$, and the co-ordinate ξ is introduced by the relation

$$\xi = \begin{cases} \sqrt{2\sin^2\alpha - (\eta - \sin^2\alpha)} & \text{for} \quad 0 \le \xi \le \bar{b} \\ \cot(\pi/4 - \alpha) + \sin 2\alpha & \text{for} \quad \bar{b} \le \xi \le \bar{a} \end{cases} \tag{4.5}$$

The transversal stress component will be calculated from the following relation, which is valid under the plane strain conditions

$$\sigma^e_{zz} = \nu(\sigma^e_{xx} + \sigma^e_{yy}) \tag{4.6}$$

where ν is the Poisson ratio.

5. Residual stresses

Analysis of equilibrium equations for plane strain leads to the conclusion (cf Johnson (1985)), that the only possible residual stress components are: longitudinal σ^r_{xx} and transversal σ^r_{zz}. A steady state of plastic yield during the rolling process assures that the residual stresses are independent of the co-ordinate x. Taking the above into account, the formulae for residual stress calculations are

$$\begin{aligned} \sigma^r_{xx} &= \sigma^p_{xx} - \sigma^e_{xx} \\ \sigma^r_{zz} &= \sigma^p_{zz} - \sigma^e_{zz} \end{aligned} \tag{5.1}$$

for σ^p_{xx}, σ^p_{zz} given by Eqs (2.1) $\div$ (2.6), and σ^e_{xx}, σ^e_{zz} by Eqs (4.2) $\div$ (4.6). Recall, that all the quantities are calculated along the line BDC.

As it was mentioned, the analysis of residual stresses made by Merwin and Johnson (1963), and next by Pomeroy and Johnson (1969), was limited to the cases when the maximum contact pressure p_0 did not greatly exceed the value $4k$. According to the Hertz theory (cf Johnson (1985))

$$p_0 = \frac{4}{\pi} p_m \tag{5.2}$$

where p_m is the mean pressure acting on the rolled surface. In our case, $p_m = W/a$, where $a = R\sin 2\alpha$ is the width of contact zone. From Eqs (1.1) $\div$ (1.2), it follows

$$p_m = \frac{2+\pi}{\cos^2\alpha} k = [5.1 + 5.1\alpha^2 + O(\alpha^3)]k \tag{5.3}$$

The present analysis is valid for a positive angle the contact chord makes with the contact surface, i.e. for $\alpha \geq 0$. When α is close to zero, an amplitude of plastic wave appears and increases proportionally to α. It takes place for

$$p_0 \geq \frac{4}{\pi}(2+\pi)k \approx 6.5k \tag{5.4}$$

On the other hand, the angle α cannot be too large, and when values of α increase, increments of p_0 are negligibly small (see Eq (5.3)). Then, one can assume that Eq (5.1) hold for values of p_0 close to $6.5k$.

Results of calculations of σ^r_{xx}/p_0 and σ^r_{zz}/p_0 as functions of y/a, for the values α increasing from 0.02 to 0.2 with the step 0.02, are shown in the Fig.5a,b. The lowest plot is for $\alpha = 0.02$, while the highest for $\alpha = 0.2$. Each

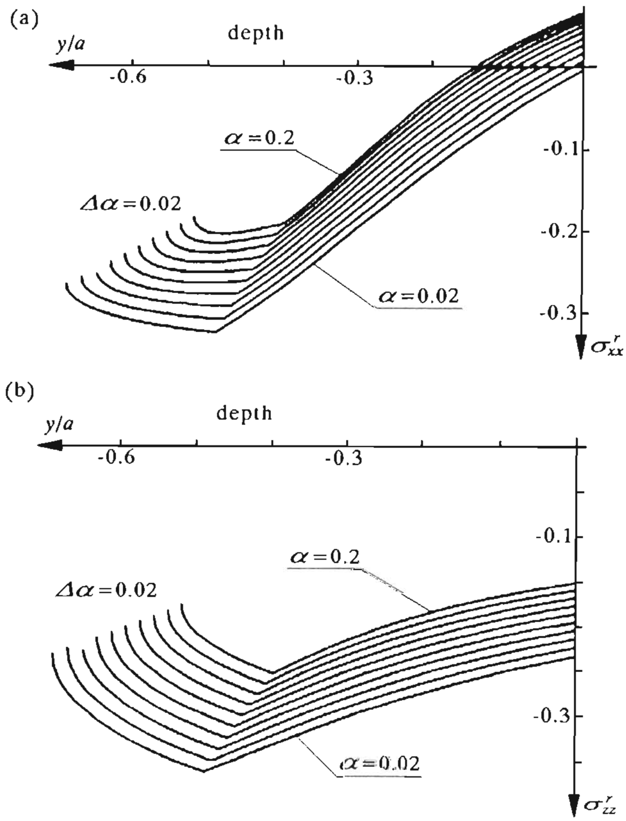

Fig. 5. Residual stresses induced by rolling with high loads: (a) – longitudinal stress σ^r_{xx}, (b) – transversel stress σ^r_{zz}

of them is cut at the depth that is equal to the thickness of corresponding plastic layer. Notice a tendency to arising of tensioned residual stresses in the direct contact zone, for an increasing amplitude of plastic wave.

The obtained results give only an estimation of the residual stress field. Recall, that the rigid-plastic model has been used during the loading, while the elastic one – during the unloading process. For that reason the stress components σ^r_{yy} and σ^r_{xy} do not vanish identically. Their maximum absolute value does not exceed 20% of the value $\max|\sigma^r_{xx}|$. The above may be a measure of accuracy of the estimation.

6. Discussion

Up to now, the residual stresses arising at this mode of deformation have not been studied and there is lack of comparable results in the literature. For lower loads ($p_0 = 4.8k$), such calculations and measurements have been made by Pomeroy and Johnson (1969). The results are shown in Fig.6, where a denotes the width of contact region.

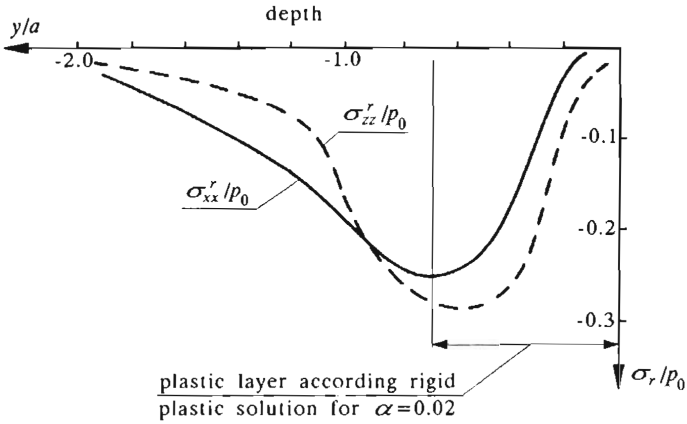

Fig. 6. Residual stresses after rolling with $p_0 = 4.8k$ according Pomeroy and Johnson (1969)

According Pomeroy and Johnson (1969), the maximum residual stresses are reached at the depth

$$|y| = \frac{1}{2}a \tag{6.1}$$

In our case $a = R\sin 2\alpha$, and maximum residual stresses appear at the depth (see Fig.2)

$$|y| = h_1 = \frac{1}{2}a + \tan 2\alpha \tag{6.2}$$

Introducing Eq (6.2) into Eq (5.1) and taking linear terms with respect to α, the following estimation of maximum residual stresses is obtained

$$\begin{aligned} \max\frac{|\sigma^r_{xx}|}{k} &= 2.2 - 5.2\alpha + O(\alpha^2) \\ \max\frac{|\sigma^r_{zz}|}{k} &= 2.4 - 4.2\alpha + O(\alpha^2) \end{aligned} \tag{6.3}$$

or taking into account Eqs (5.2) and (5.3)

$$\max \frac{|\sigma^r_{xx}|}{p_0} = 0.34 - 0.80\alpha + O(\alpha^2)$$
$$\max \frac{|\sigma^r_{zz}|}{p_0} = 0.37 - 0.63\alpha + O(\alpha^2) \tag{6.4}$$

Because the elastic response of the material was neglected in the analysis of loading process, the above values are higher than those obtained from elasto-plastic analysis (see Fig.6).

The most interesting is a material behaviour under a repeated loading. Let us consider a cyclic rolling with a fixed vertical force W. Up to now, we have assumed the rolled material to be perfectly plastic. A real material undergoes hardening during the rolling process. Denote by $k^{(i)}$ the yield shear stress after the ith pass of the roller. Then, the quantities

$$F^{(i)} = \frac{W^2}{2(2+\pi)k^{(i)}R} \tag{6.5}$$

and

$$\alpha^{(i)} = \arctan\frac{F^{(i)}}{W} \approx \frac{F^{(i)}}{W} \tag{6.6}$$

decrease during the process. A thickness of plastically deformed layer (see Fig.2a) decreases too, following the rule

$$h^{(i)} = 2R\sin\alpha^{(i)}\Big(\frac{\sqrt{2}}{2} - \sin\alpha^{(i)}\Big) \approx \sqrt{2}R\frac{F^{(i)}}{W} \tag{6.7}$$

Then, the horizontal driving force F and the amplitude of plastic wave undergo a reduction after each pass of the roller. On the other hand, from Eqs (6.3), (6.5) and (6.6) it yields that the maximum residual stresses increase as the current yield shear stress increases

$$\max|\sigma^r_{xx}{}^{(i)}| \approx 2.2k^{(i)} - 5.2\frac{W}{2(2+\pi)R}$$
$$\max|\sigma^r_{zz}{}^{(i)}| \approx 2.4k^{(i)} - 4.2\frac{W}{2(2+\pi)R} \tag{6.8}$$

For materials with a saturated hardening, the quantities: $F^{(i)}$, $\alpha^{(i)}$, $h^{(i)}$ and the maximum residual stresses are tending to fixed values. For more hardened materials, there is no limit on $k^{(i)}$ and after many cycles of rolling damage of the material may appear. In practice, it is necessary to avoid high vertical loads or to decrease a number of rolling cycles.

7. Conclusions

Residual stresses induced in a material by the roller burnishing process have been studied. The considerations were limited to the case of high loads, when a plastic wave ahead of the contact region has appeared. As a result, analytical expressions for estimation of residual stresses induced in the process were proposed. Their analysis suggests, that during the burnishing with a fixed vertical load the following quantities decrease:

- horizontal driving force
- plastic wave amplitude
- thickness of plastically deformed layer.

However, the maximum residual stresses increase at the same time. Moreover, for large plastic wave amplitudes, tensile residual stresses appear in the direct contact zone. As a consequence, checking and peeling of material during the burnishing process may occur (cf Przybylski (1987)). To avoid these undesirable effects, it is necessary to decrease a number of rolling cycles or to perform the process with lower vertical loads.

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Oszacowanie naprężeń własnych w warstwach wierzchnich metali poddanych procesowi nagniatania

Streszczenie

Proces nagniatania tocznego sprowadzono do zagadnienia toczenia sztywnego walca po odkształcalnej pólpowierzchni. Skoncentrowano się na przypadku znacznych obciążeń, podczas których przed walcem pojawia się fala plastyczna. Przyjęto, że pólprzestrzeń jest sztywno-idealnie plastyczna podczas obciążenia, a sprężysta podczas odciążenia. Umożliwiło to oszacowanie naprężeń własnych w nagniatanym materiale. Oszacowanie podano w postaci prostych wzorów. Wyniki porównano z obliczeniami Merwina i Johnsona (1963) oraz Pomeroya i Johnsona (1969).

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