Estimation of soil suction from the soil-water characteristic curve

Delwyn G. Fredlund, Daichao Sheng, and Jidong Zhao

Abstract: Soil-water characteristic curves (SWCCs) are routinely used for the estimation of unsaturated soil property functions (e.g., permeability functions, water storage functions, shear strength functions, and thermal property functions). This paper examines the possibility of using the SWCC for the estimation of in situ soil suction. The paper focuses on the limitations of estimating soil suctions from the SWCC and also suggests a context under which soil suction estimations should be used. The potential range of estimated suction values is known to be large because of hysteresis between drying and wetting SWCCs. For this, and other reasons, the estimation of in situ suctions from the SWCC has been discouraged. However, a framework is suggested in this paper for estimating the median value for in situ soil suction along with a likely range of soil suction values (i.e., maximum and minimum values). The percentage error in the estimation of soil suction from the SWCC is shown to be lowest for sand soils and highest for clay soils.

Key words: soil suction, matric suction, osmotic suction, soil-water characteristic curve (SWCC), hysteresis, water content.

Résumé : Les courbes de rétention d'eau (CRE) sont très souvent utilisées pour estimer les fonctions de différentes propriétés des sols non saturés (ex : fonction de perméabilité, fonction d'emmagasinage de l'eau, fonctions de résistance au cisaillement et fonctions de propriétés thermiques). Cet article étudie la possibilité d'utiliser la CRE pour estimer la succion d'un sol in situ. L'article se penche sur les limites lors de l'estimation de la succion d'un sol à partir de la CRE et suggère un contexte dans lequel les estimations de la succion devraient être utilisées. L'éventail potentiel de valeurs de succion estimées est large en raison des effets d'hystérésis entre les CRE en mouillage et en drainage. Pour cette raison, et plusieurs autres, l'estimation des succions in situ à partir de la CRE n'a pas été encouragée. Cependant, cet article suggère un cadre pour estimer la valeur médiane de la succion in situ en plus d'offrir une gamme de valeurs de succion possibles (c'est-à-dire des valeurs maximales et minimales). Le pourcentage d'erreur lors d'estimations de la succion d'un sol à partir de la CRE atteint sa plus faible valeur pour des sols sablonneux et est le plus élevé pour des sols argileux.

Mots-clés : succion du sol, succion matricielle, succion osmotique, courbe de rétention d'eau (CRE), hystérésis, teneur en eau.

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Introduction

Soil-water characteristic curves (SWCCs) are used extensively for the estimation of unsaturated soil property functions (Fredlund 1995, 2000). The SWCCs have become pivotal to the implementation of unsaturated soil mechanics into geotechnical engineering practice. Estimation procedures for unsaturated soil property functions have been proposed for virtually every physical process where soils become unsaturated (Fredlund et al. 1997; Fredlund 2006). However, SWCCs have not proven to be a reliable means for estimating in situ soil suctions, and their usage for this purpose has been discouraged (Fredlund et al. 2001; Fredlund 2002).

The use of SWCCs for the estimation of in situ soil suction has been discouraged primarily because of the hysteretic nature associated with the drying (desorption) and wetting (adsorption) SWCCs. Only the drying portion of the SWCCs is generally measured in the laboratory. The asymptotic nature of most empirical equations used to represent the SWCC makes the calculation of suction only possible between the air-entry value and the residual value of a soil (Fredlund 2007). The retrieval of a water content sample from the field does not provide an indication of whether the in situ stress state is on the drying curve, the wetting curve, or somewhere in between these two limiting curves. Soil suction varies on a log scale with water content, and this behavior along with hysteresis makes it difficult to obtain a reliable estimation of in situ soil suction from the SWCC.

The objectives of this paper are to

(1) Illustrate how the natural water content of a soil can be used to provide an indication of the "median" soil suction and the likely range of in situ soil suctions through use of SWCCs.
(2) Illustrate the limitations associated with using the SWCC for the estimation of soil suction.
(3) Propose a procedure for the estimation of the likely in situ soil suction and the likely range of soil suction values. The proposed methodology must be used with proper care and engineering discretion.

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D.G. Fredlund.[1] Golder Associates Ltd., 1721 8th Street East, Saskatoon, SK S7H OT4, Canada.
D. Sheng. The University of Newcastle, School of Engineering, Callaghan, NSW-2308, Australia.
J. Zhao. Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong.

[1] Corresponding author (e-mail: unsaturatedsoil@yahoo.com).

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The assumption is made in this study that the geotechnical engineer has either measured or estimated the *drying* or *desorption curve*, SWCC, for a particular soil deposit. Measured SWCCs are usually obtained by placing an undisturbed soil specimen on the high-air-entry disk of a pressure plate apparatus while the soil water content is allowed to come to equilibrium under several applied soil suctions up to 1500 kPa (ASTM 2008). Estimated drying SWCCs can be obtained either from grain-size distribution curves (Fredlund et al. 2002) or from average SWCCs compiled from a database (Zapata et al. 2000). Estimated drying SWCCs are considered less reliable than measured laboratory results (Fredlund 2007).

The saturated water content of the soil must also be known, since this is the starting point for the drying SWCC. The wetting (or adsorption) SWCC branch can either be measured in the laboratory or estimated based on previously suggested empirical rules (Pham et al. 2002). The scope of this paper is also limited to the consideration of unimodal SWCCs. The assumption is also made that there is no error associated with our knowledge of the drying SWCC.

Nature of the SWCC

Data associated with the SWCC is commonly plotted as (gravimetric) water content versus the logarithm of soil suction. The general shape of desorption SWCCs is shown in Fig. 1. There are two distinct changes in slope along the SWCC. The changes in slope define two points that are pivotal to describing the SWCC. The first point is termed the "air-entry value" of the soil, where the largest voids start to desaturate as suction is increased. The second point is termed "residual conditions", and it defines the point where the removal of water from the soil becomes significantly more difficult (i.e., requires significantly more energy for water removal). The changes in slope subdivide the SWCC into three distinct zones, namely, the "boundary effect zone" in the lower suction range, the "transition zone" between the air-entry value and the residual value, and the "residual zone" at high soil suctions reaching up to 1 000 000 kPa. Likewise, there are similar distinct changes in slope along the wetting SWCC.

The SWCC is not a single-valued, unique relationship but rather is hysteretic in character as shown in Fig. 2. Consequently, it is not possible to determine a single stress state designation for a soil based solely on a water content measurement. In other words, it is not possible to know whether the soil is presently on the drying curve, the wetting curve, or somewhere in between the two bounding curves along what is known as a scanning curve.

Figure 3 shows the hysteresis loops associated with the drying and wetting curves for a silt and sand soil, respectively (Pham et al. 2002). The results of three tests on the same soil show the laboratory reproducibility and reliability of SWCC measurements. The hysteresis between the drying and wetting curves was measured throughout three cycles, namely, an initial drying from a completely saturated state, wetting from a suction well above residual suction, and a further drying curve after the soil had been wetted to near-zero suction.

Fig. 1. Desorption branch with definition of variables for a SWCC.

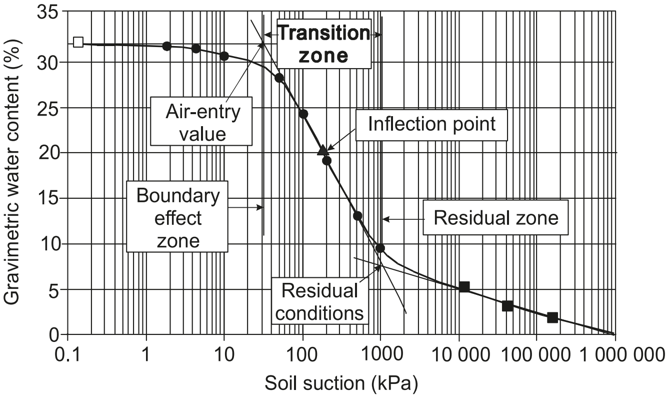
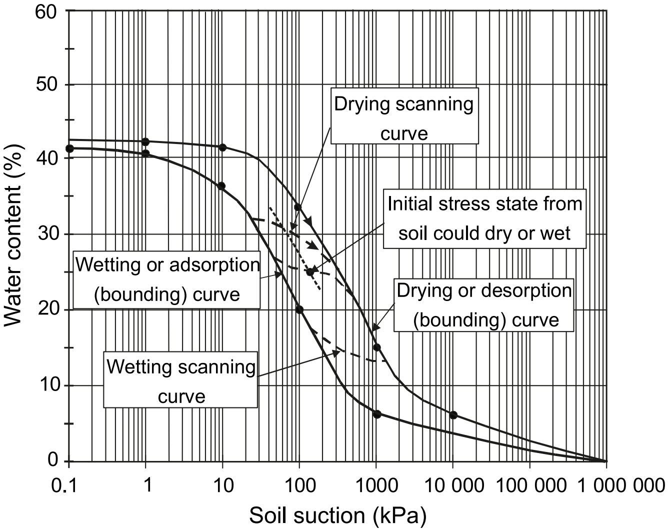


Fig. 2. Illustration of the hysteresis loops comprising the SWCC for a soil (Fredlund 2000).



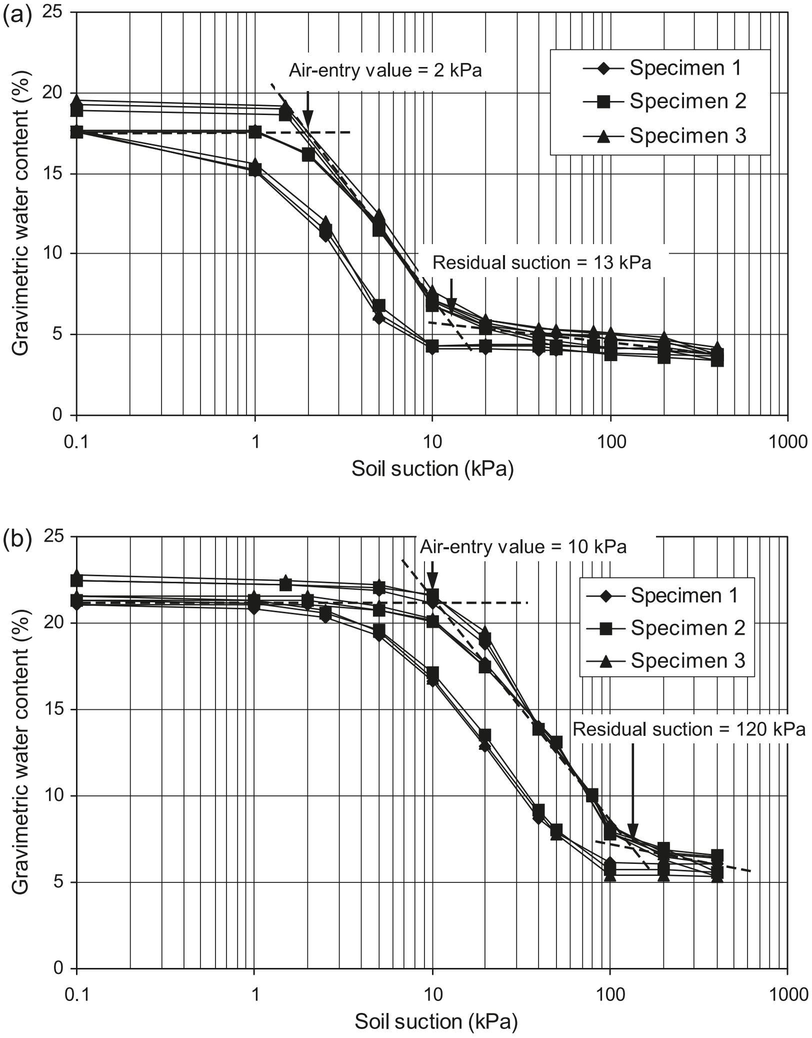
Several observations can be made from the test results. First, the hysteresis loops appear to be reproducible. Second, some air becomes entrapped in the soil after it is wetted from a stress state in excess of residual suction. Third, the slope of the desorption curve is approximately parallel to that of the adsorption curve. Similar behavior has been observed from tests on other soils, provided the soil is dried to a point beyond residual suction. Each of the bounding SWCCs has an inflection point of maximum slope on a logarithmic scale. The inflection point is a reasonable point at which to define the lateral shift between the drying and wetting SWCCs. The above characteristics of SWCCs were taken into consideration when developing a procedure for the estimation of in situ soil suction.

Accommodation of hysteretic of SWCCs

There are a number of assumptions that could be made with respect to the usage of SWCCs for the estimation of in situ suction. It is possible to

(1) Ignore the effect of hysteresis and use only the *desorption* SWCC for the estimation of soil suction (i.e., the upper bounding branch of the SWCCs shown in Fig. 2).

Fig. 3. Drying and wetting SWCCs measured on (*a*) a silt soil; (*b*) a sand soil (Pham 2002).



This would provide an estimate of the *maximum* likely soil suction corresponding to any measured water content.

(2) Measure the *desorption* SWCC and approximate the *adsorption* SWCC by estimating the magnitude of the hysteresis loop at the inflection point on the *drying* SWCC. This would allow for an estimation of the *maximum* and *minimum* soil suction values, with the maximum value likely to be more accurate.

(3) Measure both the *desorption* branch and the *adsorption* branch of the SWCCs. Often this type of laboratory test is considered too costly because of the increased time associated with measuring the adsorption SWCC. However, it is possible to measure both bounding curves of the SWCC, and this would provide the basis for an estimation of the maximum and minimum soil suction values.

(4) Determine a *median* SWCC halfway between the drying and wetting SWCCs (on a logarithmic scale). It is suggested that the water content corresponding to the inflection point on the drying curve be used as the reference point for performing the lateral shift of the SWCC. The median SWCC can then be used to determine a median value for soil suction. The median suction value is not the arithmetic average of the minimum and maximum suction values corresponding to the adsorption and adsorption curves because of the logarithmic scales.

(5) Use a more rigorous mathematical equation that describes desorption, adsorption, and scanning SWCCs (Pham et al. 2003). Such an approach would appear to be of little additional value, since it is not possible to know whether the soil is on one of the scanning curves or on one of the bounding curves (i.e., on the desorption or adsorption curves in Fig. 2). This procedure would require additional information on soils.

Attention is focused in this paper on the use of a measured desorption SWCC, along with the initial saturation water content for the soil under consideration. The adsorption SWCC and median SWCC would generally be estimated (i.e., assumption 2 from the above options).

Some of the common empirical SWCC equations

There are several empirical equations that have been proposed to describe SWCCs (Sillers et al. 2001). Some of the commonly used SWCC equations take the form of a continuous function that is asymptotic at the extremities. It is the zone between the air-entry value and "residual suction" where the curve has sufficient slope for the calculation of soil suction. The same limitation applies when the SWCC equations have been inverted to solve for soil suction as the dependent variable.

Figure 4 lists some of the commonly used SWCC equations along with more recent SWCC equations developed within the geotechnical engineering discipline. The equations are placed in one of three categories, depending upon the range of soil suction that is reasonably well defined by the SWCC equation. Below the air-entry value, some of the proposed SWCC equations asymptotically approach a horizontal line. This is indicating that the soil is going towards a "no water storage" condition. A "zero water storage" condition is an unacceptable condition when modeling transient water flow through a soil. Consequently, the derivative of the SWCC equation that defines the water storage value needs to be intercepted before it becomes too close to zero (or unreasonably small).

A similar condition is encountered at suctions greater than "residual" suction, where the proposed SWCC equation may once again become asymptotic to a horizontal line. Experimental results have repeatedly shown that a reasonably straight-line relationship exists (on a semi-log plot) between residual suction and the case of a completely dry soil (i.e., 1 000 000 kPa and zero water content). This condition has been incorporated into the equation proposed by Fredlund and Xing (1994).

It must be recognized that most proposed SWCC equations have been developed and applied in agriculture-related disciplines. However, the mathematical and physical modeling requirements in geotechnical and geoenvironmental engineering applications are somewhat different than those in agriculture.

Rewriting SWCC equations, with soil suction as the dependent variable

Most of the empirical equations that have been proposed to best fit water content versus soil suction data can be rearranged such that soil suction can be computed if the water content of the soil is known. Let us first consider the drying or desorption SWCC and then consider how best to accommodate the hysteretic nature of SWCCs. Several proposed SWCC equations are rearranged for the solution of soil suction when the water content is known. All equations are written in terms of gravimetric water content; however, each of the rearranged equations would have the same form if written in terms of volumetric content.

The following soil information is required when using a SWCC to compute soil suction. The soil parameters for the SWCC must be known. There is generally an a type parameter that is related to the air-entry value of the soil but may not be the actual air-entry value in all cases. There is an n type parameter that is primarily related to the rate of desaturation of the soil as suction exceeds the air-entry value, and in some cases, a further m type parameter is used to give greater flexibility in best fitting the SWCC data. In addition, the saturated gravimetric water content, w_s, must be known.

Gardner (1958)

The Gardner (1958) equation was originally proposed to describe the coefficient of permeability function for an unsaturated soil. However, the mathematical form proposed for the permeability equation has been inferred to apply for the water content versus soil suction relationship. In this case, the Gardner equation is written as follows:

$$[1] \qquad w(\psi) = \frac{w_s}{1 + a\psi^n}$$

where $w(\psi)$ is the water content at any soil suction, w_s is the saturated water content, and a and n are fitting soil parameters associated with the SWCC. Equation [1] can be rearranged such that soil suction, ψ, is dependent upon water content, w.

$$[2] \qquad \psi = \left[\frac{1}{a}\left(\frac{w_s}{w} - 1\right)\right]^{1/n}$$

If the two fitting parameters, a and n, for the Gardner equation (Gardner 1958) are known along with the saturated water content, the soil suction can be calculated. It should also be noted that Gardner (1958) also proposed a one-parameter equation for the SWCC. The one-parameter equation provides an indication of the rate of desaturation of a soil but assumes that desaturation commences as soon as suction is applied. This form of Gardner's equation has not found wide acceptance in geotechnical engineering because of its basic limitations and is not presented in this paper.

Brooks and Corey (1964)

Brooks and Corey (1964) divided the SWCC into two zones: one zone where the soil suctions are less than the air-entry value, and the other where soil suctions are greater than the air-entry. This gives rise to two equations of the following form:

$$[3] \qquad \begin{cases} w(\psi) = w_s & \psi < \psi_{ae} \\ w(\psi) = w_s\left(\dfrac{\psi}{a}\right)^{-n} & \psi \geq \psi_{ae} \end{cases}$$

where ψ_{ae} is the air-entry suction.

It is not possible to use the proposed SWCC equation prior to the air-entry value to calculate a soil suction value. Once the air-entry value is exceeded, the Brooks and Corey (1964) equation can be rearranged to compute the soil suction corresponding to the measured water content:

$$[4] \qquad \psi = a\left(\frac{w_s}{w}\right)^{1/n}$$

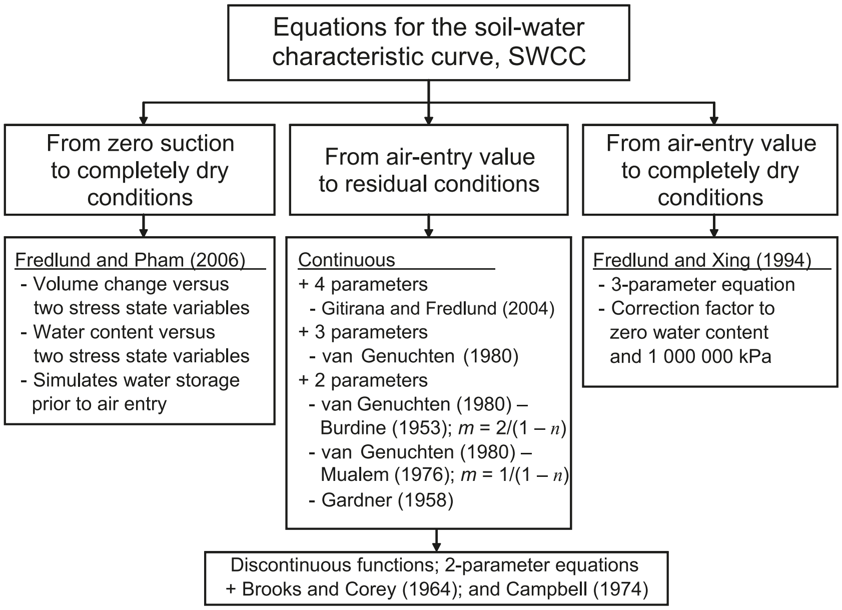
Two fitting parameters, a and n, and the saturated water content, w_s, are required along with the measured water content for the calculation of soil suction.

Brutsaert (1966)

The Brutsaert (1966) equation for the SWCC is similar to the Gardner (1958) equation, with the exception that the a parameter related to the air-entry value is the inverse of that used by Gardner (1958).

$$[5] \qquad w(\psi) = \frac{w_s}{1 + (\psi/a)^n}$$

Fig. 4. Categorization of SWCC equations based upon the range of soil suction conditions that are reasonably well defined by each of the empirical equations. *m*, *n*, fitting soil parameters associated with the SWCC.



The Brutsaert SWCC equation is rearranged to solve for the suction in terms of the water content:

$$[6] \qquad \psi = a\left(\frac{w_{\mathrm{s}}}{w} - 1\right)^{1/n}$$

If the two fitting parameters, *a* and *n*, for the Brutsaert (1966) equation are known along with the saturated water content, w_{s}, the soil suction can be calculated.

van Genuchten (1980)

The van Genuchten (1980) equation is one of the most commonly used SWCC equations. It was in 1980 that van Genuchten showed how his SWCC equation could be used to estimate a permeability function. Consequently, this is the date generally attached to the empirical van Genuchten SWCC equation. It is referred to as a three-parameter equation and takes the following form for representing water content as a function of soil suction:

$$[7] \qquad w(\psi) = \frac{w_{\mathrm{s}}}{[1 + (a\psi)^n]^m}$$

The van Genuchten (1980) equation can be rearranged to solve for soil suction in terms of water content.

$$[8] \qquad \psi = \frac{1}{a}\left[\left(\frac{w_{\mathrm{s}}}{w}\right)^{1/m} - 1\right]^{1/n}$$

If the three fitting parameters, *a*, *m*, and *n*, for the van Genuchten equation (van Genuchten 1980) are known along with the saturated water content, w_{s}, the soil suction can be calculated. The usage of this equation is limited to the range between the air-entry value and the residual suction of a soil because of the asymptotic nature of the equation.

van Genuchten (1980) – Mualem (1976)

In 1976, Mualem suggested that the *n* and *m* soil parameters in the SWCC equation could bear a fixed relationship with $m = (n - 1)/n$. This suggestion reduces the three-parameter equation of van Genuchten (1980) to a two-soil-parameter SWCC equation:

$$[9] \qquad w(\psi) = \frac{w_{\mathrm{s}}}{\left[1 + (a\psi)^n\right]^{(1-1/n)}}$$

The van Genuchten (1980) – Mualem (1976) equation can be rearranged to solve for soil suction in terms of water content:

$$[10] \qquad \psi = \frac{1}{a}\left[\left(\frac{w_{\mathrm{s}}}{w}\right)^{n/(n-1)} - 1\right]^{1/n}$$

The limitations associated with the usage of the van Genuchten (1980) equation also apply to the van Genuchten (1980) – Mualem (1976) equation.

van Genuchten (1980) – Burdine (1953)

In 1953, Burdine suggested that the *n* and *m* soil parameters for an equation representing the SWCC equation could bear a fixed relationship with $m = (n - 2)/n$. This suggestion resulted in a two-soil-parameter equation that is now commonly referred to as the van Genuchten (1980) – Burdine (1953) equation:

$$[11] \qquad w(\psi) = \frac{w_{\mathrm{s}}}{\left[1 + (a\psi)^n\right]^{1-2/n}}$$

The van Genuchten (1980) – Burdine (1953) equation can be rearranged to solve for soil suction in terms of water content:

$$[12] \qquad \psi = \frac{1}{a}\left[\left(\frac{w_s}{w}\right)^{n/(n-2)} - 1\right]^{1/n}$$

The limitations associated with the usage of the van Genuchten (1980) equation also apply to the van Genuchten (1980) – Burdine (1953) equation.

McKee and Bumb (1984) (Boltzmann distribution)

McKee and Bumb (1984) used the Boltzmann mathematical function (Boltzmann 1871) and proposed an exponential type equation for the SWCC:

$$[13] \qquad w(\psi) = w_s \exp\left(\frac{a-\psi}{n}\right)$$

The McKee and Bumb (1984) equation uses the Boltzmann mathematical function and can be rearranged to solve for soil suction in terms of water content:

$$[14] \qquad \psi = a - n\ln\left(\frac{w}{w_s}\right)$$

If the two fitting parameters, a and n, for the McKee and Bumb (1984) equation are known along with the saturated water content, w_s, soil suction can be calculated for any measured water content.

McKee and Bumb (1987) (Fermi distribution)

McKee and Bumb (1987) also suggested using the Fermi mathematical function (Fermi 1926) to normalize the degree of saturation versus soil suction relationship. The Fermi distribution has been extensively used in quantum physics, quantum statistics, and other disciplines. Normalization was applied between maximum saturation and the degree of saturation corresponding to residual conditions. In this study, residual saturation has been assumed to be zero. The resulting SWCC equation is a two-soil-parameter expression, with the a parameter related to the air-entry value and the n parameter related to the rate of desaturation of the soil:

$$[15] \qquad w(\psi) = \frac{w_s}{1 + \exp[(\psi - a)/n]}$$

Solving eq. [15] for soil suction gives,

$$[16] \qquad \psi = a + n\ln\left(\frac{w_s}{w} - 1\right)$$

If the two fitting parameters, a and n, for the McKee and Bumb (1984) [Fermi distribution] equation are known along with the saturated water content, w_s, soil suction can be calculated for any measured water content.

Fredlund and Xing (1994) — without the correction factor

The Fredlund and Xing (1994) equation for the SWCC has a correction factor, $C(\psi)$, that extends the range of suctions beyond residual suction to completely dry conditions:

$$[17] \qquad w(\psi) = C(\psi)\frac{w_s}{\{\ln[e + (\psi/a)^n]\}^m}$$

where $w(\psi)$ is the water content at any soil suction; w_s is the saturated water content; and a, n, and m are fitting soil parameters associated with the SWCC. The variable e is the base of the natural logarithm. The correction factor, $C(\psi)$, is written as follows:

$$[18] \qquad C(\psi) = 1 - \frac{\ln(1 + \psi/\psi_r)}{\ln[1 + (1\,000\,000/\psi_r)]}$$

where ψ is any soil suction value and ψ_r is soil suction at residual conditions. Both have a unit of kPa.

It appears that it is not easy to rearrange the Fredlund and Xing (1994) equation and solve for soil suction when the "correction factor" is kept within the equation. Therefore, the correction factor for suctions greater than residual suction is first set to 1.0 in this study. In this way, the Fredlund and Xing (1994) equation for calculating soil suction in terms of water content can be written as follows:

$$[19] \qquad \psi = a[e^{(w_s/w)^{1/m}} - e]^{1/n}$$

It is also possible to set the m variable to 1.0 for purposes of this study, since the focus is not on the extreme limits of function. The Fredlund and Xing (1994) equation will be used to illustrate how the SWCC equations can be used to estimate soil suctions from a water content measurement.

If the entire Fredlund and Xing (1994) equation, along with the correction factor is used, then it is necessary to use a numerical method to compute soil suction from a water content measurement.

Pereira and Fredlund (2000)

The Pereira and Fredlund (2000) equation is a three-parameter equation, with parameters c, b, and d having meanings similar to the a, n, and m parameters used in other SWCC equations, respectively:

$$[20] \qquad w(\psi) = w_r + \frac{w_s - w_r}{[1 + (\psi/c)^b]^a}$$

Equation [20] can be rearranged to give soil suction in terms of water content:

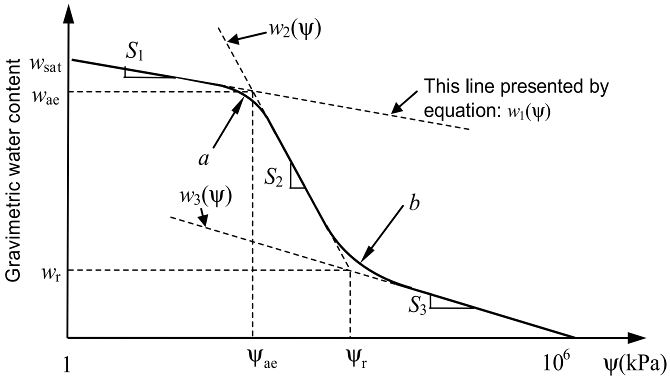
$$[21] \qquad \psi = c\left[\left(\frac{w_s - w_r}{w - w_r}\right)^{1/a} - 1\right]^{1/b}$$

Fredlund and Pham (2006)

Fredlund and Pham (2006) divided the SWCC into three zones, namely (*i*) a low suction portion from a small suction (e.g., 1 kPa) to the air-entry suction, ψ_{ae}, (*ii*) an intermediate portion from the air-entry suction, ψ_{ae}, to the residual suction, ψ_r, and (*iii*) a high suction portion from the residual suction, ψ_r, to 1 000 000 kPa as shown in Fig. 5. The water contents corresponding to the air-entry value and residual suction must be known as well as the saturated water content under low suction conditions (e.g., 1 kPa) to use these equations over the entire suction range.

Each section has a similar form of equation. The three equations cover the entire water content range from completely saturated conditions to completely dry conditions, with each portion having a meaningful slope representing water content versus suction. It is necessary to know the water contents at

Fig. 5. Fredlund and Pham (2006) SWCC equations for three zones of desaturation.



the start and end points where each line segment applies. The slope of the straight line portions of the three zones are defined as S_1, S_2, and S_3, when going from the low suction range of the SWCC curve to the high suction range.

The SWCC equations for the low suction range, intermediate suction range, and high suction range can be written as follows:

$$[22]\quad \begin{cases} w_1(\psi) = w_{\mathrm{u}} - S_1 \log(\psi) & 1 \leq \psi < \psi_{\mathrm{ae}} \\ w_2(\psi) = w_{\mathrm{ae}} - S_2 \log\left(\dfrac{\psi}{\psi_{\mathrm{ae}}}\right) & \psi_{\mathrm{ae}} \leq \psi < \psi_{\mathrm{r}} \\ w_3(\psi) = S_3 \log\left(\dfrac{10^6}{\psi}\right) & \psi_{\mathrm{r}} \leq \psi < 10^6 \text{ kPa} \end{cases}$$

where w_{u} is the water content corresponding to a suction of 1 kPa, and w_{ae} is the water content corresponding to the air-entry value. Each of the above equations can be rearranged to solve for suction in terms of water content. The equations for the low suction zone, the intermediate zone, and the high suction zone of the SWCC are, respectively,

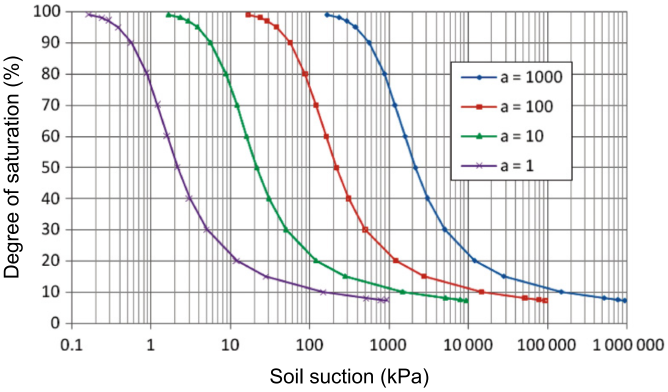
$$[23]\quad \begin{cases} \psi = (10)^{(w_{\mathrm{u}}-w)/S_1} & 1 \leq \psi < \psi_{\mathrm{ae}} \\ \psi = \psi_{\mathrm{ae}}(10)^{(w_{\mathrm{ae}}-w)/S_2} & \psi_{\mathrm{ae}} \leq \psi < \psi_{\mathrm{r}} \\ \psi = (10)^{(6-w/S_3)} & \psi_{\mathrm{r}} \leq \psi < 10^6 \text{ kPa} \end{cases}$$

The three equations proposed by Fredlund and Pham (2006) cover the entire range of suctions, from a small value to 1 000 000 kPa. These equations extend the range of possible suction predictions below the air-entry value of the soil and also above residual conditions (Pham 2005).

Percent error in soil suction, depending on the hysteresis shift in the SWCC

The desorption (or drying) SWCC is generally the first curve determined for a soil (Fredlund et al. 2001; Fredlund 2007). Then, the adsorption (or wetting) curve is usually estimated by assuming an appropriate lateral shift for the bounding SWCC. It is also usually assumed that the drying and wetting curves are congruent or parallel to one another in the inflection point portion of the curves. The mentioned assumptions appear to be commonly applied in geotechnical and geoenvironmental engineering.

Fig. 6. Effect of changing the *a* fitting parameter on the lateral shift of the SWCC.



The *adsorption* and median SWCC is always shifted to the left on a plot of the type shown in Fig. 2. The *a* fitting parameter in the SWCC equations generally controls the lateral shift of the boundary SWCCs.

The Fredlund and Xing (1994) SWCC equation will be used to illustrate the translation of the SWCCs. Figure 6 shows the effect of changing the *a* fitting parameter. The *n* and *m* fitting parameters are kept constant for all curves.

The "percent shift" of the SWCC boundary curves, ξ, is defined on a logarithmic scale. Therefore, a 100% shift corresponds to one log scale of change. This means that the *a* fitting parameter will have to change by one order of magnitude. Similarly, a 50% shift means that the *a* fitting parameter is shifted 1/2 log cycle to the left (i.e., to a lower value). The "percent lateral shift" of the SWCC boundary curves, ξ, can be written as follows:

$$[24]\quad \xi = 100[\log(\psi_{\mathrm{ad}}) - \log(\psi_{\mathrm{aw}})]$$

where ψ_{ad} is the suction at the point of inflection, a_{d}, on the drying curve, and ψ_{aw} is the suction at the point of inflection, a_{w}, on the wetting curve.

The same equation can be used when moving from the drying SWCC to the median SWCC.

$$[25]\quad \xi_{\mathrm{m}} = 100[\log(\psi_{\mathrm{ad}}) - \log(\psi_{\mathrm{am}})]$$

where ξ_{m} is the percent lateral shift between the point of inflection on the drying curve and the point of inflection on the median SWCC, ψ_{ad} is the suction at the point of inflection on the drying curve, and ψ_{am} is the suction at the point of inflection, a_{m}, on the median curve.

Since the drying SWCC and the wetting SWCC (and the median SWCC) are assumed to be congruent, the lateral shift defined by eq. [24] (and eq. [25]), ξ, applies not only at the inflection points but at all points along the SWCCs. Therefore, eq. [24] can be written as follows:

$$[26]\quad \xi = 100[\log(\psi_{\mathrm{d}}) - \log(\psi_{\mathrm{w}})]$$

where ψ_{d} is the suction at any point along the drying SWCC, and ψ_{w} is the suction at any corresponding water content on the wetting SWCC. Equation [26] also applies for moving from the drying curve to the median curve.

The equations representing the lateral shift of the SWCC can be rearranged such that the suction on a congruent

Table 1. Calculation of soil suctions on the drying and wetting (or median) SWCCs.

Water content (%)	Dimensionless water content	Suction on drying curve (kPa)	Suction on wetting curve (kPa)	Change in suction (%)
35.50	0.986	11.4	3.61	68.38
33.50	0.931	35.4	11.2	68.38
31.50	0.875	55.8	17.7	68.38
29.50	0.819	76.6	24.2	68.38
27.50	0.764	99.0	31.3	68.38
25.50	0.708	124.0	39.3	68.38
23.50	0.653	154.0	48.7	68.38
21.50	0.597	190.0	60.1	68.38
19.50	0.542	236.0	74.5	68.38
17.50	0.486	296.0	93.8	68.38
15.50	0.431	383.0	120.0	68.38
13.50	0.375	515.0	163.0	68.38
11.50	0.319	741.0	234.0	68.38
9.50	0.264	1199.0	379.0	68.38
7.50	0.208	2417.0	764.0	68.38
5.50	0.153	7834.0	2477.0	68.38
3.50	0.097	95 059.0	30 060.0	68.38
2.50	0.069	1 476 477.0	466 900.0	68.38

Note: Drying curve, inflection point, a_d = 100 kPa; wetting curve, inflection point, a_w = 31.623 kPa; n parameter, 1.5; m parameter, 1.0; 50% lateral shift.

SWCC can be computed from the drying curve. Then, eq. [26] can be written as follows:

$$[27] \qquad \psi_{\mathrm{w}} = 10^{\left(\log\psi_{\mathrm{d}} - \xi/100\right)}$$

Likewise, the suction on the median curve can be written as

$$[28] \qquad \psi_{\mathrm{m}} = 10^{\left(\log\psi_{\mathrm{d}} - \xi_{\mathrm{m}}\big/100\right)}$$

Let us define the percent change in suction between any two congruent SWCCs, ζ, as the suction difference between any two curves referenced to the suction on the drying curve,

$$[29] \qquad \zeta = 100(\psi_{\mathrm{d}} - \psi_{\mathrm{w}})/\psi_{\mathrm{d}}$$

where ψ_d is the suction on the desorption (or drying) curve, and ψ_w is the suction on the adsorption (or wetting) curve. The same equation applies for a shift to the median curve at the same water content. Equation [27] (or eq. [28]) can be substituted into eq. [29] to provide a relationship between the percent change in suction between two congruent SWCCs, ζ, and the percent shift in the curves, ξ, as shown in eq. [30].

$$[30] \qquad \zeta = 100\left[\psi_{\mathrm{d}} - 10^{\left(\log\psi_{\mathrm{d}} - \xi/100\right)}\right]\psi_{\mathrm{d}}$$

Equation [30] shows that there is a fixed relationship between the percent change in suction, ζ, and the percent lateral shift between the drying and the wetting (or median) curves, ξ, as shown in Fig. 7.

Figure 7 shows that for a lateral shift of 25%, the percent error in suction would be 43.8% for a particular water content. Similarly, for a lateral shift of 50%, the percent change in suction would be 68.4%, and for a lateral shift of 100%, the percent error in suction would be 90%. Figure 7 can also be used to compare the suction value computed from the median SWCC and the drying curve. In this case, a percent shift between the drying and wetting curves of 25% would correspond to a percent shift of 12.5% in going from the drying curve to the median SWCC. Consequently, the median SWCC significantly reduces the percent error in the estimated soil suction. Table 1 provides example calculations for cases where the lateral shift is 50% between the drying and wetting curves. The results show that the soil suctions on each drying curve produces a 68.4% change (reduction) in the suction on the wetting curve for every water content condition.

Fig. 7. Percent change in calculated suction as a function of the translation between the drying and wetting (or median) SWCCs.

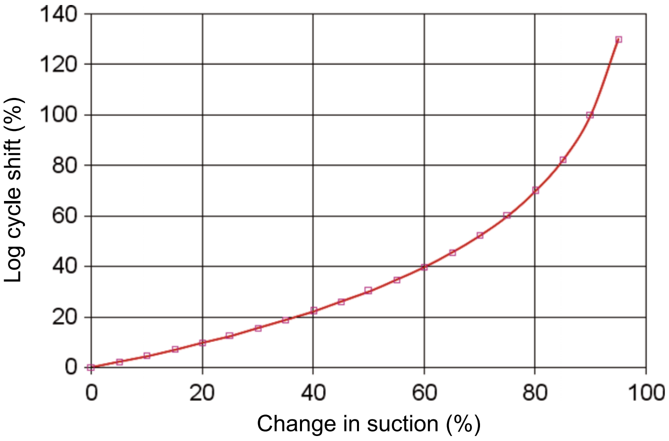
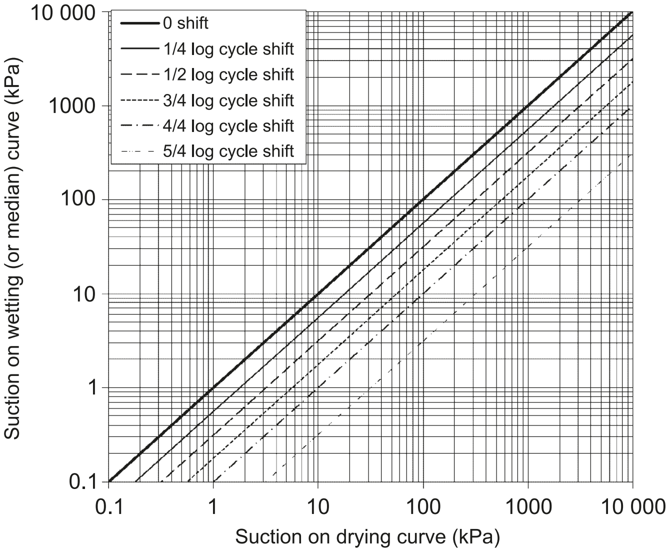


Figure 8 illustrates another way to visualize that relationship between suctions on the drying curve and any other congruent curve. The log–log plot provides a quick estima-

Fig. 8. Relationship between the suction on the drying curve and the suction on any other congruent SWCC.



tion of the relationship between the suction on the drying curve and any other SWCC if the percent shift between the two curves is known. The plot applies for any two congruent SWCCs regardless of the *a* and *n* fitting parameters. Table 2 provides a summary of the percent reduction in moving from the drying curve to the wetting (or median) curve for each percent lateral shift between the SWCCs.

Table 2. Percent change in soil suction for each lateral shift between the drying and wetting (or median) SWCCs.

Lateral shift (%)	Change (reduction) in suction (%)
0.0	0.00
10.0	20.57
20.0	30.90
25.0	43.77
30.0	49.88
40.0	60.19
50.0	68.38
60.0	74.88
70.0	80.05
75.0	82.22
80.0	84.15
90.0	87.41
100.0	90.00
120.0	93.69
150.0	96.84

Table 3. Suggested shifts of the inflection point between the drying and wetting curves for various soils.

Soil type	Range of typical shifts (% of a log cycle)	Average shift (% of a log cycle)
Sand	15–35	25
Silt and loam	35–60	50
Clay	—	Up to 100

Approximate shift between the drying and wetting SWCCs

Usually only the drying SWCC is measured in the laboratory, and an estimate is made of the difference between the drying and wetting curves at the inflection point. In this way, it is possible to estimate the maximum suction, the minimum suction, and the median likely suction for soil suction. Pham (2002) analyzed the lateral shift between the drying and wetting SWCCs for published data from various researchers. It was found that the largest shift between the drying and wetting curves existed for clay soils, and the smallest shift occurred for uniform sand soils. Table 3 summarizes typical (average) lateral shifts at the inflection point of the SWCC for various soils (Pham et al. 2002, 2003).

The mean values of shift for three soil categories (Pham et al. 2003) are used to illustrate the range of soil suctions that might be anticipated on the basis of the measured water contents.

Suggested estimation procedure for obtaining the desorption suction, median suction, and adsorption suction from SWCCs

The following soil properties and information must be available to estimate soil suction from natural water content measurements. The fitting parameters for the drying branch of the SWCC must be either measured or estimated. Any one of several SWCC equations could be used to illustrate the computation of soil suction; however, only the Fredlund and Xing (1994) three-parameter equation will be used in this study.

It is necessary to know the following empirical soil parameters for the drying SWCC, namely (*i*) the saturated water content for the soil, w_s, (*ii*) the *a* soil parameter, (*iii*) the *n* soil parameter, and (*iv*) the *m* soil parameter. The *m* parameter will be set to 1, and the correction factor is also set to 1.0. Therefore, estimates of suction are limited between the air-entry value and residual conditions.

The classification properties of the soil are used to obtain an estimate for the magnitude of lateral shift between the desorption SWCC and the adsorption SWCC. The following values for "lateral shift" at the inflection point are assumed, namely (*i*) 25% shift for sands, (*ii*) 50% shift for silt, (*iii*) 100% shift for clays.

The only fitting parameter that changes during the lateral shift of the SWCCs is the *a* parameter. The *a* parameter for the drying curve is assumed to be known, and the corresponding *a* parameter is calculated from the estimated lateral shift between two SWCCs. As an example, let us assume that the soil under consideration is sand, with the *a* parameter on the drying curve equal to 10.0 kPa. Let us also select a lateral shift of 25% of a log cycle to get to the wetting SWCC. The *a* parameter for the *wetting* SWCC can be calculated using eq. [24].

$$[31] \qquad 0.25 = \log(10) - \log(\psi_{\mathrm{aw}})$$

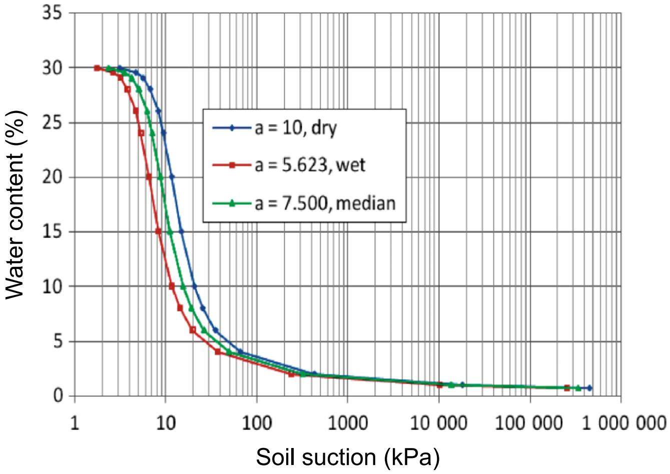
Therefore, the suction corresponding to the *a* parameter at the inflection point on the wetting curve is

Table 4. Soil properties associated with the drying and wetting SWCCs for three soils.

Soil type	Saturated water content	a_d drying SWCC	n_d drying SWCC	% shift	a_w wetting SWCC	a_m median SWCC
Sand	30.0	10.0	4.0	25	5.623	7.500
Silt (loam)	40.0	200.0	2.0	50	63.25	112.5
Clay	60.0	3000.0	1.5	100	300.0	948.7

Note: SWCC variable m = 1.0 and correction factor $C(\psi)$ = 1.0. a_d, point of inflection on drying curve; n_d, n parameter for drying curve; a_w, point of inflection on wetting curve; a_m, point of inflection on median curve.

Fig. 9. Comparison of the drying, wetting, and median curves for a soil (i.e., sand), with a lateral shift of 25% between the drying and wetting curves, and a saturated gravimetric water content of 30%.



$$\psi_{aw} = 10^{0.75} = 5.6 \quad [32]$$

The suction at the inflection points went from 10 kPa on the drying curve to 5.623 kPa on the wetting curve. The suction at the inflection point on the median SWCC can be calculated by using a lateral shift of 12.5% (i.e., 0.125). The n fitting parameter remains the same for all SWCCs because of congruency.

Suggested representation of the maximum, median, and minimum soil suction corresponding to a measured water content

It is suggested that three soil suction values be computed for any measured water content. The following format is suggested for representing soil suction values corresponding to a measured water content.

$$\text{Wetting curve suction} < [\text{median suction}] > \text{drying curve suction} \quad [33]$$

For the example shown above (i.e., eq. [32]), the suctions can be written as follows: 5.62 < [7.5] > 10.0, and read as

Most likely (median) suction = 7.5
Maximum estimated suction = 10.0
Minimum estimated suction = 5.62
Estimated range of suctions = 5.62–10

The above format provides a representation of the range and central tendencies that can be anticipated. The geotechnical engineer is asked to view all three suction values and ask himself, "Does an understanding of the suction range and central tendencies assist me in making engineering judgements?" In some cases, the answer may be yes, while in other cases it may be no.

Fig. 10. Comparison of the drying, wetting, and median curves for a soil (i.e., silt or loam), with a lateral shift of 50% between the drying and wetting curves, and a saturated gravimetric water content of 40%.

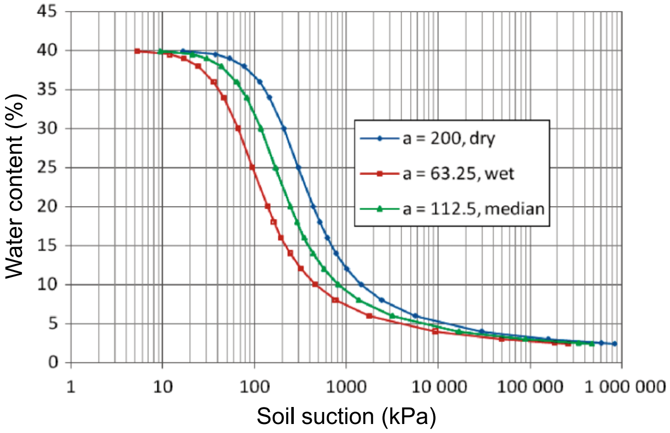


Fig. 11. Comparison of the drying, wetting, and median curves for a soil (i.e., clay), with a lateral shift of 100% between the drying and wetting curves, and a saturated gravimetric water content of 60%.

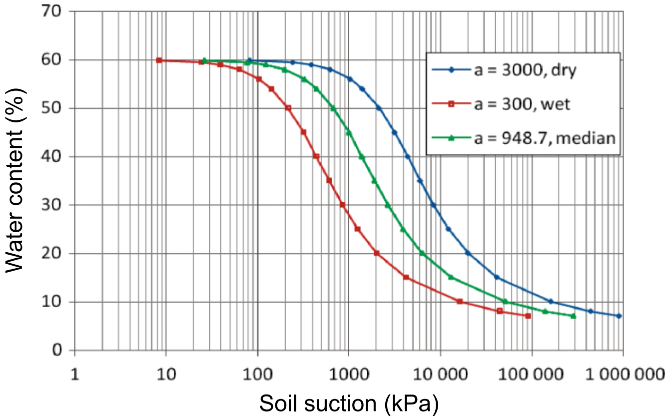
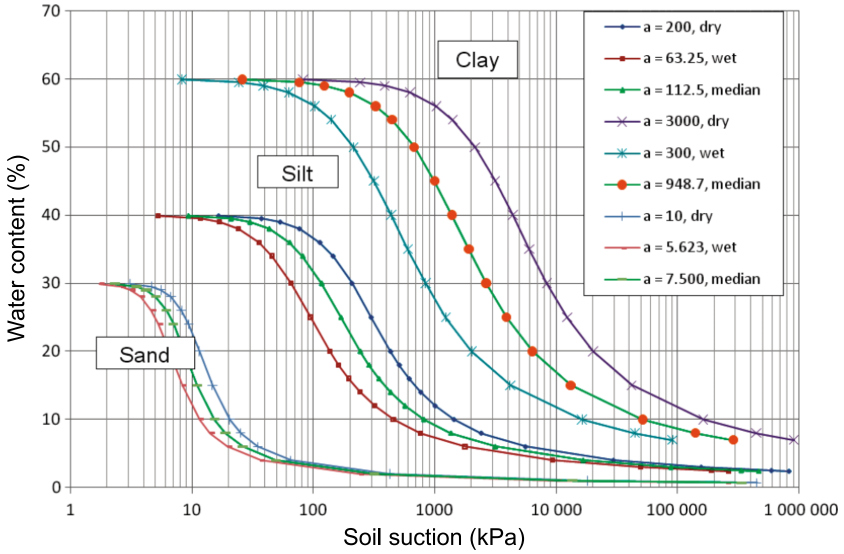


Fig. 12. Comparison of three sets of drying, wetting, and median curves for three typical soils.



Example problem to illustrate typical values and ranges of values from the proposed procedure to obtain soil suction

Let us assume the soil parameters shown in Table 4 for sand soil, silt (loam) soil, and clay soil. The Fredlund and Xing (1994) equation is used to illustrate typical differences between various soil classifications.

Drying, wetting, and median SWCCs are computed for three different soil types. The soil parameters are typical values for a sand, silt, and clay but should not be taken as fixed values for these soil classifications. Figure 9 illustrates the relationship amongst the three SWCCs (i.e., drying curve, median curve, and wetting curve) for a soil with a lateral shift of 25%. The *n* parameter was set to 4 to show rather steep curves typical of uniform sand.

Figures 10 and 11 illustrate the relationship amongst the three SWCCs for soils with a lateral shift of 50% and 100%, respectively. The *n* parameters were reduced to 2.0 and 1.5, respectively. All three sets of SWCCs are shown in Fig. 12 to show the wide range of SWCCs that can occur for various soils.

Suggestions and recommendations for using the SWCC to estimate in situ soil suction in geotechnical engineering practice

The following guidelines are suggested for usage of the SWCCs for the estimation of in situ suction:

(1) The *desorption SWCC* will provide an estimate of the *maximum* value for in situ soil suction.

(2) The estimated *adsorption SWCC* will provide an estimate of the *minimum* value for in situ soil suction.

(3) The estimated *median SWCC* will provide an estimate closest to the *most likely* (or *middle*) in situ soil suction.

(4) The *percent error* in the estimated soil suction should be referenced to the drying SWCC.

(5) Because of the logarithmic nature of the SWCCs, it is possible for the errors in the estimated soil suction to be quite large. The errors are much smaller for sand soils than for soils with high clay content.

The analysis presented in this paper shows that it is difficult to obtain an accurate indication of the in situ soil suction through use of SWCCs and the measured natural water content of the soil. Rather, the proposed procedure merely allows the engineer to obtain a crude approximation of in situ suction conditions.

This study was undertaken for two primary reasons: first, as a warning to geotechnical engineers who desire to use the SWCC and the measured natural water content to determine in situ suction; second, as a guide to assessing the likely range of in situ suctions that might correspond to a single water content measurement. The authors are not saying that SWCCs should *not* be used to estimate in situ soil suction but rather that the users of this approach should be aware of the wide spread of suction values that might actually exist in situ.

The authors have not taken into consideration other potential errors such as those associated with the measurement (or estimation) of the SWCCs. These are added sources of error.

It is interesting to note that while the SWCCs have become “key” to the implementation of unsaturated soil mechanics in engineering practice, the SWCCs have *not* proven to be of much value in the estimation of in situ soil suctions. Put another way, the SWCCs are effective for certain applications in unsaturated soil mechanics but are relatively ineffective for other applications.

Conclusions

Geotechnical engineers have long desired to use the SWCC for the estimation of in situ soil suction. This practice has been discouraged because of hysteresis associated with the SWCCs. This paper provides a means of quantifying the range of soil suctions that might be inferred from the usage of measured natural water contents and SWCCs. Specific conclusions from this study can be summarized as follows:

(1) Proposed empirical equations for SWCCs can be rearranged such that the desired soil suction value can be calculated from water content.

(2) It has become quite common in engineering practice for the drying SWCC to be measured in the laboratory, while the wetting SWCC is estimated based on soil classification. These conditions were applied in this study to obtain two congruent curves: one corresponding to the drying SWCC and the other corresponding to the wetting SWCC. This procedure provided a means of estimating the maximum soil suction, the minimum soil suction, and the median soil suction.

(3) Congruent hysteretic curves for the drying and wetting curves can be drawn by changing the variable related to the inflection point on the SWCCs. For most empirical SWCCs, it is the "*a*" variable that changes between the drying and wetting curves. Consequently, only one variable needs to be changed to move from the drying SWCC to the wetting SWCC.

(4) If measured values are not available for the translation of the drying SWCC to the wetting SWCC, then the following values are suggested for engineering usage — namely, sand soils use an *a* shift of 25%, silt (loam) soils use an *a* shift of 50%, and clay soils use an *a* shift of 100%.

(5) Values of soil suction computed from the median SWCCs should also record plus and minus values corresponding to the drying and wetting SWCCs.

The authors would encourage further research in this area so that the estimation of soil suction from SWCCs can become a more reliable methodology for use in engineering practice.

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