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Estimation of TFP Growth:
A Semiparametric Smooth Coefficient Approach

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Abstract This paper uses a semiparametric smooth coefficient model (SPSCM) to estimate TFP growth and its components (scale and technical change). The SPSCM is derived from a nonparametric specification of the production technology represented by an input distance function, using a growth formulation. The functional coefficients of the SPSCM come naturally from the model and are fully flexible in the sense that no functional form of the underlying production technology is used to derive them. Another advantage of the SPSCM is that it can estimate bias (input and scale) in technical change in a fully flexible manner. We also used a translog input distance function framework to estimate TFP growth components. A panel of U.S. electricity generating plants for the period 1986 – 1998 is used for this purpose. Comparing estimated TFP growth results from both parametric and semiparametric models against the Divisia TFP growth, we conclude that the SPSCM performs the best in tracking the temporal behavior of TFP growth.

Keywords TFP growth · Semiparametric smooth coefficient model · Input distance function
JEL Classification C13 · C14 · D24

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1 Introduction

Measurement of total factor productivity (TFP) growth has been the subject of investigations in many empirical studies. Various approaches have been used for this in the TFP growth literature. These approaches are classified by Diewert (1981) into parametric estimation of production/cost/distance functions, nonparametric indices, exact index numbers, and nonparametric methods using linear programming.[1] In the nonparametric approach, the Divisia index has been widely used as a convenient measure of TFP growth because it can be computed directly from the data without estimating anything. An important assumption of the Divisia index of TFP growth is that the underlying technology is homogeneous of degree one so that TFP growth coincides with technical change (TC). In the case of non-constant returns to scale technology, TFP growth can be decomposed into TC and scale effects for which estimation of the underlying production technology is necessary.

In parametric models using production/cost/distance functions, a flexible functional form (mostly translog) is chosen first. Algebraic formulae for TC and scale components are then derived from it. Estimates of these components are then obtained using the estimated parameter values and data. To model TC a time trend (TT) variable is usually used as a regressor in the production/cost/distance function. This is called the TT model which makes TC a linear function of time and input variables (input prices and output) if a translog production (cost) function formulation is used. Stevenson (1980) considered third order terms in the translog model to make TC in the TT model more flexible. Baltagi and Griffin (1988) have shown that in a translog model TC can be modeled in a much more flexible manner by replacing the time trend variable with an index which is a linear function of time dummies. Different generalizations of TT models of TC have been developed and their performance and sensitivity using different data sets have been evaluated (see Kumbhakar et al, 1999; Kumbhakar, 2000; Oh et al, 2009, among others).

Similarly, given a parametric production/cost/distance function one can easily derive the scale component. Assuming that the markets are competitive and producers are efficient, one can get an estimate of TFP growth simply by adding the TC and scale components.[2] If TFP growth is computed from data, the observed (Divisia) TFP growth is likely to deviate from the estimated TFP growth, especially if the estimated TFP growth is not linked to the observed TFP growth in the econometric model (Kumbhakar and Lozano-Vivas, 2005). This residual component can be large if the functional form used to represent the underlying technology is wrong. However, this residual component can only be obtained if one can compute TFP growth directly from data. This is possible if price information is available. If price information is not available, the econometric techniques come handy because one can simply add the estimated TFP growth components to get the estimated TFP growth. The only drawback is that one cannot be sure whether estimated TFP growth obtained this way comes close to the Divisia TFP growth. Since the estimated TFP growth depends on the choice of functional form of the underlying technology, it is important to specify the technology in a flexible manner (Baltagi and Griffin, 1988).

In estimating TFP growth and its components, our objective, in this paper, is to specify the technology as flexible as possible. Parametric model like translog can sometimes satisfy this objective. However, many researchers are attracted by more flexible specifications, such as kernel-based nonparametric or semiparametric models, which can capture heterogene-

[1] For references up to the mid 1990s, see Jorgenson's (1995) volumes on productivity. Some of the recent references are: Chun and Nadiri (2008), Key et al (2008), Brümmer et al (2002), Karagiannis et al (2004), among others.

[2] If producers are inefficient additional components associated with inefficiency can be obtained (Kumbhakar and Lovell, 2000).

ity in the underlying technology much better. This is because kernel functions generate observation-specific conditional mean estimates (Li and Racine, 2006). A purely nonparametric model is attractive when there are not too many continuous regressors or when there are many observations to fit the relationship. One popular semiparametric specification is a partially linear model proposed by Robinson (1988), where the intercept is allowed to be an unknown smooth function of some variables, which are different from the other regressors in the model. Li et al (2002) generalized this model by making the slope coefficients also unknown smooth functions of the same variables, so that researchers can obtain both heterogeneous intercept and slope coefficients. This model is coined as semiparametric smooth coefficient model (SPSCM). The SPSCM allows more flexibility than a parametric model and the sample size required to obtain a reliable estimation is not as large as required for estimating a purely nonparametric model, while bandwidth can be selected via least-squares cross-validation (LSCV) method (Li and Racine, 2010). When it comes to panel data modeling, the standard practice is to make parameters heterogeneous by adding fixed or random effect or making all the coefficients random. However, these models do not capture heterogeneity as well as a SPSCM does in the sense that the SPSCM captures heterogeneity in intercept and slopes through some covariates.

In this paper we consider estimating the TFP growth model based on a nonparametric production technology represented by an input distance function (IDF). The growth formulation of the model fits naturally into a SPSCM. That is, we do not start from a SPSCM, it is the outcome from a growth representation of the model. The variables in the functional coefficients (outputs, inputs and time) of the SPSCM comes naturally from the distance function formulation of the technology. The growth formulation automatically controls for fixed individual effects which are usually ignored in the TFP growth literature. Furthermore, the functional coefficients are uniquely related to TC and scale components. Thus, both TC and scale components become nonparametric functions of time, inputs and outputs. Therefore, these components are completely flexible. Since TC is estimated in a fully flexible manner, we can also measure bias in TC following Stevenson (1980) in a fully flexible manner. That is, no additional assumptions are to be made in estimating input biases in TC.

Empirically, we examine TFP growth of the U.S. electricity generating plants during the period 1986 – 1998. For this we use a growth formulation of IDF which was earlier proposed by Kumbhakar et al (2008). However, they assumed the coefficients to be linear parametric functions of all other variables in the model. The contribution of this paper is to relax the functional form assumption on these coefficients which in our model are completely nonparametric.[3]

An additional feature of our application is that we have price information, which enables us to compute the observed (the Divisia) TFP growth. The Divisia TFP growth is then used as the benchmark against which estimated TFP growth from SPSCM and parametric models are compared. We find that estimated TFP growth from the SPSCM comes very close to the Divisia index, thereby confirming the conventional wisdom that the residual component (the unexplained part of TFP growth) will be smaller if a flexible functional form is used. The other lesson we draw from the application is that if the objective is to estimate TFP growth from the estimated components, it is better to use a growth formulation because it ties up TFP growth with its components thereby reducing the unexplained component which is captured by a zero mean noise term in the regression. This is true whether one uses a parametric or semiparametric function.

[3] In the parametric model these coefficients are functions of the parameters of the underlying technology (production/distance/cost functions) and data. For example, Kumbhakar et al (2008) used an input distance function to estimate the growth model in which the parameters are linear function of data and parameters of the input distance function.

The rest of the paper is organized as follows. Section 2 discusses TFP growth and its decomposition in both nonparametric and parametric IDF framework and shows how to measure bias in TC. Data and results are discussed in section 3 and 4, respectively. Section 5 concludes the paper.

2 TFP growth formulation

In a multiple output case TFP growth is defined as $T\dot{F}P = \sum_{q=1}^{Q} R_q \dot{Y}_q - \sum_{k=1}^{K} S_k \dot{X}_k$, where Y is a vector of Q outputs, X is a vector of K inputs, $S_k = W_k X_k / \sum_k W_k X_k$ is the share of input k in the total cost, W_k being the price of input X_k. The share of output q in the total revenue is $R_q = P_q Y_q / \sum_q P_q Y_q$, P_q being the price of output Y_q, and a dot over a variable indicates its annual rate of change. Using the above definition, the TFP growth can be computed from the observed data without any estimation. The resulting measure is called the Divisia index of TFP growth. It gives us information about output growth that is not explained by the growth of inputs used (often called the Solow residual). The Divisia index is nonparametric in the sense that it can be directly computed from data without any econometric estimation. However, it cannot provide any information on the factors affecting productivity growth. Furthermore, without price information the Divisia index of TFP growth cannot even be computed. The main advantage of the econometric approach is that we can both estimate and decompose TFP growth. It does not require information on prices, if the econometric approach is based on production/distance functions. Furthermore, it allows for non-constant returns to scale so that we can estimate the contribution of scale economies/diseconomies in the overall TFP growth. We show that the non/semiparametric models can do all these in a much more flexible manner compared to the flexible (translog) parametric models.

2.1 TFP growth as a semiparametric smooth coefficient model

Instead of starting from the typical route of specifying the technology in terms of a production/cost function, we start from the transformation function. The advantage of doing so is that all the primal formulations can be derived from it by using different normalizing (identifying) restrictions. We write the transformation function as, $A \cdot T(X, Y, t) = 1$ where X is a vector of inputs, Y is a vector of outputs, t is the time trend variable, and $T(\cdot)$ is the transformation function. One needs some identifying restrictions to estimate the transformation function $A \cdot T(\cdot) = 1$ parametrically or nonparametrically. For example, if one assumes $T(\cdot)$ to be separable in Y and Y is a scalar, then we can write it as $Y = B \cdot f(X, t)$ which is the production function. On the other hand, if one of the input (say X_1) is separable (identifying restrictions) from others, we can express the transformation function as $X_1 = C \cdot g(X_{-1}, Y, t)$ which is an input requirement function (Diewert, 1974). In the above formulation X_{-1} is the X vector excluding X_1. Finally, if the identifying restrictions are such that $T(\cdot)$ is homogeneous of degree one in X, then we can rewrite it as $X_1^{-1} = \Lambda \cdot H(\tilde{X}, Y, t)$ where X_1 is the numeraire input and $\tilde{X}$ is a vector of input ratios, with $\tilde{X}_k = X_k / X_1$, $\forall k = 2, \ldots, K$. The transformation function written in this form is nothing but the IDF which was introduced by Shephard (1953) and is extensively used for modeling inefficiency. Note that A, B, C, and Λ are the efficiency parameters/functions in various representations of the technology.

In this paper we use the IDF representation of the transformation function. This formulation is economically appropriate because for electricity generating plants (which are the units of observation in our data) inputs are endogenous and output (electricity generated) is exogenous (Nerlove, 1965). In other words, the plants minimize cost to produce the exogenously given (determined by demand) output. It can be shown that under cost minimization input ratios are exogenous (Das and Kumbhakar, 2010). Furthermore, IDF is dual to the cost function (Färe and Primont, 1995) and therefore, IDF is ideal to use when either input prices are not available or prices do not vary much. Since we focus on TFP growth and its components, first we derive an algebraic expression for TFP growth starting from the IDF.

To simplify notations and ease derivation of the growth formulation of the IDF, we rewrite it as:

$$-\ln X_1 = \ln \Lambda + m(\ln Y, \ln \tilde{X}, t) \tag{1}$$

where $m(\ln Y, \ln \tilde{X}, t) = \ln H(\tilde{X}, Y, t)$. Differentiating (1) with respect to t gives,

$$-\frac{d\ln X_1}{dt} = \frac{\partial \ln \Lambda}{\partial t} + \sum_{q=1}^{Q} \frac{\partial m(\ln Y, \ln \tilde{X}, t)}{\partial \ln Y_q} \cdot \frac{\partial \ln Y_q}{\partial t} + \sum_{k=2}^{K} \frac{\partial m(\ln Y, \ln \tilde{X}, t)}{\partial \ln \tilde{X}_k} \cdot \frac{\partial \ln \tilde{X}_k}{\partial t} + \frac{\partial m(\ln Y, \ln \tilde{X}, t)}{\partial t} \tag{2}$$

For estimation purposes, we define:

$$\frac{d\ln X_1}{dt} = \dot{X}_1 = \frac{X_{1,t} - X_{1,t-1}}{0.5(X_{1,t} + X_{1,t-1})}$$

$$\frac{\partial \ln \tilde{X}_k}{\partial t} = \dot{\tilde{X}}_k = \frac{\tilde{X}_{k,t} - \tilde{X}_{k,t-1}}{0.5(\tilde{X}_{k,t} + \tilde{X}_{k,t-1})}, \ \forall k = 2, \ldots, K$$

$$\frac{\partial \ln Y_q}{\partial t} = \dot{Y}_q = \frac{Y_{q,t} - Y_{q,t-1}}{0.5(Y_{q,t} + Y_{q,t-1})}, \ \forall q = 1, \ldots, Q$$

Furthermore, we let

$$\beta_k(\ln Y, \ln \tilde{X}, t) = -\frac{\partial \ln X_1}{\partial \ln \tilde{X}_k} = \frac{\partial m(\ln Y, \ln \tilde{X}, t)}{\partial \ln \tilde{X}_k}, \ \forall k = 2, \ldots, K \tag{3}$$

$$\gamma_q(\ln Y, \ln \tilde{X}, t) = -\frac{\partial \ln X_1}{\partial \ln Y_q} = \frac{\partial m(\ln Y, \ln \tilde{X}, t)}{\partial \ln Y_q}, \ \forall q = 1, \ldots, Q \tag{4}$$

$$\beta_0(\ln Y, \ln \tilde{X}, t) = -\frac{\partial \ln X_1}{\partial t} = \frac{\partial m(\ln Y, \ln \tilde{X}, t)}{\partial t} \tag{5}$$

Using the above definitions, (2) can be simplified as:

$$-\dot{X}_1 = \beta_0(\ln Y, \ln \tilde{X}, t) + \sum_{k=2}^{K} \beta_k(\ln Y, \ln \tilde{X}, t)\dot{\tilde{X}}_k + \sum_{q=1}^{Q} \gamma_q(\ln Y, \ln \tilde{X}, t)\dot{Y}_q + u \tag{6}$$

where $u = \partial \ln \Lambda / \partial t$ is the rate of change in the efficiency parameter Λ. It also captures the effect of unobserved variables that are time-varying. We treat it as the residual (noise) component, which is assumed to be a zero mean random variable. Economic theory tells us that: (i) β_0 should be positive (i.e., $\frac{\partial \ln X_1}{\partial t} < 0$) so that producers do not require more inputs going forward from say period t to $t+1$, holding output and other input quantities constant; (ii) β_k, $\forall k = 2, \ldots, K$ should be positive (i.e., $\frac{\partial \ln X_1}{\partial \ln \tilde{X}_k} < 0$), thereby meaning that when an input is increased producers do not need more of any of the other inputs to produce the same amount of outputs; (iii) γ should be negative (i.e., $\frac{\partial \ln X_1}{\partial \ln Y} > 0$) so that producers cannot produce more outputs by decreasing the amount of inputs, ceteris paribus. If $m(\cdot)$ in (1) is an unknown smooth function of $\ln Y$, $\ln \tilde{X}$, and t, then its gradients, i.e., β_0, β_k, $\forall k = 2, \ldots, K$, and γ_q, $\forall q = 1, \ldots, Q$, are also unknown smooth functions of these variables.

Under these circumstances, (6) can be viewed as the SPSCM of Li et al (2002) where the model is linear in the $\dot{\tilde{X}}_k$, $\forall k = 2, \ldots, K$, and $\dot{Y}_q$, $\forall q = 1, \ldots, Q$, variables. Note that the covariates in the functional coefficients are $\ln Y, \ln \tilde{X}$ and t and we do not have to introduce them in an *ad hoc* fashion. They come naturally from the model.

Next we rewrite (6) (after adding the subscript i for observation) as

$$\mathcal{Y}_i = \mathcal{X}_i' \Psi(\mathcal{Z}_i) + u_i \tag{7}$$

where $\mathcal{Y}_i = -\dot{X}_{1i}$; $\mathcal{X}_i' = [1, \dot{\tilde{X}}_{2i}, \ldots, \dot{\tilde{X}}_{Ki}, \dot{Y}_{1i}, \ldots, \dot{Y}_{Qi}]$; $\mathcal{Z}_i' = [\ln \tilde{X}_{2i}, \ldots, \ln \tilde{X}_{Ki}, \ln Y_{1i}, \ldots, \ln Y_{Qi}, t_i]$; $\Psi'(\cdot) = [\beta_0(\cdot), \beta_2(\cdot), \ldots, \beta_K(\cdot), \gamma_1(\cdot), \ldots, \gamma_Q(\cdot)]$. Following Li et al (2002) and Li and Racine (2006), the local-constant least squares estimator for $\Psi(z)$ is expressed as (see Li and Racine, 2006, Chap. 9, pg. 302):

$$\hat{\Psi}(z) = \Big[\sum_{i=1}^{n} \mathcal{X}_i \mathcal{X}_i' K(\frac{\mathcal{Z}_i - z}{h}) \Big]^{-1} \sum_{i=1}^{n} \mathcal{X}_i \mathcal{Y}_i K(\frac{\mathcal{Z}_i - z}{h}) \tag{8}$$

where n denotes sample size, h is a $(K + Q)$ vector with each element a selected bandwidth for each z variable and $K(\cdot)$ is the product Gaussian kernel function. The basic idea behind local-constant estimator is that $\hat{\Psi}(z)$ is a simple local average of data.[4] It differs from simple ordinary least squares (OLS) estimator only in the additional kernel function: elimination of the kernel function in (8) reduces the estimator from a smooth coefficient to its OLS counterpart. Note that the SPSCM nests the partially linear model proposed by Robinson (1988) as a special case, which makes only the intercept as an unknown smooth function of $\mathcal{Z}$ variables and keeps all the slopes constant.

Following Li and Racine (2010), we employ the most commonly used least-squares cross-validation (LSCV) method, which is a fully automatic data-driven approach, to select the bandwidth vector h, i.e.,

$$CV_{lc}(h) = \min_h \; n^{-1} \sum_{i=1}^{n} [\mathcal{Y}_i - \mathcal{X}_i' \hat{\Psi}_{-i}(\mathcal{Z}_i)]^2 M(\mathcal{Z}_i) \tag{9}$$

where $CV_{lc}(h)$ determines the cross-validation bandwidth vector h for local constant estimator, $\mathcal{X}_i' \hat{\Psi}_{-i}(\mathcal{Z}_i) = \mathcal{X}_i' \big[\sum_{j \neq i}^{n} \mathcal{X}_j \mathcal{X}_j' K(\frac{\mathcal{Z}_j - z_i}{h}) \big]^{-1} \sum_{j \neq i}^{n} \mathcal{X}_j \mathcal{Y}_j K(\frac{\mathcal{Z}_j - z_i}{h})$ is the leave-one-out local-constant kernel conditional mean, and $0 \leq M(\cdot) \leq 1$ is a weight function that serves to avoid difficulties caused by dividing by zero. The bandwidth for the $\mathcal{Z}$ variables and the smooth coefficients can be estimated by the *np* package (Hayfield and Racine, 2008) in R.

[4] Consistency requires that the size of local sample, nh, must increase with the overall sample size, n, while at the same time the bandwidth, h, must shrink to zero in the limit (Li and Racine, 2006).

2.2 TFP growth components

Adding $T\dot{F}P = \sum_{q=1}^{Q} R_q \dot{Y}_q - \sum_{k=1}^{K} S_k \dot{X}_k$ to both sides of (6) and moving $\dot{X}_1$ to the right-hand-side gives:

$$\begin{aligned} T\dot{F}P &= \sum_{k=2}^{K} \beta_k(\cdot)\dot{\tilde{X}}_k + \sum_{q=1}^{Q} \gamma_q(\cdot)\dot{Y}_q + \beta_0(\cdot) + \dot{X}_1 + \sum_{q=1}^{Q} R_q \dot{Y}_q - \sum_{k=1}^{K} S_k \dot{X}_k + u \\ &= \beta_0(\cdot) + \sum_{q=1}^{Q} \dot{Y}_q (R_q + \gamma_q(\cdot)) + \dot{X}_1 + \sum_{k=2}^{K} \beta_k(\cdot)\dot{\tilde{X}}_k - \sum_{k=1}^{K} S_k(\dot{\tilde{X}}_k + \dot{X}_1) + u \\ &= \beta_0(\cdot) + \sum_{q=1}^{Q} \dot{Y}_q (R_q + \gamma_q(\cdot)) + \dot{X}_1 + \sum_{k=2}^{K} \beta_k(\cdot)\dot{\tilde{X}}_k - (\sum_{k=2}^{K} S_k \dot{\tilde{X}}_k + \dot{X}_1 \sum_{k=1}^{K} S_k) + u \\ &= \beta_0(\cdot) + \sum_{q=1}^{Q} \dot{Y}_q (R_q + \gamma_q(\cdot)) + \sum_{k=2}^{K} (\beta_k(\cdot) - S_k)\dot{\tilde{X}}_k + u \end{aligned} \tag{10}$$

using the fact that $\sum_{k=1}^{K} S_k = 1$ (i.e., the sum of the cost shares is unity).

Thus, the TFP growth has four components: the first component is TC given by $\beta_0(\cdot)$; the second component is the scale component given by $(\sum_{q=1}^{Q} \dot{Y}_q (R_q + \gamma_q(\cdot)))$ because returns to scale (RTS) is $-1/\sum_{q=1}^{Q} \gamma_q(\cdot)$. The third component is often labeled as the allocative component and is non-zero if producers fail to minimize cost or maximize profit. In other words, if producers allocate their inputs without any mistakes this allocative component will be zero.[5] With allocative errors predicted TFP growth is the sum of TC, scale, and allocative components. Finally, the last component is the residual (unexplained) which is not explained by the model. Note that the allocative component can be computed if price information is available (i.e., data on cost shares are available). This means that one can compute $T\dot{F}P$ and therefore predict the last (residual) component. Thus, prediction of the last two components require information on cost shares.

2.3 TFP growth components in a parametric model

We can also impose a particular parametric functional form on $m(\ln Y, \ln \tilde{X}, t)$, for example, a flexible translog specification. Then (1) can be rewritten as:

$$\begin{aligned} -\ln X_1 =& \alpha_0 + \sum_{k=2}^{K} \theta_k \ln \tilde{X}_k + \frac{1}{2}\sum_{k=2}^{K}\sum_{m=2}^{K} \theta_{km} \ln \tilde{X}_k \ln \tilde{X}_m + \sum_{q=1}^{Q} \alpha_q \ln Y_q + \frac{1}{2}\sum_{q=1}^{Q}\sum_{o=1}^{Q} \alpha_{qo} \ln Y_q \ln Y_o \\ &+ \sum_{q=1}^{Q}\sum_{k=2}^{K} \delta_{qk} \ln Y_q \ln \tilde{X}_k + \alpha_t\, t + \frac{1}{2}\alpha_{tt}\, t^2 + \sum_{q=1}^{Q} \lambda_{qt} \ln Y_q\, t + \sum_{k=2}^{K} \delta_{kt} \ln \tilde{X}_k\, t \end{aligned} \tag{11}$$

where $\theta_{km} = \theta_{mk}, \forall k, m = 2, \ldots, K$ and $\alpha_{qo} = \alpha_{oq}, \forall q, o = 1, \ldots, Q$. We call this the parametric log IDF model. From this specification, we can derive the expressions for the coefficients in (6):

$$-\frac{\partial \ln X_1}{\partial \ln \tilde{X}_k} = \beta_k(\cdot) = \theta_k + \sum_{m=2}^{K} \theta_{km} \ln \tilde{X}_m + \sum_{q=1}^{Q} \delta_{qk} \ln Y_q + \delta_{kt} t,\ \forall k = 2, \ldots, K \tag{12}$$

[5] See appendix A for a proof of this.

$$-\frac{\partial \ln X_1}{\partial \ln Y_q} = \gamma_q(\cdot) = \alpha_q + \sum_{o=1}^{Q} \alpha_{qo} \ln Y_o + \sum_{k=2}^{K} \delta_{qk} \ln \tilde{X}_k + \lambda_{qt}\, t,\ \forall q = 1, \ldots, Q \tag{13}$$

$$-\frac{\partial \ln X_1}{\partial t} = \beta_0(\cdot) = \alpha_t + \alpha_{tt} t + \sum_{q=1}^{Q} \lambda_{qt} \ln Y_q + \sum_{k=2}^{K} \delta_{kt} \ln \tilde{X}_k \tag{14}$$

Plugging these parametric functional coefficients into (6) gives us the equation to be estimated for the parametric model.[6] We rewrite it here for convenience:

$$\begin{aligned} -\dot{X}_1 =& \alpha_t + \alpha_{tt}\, t + \sum_{q=1}^{Q} \lambda_{qt} (\ln Y_q + \dot{Y}_q\, t) + \sum_{k=2}^{K} \delta_{kt} (\ln \tilde{X}_k + \dot{\tilde{X}}_k\, t) + \sum_{k=2}^{K} \theta_k\, \dot{\tilde{X}}_k + \sum_{k=2}^{K} \sum_{m=2}^{K} \theta_{km} \ln \tilde{X}_m\, \dot{\tilde{X}}_k \\ &+ \sum_{k=2}^{K} \sum_{q=1}^{Q} \delta_{qk} (\ln Y_q\, \dot{\tilde{X}}_k + \ln \tilde{X}_k\, \dot{Y}_q) + \sum_{q=1}^{Q} \alpha_q\, \dot{Y}_q + \sum_{q=1}^{Q} \sum_{o=1}^{Q} \alpha_{qo} \ln Y_o\, \dot{Y}_q + u \end{aligned} \tag{15}$$

We call this the parametric growth IDF model. The idea is to estimate (15) and use the estimated parameters to compute βs and γ defined in (12), (13), and (14). Finally, these are used in (10) to obtain the TFP growth components. Note that although we are using the same equation to decompose TFP growth, the estimates of the βs and γ from the parametric model are different from those in the SPSCM. Since no functional forms are used for the β and γ functions, the estimated TFP growth components in the SPSCM are more flexible.[7]

2.4 Biases in technical change

Technical change can be neutral and/or biased towards some inputs. Following Stevenson (1980) we measure bias in TC for input k as $IB_k = \partial S_k / \partial t = \partial \beta_k / \partial t$. If $IB_k > 0$, TC is relatively kth input-using. On the other hand, it is kth input-saving if $IB_k < 0$ and neutral to input k if $IB_k = 0$. Similar to input bias, one can also measure scale bias in TC from $SB_q = -\partial \gamma_q / \partial t$. A positive (negative) value of SB_q indicates decreasing (increasing) minimum efficient scale over time (Stevenson, 1980). TC is neutral if it is neither input nor scale biased, i.e., $IB_k = 0$, $\forall k$ and $SB_q = 0$, $\forall q$. This means that β_0 is a function of only t.

In the parametric model (translog) the neutral component of TC is $\alpha_t + \alpha_{tt}\, t$, and therefore its change over time is constant. Input and scale biases are constant for all producers and for every year. This is, however, not the case in the SPSCM. Thus, one advantage of the SPSCM is flexibility in measuring biases in TC. This does not involve anything new other than computing derivatives of the nonparametric functions.[8]

[6] The usual practice in the literature, going back to Denny et al (1981), is to estimate the cost/distance function and use the estimated parameters to compute TC and scale components. This procedure often leads to substantial differences between the estimated and actual TFP growth rates (Kumbhakar and Lozano-Vivas, 2005). Here we avoid this problem by estimating the model in rates of change in (6). Furthermore, the growth model in (6) fits nicely into the SPSC formulation. As shown in (15) the growth model in (6) can be used for parametric models as well.

[7] It is worth noting that TFP growth components in the parametric model can also be computed by estimating (11). We discuss this later in the result section.

[8] See Appendix B for a detailed derivation. R codes for estimating partial derivatives of the smooth coefficients are available from the authors upon request.

3 Data

We used an unbalanced panel data on 82 U.S. major investor-owned utilities (IOUs) observed for the time period of 1986−1998. The production technology is represented by one output (Y), measured by net steam electric power generation in megawatt-hours, and three inputs, viz., the aggregate of labor and maintenance (X_1), fuel (X_2), and capital (X_3). The data set is obtained from *Journal of Applied Econometrics* data archive (http://www.econ.queensu.ca/jae/) and is described in Rungsuriyawiboon and Stefanou (2007) and Kumbhakar and Tsionas (2010). For the electricity industry, the output (electric power generated) is usually exogenously given (demand determined). Therefore, it would be inappropriate to estimate the production function which assumes output to be endogenous. If output is exogenously given, one can either use the cost function or the IDF where the choice (endogenous) variables are inputs. Here we use the IDF because it does not require price information which are often not directly available. If the prices are computed from some other information it is likely that they will be contaminated by measurement errors. That is the 'observed' prices might not be the same as the 'true' prices. In such a case cost function estimation might not be appropriate, although the unobserved true prices are exogenous.

4 Results

In estimating the growth IDF in (6) and (15) and log IDF in (11) we used labor (X_1) as the numeraire input. It is worth noting that the results (estimated $T\dot{F}P$ and its components) are invariant to the choice of the numeraire input. In this section we report results related to functional coefficients; TFP growth and its components; and biases in TC.

4.1 Functional coefficients in parametric and semiparametric models

The SPSCM that we estimated is given in (6). To make it comparable the parametric model that we estimated is (15), which is based on (6) but used (12), (13) and (14) for $\beta_k(\cdot)$, $\gamma_q(\cdot)$ and $\beta_0(\cdot)$, respectively. That is, the IDF in (11) should not be directly estimated.[9] According to microeconomic theory, TC, under normal circumstances, should be non-negative (technical progress) so that $\hat{\beta}_0 \geq 0$. This means that requirement of inputs to produce a given level of output should not increase over time. Similarly, $\hat{\beta}_2$, $\hat{\beta}_3$ should be positive so that holding output constant, requirement of an input (here X_1) will be lower when other inputs are increased (i.e., input ratios X_2/X_1 and X_3/X_1 are increased). Similarly $\hat{\gamma}$ should be negative since $-\hat{\gamma}$ shows the percentage by which all inputs (and therefore cost) are to be increased when output is increased by one percent. That is, $-\hat{\gamma}$ has a cost elasticity of output interpretation. Thus violations occur when either $\hat{\beta}_0 < 0$ or $\hat{\beta}_2 < 0$ or $\hat{\beta}_3 < 0$ or $\hat{\gamma} > 0$ (see section 2.1 for the detailed explanation of these coefficients). For the parametric model, we can calculate values of these functional coefficients in (12), (13) and (14) using the estimated parameters in (15) which are summarized in Table 1. We also checked the percentage of violations in the functional coefficients. It can be seen from Table 2 that for the SPSCM there are some violations for all the coefficients. For example, 4.48% of the producers experienced technical regress. For the other three smooth coefficients, the percentage of violations are less than

[9] Since the TFP growth decomposition in the empirical literature is based on estimated cost/distance functions (see the references cited in Kumbhakar et al (2008)), we have also estimated the IDF in (11). TFP components from the estimated log IDF will be reported alongside the results from the growth formulation in (15).

1%. The bandwidth summary in Table 2 indicates that the smooth coefficients are nonlinear functions of the $\mathcal{Z}$ variables because the bandwidths are small enough (less than two times the standard deviations of the corresponding $\mathcal{Z}$ variables as a rule of thumb). For the parametric model, however, the percentage of violations are not that bad compared to the SPSCM. Although there are more violations in terms of TC in the parametric model, no violation is observed in the estimated input elasticities. Table 3 reports the quartile values (Q_1, Q_2 and Q_3) for the functional coefficients derived from both the semiparametric and parametric models. All the functional coefficients in both SPSCM and parametric model have the expected signs across the quartiles.

Figure 1 plots kernel density functions of the functional coefficients in semiparametric and parametric growth models. From these plots, one can get a good idea about closeness of the distributions of various functional coefficients from the two competing models. For example, it can be seen that there are fewer violations in the semiparametric model for $\hat{\beta}_0$ (upper left panel) because of its thinner tail in the negative region. The rest of the three panels show that the distributions of $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\gamma}$ from the semiparametric model are all skewed while those from the parametric model are symmetric. Furthermore, the spread of these distributions from the semiparametric model are wider than those from its parametric counterpart. These may stem from the fact that restrictive functional form assumption on the coefficients in the parametric models makes the distributions tighter. Once the functional form assumption is relaxed in the semiparametric model, the spread of the distributions gets bigger. A nonparametric test for equality of distributions (Li et al, 2009) rejects the equality of distributions of the functional coefficients across the semiparametric and parametric models at the 1% level.

Figure 2 shows scatter plots of each functional coefficient. Coefficients from the semiparametric model are measured on the x-axis and those in parametric model are measured on the y-axis. These plots give a visual picture of how closely each functional coefficient in the two models are related. A 45 degree line is drawn to compare their closeness visually. It can be seen that none of the functional coefficient is highly correlated between the two models. This is because most of the points do not cluster along the 45 degree line. The correlation coefficients for the four functional coefficients, $\hat{\beta}_0$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\gamma}$, between the two competing models are: 0.1924, 0.2654, 0.5043, and 0.3274, respectively. These results indicate that SPSCM results are different from their parametric counterparts. For this particular dataset, SPSCM is preferred since a consistent model specification test (Hsiao et al, 2007) rejects the parametric specification at the 1% level.

Because the coefficients in our model are observation-specific, the standard errors for these coefficients are also observation-specific. We used wild bootstrap (Härdle and Mammen, 1993) to calculate these standard errors for both models. Figures 3 and 4 show a convenient way of reporting the observation-specific estimates along with their 95% confidence intervals. To understand these plots consider the plot for any functional coefficient, say $\hat{\beta}_0$ in Figure 3. First, we plot $\hat{\beta}_0$ against $\hat{\beta}_0$ so that all the $\hat{\beta}_0$ observations lie along the 45 degree line. Then we plot both the upper and lower confidence bounds for each $\hat{\beta}_0$ observation. All the points above the 45 degree line are upper bounds and those below it are lower bounds. Thus, for every point estimate of $\hat{\beta}_0$ placed on the 45 degree line, we can also see an observation-specific confidence interval. If the horizontal line at zero passes inside of the confidence bounds for any given observation, then $\hat{\beta}_0$ for this observation is statistically insignificant. Conversely, if the horizontal line at zero passes outside of the confidence bounds, then $\hat{\beta}_0$ for this observation is statistically significant. Furthermore, if the lower (upper) bound lies above (below) zero, then this observation is significantly positive (negative). If we define percentage of violations reported in Table 2 in terms of interval rather than point estimates, i.e., violations occur when the estimates are significantly positive or negative, then such violations in the semiparametric (parametric) model will be 0.61% (2.03%), 0.1% (0%), 0.1% (0%), and 0% (0%) for $\hat{\beta}_0$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\gamma}$, respectively.

We can see from Figure 4 that for $\hat{\beta}_0$ in the SPSCM, the confidence interval becomes wider near the tails when the estimates are either too small or too large. However, for $\hat{\beta}_2$, the confidence interval is wider at the upper tail when the estimates are too large, whereas for $\hat{\beta}_3$ and $\hat{\gamma}$, the confidence interval is wider at the lower tail when the estimates are too small. These correspond to what are shown in the kernel density plots in Figure 1: the distribution of $\hat{\beta}_2$ in the SPSCM is skewed to the right, meaning that there are not many observations in the right tail (when the estimates are large), whereas the distributions of $\hat{\beta}_3$ and $\hat{\gamma}$ are skewed to the left, meaning that there are not many observations in their left tails (when the estimates are small). Figure 3 suggests that the distribution of each coefficient in the parametric model is less skewed relative to their semiparametric counterpart so that the confidence interval gradually becomes wider from the center of the 45 degree line to the two ends of the line. This is also evidenced by the kernel density plots of Figure 1.

4.2 TFP growth/index and its components

So far our focus was on functional coefficients. We now use their estimates in computing TFP growth and its components for both the SPSC and parametric models. Instead of reporting the observation-specific values we computed weighted average of TFP growth over time where the weights are the ratio of output for plant i at time t to the total industry output at time t (Baltagi and Griffin, 1988). Thus a plant with lower output will have smaller influence on TFP growth. Three types of TFP growth are calculated. These are the Divisia, semiparametric, and parametric (based on the growth IDF as well as the log IDF model). These TFP growth rates are used to define TFP indices from $TFP_t = TFP_{t-1}(1 + T\dot{F}P)$, when $TFP_{1986} = 100$. By doing this we focus on the temporal behavior of TFP from the four different models. The results are reported in Figure 5. All four measures of TFP index suggest a pattern of appreciable productivity growth from the year 1986 to 1990 and from 1993 to 1996, although the Divisia index shows more uneven trend than the rest of the three econometric models. From 1987 to 1991 the parametric growth model keeps underestimating TFP index relative to SPSCM and the Divisia. In the parametric literature estimates of TFP growth and its components are mostly calculated from estimated IDF (Brümmer et al, 2002; Karagiannis et al, 2004), whereas in the present model we estimated an IDF in a growth formulation. Thus it is appropriate to look at the TFP growth components from the log IDF as well.[10] It can be seen from Figure 5 that the TFP index based on the log IDF underestimates the Divisia TFP index for most of the years. Using the TFP observed (Divisia) growth/index as the benchmark, we can say that the SPSC model traces the observed TFP growth/index much more closely than any other models (as shown in the top two panels of Figure 5). A closer look at the top left panel of Figure 5 shows that TFP growth from the parametric growth IDF model performs better than the parametric log IDF model. The reason for this is that the growth models (both parametric and semiparametric) tie up TFP growth with its components econometrically (see (6) and (15)). On the other hand if one uses a log IDF, the TFP growth components come from the econometric model and these components are not tied up with the TFP growth that comes from data. That is, there is no link between observed and estimated TFP growth empirically, and consequently the residual component (represented by the u term in either (6) or (15)) might not have zero mean, while the zero mean assumption is empirically used in the growth formulation.

[10] Results from the log IDF are also reported in Table 3.

Similar to TFP index we can also compute TC and Scale indices from the SPSC and parametric models. Since we cannot get the TC and Scale components directly from the Divisia index, there is no benchmark to compare for these components. Nonetheless it is interesting to see whether these indices differ across competing models. The results are also plotted in Figure 5 (lower two panels). It can be seen that the TC index based on SPSCM differs markedly from its parametric counterparts, especially after 1990. Both the parametric models overestimates TC index compared to its semiparametric counterpart. The overestimation is somewhat less for the log IDF model. When it comes to the Scale index, we find that SPSC and parametric growth models give almost identical estimates. The parametric log IDF model underestimates Scale index for all the years.

We can see from (10) that the decomposition of TFP growth from any econometric model naturally carries an unexplained component, which comes from the error term u in (6). This error term, by assumption, is centered around zero with some spread. The growth models (parametric and semiparametric) based on (6) use this information explicitly. This is, however, not the case with the IDF model in (11). That is, the error term implicitly used in estimating this function is not related to u (in either (6) or (15)) in a formal way. To examine the implication of this, we plotted weighted average of residuals (using varying output weights) from the three econometric models over time in Figure 6. The residual component from the SPSCM is much closer to zero compared to the other two parametric models. The residual component from the log IDF model in (11) shows large departure from zero. This is because of the fact that this model does not include the residual component the same way as the other two models do. In other words, the SPSCM fits the data better and therefore lower residual component. The R^2 (i.e. squared correlation coefficient between actual and predicted dependent variable) for the parametric and semiparametric growth models are 0.9027 and 0.9606, respectively.[11]

4.3 Biases in technical change in parametric and semiparametric models

One advantage of the SPSCM is that the bias in TC measures are observation-specific because these are based on the nonparametric functions of smooth coefficients. The derivative of the smooth coefficients with respect to time is a function of kernel function (see Appendix B), which yields observation-specific estimates. In contrast, parametric model can only yield one estimate for bias in TC for all the observations because the functional coefficients in (12), (13), and (14) are linear parametric functions of time. The derivative of each coefficient with respect to time is a single number. These results are reported in Table 4. It can be seen that sign on scale bias measure is not the same across all observations in SPSCM. The $Q_1 - Q_3$ values are $-.0366, -.0102$ and .0031, respectively, and are statistically significant. The corresponding value from the parametric model ($-\lambda_{1t}$) is $-.0025$ and is statistically insignificant (showing no scale bias). Thus, although the SPSCM shows that scale bias is negative for 70% of the plants, the parametric models show negative (but insignificant) bias for all plants. In other words, the nonparametric model is predicting that about 70% of the plants are operating below their efficient scale and about 30% are operating above their efficient scale. On the other hand, the parametric model is predicting that plants are operating at their efficient level.

TC is found to be fuel-saving (relative to labor) for all firms in the parametric model (the coefficient (δ_{2t}) is negative and significant). On the contrary, the SPSCM shows that TC is fuel-using for about 35% and fuel-saving for 65% of

[11] Note that the parametric log IDF model has a different dependent variable and therefore R^2 from this model cannot be compared with R^2 from the growth models.

the observations. Finally, TC is found to be capital-using (relative to labor) in the parametric model whereas in the SPSCM, TC is found to be capital-using for 73% (and capital-saving for 27%) of the observations. In addition to bias we can also examine whether the neutral component of TC, β_0, is time-varying or not. It is time-invariant if the time derivative of β_0 is constant for all observations (which is the case for the parametric model). The SPSCM shows that the neutral component of TC is time-varying in the sense that the Q_1, Q_2, Q_3 values of $\partial\hat{\beta}_0/\partial t$ are $-.0015, -.0002$ and $.0009$, respectively, and are statistically significant. Thus, given everything else the neutral component of TC increased (over time) for 43% of the observations. The value of the parameter (α_{tt}) from the parametric model is $-.0004$ and is statistically insignificant (thereby showing that neutral component of TC is invariant with respect to time). Thus, we find some differences in the results between the parametric and SPSCM so far as biases in TC are concerned.

Figure 7 plots observation-specific estimates of bias measures along with their 95% confidence intervals (following the same plotting technique as before). This plot is more informative than Table 4 in that it reports bias for every observation, instead of a particular quartile from the whole distribution, and tests whether it is statistically significant. While the parametric model predicts that neutral component of TC is invariant with respect to time for all observations, the SPSCM shows (in the upper left panel) it to be significantly increasing over time for 4.48% of the observations, i.e., those whose lower bounds are above zero; and significantly decreasing over time for 10.27% of the observations, i.e., those whose upper bounds are below zero. However, most of the observations do have a neutral TC (i.e. TC is invariant with respect to time), which is what the parametric model predicts. While the parametric model predicts that TC is significantly fuel-saving for all the observations, the SPSCM (upper right panel) finds that most observations (66.94%) do not have statistically significant bias towards fuel (TC is fuel-neutral), although TC is significantly fuel-using (fuel-saving) for 10.58% (22.48%) of the observations (i.e., observations with their lower(upper) bounds above (below) zero). TC is significantly capital-using in the parametric model for all the observations. This is true for those observations whose lower bounds are above zero in the SPSCM (lower left panel), although we do find that TC is significantly capital-saving for 11.80% of the observations whose upper bounds are below zero and that 59.51% have no significant bias towards capital (TC is capital-neutral). There is no significant scale bias in the parametric model, whereas in the SPSCM (lower right panel) there are 7.63% (22.38%) observations that have significantly positive (negative) scale bias (i.e., observations with their lower (upper) bounds above (below) zero). Most of the observations (69.99%) in the SPSCM do not have significant scale bias, which is what the parametric model predicts.

5 Conclusion

This paper used both the parametric and semiparametric smooth coefficient modeling approaches to estimate TFP growth and its components. The semiparametric TFP growth model is derived from a nonparametric input distance function representation of the underlying technology. The functional coefficients of the semiparametric smooth coefficient model came naturally from the model and are nonparametric functions of inputs, outputs and time. Since TFP growth components are functions of these functional coefficients, these are fully flexible. Another advantage of this approach is that one can obtain measures of bias (input and scale) in technical change which are observation-specific. This came as a by-product of making the functional coefficients of the estimated model fully nonparametric. We use firm-level data on U.S. electricity generation plants as an application of this methodology. Since output for the generation plants is assumed to be exogenous we used an input distance function instead of a production function. The method can, however, be used for production, cost and profit functions. The estimated model is expressed in growth (rates of change) form which automatically removes individual (plant-specific) effects (which are often ignored in empirical applications). To check superiority of the semiparametric model we compare results from a fully parametric flexible (translog) input distance function. Furthermore the parametric model is estimated with and without growth formulation. We find that TFP growth results from the semiparametric model is better than their parametric counterparts in the sense that it traces the observed TFP growth better. The differences are, however, smaller when the models based on growth formulation are compared.

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References

Baltagi B, Griffin J (1988) A general index of technical change. J Polit Econ 96(1):20–41
Brümmer B, Glauben T, Thijssen G (2002) Decomposition of productivity growth using distance functions: the case of dairy farms in three European countries. Am J Agr Econ 84:628–644
Chun H, Nadiri M (2008) Decomposing productivity growth in the U.S. computer industry. Rev Econ Stat 90(1):174–180
Das A, Kumbhakar S (2010) Productivity and efficiency dynamics in Indian banking: An input distance function approach incorporating quality of inputs and outputs. J Appl Econom n/a. doi: 10.1002/jae.1183
Denny M, Fuss M, May J (1981) Intertemporal changes in regional productivity in Canadian manufacturing. Can J Economics 14(3):390–408
Diewert W (1974) Functional forms for revenue and factor requirements functions. Int Econ Rev 15(1):119–130
Diewert W (1981) The economic theory of index numbers: A survey. Cambridge University Press, London
Färe R, Primont D (1995) Multi-output Production and Duality: Theory and Applications. Kluwer Academic Publishers, Boston
Härdle W, Mammen E (1993) Comparing nonparametric versus parametric regression fits. Ann Stat 21:1926–1947
Hayfield T, Racine J (2008) Nonparametric econometrics: The np package. J Stat Softw 27(5)
Hsiao C, Li Q, Racine J (2007) A consistent model specification test with mixed discrete and continuous data. J Econometrics 140:802–826
Jorgenson D (1995) Productivity. MIT Press, Cambridge, Mass.
Karagiannis G, Midmore P, Tzouvelekas V (2004) Parametric decomposition of output growth using stochastic input distance function. Am J Agr Econ 86:1044–1057
Key N, McBride W, Mosheim R (2008) Decomposition of total factor productivity change in the U.S. hog industry. J Agr Appl Econ 40(1):137–149
Kumbhakar S (2000) Estimation and decomposition of productivity change when production is not efficient: A panel data approach. Economet Rev 19:425–460
Kumbhakar S, Lovell K (2000) Stochastic Frontier Analysis. Cambridge University Press, Cambridge
Kumbhakar S, Lozano-Vivas A (2005) Deregulation and productivity: The case of Spanish banks. J Regul Econ 27:331–351
Kumbhakar S, Tsionas E (2010) Stochastic error specification in primal and dual production systems. J Appl Econom n/a. doi: 10.1002/jae.1100
Kumbhakar S, Heshmati A, Hjalmarsson L (1999) Parametric approaches to productivity measurement: A comparison among alternative models. Scand J Econ 101:405–424
Kumbhakar S, Lien G, Flaten O, Tveterås R (2008) Impacts of Norwegian milk quotas on output growth: A modified distance function approach. J Agr Econ 59(2):350–369
Li Q, Racine J (2006) Nonparametric Econometrics: Theory and Practice. Princeton University Press, Princeton
Li Q, Racine J (2010) Smooth varying-coefficient estimation and inference for qualitative and quantitative data. Economet Theor 26:1607–1637
Li Q, Huang C, Li D, Fu T (2002) Semiparametric smooth coefficient models. J Bus Econ Stat 20(3):412–422
Li Q, Maasoumi E, Racine J (2009) A nonparametric test for equality of distributions with mixed categorical and continuous data. J Econometrics 148:186–200

Nerlove M (1965) Estimation and identification of Cobb-Douglas production functions. Rand McNally & Company, Chicago

Oh D, Heshmati A, Lööf H (2009) Total factor productivity of Korean manufacturing industries: Comparison of competing models with firm-level data, unpublished manuscript

Robinson P (1988) Root-n-consistent semiparametric regression. Econometrica 56(4):931–954

Rungsuriyawiboon S, Stefanou S (2007) Dynamic efficiency estimation: An application to U.S. electric utilities. J Bus Econ Stat 25:226–238

Shephard R (1953) Cost and Production Functions. Princeton University Press, Princeton

Stevenson R (1980) Measuring technological bias. Am Econ Rev 70:162–173

Appendix A: Proof of $S_k = \beta_k,\ k = 2, \ldots, K$

This appendix proves that the allocative component in equation (10) is equal to zero when firms minimize cost subject to input distance function. The constrained optimization problem can be written as:

$$\min_X W'X \text{ subject to } \; -\ln X_1 = m(\ln \tilde{X}, \ln Y, t) \tag{16}$$

where W is a vector K input price vector; X is the vector of K inputs and $\tilde{X} = [\tilde{X}_2, \ldots, \tilde{X}_K]$. The Lagrangian of the above problem can be written as:

$$L = W'X + \lambda[\ln X_1 + m(\ln \tilde{X}, \ln Y, t)] \tag{17}$$

where λ is the Lagrange multiplier. The first-order conditions are:

$$\frac{\partial L}{\partial X_1} = W_1 + \lambda(\frac{1}{X_1} + \sum_{k=2}^{K} \frac{\partial m}{\partial \ln \tilde{X}_k} \cdot \frac{\partial \ln \tilde{X}_k}{\partial \ln X_1} \cdot \frac{\partial \ln X_1}{\partial X_1}) = 0 \tag{18}$$

and

$$\frac{\partial L}{\partial X_k} = W_k + \lambda(\frac{\partial m}{\partial \ln \tilde{X}_k} \cdot \frac{\partial \ln \tilde{X}_k}{\partial \ln X_k} \cdot \frac{\partial \ln X_k}{\partial X_k}) = 0 \; \forall k = 2, \ldots, K \tag{19}$$

The above first-order conditions can be simplified as:

$$W_1 X_1 + \lambda(1 - \sum_{k=2}^{K} \beta_k) = 0 \tag{20}$$

and

$$W_k X_k + \lambda \beta_k = 0 \; \forall k = 2, \ldots, K \tag{21}$$

respectively. Furthermore, we can simplify them as

$$\frac{W_k X_k}{W_1 X_1} = \frac{S_k}{S_1} = \frac{\beta_k}{1 - \sum_{k=2}^{K} \beta_k} \; \forall k = 2, \ldots, K \tag{22}$$

If we add these $K-1$ equations first and then add 1 to both sides, we get $S_1 = 1 - \sum_{k=2}^{K} \beta_k$, since $\sum_{k=1}^{K} S_k = 1$. Thus, $S_k = \beta_k, k = 2, \ldots, K$.

Appendix B: Derivation of partial derivatives of smooth coefficients

The appendix derives the partial derivative of the smooth coefficient vector $\hat{\Psi}(z)$ with respect to the l-th continuous variable Z_l.

$$\frac{\partial\hat{\Psi}(z)}{\partial Z_l} = \sum_{i=1}^{n} \frac{\partial A_i(z)}{\partial Z_l}\mathcal{Y}_i \tag{23}$$

where $A_i(z) = A^{-1}\mathcal{X}_i K(\frac{\mathcal{Z}_i - z}{h}) = (K^{-1}(\frac{\mathcal{Z}_i - z}{h})A)^{-1}\mathcal{X}_i$, and $A = \sum_{i=1}^{n} \mathcal{X}_i\mathcal{X}_i' K(\frac{\mathcal{Z}_i - z}{h})$.

From what follows, we let $K(\cdot)$ to represent $K(\frac{\mathcal{Z}_i - z}{h})$, in order to simplify notation. $K(\cdot)$ is a product (scalar) kernel function:

$$K(\cdot) = \prod_{s=1}^{q} K(\frac{Z_{si} - z_s}{h_s}) \tag{24}$$

For the lth continuous variable Z_l,

$$K(\frac{Z_{li} - z_l}{h_l}) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}(\frac{Z_{li} - z_l}{h_l})^2) \tag{25}$$

$$\begin{aligned}\frac{\partial A_i(z)}{\partial Z_l} &= -(K^{-1}(\cdot)A)^{-1}\frac{\partial K^{-1}(\cdot)A}{\partial Z_l}(K^{-1}(\cdot)A)^{-1}\mathcal{X}_i \\ &= -K(\cdot)A^{-1}\Big(\frac{\partial K^{-1}(\cdot)}{\partial Z_l}A + \frac{\partial A}{\partial Z_l}K^{-1}(\cdot)\Big)K(\cdot)A^{-1}\mathcal{X}_i\end{aligned} \tag{26}$$

where

$$\frac{\partial A}{\partial Z_l} = \sum_{i=1}^{n} \mathcal{X}_i\mathcal{X}_i'\frac{\partial K(\cdot)}{\partial Z_l} = \sum_{i=1}^{n} \mathcal{X}_i\mathcal{X}_i'(\frac{Z_{li} - z_l}{h_l^2})K(\cdot) \tag{27}$$

and

$$\frac{\partial K^{-1}(\cdot)}{\partial Z_l} = \frac{\partial K^{-1}(\cdot)}{\partial K(\cdot)} \cdot \frac{\partial K(\cdot)}{\partial Z_l} = -(\frac{Z_{li} - z_l}{h_l^2})K^{-1}(\cdot) \tag{28}$$

Table 1 Parameter estimates (parametric model)

α_t	-0.0987***	δ_{2t}	-0.0114***	θ_2	-0.4167**	θ_{33}	-0.0571***
	(0.0286)		(0.0026)		(0.1867)		(0.0220)
α_{tt}	-0.0004	δ_{3t}	0.0197***	θ_{22}	0.0136	λ_{1t}	0.0025
	(0.0005)		(0.0030)		(0.0209)		(0.0016)
α_1	0.6173***	δ_{12}	0.0222**	θ_3	1.5369***	R^2	0.9027
	(0.2201)		(0.0107)		(0.2140)		
α_{11}	-0.0358***	δ_{13}	-0.0513***	θ_{32}	0.0447**		
	(0.0137)		(0.0115)		(0.0183)		

1. The parameters are estimated from (15).
2. The numbers in the parentheses are the asymptotic standard errors.
3. $*$,$**$,and $***$ means significance at 10%, 5%, and 1% level, respectively.
4. δ_{2t} and δ_{3t} measures input bias; $-\lambda_{1t}$ measures scale bias; α_{tt} measures technical neutrality.

Table 2 Bandwidth and percentage of violations

$\mathcal{Z}$ **Variable**	t	$\ln \tilde{X}_2$	$\ln \tilde{X}_3$	$\ln Y$
Bandwidth	1.419598	0.4221742	0.3770268	0.5959304
$\mathcal{X}$ **Variable**	Intercept	$\dot{\tilde{X}}_2$	$\dot{\tilde{X}}_3$	$\dot{Y}$
Coefficient	$\hat{\beta}_0$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\gamma}$
Percentage of violations: Semiparametric	4.48%	0.92%	0.31%	0.61%
Percentage of violations: Parametric	11.39%	0%	0%	0%

Table 3 Summary statistics of functional coefficients

	$\hat{\beta}_0$ (TC)	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\gamma}$
Semiparametric				
Q1	0.0044	0.0765	0.6602	-0.2329
	(0.0002)	(0.0017)	(0.0085)	(0.0091)
Q2	0.0065	0.1214	0.7946	-0.1188
	(0.0002)	(0.0036)	(0.0059)	(0.0060)
Q3	0.0097	0.2110	0.8613	-0.0747
	(0.0003)	(0.0079)	(0.0038)	(0.0017)
Parametric (Growth IDF) from (15)				
Q1	0.0040	0.1301	0.6357	-0.1827
	(0.0003)	(0.0019)	(0.0045)	(0.0012)
Q2	0.0091	0.1651	0.6949	-0.1568
	(0.0003)	(0.0019)	(0.0042)	(0.0023)
Q3	0.0130	0.1964	0.7573	-0.1202
	(0.0004)	(0.0020)	(0.0043)	(0.0017)
Parametric (Log IDF) from (11)				
Q1	0.0077	0.2624	0.4759	-0.4625
	(0.0001)	(0.0039)	(0.0025)	(0.0038)
Q2	0.0093	0.3378	0.5207	-0.3947
	(0.0001)	(0.0039)	(0.0030)	(0.0033)
Q3	0.0108	0.3890	0.6033	-0.2940
	(0.0001)	(0.0038)	(0.0069)	(0.0041)

The numbers in the parentheses are the bootstrapped standard errors.

Table 4 Biases in technical change

	Neutrality	Input bias		Scale bias
	$\partial\hat{\beta}_0/\partial t$	$\partial\hat{\beta}_2/\partial t$	$\partial\hat{\beta}_3/\partial t$	$-\partial\hat{\gamma}/\partial t$
Semiparametric				
Q1	-0.0015	-0.0360	-0.0055	-0.0366
	(0.0001)	(0.0021)	(0.0029)	(0.0023)
Q2	-0.0002	-0.0076	0.0178	-0.0102
	(0.0001)	(0.0009)	(0.0011)	(0.0008)
Q3	0.0009	0.0066	0.0521	0.0031
	(0.0001)	(0.0013)	(0.0025)	(0.0010)
Parametric				
	-0.0004	-0.0114	0.0197	-0.0025
	(0.0005)	(0.0026)	(0.0030)	(0.0016)

The numbers in the parentheses are the standard errors.

Fig. 1 Kernel density plots of functional coefficients

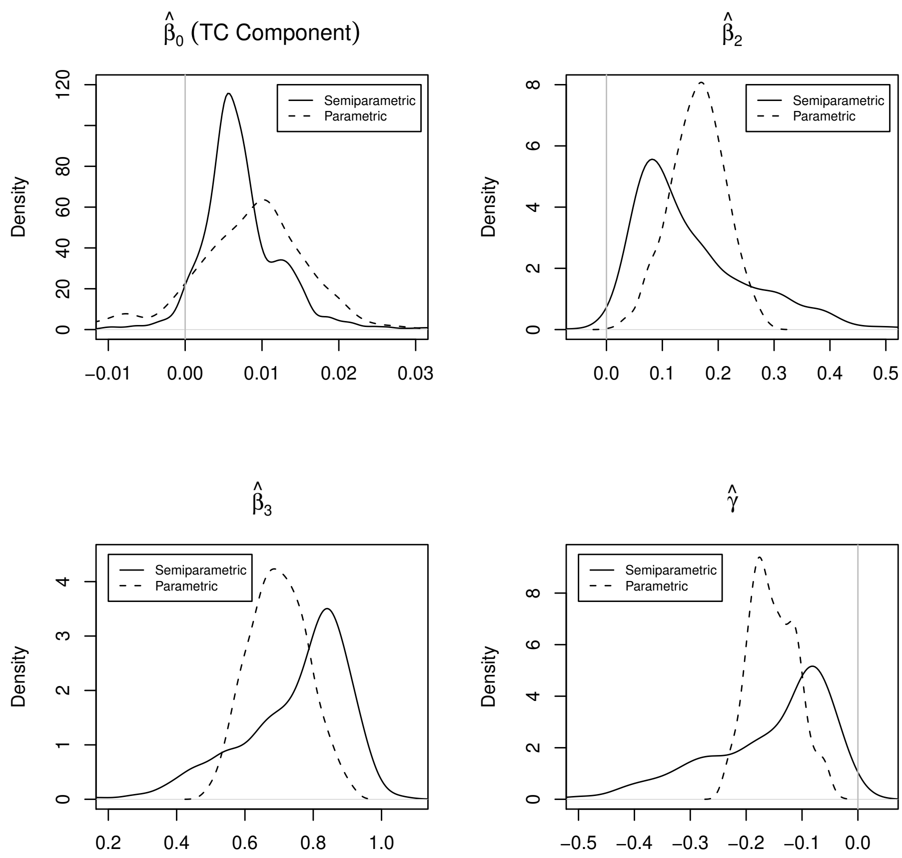

Fig. 2 Scatter plots of functional coefficients

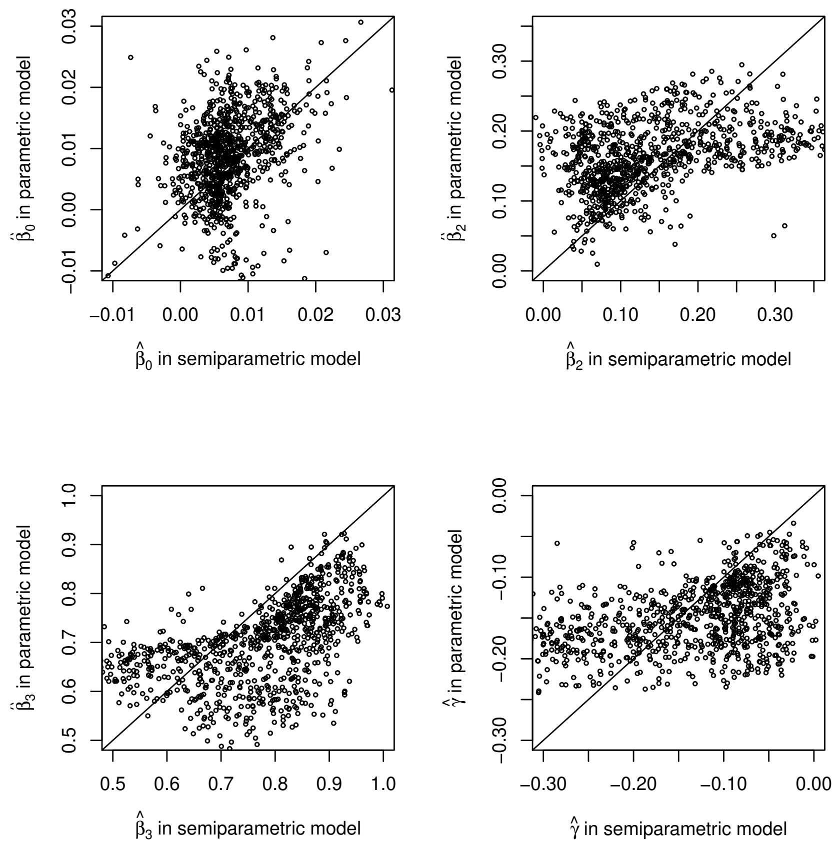

Fig. 3 Functional coefficients in parametric model

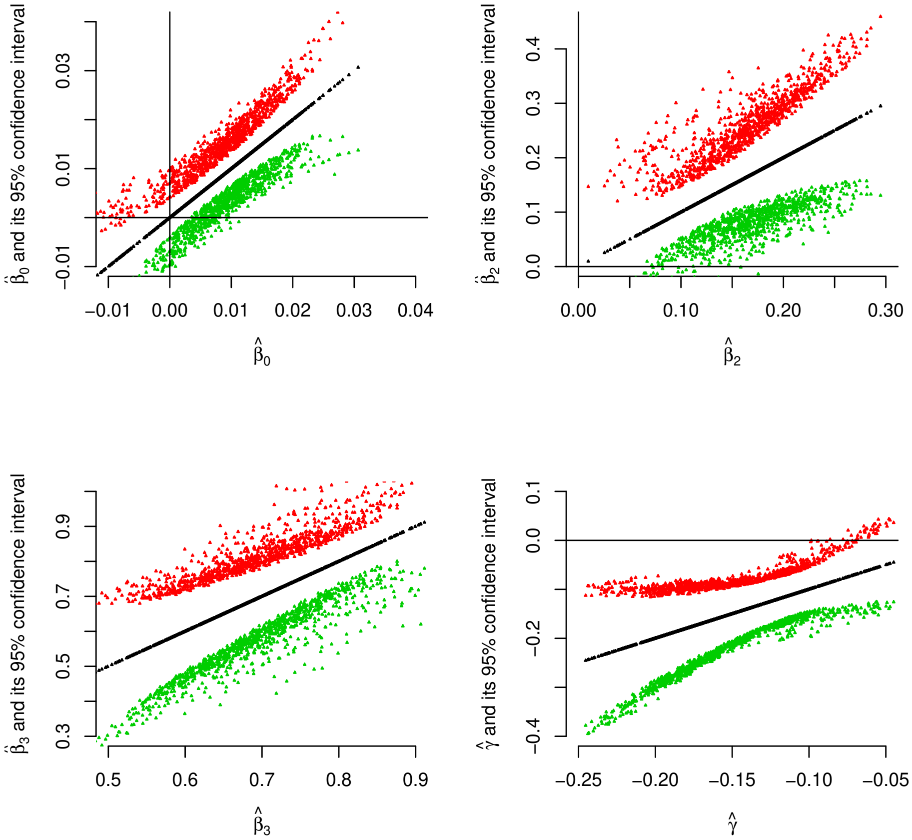

Fig. 4 Functional coefficients in semiparametric model

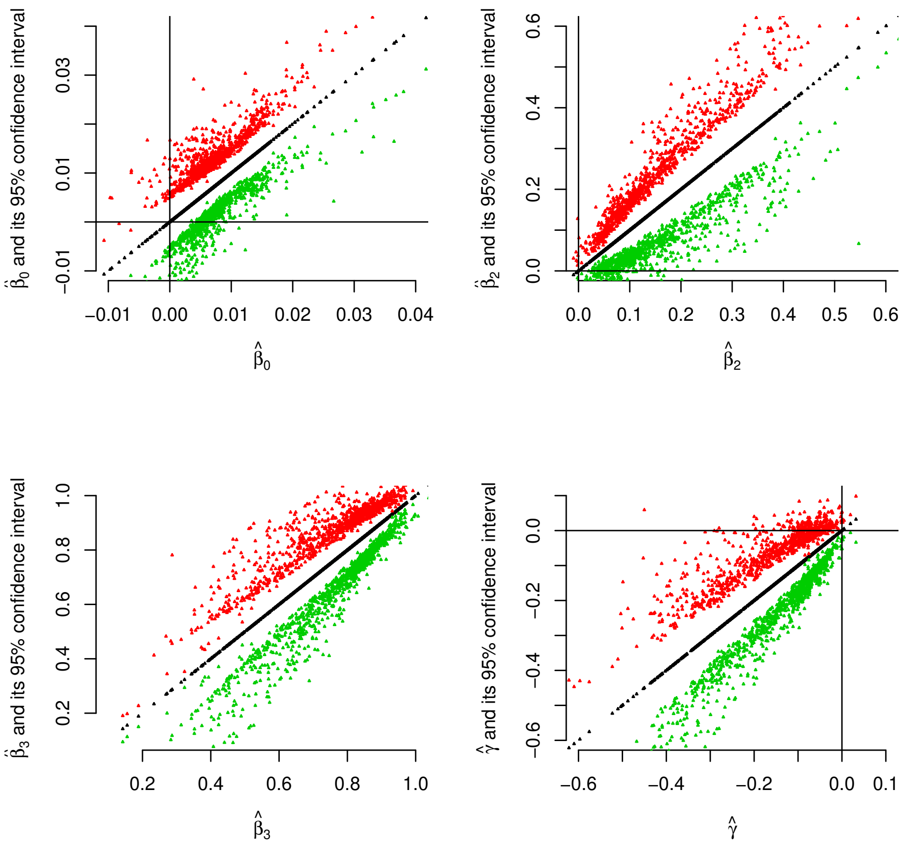

Fig. 5 TFP growth/indices over time

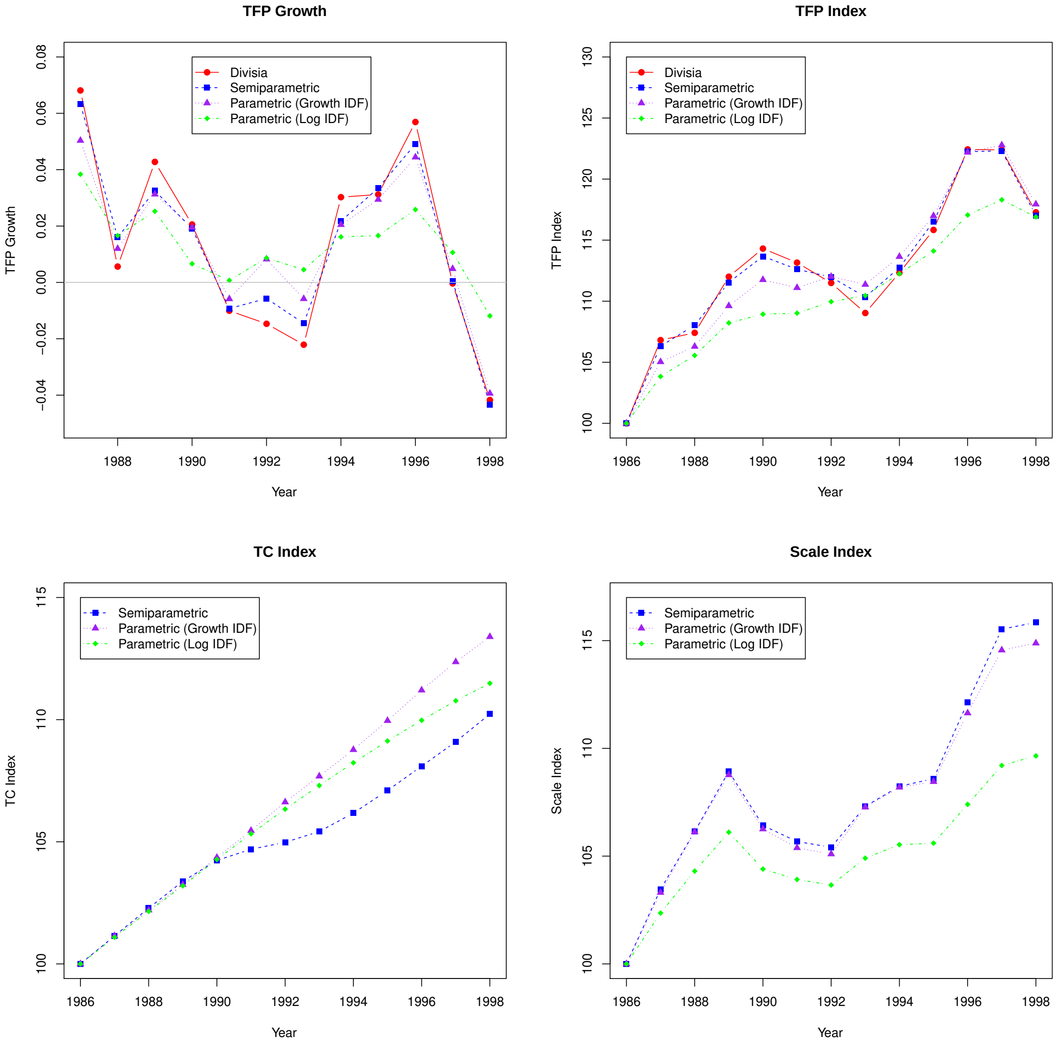

Fig. 6 Residual (unexplained) component of TFP growth

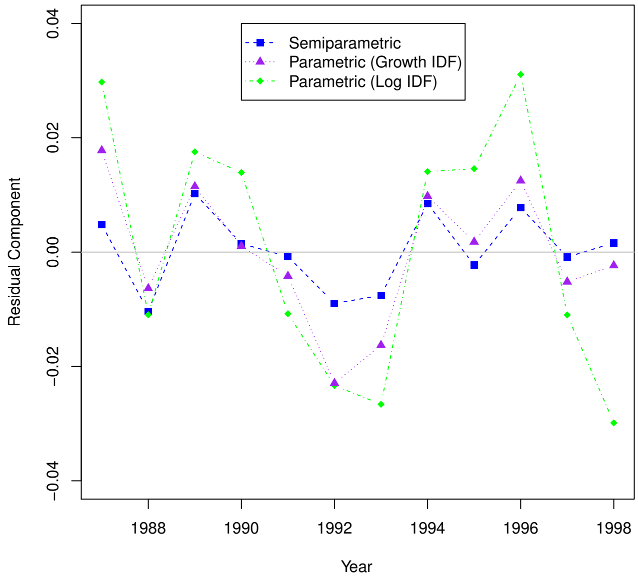

Fig. 7 Biases in technical change