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# ESTIMATION OF THE FISHER EFFECT ON THE TERM STRUCTURE OF INTEREST RATES EMPLOYING A TERM STRUCTURE OF INFLATIONARY EXPECTATIONS

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# ESTIMATION OF THE FISHER EFFECT ON THE TERM STRUCTURE OF INTEREST RATES EMPLOYING A TERM STRUCTURE OF INFLATIONARY EXPECTATIONS

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## DEDICATION

This dissertation is dedicated to William B. Stronge -- teacher, mentor, colleague, and, above all, friend. His encouragement and support have served as a constant source of strength. This dissertation is also dedicated to Edwin L. Marsden, Jr. Without his patience, understanding, and continuing support, the frustrations and difficulties would have been impossible for me to bear.

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ABSTRACT

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This study deals with the estimation of the effect of a term structure of inflationary expectations on the term structure of interest rates. By estimating the Fisher effect over the entire term structure, this analysis captures the associational effects between values of the interest rates along a term structure and values of inflationary expectations along a term structure.

The study contains a discussion of the Fisher hypothesis as it was developed and tested by Irving Fisher. Models of inflationary expectations and previous applications of these to test the Fisher effect are dicussed.

A method of constructing a term structure of inflationary expectations is developed. Inflationary expectations are estimated and proof is offered to

demonstrate that these expectations are statistically rational.

A model for estimating nominal interest rates from yields on Treasury notes and bank discounts on Treasury bills is developed. The technique is applied to obtain estimates of the term structure of nominal interest rates, monthly, for the period of January 1970 through November 1982.

In order to summarize the term structures of inflationary expectations and the term structures of nominal interest rates as functions, this study estimates empirical term structures using cubic exponential spline functions. There is a discussion of spline methodology from a modeling perspective and from an econometric perspective.

The Fisher effect of the term structure of inflationary expectations on the term structure of interest rates is estimated by pooling the cross-section data described by the coefeficients of the cubic exponential splines, and the time-series data. The evidence does not reject the Fisher hypothesis of a complete pass-through of inflationary expectations to nominal interest rates in a world of taxes. The evidence also suggests that associational effects along the term structures are present.

# CHAPTER I

## INTRODUCTION

The proposition that the nominal interest rate consists of two parts, the real rate of interest and the expected rate of inflation, was presented most effectively by Irving Fisher in a series of works between 1896 and 1930. Specifically, Fisher hypothesized that a change in the expected rate of inflation over a given time horizon will produce an equivalent change in nominal interest rates of financial assets of equivalent maturity. This hypothesis, known as the Fisher effect, has been and continues to be the subject of wide-spread theoretical and empirical interest. Fisher's initial interest in examining the relationship between inflation and nominal interest rates seems to have originated from an interest in the controversy over the wealth re-distribution effects associated with the bimetallic era in the United States. His hypothesis extended to the argument that there would be no net re-distribution of wealth from lenders to borrowers (borrowers to lenders) if inflation (deflation) were correctly anticipated.

During his life-time Fisher found no empirical support for the Fisher effect. In the 1970's a number of simple regression tests of the Fisher effect did seem to find evidence which supported the full pass through of expected inflation to nominal interest rates.[1] Support for the hypothesis seems, however, to have been limited to sample data for the period of 1953 through 1971.[2] For the post 1971 period, models of the Fisher effect became more complex. Rather than regressing nominal interest rates on some measure of expected inflation alone, the models included additional independent variables. Some of these variables were included to account for trend or cycle in the real economy. Some of these variables were included to account for the effects of monetary policy and fiscal policy. Some of these variable were included to account for the impact of wealth on nominal interest rates. Some versions of the estimates of the Fisher effect also included estimates of the impact of tax-rate effects and the effects of supply-side shocks.

## I. HYPOTHESIS

In this study we re-examine the Fisher effect focusing on a different missing-variables problem. At any point in time there is a spot interest rate for every maturity. Sargent (1979b) has shown that there is significant information about any one of these spot interest rates contained in its relationship to the other spot interest rates along the term structure. These relational or associational effects are ignored when one regresses the nominal interest rate for some given maturity on expected inflation. We also maintain that there is a term structure of expected inflation since there is no a priori reason to believe that economic agents would hold the same estimate of expected inflation at time t for all maturity horizons t+m into the future. In fact it is not only the relationship between the nominal interest rates across the term structure that is important in measuring the Fisher effect but it is also the relationship between the expected inflation rates as well. This thesis proposes a test of the Fisher effect over the entire term structure. A full test of the Fisher effect requires that we develop a method of generating a term structure of spot rates of interest. It requires that we develop a method of generating a term structure of inflationary expectations. Finally, it requires that we estimate the effect of the entire term structure of inflationary expectations on the entire term structure of nominal interest rates.

In order to distinguish between the usual representation of the Fisher equation and our version of the Fisher effect we need to distinguish between a particular spot rate and a set of spot rates. We write the usual Fisher identity as:

(1.1)

$$ {}_m i_t = {}_m r^e_t + {}_m I^e_t $$

where:

$_m i_t$ = a single nominal interest rate at time t, for maturity m

$_m r^e_t$ = a single expected real rate of interest at time t, for maturity m

$_m I^e_t$ = a single expected rate of inflation at time t, for maturity m

We write our term structure version of the Fisher identity as:

(1.2)

$$ {}_{t+m} i_t = {}_{t+m} r^e_t + {}_{t+m} I^e_t $$

where:

$_{t+m} i_t$ = the complete set of nominal interest rates at time t, for term to maturity running from m=1 to m=n

$_{t+m} r^e_t$ = the complete set of expected real rates of interest at time t, for term to maturity running from m=1 to m=n

$_{t+m}I^e_t$ = the complete set of expected rates of inflation at time t, for term to maturity running from m=1 to m=n

Our estimating equation of the Fisher effect is:

(1.3)

$$_{t+m}i_t = {}_{t+m}A_t + {}_{t+m}B\left({}_{t+m}I^e_t\right) + {}_{t+m}u_t$$

where A and B are the parameters to be estimated. We would support the hypothesis of the Fisher effect that a change in expected inflation would produce an equivalent change in nominal interest rates if:

(1.4)

$$_{t+m}B_t = 1.$$

This is the appropriate hypothesis in a world with no taxes.

## II. OUTLINE OF STUDY

We will begin this study by presenting the development of the Fisher hypothesis in Chapter II. We do this not only to provide essential theoretical background, but also to illustrate the difficult task of the empirical evaluation of the Fisher effect. In the discussion of Fisher's development of the hypothesis we will present arguments which support our approach to estimating the Fisher effect across the term structure of nominal interest rates, where inflationary expectations are also expressed as a term structure.

While the computational technology of his day was limited, Fisher's empirical work stands as a cornerstone of modern econometrics. Even given the sophistication of his empirical work, Fisher never found empirical evidence to support his hypothesis. Finding no support for the hypothesis, Fisher devoted extensive work to the development of a hypothesis of agent behavior in borrowing and lending decisions which would be consistent with the evidence. He spent considerable effort in developing a more complete model of interest.

The wide-spread application of the Muth concept of rational expectations re-generated interest in simple regression estimates of the Fisher effect. One contention of the rational expectations approach was that failure to find support for the full Fisher effect rested with incorrect measurement of inflationary expectations.[3] The essential attack here was that distributed lags of past prices were not the best estimates of expected prices. The use of the Livingston Survey Data was also severely criticized on grounds that it contained measurement errors which produced biased expectations.

Our model of expected inflation (in Chapter III) is based on the criterion of rationality. We discuss alternative approaches to estimating models of expectations formation which meet the rationality criterion. We develop a rational expectations model based on passive learning. This model is used to generate a data base of term structures of both actual annualized rates of inflation and expected annualized rates of inflation. We present statistical evidence, which supports our contention that the model produces rational expectations.

In order to test our stated hypothesis, we have to construct both a term structure of the expected rate of inflation and a term structure of spot interest rates. The development of these two data bases is a major task of this thesis and we discuss at length the manner in which these data are estimated.

Our term structure of spot interest rates is derived from the bank discount on Treasury bills and from the coupon and yield on Treasury notes. In order to generate this data base we estimate the term structure of spot interest rates by adapting (in Chapter IV) a technique developed by Carleton and Cooper (1976). This technique requires estimation of the present value and the computation of time, in months to maturity, for each bill and note. The spot interest rate for each maturity is estimated from the slope coefficient of the regression of present value on term to maturity.

Our time series of term structures run from January 1970 through November 1982. Extending the sample back in time would have limited the number of terms to maturity in the term structure. Even the inclusion of the early 1970s caused us to reduce the number of terms to maturity to seventy-six months, because of the sparceness of Treasury notes for longer maturities. We end our sample with November 1982, after which the Treasury Bulletin switched from a monthly publication schedule to a quarterly publication schedule.

With 76 terms to maturity in each term structure and a sample of 155 months, each data set contains 11,780 observations. In order to estimate the effect of the term structure of expected inflation on the term structure of nominal interest rates we are required to pool the cross-section and the time-series data. Under existing computational technology, both in terms of available hardware and software, this is not a tractable problem. In order to resolve this technical constraint we represent each of our monthly term structures by means of spline functions.

A spline function is a piecewise, continuous function. The pieces of the continuous function are joined at knots, or joint points. Each segment of the spline is itself a function. The entire spline over the segments is continuous by virtue of continuity restrictions imposed on the joined segments at the knot points. Since an understanding of how these spline functions compactly summarize entire term structures is essential to an understanding of our estimates of the Fisher effect, we present extensive discussion of spline methodology in Chapter V.

The application of splines to term structure modelling has gained wide-spread acceptance, the most notable examples being Huston McCulloch's use of splines to produce a tax adjusted yield curve (1975a) and to estimate the liquidity premium (1975b). Other applications may be found in Vasicek and Fong (1982), Langetieg and Smoot (1981), and Thies (1982). These applications fit splines to the present value function where term to maturity is the independent variable. Our application of splines is unique in that we apply the spline to curve-fit the term structure of our estimates of the spot interest rates.

We have chosen a cubic exponential spline to curve-fit our term structures and we have divided our term to maturity into five intervals with four knot points. Within each segment of our spline function, that is, in each interval, we estimate a cubic exponential function. Each function within each interval is described by four estimated coefficients: one on the dummy value of being in the interval, and three on the three values of the cubic exponential. Given five intervals, we can compactly summarize all 76 terms to maturity for a given term structure with the 20 estimated spline coefficients. We will denote the spline function by Q(x) and the pieces of the spline function by p(x). We will denote the spline function for the term structure of nominal interest rates as, ${}_{t+m}i_{Q(x)}$. Similarly, we will denote the spline function for the term structure of expected inflation as ${}_{t+m}I^{e}_{Q(x)}$.

Using our spline notation the Fisher model can be written as:

(1.5)

$${}_{t+m}i_{Q(x)} = {}_{t+m}\alpha_{Q(x)} + {}_{t+m}\beta\left({}_{t+m}I^{e}_{Q(x)}\right) + {}_{t+m}v_{Q(x)}$$

Using our spline notation we can rewrite our estimating equation (1.3) as:

(1.6)

$${}_{t+m}i_{Q(x)} = {}_{t+m}A_{Q(x)} + {}_{t+m}B\left({}_{t+m}I^{e}_{Q(x)}\right) + {}_{t+m}u_{Q(x)}$$

Similarly, we can restate our null hypothesis, equation (1.4), as follows:

(1.6)

$${}_{t+m}B_{Q(x)} = 1.$$

The Fisher effect of a full adjustment of the term structure of nominal interest rates, splined, to the term structure of expected inflation, splined, would be supported if the estimated relationship between the two functions has a slope of one.

Examining the Fisher effect over the entire term structure as described above has advantages over the usual method of estimating the effect. The usual approach is to regress a nominal interest rate on expected inflation of which both are for a given maturity. The adjustment of the nominal interest rate to variation in associated near-term maturities is omitted from the usual estimating equation and therefore contributes to omitted variables bias. By pooling the cross-section elements with the time series of the spline functions, our estimates of the Fisher effect avoids this source of omitted variables bias.

Two types of pooled regressions are run (in Chapter VI) to estimate the Fisher effect of a term structure of expected inflation on the term structure of nominal interest rates. The first estimates the pooled regression using dummy variables to control for time. The second estimates the pooled regression using a joint estimation technique similar to GLS (Generalized Least Squares), as outlined by Theil (1971). In this latter procedure the OLS (Ordinary Least Squares) estimates of paired maturities of nominal interest rates, splined, and expected inflation, splined, are re-estimated employing the information contained in the pooled OLS residuals. The regression parameters in the re-estimated equations are parameter estimates for the spline pairs. These re-estimated parameters are conditioned by the entire pooled set of term structure information. The use of dummy variables in the first procedure treats the entire data set as one equation whereas the joint estimation technique views the entire data set as a system of equations. Each equation is dependent on all the other equations in the system.[4]

Theil's, joint-estimation procedure has two interesting extensions. First, we can conduct a chi-square test of the sets of linear restrictions, making it possible on the basis of this test, to determine whether pooling improves the estimation of the Fisher effect across the term structure as compared to estimating the Fisher effect one maturity at a time. Second, one can use the estimated slope coefficients to judge the extent to which the size of the Fisher effect is different for different maturities in the term structure.

We conclude (in Chapter VII) that the evidence does not support the hypothesis that ${}_{t+m}B = 1$. The results of the regressions with dummy variables provide estimates of ${}_{t+m}B > 1$. The results of the estimates of the Fisher effect using the joint estimation technique provide estimates of ${}_{t+m}B < 1$. The chi-square test does indicate that the associational effects along the term structure are significant omitted-variables.

# CHAPTER I

## CHAPTER NOTES

1. See Gibson (1974), Pyle (1972) and Lahiri (1976). Other papers frequently cited are Yohe and Karnosky (1969) and Lucas (1980).

2. See Robert J. Shiller (1979) and Lawrence H. Summers (1983).

3. See Pearce (1979), Tanzi (1980), and Wisley (1982).

4. The conditioning of each regression estimate can be viewed as imposing sets of linear restrictions in the sense of Sims (1980) identification by including cross equation restrictions.

# CHAPTER II

## THE FISHER EFFECT

While Fisher was not the first economist to describe the systematic relationship between inflation and nominal interest rates, his description has been the most enduring (Humphrey, 1983). His discussion of this relationship was thorough and based on a complete theory of interest. Fisher derived his theory of interest from the theory of consumer behavior in an inter-temporal framework.

He never failed to focus on the inter-temporal nature of the problem. As long as there existed some durable commodity, the market value of that commodity would be derived from the value of its expected future income. It was Fisher's willingness to declare any commodity as durable, if it was not instantaneously consumed, that permitted him to develop a term structure of interest rates which discounted these time dated streams of income (services).

## I. THE THEORY OF INTEREST AND EMPIRICAL EVIDENCE: IRVING FISHER - 1896 TO 1930

To describe more carefully Fisher's theory of interest as it was developed, one must consider in chronological order three major works by Fisher: "Appreciation and Interest," Publications of the American Economic Association, 1896; The Nature of Capital and Income, 1906; and The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It, 1930 (The Last of these will hereafter be referred to as The Theory of Interest).

The first of these was not a theory of interest but rather a meticulous development of the mathematical rules which govern the inter-temporal valuation of commodities where the measure of commodity value was varying. It was in "Appreciation and Interest" that Fisher outlined the relationship between yields, 'actuarial average', and spot interest rates n periods into the future. He allowed for time varying rates of interest and for time varying rates of inflation (Fisher, 1896, p. 26). He assumed that the rate of appreciation (inflation), though varying, was foreknown for each of the time periods. (This time varying aspect of the Fisher effect seems to have been neglected both in the subsequent literature and in empirical studies of the Fisher effect.)

After presenting the mathematical relationships between spot interest rates, yields, nominal interest, real interest, and inflation, Fisher turned to the task of measuring the relationships empirically. His conclusions, after methodically examining three time series cases and a cross-section of seven cases over time, were stated as four "facts" which systematically emerged from these empirical relationships:

> 1) High and low prices are directly correlated with high and low rates of interest;
> 2) Rising and falling prices and wages are directly correlated with high and low rates of interest;
> 3) The adjustment of interest to price (or wage) movements is inadequate;
> 4) This adjustment is more nearly adequate for long rather than for short periods. (Fisher, 1896, p.75)

He also concluded that the failure of nominal interest to adjust fully was because of lack of foresight (Fisher, 1896, p.67).

Fisher began The Theory of Interest with a summary of his earlier text The Nature of Capital and Income, 1906. The Nature of Capital and Income is often neglected in the study of Fisher's theory of interest. However, it offers a great deal of insight into Fisher's methodology and in reading it one sees why Fisher felt compelled to review it before proceeding to The Theory of Interest. The Nature of Capital and Income prepared the framework for Fisher's theory of interest. He carefully defined capital as any commodity which yields a stream of services. He put aside the arguments about durability, arguing instead that anything is capital which is not instantaneously consumed. Income was the stream of services from a stock of capital. He defined interest as the expected rate of return on capital.

> The value of capital at any instant is derived from the value of the future income which that capital is expected to yield. . . . The principal of present worth is of fundamental importance in the theory of value and prices. It means that the value of an article of wealth or property is dependent alone on the future, not the past" (Fisher 1906, p.88).

In developing his definition of income Fisher was relentless in separating stocks from flows. But more relevant for the discussion here, was his development of a definition of purchasing power. True to his definition of capital, purchasing power included not only the ability to buy a stock of goods but also the ability to purchase a flow of income. He used this concept to define and differentiate between the two common usages of the term interest: interest as the price of capital and interest as a premium for inter-temporal exchanges of goods. The rate of interest was defined as the price of capital in the sense that X dollars of capital could be purchased for Y dollars per year of income. The rate of interest was defined as a premium in the sense that the price of X dollars in one year's goods is Y dollars in the next year's goods. 'Income value' and the 'Capital value' as Fisher defined them were linked together by the rate of interest.

In The Nature of Capital and Income, Fisher also continued the work he began in "Appreciation and Interest" by clarifying, detailing, and expanding upon the mathematical rules for computing yield and interest under various assumptions. He began again with the assumption of certainty about the flow of the future stream of income, and developed the discount curve as an exponential function, considering both the case of discrete and the case of continuous compounding. He discussed why market value and book value of stocks were likely to differ. He considered the accrual problem and demonstrated the changes that occur in present value around installment payments. He discussed the valuation problems associated with realized income and earned income as they affected appreciation and depreciation of capital.

By the time Fisher wrote The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It, he had developed a framework of definitions, a set of mathematical relations, mathematically descriptive tools, and a whole complex of empirical examples, based on different underlying assumptions specific to real problems in valuation. Fisher's formal theory of interest, which was essentially deductive, was also rooted in induction and empiricism. His theoretical framework was based on intertemporal utility maximization by individuals. The sum of the collective decisions by individuals regarding borrowing and lending determined the market clearing rate of interest.

At the end of his discussion of the theory of interest, Fisher returned to the question of the relationship between interest and changing prices. He expanded his empirical work to include more examples than he had presented in 1896, and he applied more sophisticated empirical tests. However, as in 1896, he did not find evidence of full adjustment of nominal interest to changing prices. The real interest rate showed substantial variability as a result of lack of foresight:

> If perfect foresight existed, continuously rising prices would be associated not with a continuously rising rate of interest but with a continuing high rate of interest, falling prices would be associated not with a continuously falling rate of interest, but with a continuing low rate of interest, and a constant price level would be associated with a constant rate of interest-assuming, in each case, that other influences than price change remained the same. . . . One obvious result of such an ideally prompt and perfect adjustment would undoubtedly be that money interest would be far more variable than it really is and that it was translated into real interest this real interest would be comparatively steady. What we actually find, however, is the reverse -- a great unsteadiness in real interest when compared with money interest. (Fisher, 1930, pp. 411-413)

Based on the changes in the real interest rate that resulted from the incomplete adjustment of nominal interest rates to expected inflation, Fisher proposed an explanation of the business cycle (Fisher, 1896, p. 66). He suggested an explanation of the stickiness of nominal interest rates based on inequality of foresight. The "captains of industry," those who borrow, had superior foresight to lenders. These borrowers saw rising prices before lenders saw the change in price. They were, therefore, willing to pay higher interest for the same amount of loans while lenders where willing to supply the same amount of loans for the same interest (Fisher, 1896. pp. 76-79).

Between 1896 and 1933 one finds no revisions in Fisher's empirical work of the findings of incomplete adjustment or in the conclusions he drew from those findings. During this period, Fisher continued to maintain an hypothesis of imperfect foresight between borrowers and lenders with respect to the rising prices.

However, by 1930, Fisher had added to this explanation of volatility of real interest rates a discussion of two additional sources of variation in real interest rates. First, Fisher noted that rising income would mean higher interest rates, and falling income would imply lower interest rates. Such variations in income would be observed over both secular and cyclical changes in the economy. The real rate of interest would not, therefore, be a constant either secularly or cyclically.

The second source of systematic variation in real rates was changes in the money supply because of changes in bank reserves. Fisher felt that the effect of changes in bank reserves was a short run variation in interest rates. Over the business cycle as income increased there would be an increase in the demand for goods and for borrowing. This would push interest rates up and cause bank reserves to fall. Since lending would reduce bank reserves, the rate of growth in the supply of money and credit would ultimately decrease to zero, driving interest rates up even higher. At this point the effective demand would be reduced and prices would fall. The net result would be a high real rate of interest and falling prices (Fisher, 1930, pp.444-450).

In many studies of the Fisher effect, variables such as the rate of growth in income and the rate of growth in the money supply are included to model shifts in the real rate of interest.[1] These variables are not included in our estimates of the Fisher effect.

## II. THE FISHER HYPOTHESIS: MORE RECENT STUDIES

A number of empirical tests of the simple model of the Fisher effect have been done. Some of the reported estimates, Gibson (1974), Pyle (1974), Lahiri (1976) and Yohe and Karnosky (1969) tend to support the Fisher hypothesis. However, studies of more complex models, those including variables assumed to effect real interest rates, tend to provide evidence suggesting that the simple regression model of the Fisher effect is incorrectly specified. Examples would include Summers (1983), Wilcox (1983), Tanzi (1980) and Mullineaux (1980).

Additionally, studies have shown that there is a positive effect on nominal interest rates associated with income taxes. However, in a world of taxes the slope coefficient in a simple regression model of the Fisher effect would have to be larger than one. A number of empirical studies have not found coefficients significantly greater than one. Examples can be found in Cargill (1977); McCulloch (1975a); and Carr, Pesando, and Smith (1982). The implication is that the simple regression studies which have reported a slope coefficient equal to one, do not support a full pass through of expected inflation to nominal interest rates in a world of taxes.[2]

A number of empirical studies have derived and estimated modified versions of the Fisher hypothesis from macroeconomic models. Recent examples include Peek (1982), Taylor (1982), and Mehra (1984). Such models incorporate tax effects, but also incorporate Mundell (1963) and Tobin (1965) effects. In these models the estimated coefficients on expected inflation rate need not exceed one in a world of taxes, if other economic factors offset the influence of expected inflation.

The theoretical attack on the simple regression method of estimating the Fisher effect is that the nominal interest rate reflects systematic variation in the real rate of interest as well as expected inflation. Simple regression estimates of the Fisher effect are thought to be biased by missing variables.

Beyond the issue of the appropriate specification of the Fisher model, a large body of literature has addressed the problem of modeling inflationary expectations. In Fisher's empirical analysis he employed a pure auto-regressive model of the distributed lag of past rates of change in prices to describe inflationary expectations (Fisher 1896 and 1930). Subsequently, many of the empirical estimates of the Fisher effect tended to follow Fisher using lagged prices to measure expected inflation. A number of these studies applied a restriction on the weights assigned to the lag structure and these weights were implicitly required to sum to one. This is equivalent to requiring that inflation is eventually fully anticipated. The use of the distributed lag and linearization by a priori restrictions not generated from a theoretical model of expectations are subject to Muth's (1961) criticism.

A number of alternative inflation expectations models have been proposed and applied to tests of the Fisher effect. These generally fall into four categories: (1) adaptive, (2) extrapolative, (3) a mixed model attributed to Frankel (1975) which combines regressive and adaptive expectations, and (4) error learning models. Only the adaptive (in certain cases) and the error learning models, when properly estimated, satisfy the Muth criteria for parameterization under rational expectations.[3] In general, both on a theoretical and empirical basis, adaptive and error learning models are considered to be superior methods for modeling inflationary expectations.[4]

Tests of the Fisher effect using the four models of inflationary expectations have included data based both on various price indices and expectations data. The expectations data has generally been the Livingston Survey.[5] Studies which have been based on survey data of inflationary expectations are generally considered to contain measurement error. The Livingston Survey has been found to utilize past information on price changes inefficiently. Such survey data demonstrates a systematic bias to underestimation of expected inflation.[6] A direct consequence of this bias is that estimates of the Fisher effect using these data perform poorly relative to rational expectations models of inflationary expectations.

Jacob and Jones (1980) have shown, however, that previous studies defined the rational expectations generating process in too narrow a form. They have demonstrated both theoretically and empirically that error learning models can capture the inflationary expectations described by the Livingston data in a manner consistent with rational expectations.

# CHAPTER II

## CHAPTER NOTES

1. Examples of studies which incorporate some of these variables are Tanzi (1980), Mullineaux (1980) and Sargent (1981).

2. Peek (1982) notes that this tax-adjusted Fisher effect hypothesis is overly stringent and that empirical studies have not been able to reject either a properly specified tax-adjusted null hypothesis or a properly specified non-tax-adjusted null hypothesis. Peek proposed an alternative approach of testing the Fisher effect on a hypothesis derived from a full macro model in which the real and nominal interest rates are after-tax rates.

3. The issue of proper estimation of an unobservable variable is discussed extensively in both Chow (1981) and Wallis (1981). Wallis (1981) contains the more extensive discussion of some of the cases where Muth's adaptive scheme is not sufficient to model rational expectations.

4. As suggested earlier a theoretical discussion of rational expectations modeling schemes can be found in Chow (1981) and Wallis (1981). An application of a two stage learning model can be found in Levy (1981) and an application of a three stage model can be found in Jacob and Jones (1980).

5. For a sampling of tests employing the Livingston data see Frankel (1975), Gibson (1976), Jacob and Jones (1980), Lahiri (1976), Mussa (1975), Pyle (1972), Roll (1972), Tanzi (1980), Turnovsky (1970), and Wisely (1982).

6. Carlson (1977) discussed a correction for measurement error. Pearce (1979) however, contends that such corrections are not adequate.

# CHAPTER III

## CONSTRUCTION AND ESTIMATION OF THE TERM STRUCTURE OF INFLATIONARY EXPECTATIONS

Since the term structure measures the return at time t associated with assets dated to mature in future time periods, t+1, t+2,..., t+n then expectations variables will have to be constructed which correspond to this structure. Two steps will be required in order to develop this time dated structure. Let X denote an expectations variable and consider the case of a two period maturity. The first step in estimation is the construction of all two period moving averages of $\bar{X}$, denoted $\bar{X}_{(2t)}$. Using this notation, if $(\bar{X}_{(2t)})$ is the two period moving average, then the term structure is represented as ${}_{t+m}\bar{X}_{(2t)}$ for m = 1 to m = infinity. The second step is to estimate

$$E(\bar{X}_{(2t)}) = (E({}_{t+m}\bar{X}_{(2t)}) \mid M_{t-1})$$

where $M_{t-1}$ is the information set over which the expectations variable is estimated.

For our purposes, the only expectations variable which we model is expected inflation. The information set which serves as the basis of our estimates of a term structure of inflationary expectations is the monthly Consumer Price Index (CPI) from January 1950 through December 1983.[1] From this data, a database is constructed under the assumption that the information set to which an economic agent might refer in forecasting inflation for a particular maturity would contain the average of the CPI for the maturity to be forecasted. That is, the reference set for two month inflation, three month inflation . . ., m month inflation would be two month averages of the CPI, three month averages of the CPI, . . ., m month averages of the CPI respectively. The averages as constructed in the database are moving averages.

The estimation methodology employed to generate a term structure of inflationary expectations is a passive learning model, based on work by A. W. Phillips (1957), Gregory Chow (1975), and Jacob and Jones (1980). The estimation procedure produces a one-step-ahead unconditional forecast of inflationary expectations which reflects passive learning about the model's parameters. This technique will be applied to the 84 different data structures to generate a term structure of one to 84 month horizons of inflationary expectations. While the CPI serves as the essential database and the forecasting model itself is a model of the expected price level, the forecasts of expected prices are converted to annualized expected inflation by $(1.0 + (\ln P_t - \ln P_{t-1})$ ** 12 ) - 1.0.

## I. MEASURING INFLATIONARY EXPECTATIONS

As noted previously, much of the early empirical work on inflationary expectations followed Irving Fisher (1930) and assumed that a distributed lag of past prices could adequately represent anticipated inflation. More recently however, studies of inflationary expectations have been dominated by the concept of rationality introduced by Muth (1981a and 1981b). The focus of these studies has been to examine the deficiencies of the distributed lag technique as a model of expectations (Fama, 1983) and to propose various empirical models of the manner in which economic agents formulate expectations which may be classed as rational in the sense of Muth.

In addition to the various empirical models for forecasting inflation, a measure of inflationary expectations is provided by the Livingston Survey data. While the series has often been used in empirical studies, its validity as a measure of inflationary expectations is questionable. Tests by Pesando (1975), Carlson (1977), Pearce (1979), and Turnovsky (1970) find that the survey data are inconsistent with the rational expectations hypothesis. Other findings suggest that if corrections are made for the information available at the time respondents are surveyed, then the Livingston series is consistent with rational expectations (Mullineaux, 1978).[2]

Irrespective of whether the Livingston series is, or can be rendered, consistent with rational expectations, the data are inadequate for generating a term structure of inflationary expectations beyond one year.[3] Our test of the Fisher hypothesis requires a term structure of expected inflation and the purpose of this study is to derive a term structure of inflationary expectations over a time horizon of longer than one year with an empirical model which is consistent with rational expectations.

A widely accepted empirical definition of rational expectations is that such expectations are optimal forecasts conditional on the information that is available at the time of the forecast.[4] Alternatively, if expectations are rational, then there should be no systematic forecast error. The forecast error should have a mean of zero so that on average forecasts should differ from actual values only by random error.

## II. MODELS OF INFLATIONARY EXPECTATIONS

There are two predominant methods of including learning processes in models of inflationary expectations. One, the multistage error learning models, are essentially a modification of Muth's technique. These are adaptive methods which focus on distributing the observed forecast error to various stages of the learning process. The second method, that of passive learning models, depends on the optimal revision of the parameter estimates where such revisions are based on the most recently observed information. This second technique is largely the result of work based on Gregory Chow's (1975) solution to control problems in the presence of uncertainty.

Muth's (1981a) initial example of a 'rational expectations' process, which could lead one to the conclusion that adaptive expectations are rational, has subsequently been shown to be a special case of a more general class of rational expectations models (Wallis 1981).[5] This more general class of rational expectations models is often referred to as that of error learning models. The model within this class which will produce rational expectations forecasts depends upon the nature of the underlying generating process. Following this insight, Jacob and Jones (1980) describe a three stage error learning model of price expectations. Since the logic of their multistage error learning model serves as inspiration for the model developed here, the short discussion presented below will provide some essential background.

Assume that the true underlying generating process producing realizations of the price level is described as:

(3.1)

$$P_t = p_t + u_t$$

$$p_t = p_{t-1} + I_t + v_t$$

$$I_t = I_{t-1} + d_t + w_t$$

$$d_t = d_{t-1} + z_t$$

where:

$P_t$ is the observed price level,

$u_t$, $v_t$, $w_t$, and $z_t$ are independent white noise processes,

$p_t$ is the true underlying price level,

$I_t$ is the true underlying inflation rate, to be measured as

$(P_t - P_{t-1})$, given $P_t$ and $P_{t-1}$ are the logarithm of the prices at time t and t-1.

and

$d_t$ is the drift in the underlying inflation rate, to be measured as $(I_t - I_{t-1})$, given $I_t$ and $I_t - 1$ are the logarithms of the observed rate of inflation at time t and t-1.

Given the above generating process an optimal forecast of the price level would be produced by:

(3.2)

$$P^{e}_{t+1} = P^{e}_{t} + L_1 (P_t - P^{e}_{t}) + I^{e}_{t+1}$$
$$I^{e}_{t+1} = I^{e}_{t} + L_2 (P_t - P^{e}_{t}) + d^{e}_{t+1}$$
$$d^{e}_{t+1} = d^{e}_{t} + L_3 (P_t - P^{e}_{t})$$

where:

$P^{e}_{t+1}$, $I^{e}_{t+1}$, and $d^{e}_{t+1}$

are the one step ahead expected price level, rate of inflation, and the drift in the rate of inflation

$P^{e}_{t}$, $I^{e}_{t}$, and $d^{e}_{t}$

are the current period expected price level, rate of inflation and drift

$P_t$ is the current observed price level,

$(P_t - P^{e}_{t})$ is the observed forecast error,

and

$L_1$, $L_2$, and $L_3$ are the adaptation coefficients measuring the portion of the forecast error attributed to each of the possible types of change which drive the price level generating process.

In absence of rates of change in the rate of inflation, that is in the absence of drift the optimal forecasting model would collapse to:

(3.3)

$$P^{e}_{t+1} = P^{e}_{t} + L_1(P_t - P^{e}_{t}) + I^{e}_{t+1}$$
$$I^{e}_{t+1} = I^{e}_{t} + L_2 (P_t - P^{e}_{t})$$

In the absence of rates of change in the price level, that is in the absence of inflation, the optimal forecasting model would collapse to:

(3.4)

$$P^e_{t+1} = P^e_t + L\ (P_t - P^e_t)$$

In the presence of a trend (inflation) price forecast made with the model described by equation (3.4) would consistently over- or under-predict the price level. In the presence of drift (change in the rate of inflation) the model described by equation (3.3) price forecasts would consistently over- or under-predict the price level.

In the estimation of their model Jacob and Jones (1980) used the Carlson's (1977) corrected Livingston data as a measure of the expected price level, and derive from it expected inflation and expected drift. The CPI serves as their measure of observed prices. They employ a gradient technique to estimate $L_1$, $L_2$, and $L_3$. This multilevel error learning model explains 89% of the variation in the underlying expected inflation rates implied by the Livingston survey data. A major conclusion they draw from their results is that an individual using a relatively simple model could generate at very little cost much the same 'expectations' as those reported in the survey.

The estimation of a Jacob and Jones style error learning model requires an observed, or prior, expectations series such as the Livingston data. As suggested earlier, however, it was felt that it was undesirable to depend on the Livingston data because of its restricted maturity horizon. For this reason, it was necessary to deviate from the Jacob and Jones model. However, in so doing, it was desirable to preserve the concept of learning which Jacob and Jones describe, and the concept that changes in the price level were likely to be generated from both trend and drift. Note, however, that one can logically extend the Jacob and Jones argument about the levels of learning to additional terms such as changes in the drift. If there were periodically sudden increases or decreases in the drift, the model described by equation (3.2), above, would consistently over- or under-estimate the price level.

Gregory Chow (1975 and 1981) has argued that where there is uncertainty about a model's parameters the incorporation of learning in the solution to a control problem improves the performance of the decision maker (forecaster). He demonstrated that an optimal solution to a forecast for a T-period horizon was equivalent to T, one period forecasts if learning is incorporated prior to each prediction. The revision of the coefficients in the learning process can be considered equivalent to T changes in the Bayesian posterior density functions generating the realizations.

In the case of passive learning, one revises the estimates of the model's parameters based on new information before calculating the next forecast. Rather than incorporate learning by distributing the error in a forecast to its various possible sources in the generating process (as in the Jacob and Jones model), the model that is estimated here incorporates learning in the sense of Chow.

The formation of each forecast requires a revision of the coefficients each time new information is obtained by the economic agent. The exact manner in which this is accomplished is described below, where the estimation procedure is discussed. The single equation model proposed here is:

(3.5)

$$P_t = A + B_1 P_{t-1} + B_2 I_{t-1} + B_3 D_{t-1} + B_4 D'_{t-1} + u_t.$$

where:

$P_t$ is logarithm the current price level measured by the CPI or a moving average of the CPI as described above in the discussion of a database for the term structure.

$P_{t-1}$ is the logarithm of the lagged price level,

$I_{t-1}$ is the lagged rate of inflation, $I_t$, measured by $(P_t - P_{t-1})$

$D_{t-1}$ is the lagged drift, $D_t$, measured by $D'_t$, the logarithm of $(I_t - I_{t-1})$

$D'_{t-1}$ is the lagged change in the drift, $D_t$, measured by $(D_t - D_{t-1})$.

Equation (3.5) acknowledges the multiple sources of variation in the price level generating process as discussed above.

## III. ESTIMATION AND PREDICTION PROCEDURES

Two techniques were tested as methods for including passive learning in modelling inflationary expectations in equation (3.5) above. The first was to initialize estimation with a sample of 35 observations. (The term $D'_t$ was constant for so much of the time period under consideration that there was a problem of multicollinearity with the intercept term. The constant term was thus dropped from the estimating equation.) Then, based on an initial set of parameter estimates using the first 35 observations in the sample, a forecast for period 36 was generated using the actual values for period 35 for $P_t$, $I_t$, $D_t$, and $D'_t$ in the forecasting equation:

(3.6)

$$P_{t+1} = b_1 P_t + b_2 I_t + b_3 D_t + b_4 D'_t$$

where $b_1$, $b_2$, $b_3$, and $b_4$ are the sample estimates of the parameters $B_1$, $B_2$, $B_3$, and $B_4$ in equation (3.5).

Then the sample was extended to 36, the parameters re-estimated and a forecast for time period 37 was made based on the new parameter estimates and the actual values for time period 36. The process of simply extending the sample to reflect new information in each new set of parameters was continued for all observations.

Since the model produces an estimate of the price level and not inflation, expected annualized inflation was computed from the expected price level using,

$$(1.0 + (\ln P_t - \ln P_{t-1}) ** 12) - 1.0).$$

Theil's inequality was used to test for 'rationality' comparing actual annualized inflation with expected annualized inflation.

While Theil's inequality indicated that the forecasts were good in the sense that Theil's inequality was less than one, it was apparent from analysis of the forecast error and plots of the actual and predicted values that the model's ability to predict decreased as time passed. The more variable inflation of recent history was simply outweighted by the very long history of low, relatively stable inflation throughout the 1950's and much of the 1960's. Since it seemed that too much information was retained when the sample was simply extended, a moving window regression was the second technique tested. As in the first method, the process was initialized by estimation of equation (3.5) for a sample of 35. The forecast for period 36 was obtained by employing forecasting equation (3.6) above for the actual values of $P_t$, $I_t$, $D_t$, and $D'_t$ for period 35. The window was then moved down one observation in the series and deleting the first observation from the sample adding the observation 36. The parameters were re-estimated and the forecast for period 37 was made. This process was continued for the entire series. The same procedures were used to compute expected annualized inflation and for constructing estimates of forecast performance. The forecasting performance using this technique proved to be superior to the first technique. Since each forecast is a short run prediction, one period ahead, it is probably more reasonable to assume that the economic agents use the more recent pieces of information and discount extremely old pieces of information. Admittedly, however, there is no defense for the specific sample size selected except for its econometric convenience.

Following the definition of rationality that was presented above, Granger and Newbold (1977) show that optimal forecasts will have small values (approximately zero) for the bias and regression (variance) portions of Theil's decomposition of forecast errors, and larger values (approximately one) for the disturbance (covariance) proportion of the decomposition. This is the measure applied here to determine the 'rationality' of the model's forecast. The moving window regression was computed for 84 maturities of inflationary expectations. The result for all cases fall into the Granger and Newbold rule for rational forecasts. The detailed results are presented in Table 3.1.

TABLE 3.1

VALUES FOR THEIL'S DECOMPOSITION
FOR EIGHTY-FOUR MATURITIES
OF EXPECTED INFLATION

| Term of Maturity | Bias Portion | Variance Portion | Covariance Portion |
|---|---|---|---|
| MA 1 | .3213509E-04 | .4310119 | .7529143 |
| MA 2 | .5084492E-04 | .1491711 | .3787561 |
| MA 3 | .4158777E-04 | .12908399 | .5587130 |
| MA 4 | .1080218E-04 | .9233818E-01 | .6207178 |
| MA 5 | .1156379E-04 | .6400962E-01 | .5972150 |
| MA 6 | .4284828E-05 | .5242242E-01 | .6016281 |
| MA 7 | .6664040E-06 | .4034638E-01 | .6049629 |
| MA 8 | .2926172E-06 | .3343119E-01 | .6615812 |
| MA 9 | .2896233E-06 | .2311109E-01 | .6301864 |
| MA 10 | .1501087E-05 | .2027109E-01 | .6557719 |
| MA 11 | .3682358E-06 | .1748112E-01 | .6748148 |
| MA 12 | .5153331E-05 | .1637248E-01 | .6937996 |
| MA 13 | .6080362E-05 | .1385322E-01 | .6260667 |
| MA 14 | .2264415E-05 | .1388238E-01 | .6549026 |
| MA 15 | .2017724E-05 | .1172466E-01 | .6526506 |
| MA 16 | .5732727E-06 | .1051580E-01 | .6810850 |
| MA 17 | .3116828E-05 | .1143361E-01 | .6803034 |
| MA 18 | .5188942E-09 | .8582735E-02 | .5902615 |
| MA 19 | .1840261E-05 | .9050418E-02 | .6912536 |
| MA 20 | .3544663E-05 | .6047110E-02 | .6249155 |
| MA 21 | .7430096E-05 | .6928960E-02 | .6582814 |
| MA 22 | .7380520E-05 | .7075805E-02 | .6471862 |
| MA 23 | .1254247E-08 | .5862300E-02 | .6697337 |
| MA 24 | .1253605E-06 | .5718291E-02 | .6915423 |
| MA 25 | .2935977E-05 | .6293800E-02 | .6595063 |
| MA 26 | .1066802E-04 | .5296595E-02 | .6475913 |
| MA 27 | .2538217E-05 | .2660664E-01 | .8703021 |
| MA 28 | .6660941E-06 | .4706871E-02 | .7550348 |
| MA 29 | .3458792E-08 | .4456324E-02 | .6838915 |
| MA 30 | .2400858E-05 | .3865397E-02 | .6129582 |
| MA 31 | .8785963E-05 | .3630110E-02 | .6556293 |
| MA 32 | .9390176E-06 | .3923721E-02 | .7045852 |
| MA 33 | .5955720E-05 | .4007234E-02 | .7255952 |
| MA 34 | .1704871E-05 | .3404719E-02 | .6599681 |
| MA 35 | .3623302E-06 | .3429442E-02 | .6753596 |
| MA 36 | .4690538E-05 | .3286151E-02 | .6974572 |
| MA 37 | .1506555E-06 | .3183901E-02 | .7066284 |
| MA 38 | .8286659E-07 | .2744109E-02 | .6775693 |
| MA 39 | .3652560E-04 | .2120710E-02 | .6525694 |
| MA 40 | .1788926E-05 | .2514477E-02 | .6971620 |
| MA 41 | .5512629E-05 | .2923214E-02 | .7034525 |
| MA 42 | .2763633E-06 | .2755552E-02 | .6622411 |
| MA 43 | .8694678E-05 | .2446806E-02 | .7494373 |

TABLE 3.1 continued

| Term of Maturity | Bias Portion | Variance Portion | Covariance Portion |
|---|---|---|---|
| MA 44 | .4886520E-05 | .2276123E-02 | .6864439 |
| MA 45 | .4390657E-05 | .2402523E-02 | .6872572 |
| MA 46 | .2832524E-06 | .1796647E-02 | .6221102 |
| MA 47 | .2653513E-04 | .1500203E-02 | .6720185 |
| MA 48 | .3105086E-05 | .1506009E-02 | .6664434 |
| MA 49 | .2608149E-06 | .1678583E-02 | .6771877 |
| MA 50 | .1361558E-07 | .1641487E-02 | .6314877 |
| MA 51 | .3550102E-06 | .1557862E-02 | .6496575 |
| MA 52 | .7735410E-05 | .1387538E-02 | .6276179 |
| MA 53 | .4299452E-05 | .8975520E-03 | .4114564 |
| MA 54 | .5493967E-06 | .1617452E-02 | .6811230 |
| MA 55 | .4756851E-05 | .1511041E-02 | .6453054 |
| MA 56 | .6705727E-05 | .1478467E-02 | .6785680 |
| MA 57 | .3696536E-06 | .1187791E-02 | .6441996 |
| MA 58 | .1326478E-04 | .1147672E-02 | .6299213 |
| MA 59 | .2388891E-05 | .1398005E-02 | .6127520 |
| MA 60 | .6358145E-05 | .1191698E-02 | .5976649 |
| MA 61 | .3299119E-05 | .8298616E-03 | .6121598 |
| MA 62 | .1948579E-05 | .1128614E-02 | .5641792 |
| MA 63 | .6303508E-06 | .1249853E-02 | .6231704 |
| MA 64 | .1411628E-05 | .1149668E-02 | .6135475 |
| MA 65 | .1086598E-05 | .9843562E-03 | .6371314 |
| MA 66 | .8167771E-05 | .1002118E-02 | .6393371 |
| MA 67 | .7436788E-06 | .1472487E-02 | .6082697 |
| MA 68 | .1182820E-04 | .7140851E-03 | .5821619 |
| MA 69 | .1561385E-05 | .1017830E-02 | .5736533 |
| MA 70 | .1287823E-07 | .1326718E-02 | .6356857 |
| MA 71 | .1999514E-09 | .9357152E-03 | .6450365 |
| MA 72 | .1815500E-06 | .9041991E-03 | .6177498 |
| MA 73 | .2161824E-06 | .9210336E-03 | .6339443 |
| MA 74 | .8336088E-06 | .9430287E-03 | .6652636 |
| MA 75 | .2035502E-05 | .9816515E-03 | .6517258 |
| MA 76 | .1084070E-05 | .1158494E-02 | .7021041 |
| MA 77 | .3537942E-05 | .8462731E-03 | .5913678 |
| MA 78 | .1447567E-04 | .8079536E-03 | .6872206 |
| MA 79 | .2503409E-06 | .1065175E-02 | .5851421 |
| MA 80 | .3301367E-05 | .1035263E-02 | .6299666 |
| MA 81 | .2220971E-04 | .5960694E-03 | .5944738 |
| MA 82 | .6003454E-05 | .7921079E-03 | .6108555 |
| MA 83 | .1300070E-04 | .8076215E-03 | .7123158 |
| MA 84 | .1207311E-05 | .7103373E-03 | .6043836 |

Table A.1 in Appendix A contains the mean and variance of both the actual annualized and expected annualized rates of inflation for all 84 terms to maturity.

The estimates of the decomposition values presented in Table 3.1 were computed using the decomposition formulae recommended by Granger and Newbold (1977):

$$U^M = (\bar{P} - \bar{A})^2 \,/\, MSE$$
$$U^R = (S_P - (corr_{A \cdot P})S_A)^2 \,/\, MSE$$
$$U^D = (1 - (corr_{A \cdot P})^2)S_A^2 \,/\, MSE$$

where:

$U^M$ is the bias portion,

$U^R$ is the variance portion,

$U^D$ is the covariance portion,

$\bar{P}$ is the average of predicted annualized inflation

$\bar{A}$ is the average of actual annualized inflation,

$S_A$ and $S_P$ are the standard deviations of actual and predicted annualized inflation, respectively, and

$corr_{A \cdot P}$ is the correlation coefficient of actual and predicted annualized inflation,

$S_A^2$ is the variance of actual annualized inflation, and

MSE is the mean square error.

It would be impossible to detail the results of all of the regressions. However, an analysis of the pattern of statistical significance of the variables in the estimating equation produces results that one might expect. For example, in examining the one month ahead forecast of the one month expected price level, one finds that throughout the 1950's, except for rare occasions, only the lagged price level is statistically significant. This might be expected since there was very little inflation during the period. For the period of the 1960's, in most cases both the lagged price level and lagged inflation were statistically significant. Again, this might be expected since much of this period was characterized by mild increases in the price level. Throughout the 1970's, which was the most volatile period of rising prices, the model's estimates of the parameters were statistically significant for the lagged price level, lagged inflation, and lagged drift. During this time period the lagged change in the drift was also occasionally statistically significant. In the 1980's there was a return to more stable inflation. This was reflected in the estimation results, with only the lagged price level and lagged inflation being statistically significant. An analysis of the one month ahead forecast of the two month expected price level reveals a similar pattern of significance in the coefficients of the variables with some smoothing present. These 84 maturity structures were then sorted into monthly term structures for the period January 1970 through November 1982.[6,7]

Broadening the information set to include policy variables would, within the context of the proposed study, greatly expand the computational requirements. Note that to include variables such as monetary policy, fiscal policy, and other exogenous events would require the construction of maturity data for these variables, estimation of expectations from the maturity data, and construction of a spline function for each time period. While the computations would be a burdensome, failure to include these variables may carry a cost. To the extent that Sargent's analysis is correct, the residual matrix, resulting from estimation of nominal interest rates (splined) as a function of inflationary expectations (splined), will contain a systematic component associated with missing variables.[8]

# CHAPTER III

## CHAPTER NOTES

1. These were taken from Business Conditions Digest and are in base year 1967. Mr. Eric Dmytrow, quality assurance manager for the Bureau of Labor Statistics, recommends this base year over the 1972 base year because the 1972 revision was incomplete, and contains substantial error both in sampling and in the database.

2. Mullineaux found that other variables, such as the money growth rate, entered into the information set in forming inflationary expectations.

3. It should be noted that Cargill and Meyer (1984) have devised a method of generating a term structure of inflationary expectations from the Livingston data by estimating a time series on six and twelve month Livingston forecasts using a curve fitting technique. They were able to produce six points on a term structure of inflationary expectations.

4. See Robert J. Barro and Stanley Fisher (1976), Gregory Chow (1981), or Douglas K. Pearce (1979).

5. Wallis (1981) notes that Nelson (1969 and 1975) points out that Muth's initial two equation model of a market is a special case where the optimal extrapolative predictor and the rational expectations prediction coincide. When Muth further specialized this by assuming the disturbance term to be random walk then the rational and the adaptive expectations models predictors coincide.

6. November of 1982 ends the data set rather than December because in December 1982 the Treasury stopped publishing the data on Treasury Bills and Notes on a monthly basis. Since December of 1982 the data are available only a a quarterly basis.

7. The term structures of both actual and expected annualized data may be obtained on request from the author.

8. The use of two stage least squares estimation can compensate somewhat for missing variables. For this reason it is a commonly used technique in estimation of rational expectations models where maximum likelihood and limited information cannot be employed. The

missing information in these cases is the expected future value of the variable, which is unobservable (Wallis, 1981). For a more in-depth discussion of the econometric procedure and its applicability see Judge et alii. (1980).

# CHAPTER IV

## TREASURY YIELDS AND THE DERIVATION OF SPOT INTEREST RATES

A unique aspect of the test of the Fisher effect that is presented here is that we employ a term structure of spot interest rates rather than a term structure of yields. While practical applications increasingly rely on spot interest rates most prior research on the term structure has relied on yield data. The use of yields creates a problem of bias in empirical investigations which is an unnecessary complication. A wide variety of empirical techniques have been introduced to estimate spot interest rates. One relatively simple regression based method was introduced by Carleton and Cooper (1976). Their method is a technique for the direct estimation of spot interest.[1] Their methodology has been expanded upon and applied here. We estimate the term structure of spot interest rates from the yield data on Treasury notes and bank discount on Treasury bills. The spot interest rates are estimated in two steps from this data. First, the present value is estimated for each security. Second, a regression is performed to estimate the present value as a function of maturity. The slope coefficients from this regression are assumed to measure the rate of interest for the associated term to maturity so that the spot rate for any given maturity can be computed from:

(4.1)

$$i_{mat(t)} = (200.0 * (100.0/(b_{mat(t)}))^{t}) - 200.0$$

where:

$mat(t)$ = term to maturity

$i_{mat(t)}$ = the estimated spot rate for a security of term to maturity (t)

$t$ = 182.5/mat(t), since coupon payments on Treasury notes are semi-annual payments

$b_{mat(t)}$ = estimated slope coefficient of present values as a function of term to maturity

The actual details of this method will be elaborated below. However, one critical requirement of this methodology is that there be at least one present value observation for each term to maturity on a continuous time scale. This meant that the raw data for each of 155 months had to be sorted into a standardized structure which was consistent for regression. A number of problems resulted from the way in which the data was presented in the *Treasury Bulletin*.

The primary data employed here were taken from the *Treasury Bulletin*, "Market Quotations on Treasury Securities," Tables $MQ_1$ and $MQ_2$: Treasury Bills and Treasury Notes. The data used were bank discount quotations for Treasury bills, and coupon and yield quotations for Treasury notes. The data cover the period 1970 through 1982 monthly with the exception of December 1982, since in that month the Treasury Department switched from publishing the data on a monthly basis to a quarterly publication schedule. Each monthly term structure was measured out to a seven year (84 month) limit. The number of maturities was eventually reduced to 76 months in order to have the same sample size for 155 months.

Problems arose in the collection and coding of the Treasury Bulletin data which created difficulties in the estimation of spot interest rates. Several research assistants compiled the data set.[2] Although they carefully followed instructions regarding coding, punching and verifying, a number of data files failed scrutiny by the program which had been developed specifically to estimate the present value of the combined Treasury bills and notes. It was discovered from this exercise that for the early period of the data there were some rather odd problems. There were zero and negative yields recorded in the early 1970's. There was also a high percentage of "bad" maturity dates, such as a maturity date which occurs before the closing date for the month; hence an observation on a bill or a note that should already have matured. There were maturity dates on days that do not exist, such as the thirty-first of February. In the period of 1970 to 1975 there were also notes which appeared to be out of sequence in terms of maturity dates. These notes almost inevitably disappeared in subsequent months, suggesting the subsequent correction of errors in earlier publications. Quotations which were clearly in error were deleted from the data file, and do not appear in the estimates of the spot rates of interest. The quality of the data improved significantly after 1975.

A second set of problems arose as a result of our attempt to utilize all the available information on Treasury bills and notes. We did not want to restrict ourselves to a sample of securities which either matured or paid coupons on pre-specified dates as Carleton and Cooper (1976) had done, since such a technique discards a great deal of information which may bias results. We learned, however, that trade-offs were involved in gaining a sample which attempted to maximize the securities included. These trade-offs included distortions in estimations of interest rates because of large movement in securities further away or closer to maturity, and an inability to estimate $b_{mat(t)}$ because of improper identification for regression.

Initially, it was decided to start the procedure which created the maturity structure by selecting the Treasury bill closest to maturity to enter first, then overlaying the Treasury bill data with the Treasury note data. However, in the early dates, up through 1976, the number of notes in a month was small making the observations on the term structure thin, perhaps no more than one or two per year. In some cases this meant that the program which was employed to manipulate the primary data could not find any notes in range of its forward target date. This required not only a revision which forced the Treasury note closest to maturity to enter first (and then overlaying the notes with Treasury bills), but also a revision of the interval for admission of a security around the target date to a smaller range, from an original of fifteen days to seven days. The result of the modification was that frequently a note at the near end of the term structure is moved closer to or further away from its true maturity date. If such notes have relatively large coupons and are moved closer to maturity, the spot interest rate is driven up and in some cases the resulting interest rate is rather large. If it is moved further from its actual maturity then the spot rate tends to be low compared to other spot interest rates on the short end. This effect is not noticeable on those spot rates for longer term to maturities. Because of this problem in the first spot interest rate this observation is not used in subsequent analysis.

In "Estimation and Uses of the Term Structure of Interest Rates," Carleton and Cooper note that the present value as of a current date of for an n period coupon bond is given by:

(4.2)

$$PDV(0,j) = (1 + r(0,j)) + x(1,j) / (1 + r(1,j)) + x(2,j) / (1 + r(2,j)) + \cdots + x(n,j) / (1 + r(n,j)),$$

which can alternatively be written as:

(4.3)

$$PDV(0,j) = b(0,j) + b(1,j)x(1,j) + b(2,j)x(2,j) + \dots + b(n,j)x(n,j)$$

given that:

$$b(t,j) = (1 + r(t,j))^{-t}$$

and where:

$PDV(0,j)$ = the present value of the $j^{th}$ bond for the current time period and

$b(t,j)$ = the present value of the expected cash flow for the $t^{th}$ period from the $j^{th}$ bond. Specifically, this present value is an expected value per unit of cash flow for each of the n periods.

$x(t,j)$ = coupon or cash flow for n-1 of the t periods and coupon plus principal for the $n^{th}$ period.

Since the $b(t,j)x(t,j)$ are expected present values and not actual holding period cash flows one can rewrite (4.3) as:

(4.4)

$$PDV(0,j) = X_j b_j + e_j$$

where:

$X_j$ is a matrix of cash flows for t period for j bonds.

$b_j$ is a vector of present value of the expected cash flow for period t = 1, . . ., n and

$e_j$ is the error on the $j^{th}$ bond

Given a sample of j bonds, j = 1, . . . , J, equation (4.4) can be estimated by regression under the constraint that all securities maturing on the same date t have the same interest rate, $b_{(t,j)} = b_{(t,i)} = b_i$ for $j \neq i$ so that $r_{(t,j)} = r_{(t,i)}$. All $b_t$ are positive but less than or equal to one.

The regression formulation of the present value (or discount) function developed by Carleton and Cooper (1976) implies that there are no <u>systematic</u> arbitrage opportunities. Equivalently the implication is that the expected present value of any dollar to be received in period t is independent of the security from which it flows. The error term, $e_j$, implies that there may be quasi-arbitrage opportunities. Such quasi-arbitrage opportunities are found for example in bond trading models. These bond trading models assume that spreads between yields on bonds which are unusually large are a signal that there is potential for a trading profit. Since it is difficult (or expensive) for market participants to compare yields between long-term and short-term bonds with different coupon rates and tax effects, the market may produce profits from trading where yield spreads are 'apparently' divergent from average yield spreads. While profits from such trades are expected to be positive, they are not certain. These 'apparently' different expected returns are referred to as quasi-arbitrage opportunities. The ability to efficiently and quickly adjust bond prices for the removal of coupon effects has been one of the reasons for the significant increase in the number of empirical studies on the t… tructure in recent years.

In order to estimate equation (4.4) Carleton and Cooper select a sample of government coupon securities which formed a quarterly grid of securities paying semi-annual coupons. To fill in the gaps where no scheduled payment occurred on a quarterly target date and to increase size of their sample, Carleton and Cooper assumed that Federal Home Loan Bank securities which carry a government guarantee were risk and tax equivalents to government notes or bonds. Carleton and Cooper then use their data sample to estimate equation (4.4) and from the estimate of $b_{(n,j)}$ derive estimates of the spot interest rate.

Our method of estimating spot interest rates is an elaboration of the Carleton and Cooper method, differing primarily because our data are substantially more complicated and our sample size is much larger.

One such complication was that our data did not include the security price. The present value derived from the coupon stream of payments for Treasury notes is derived from:

(4.5)

$$PDV_N = (1 - 1/(1 + y)^{ncoup})\ c/y.$$

The accrued interest or the accretion of note price toward par is computed by:

(4.6)

$$ACC_N = 100.0/(1 + y)^{ncoup}$$

The present value for Treasury bills is computed by:

(4.7)

$$PDV_B = 100 - (y - n/360)$$

where C is the coupon to be paid, y is the reported yield to maturity, ncoup is the number of days to maturity divided by 182.5 (assuming semi-annual coupon payments), and n, in the case of bills, is the number of days to maturity. The price of Treasury notes is the sum of present value from the coupon stream plus accrued interest, from equations (4.5) and (4.6). These present values are indexed by term to maturity. The primary data sets included all bills and notes, averaging 720 observations per year up through 1978 and approximately triple that number of observations per year after 1978.

The program (see Appendix B) which computes the present values and the associated monthly index of term to maturity also sorts through the index to form a grid of terms to maturity (columns) versus the present values (rows).

As noted earlier, we require at least one observation for each term to maturity. Even though our sample for each month is large, and exhausts all bills and notes which have no special features, the retirement/payment schedule does not provide us with at least one observation for each thirty days. For this reason the present value program includes an interpolation procedure which creates place holders called 'pseudo-zero' securities. These resemble zero-coupon 'stripped' bonds which are now regularly traded. These 'pseudo-zero' securities yields' are estimated as the average yield to maturity of the securities on each end of the gap. This requires that the program compute the bond equivalent yield for discount bills.

Using the 'pseudo-zero' securities does present some problems in estimating spot interest rates. All actual securities yields' that enter into estimating nominal interest rates are assumed to be inaccurate, incorrect in the sense that their yields-to-maturity are to be corrected to produce spot rates. Hence, in estimating their present value we correct for coupon effects, and accrual or accretion of their price toward par. However, whenever we have a missing value, we employ the average yield to maturity as if it implied the correct present value. The result is that where the term structure is rising (falling) our 'pseudo-zero' securities will under-estimate (over-estimate) the actual spot rates. This creates some bias in the term structure but otherwise provides smoothness along the term structure.

Other techniques for filling the gaps in the term structure were tested. In particular, we attempted to use an average coupon method. While this works reasonably well for small gaps, it produces severe bias where the width of the gap is large. Large gap widths occur up through 1978/79. The average coupon method procedure not only creates bias, but also produces extremely jagged term structures.

The present value procedure which indexes the securities does not enter all possible actual bills and notes but only a sample. The program overlooks an actual value if that actual value does not fall in a seven day target interval when searching for the next thirty day date forward from the last maturity date in the grid. If the target interval was larger, it is likely that a monthly sample would contain more actual observations entering that present value matrix than the sample with a seven day interval. The target interval was originally set at fifteen days. However, two problems arise from a fifteen day interval. First, given that notes mature only on the fifteenth or thirtieth of each month and that up through 1976 there were numerous cases where no more than two notes may mature in a year, there were cases where the first maturity date falls just so that no notes are found by the program as it looks ahead to fill gaps. This made it impossible to proceed to the second step of estimation, the regression, because there were too few observations. A second problem with the fifteen-day target was that a particular security could be moved as much as fourteen days closer to (or further away from) its actual maturity date. While the sample of actual securities could have been increased, there was a greater potential for severe spikes in the present value function leading to over-or under-estimation of spot interest.

The sampling technique of selecting an interval of seven days around a 30 day forward target produces a ratio of actual securities to 'pseudo-zero' securities of about 2 to 3 for samples in the years 1978 through 1982. The ratio is lower, about 1 to 2, for samples in the years 1970 through 1977. Through the entire data set, however, the majority of the 'pseudo-zeros' -- particularly those cases where there were wide gaps -- occur at the far end of the term structure.

There is an advantage to our sampling technique compared to that used by Carleton and Cooper, in that our sample is created impartially. Carleton and Cooper selected a set of securities which were 'convenient.' We, therefore, do not know to what extent valuable information about variability along the term structure was eliminated in the sampling process. This is similar to the problem presented by the Durand interwar data. Durand (1942), in drawing yield curves, assumed that some yields which were 'too far away' from the general pattern, were incorrect measurements, which he chose to eliminate from his sample. Our sampling technique does not contain such subjective selection bias.

In the first step of the estimation of the spot interest rates, as discussed above, the present values of the Treasury bills and notes are estimated. These present values include correction for coupon and accrual/accretion.

We did not correct for tax effects. Each of these present values is indexed to its retirement date in terms of days to maturity. In the second step of the estimation of spot interest a mixed estimation regression produces slope coefficients which are used to compute the spot interest, rates as shown in equation (4.1).

Mixed estimation is one of a general class of regression methods which are referred to as restricted least squares. Estimates of the regression parameters are obtained subject to some type of prior information. These restrictions may come from out of sample information or from in sample information (GLS). In our case we employed a kind of reverse Theil-Goldberger mixed estimation. Mixed estimation of the Theil-Goldberger type is generally designed for situations in which there is prior knowledge about the range of values within which the population parameters are most likely to fall. This implies that there is prior information about the variance of the sampling distribution around a point estimate of the parameters. This may come from theory or from the sample information.

The prior information in the case of the term structure is that we expect some smoothness along the term structure. Shiller (1973) introduced smoothness restrictions as an alternative to the Almon distributed lag for the purpose of estimating functions which have both flexibility and continuity. The method essentially involves placing prior distributions on linear combinations of coefficients. The variance-covariance matrix is restricted by a tightness prior, so that the estimated coefficients' variance-covariance matrix has an expected value scaled by the tightness prior.

Unlike the usual Theil-Goldberger case, we do not have point estimates, or a degree of certainty about the probable range within which the population values are likely to occur (except that they must be in the unit interval). Rather, the prior information we have is based on hypotheses about the relationships of term to maturity values along the term structure. As in Shiller's case, we want to produce estimates of spot interest rates which are smooth along the term structure, for we believe that the efficiency of the markets for government securities will generally create a smooth discount function. In our case, we wish to modify the initial estimates of the regression parameters by expressing little confidence in the estimates. In the first round of estimation we obtain a point estimate of the slope coefficient of the present value as a function of maturity, retaining the variance-covariance estimates of the residuals. In the second iteration, the first round slope estimates and the residual variance-covariance estimates are employed to obtain new, restricted estimates of the slope coefficients. The restriction in the estimation procedure is in the form of a tightness prior which modifies the variance-covariance matrix of the estimated coefficients. The magnitude of the tightness prior -- which is a scalar quantity -- reflects our lack of confidence in the original OLS betas.

As noted, earlier estimates of the spot interest rate for time t are computed according to equation (4.1):

$$i_{mat(t)} = (200.0 * (100.0 / (b_{mat(t)})^{t}) - 200.0$$

where:

$i_{mat(t)}$ = the predicted spot rate of interest for term to maturity for security mat(t)

t = 182.5 / mat(t)

mat(t) = term to maturity for all securities with t periods to maturity

$b_{mat(t)}$ = estimated slope coefficient of present values as a function of term to maturity from the second iteration, the mixed beta.

Several tightness (or smoothness) priors were tested. The smaller the value of the tightness parameters, the smoother the estimates of interest rates. Even extremely small values for the tightness prior do not eliminate the jaggedness in the first interval, which approximately corresponds to the fifteen day rate in a series. This is particularly a problem as noted previously when a note with a large coupon is moved closer to maturity.

Carleton and Cooper also used a prior restriction to achieve smoothness, since they also had problems with the near end of the term structure. The restriction they used partially resolves the near term problem by forcing a zero intercept. The resulting functional form employed in their estimation was, however, more restrictive than the form we use.

Appendix B contains a listing of the program which was used to produce the estimated present values. This program is based on work done by C.F. Baum and all rights to the use of the program are reserved. Sample program output is also provided. Appendix C presents a sample of the OLS betas, the mixed betas and the computation of the spot rate of interest.[3]

## CHAPTER IV

## CHAPTER NOTES

1. Most applications of spline functions to estimation of the term structure result in the estimation of spot interest rates from the spline of the discount function. Our application of the spline differs in that we estimate the spot interest rates and then summarize the term structure of spot interest rates by means of spline functions. Our use of the spline function, then allows us to summarize 11,780 observations on spot interest rates with 3100 observations. We could fully reproduce our original sample space by simply multiplying the 3100 observations by a maturity structure. Our pooled regression on the spline coefficients is equivalent to pooling all 11,780 observations. We therefore avoid the problem of sampling in estimating the Fisher effect.

2. Mr. Wally Kulesza worked on the project for two years primarily as coordinator and supervisor. Mr. Christopher Hedges also worked on the project as did Mr. James Casale.

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# CHAPTER V

## SPLINE FUNCTIONS AND THE ESTIMATION OF TERM STRUCTURES

The Fisherian theory of interest is fundamentally founded on a model of intertemporal decision making. However, empirical estimates of the Fisher effect have not, to date, expressly taken account of this intertemporal nature. Most generally, in empirical work a point, or single maturity, on the term structure of nominal interest rates is estimated as a function of variables representing expectations and possible sources of structural shifts in the real interest rate, such as monetary policy, fiscal policy, and measures of the level of economic activity. What is ignored by the single maturity approach are the arbitrage effects on that single maturity of changes in other maturities along the term structure. Work by Sargent (1979b) indicates that these associational effects are important. In models testing the unbiased expectations hypothesis of the term structure he found significant bidirectional feedback between long term and short term interest rates. When flows of expectational information from long term securities' yields to short term securities' yields are included in estimates of the unbiased expectation hypothesis, changes in the nominal interest rate of short term securities produce significant changes in long term securities' yields. Similarly, changes in the nominal interest rate of long term securities produce significant changes in the nominal interest rate of short term securities.

A term structure of returns on a set of securities can be described by their yields to maturity, spot rates as a function of maturity, forward rates, or discount functions. All of these representations are related and any one may be derived from the others. The spot rates, $R_t$, are derived from the yields to maturity by correcting for coupon effects, tax considerations, and call features.[1] In turn the forward rate for maturity t, $F_t$, is related to the current spot rate by:

(5.1)

$$F_t = \frac{(1 + R_t)^t}{(1 + R_{t-1})^{t-1}} - 1$$

Conversely the spot rate is a continuously compounded function of forward rates such that:

(5.2)

$$(1 + R_t)^t = (1 + F_1)(1 + F_2) \ldots (1 + F_t)$$

where $F_1$, $F_2$, . . . $F_n$ are the one-period, two-period, t-period contract rates today for securities to be delivered one-period, two-periods, to t-periods in the future.[2] This may also be written as:

(5.3)

$$(1+{}_tR_n)^n = (1+{}_tR_1)(1+{}_{t+1}R_1)(1+{}_{t+2}R_1) \ldots (1+{}_{t+n-1}R_1)$$

where:

$_tR_1$ = current (observed) spot rate on a one month security

$_{t+n}R_1$ = the nth period ahead expected spot rate on a one month security.

The discount function, $D_t$, as defined by the spot rate is:

(5.4)

$$D_t = \frac{1}{(1 + R_t)^t}$$

By substitution then the discount function is related to forward rates by:

(5.5)

$$D_t = \frac{1}{(1+F_1)(1+F_2) \ . \ . \ . \ (1+F_t)}$$

Again this may be written as:

(5.6)

$$D_t = \frac{1}{(1+{}_tR_1)(1+{}_{t+1}R_1)(1+{}_{t+2}R_1) \ . \ . \ . \ (1+{}_{t+n-1}R_1)}$$

In most cases the only directly observable return on a security is the yield to maturity. The nominal interest rate must be estimated from the observed yield. As discussed in Chapter IV, we estimate nominal interest rates from the observed yields and our term structures are therefore term structures of interest rates.

A term structure is described by a continuous curve. It can assume a variety of shapes and although it is continuous it is likely to have significant curvature. The flexibility of spline functions has been found to be well suited to the modeling of term structures (McCulloch 1971, 1975, Thies 1982, Vasicek and Fong 1982).[3]

A spline is a piecewise continuous function. The pieces of the function are joined at knot points with the restriction that the function is to be continuous at each joint. In most applications of spline functions there are also smoothness restrictions at these knot or joint points. The smoothness restrictions require that one or more derivatives of the function are also continuous at the knot points. With respect to the application of spline functions employed here, these smoothness restrictions have an economic meaning and are required mathematically in order to implement our estimation procedure.[4]

Cubic polynominal splines have been fitted to a discount function by McCulloch (1971, 1975) and Thies (1982). Vasicek and Fong (1982) have fitted an exponential spline to a discount function. A spline function could, however, be fitted to any of the descriptions of the term structure. A summary of the relative strengths and weaknesses of alternative spline methods has been presented by Smoot (1983). We will fit cubic exponential spline functions directly to the nominal interest rates.

## I. ALTERNATIVE SPLINE FUNCTION FORMS FOR THE ESTIMATION OF THE TERM STRUCTURE OF INTEREST RATES

The choice of the exponential functional form of the spline used by Vasicek and Fong (1982) has several advantages over the more general polynomial spline. Consider the discrete representation of the discount function, equation (5.7). If one expands the spot rate for several terms, as in equation (5.8), the representation of the discount function can be approximated by a polynomial:

(5.7)

$$D_t = \frac{1}{(1 + R_t)^t}$$

(5.8)

$$D_1 = \frac{1}{1 + R_t}$$

$$D_2 = \frac{1}{1 + 2R_t + R_t^2}$$

$$D_3 = \frac{1}{1 + 3R_t + 3R_t^2 + 3R_t^3}$$

$$D_4 = \frac{1}{1 + 4R_t + 6R_t^2 + 4R_t^3 + R_t^4}$$

The discount function is a progressively higher order polynomial in the spot rate. This polynomial is, however, in discrete time while the term structure is conceptually in continuous time. An instantaneous discount function would be expressed by an exponential. The traditional representation of the discount function as a polynomial is simply an approximation to its exponential form. Fisher himself devotes much time to the demonstration of the relationship between discrete and continuous discount functions because of the constraints computational technology placed on his empirical work. For a detailed and careful exposition see Fisher's discussion in "Appreciation and Interest" (1896 pg. 27) and The Nature of Capital and Income (1906, pp. 203-204, and pp. 368-371). An advantage of the exponential spline form in estimating term structures of spot interest rates us the innately exponential nature of compound interest.

It can also be aruged that the polynomial functional form of the spline does not perform well in fitting term structures. When Huston McCulloch (1971) introduced the methodology of fitting spline functions to estimate the term structure, he used a polynomial spline to fit a continuous time discount function. The selection of the polynomial was motivated by the fact that it approximates the exponential, but can still be estimated by least squares. Polynomial functions, however, have a curvature that differs from the exponential function, and can only be forced to approximate the exponential by utilizing a large number of knot points (Ahlberg, 1967). However, both Poirier (1976) and Kemmsies (1983) demonstrate that an increase in the number of knots tends to produce overfitting. The polynomial over relatively large intervals, fails to fit the exponential curve. The result is that, with few knots, the polynomial approximation tends to weave around the exponential and does not tail off well for long maturities. Increasing the number of knots to produce a smoother approximation is likely to lead to greater sampling error in the coefficient estimates because of overfitting.

Langtieg and Smoot (1981) suggested several modifications of the McCulloch technique which involved estimating the log transform of the discount function. This, however, requires nonlinear estimation. Vasicek and Fong (1982) note that by using a log transform of the argument of the term to maturity function, one can employ linear estimators. They employ generalized least squares to estimate the present value function on the log transform of the term to maturity.

We have chosen to curve-fit our term structures using a cubic exponential function. The continuity and smoothness of the cubic function produced by the restrictions at the first derivative of a cubic function are naturally suited to expected interest rate relationships along a term structure. The continuity of the cubic function produced by the restrictions at the second derivative of a cubic function are also naturally suitable to term structures. The exponential functional form is justified on both theoretical and empirical grounds.

Poirier (1976) has made a strong case for the general applicability of splines as a modeling technique for economic relationships. He argues that while the structural parameters of a process may change in value because of a change in policy (the Lucas critique) or a stochastic shock, these changes do not necessarily imply the underlying motivation of agent behavior itself has changed. Structural changes represent the response to changes in the economic environment. However, the changes in these structural parameters only signify alterations in the operational rule to changes in the rules of the game. They do not represent revisions in the underlying objectives (the so-called 'deep parameters') of economic agents.[5]

The technique of spline functions is most applicable where forces periodically intrude, altering the structural parameters. In these cases of structural shifts, market realizations often appear to have been generated by a new process when, in fact, all that has occurred is a change in the operational rule for achieving the same objective. Such interruptions leave underlying participant objectives unaltered. The consistency of participants' objectives is embodied in the continuity restrictions.[6]

Applying Poirier's reasoning to the term structure of interest rates, spline continuity restrictions amount to assuming that the investor's objectives are invariant. What differs over maturities (and over time) is the investor's optimal rule for achieving objectives. The various shapes of the term structure are the result of time varying <u>perceptions</u> of the changing structural relationships. The use of splines will capture the embedded associational relationships between interest rates of various maturities that are expressed in the term structure.

This last point is crucial to the motivation for using spline functions of expected inflation and spline functions of nominal interest rates to estimate the Fisher effect. The spline function provides a means of expressing the associational information that is contained in the term structure. One way of recognizing the importance of this issue is to note that separately regressing a sequence of the nominal interest rates on rates of expected inflation is by no means equivalent to regressing the term structure of nominal interest rates on the term structure of expected inflation in a single regression. In the first case, each equation will have missing variables bias. The equation is not correctly identified (Sims 1980, Granger and Newbold 1973). Since the spline function includes the associational effects, estimates of the Fisher effect using term structures should reduce this missing variable bias.

## II. THE ESTIMATION OF SPLINE FUNCTIONS

Most discussions of the econometric practices of estimating spline functions relate the spline estimation problem to the problems of estimating finite distributed lag models. However, Suits, Mason, and Chan (1977) have pointed out that the estimation of spline functions can be viewed as a special case of piecewise regression using dummy variables. Both of these approaches will be discussed here. Our procedure for estimating splines of the term structure of interest rates and splines of the term structure of inflationary expectations is based on piecewise regression methods.

In econometric practice the estimation of spline distributed lag functions can be viewed as a generalization of the problem of estimating a finite distributed lag of the form:

(5.9)

$$y_t = \sum_{i=0}^{n} \beta_i x_{t-i} + u_t$$

where the $x_{t-i}$ are the t-i lags of the exogenous variable x. In such functions the $x_{t-i}$ are, however, likely to exhibit multicollinearity. In the presence of severe multicollinearity, estimation by ordinary least squares (OLS) is not sufficiently precise to separate the effect of the different exogenous variables. The OLS estimates of the parameters, the b's, lack precision. This lack of precision can be solved by introducing prior information in the form of restrictions on the parameters. The various lag distributions such as the Koyck and the Almon lag are methods of introducing such restrictions.

Within the class of distributed lag estimation, the spline function is a generalization of the Almon lag technique. In the Almon technique the lag weights, $\beta_i$ are assumed lie on a polynomial of some order and may be written as:

(5.10)

$$\beta_i = \alpha_o + \alpha_1 i + . . . + \alpha_q i^q$$

In matrix form this is:

(5.11)

$$\beta = H\alpha$$

where:

$$\alpha' = (\alpha_o, \alpha_1, . . . , \alpha_q)$$

and where:

$$H = \begin{bmatrix} 1 & 0 & 0 & . . . & 0 \\ 1 & 1 & 1^2 & . . . & 1^q \\ 1 & 2 & 2^2 & . . . & 2^q \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ 1 & n & n^2 & . . . & n^q \end{bmatrix}$$

The distributed lag function in equation (5.9), by substitution is:

(5.12)

$$y = XH\alpha + e$$

This equation can be estimated directly by OLS; $a = (Z'Z)^{-1}Z'y$ where $Z=XH$. Once $\alpha$ is estimated, the estimator of $\beta$ is calculated by $b=Ha$. While the estimation is straightforward and is parsimonious in terms of the number of parameters estimated, the properties of the sampling distribution are unknown since their selection is subject to pretesting strategy (the specification of lag length and polynomial order).

An alternative way of viewing the problem of estimating $b=Ha$ where $a = (Z'Z)^{-1}Z'y$ is to note that the a's are a set of restricted least squares estimators. These are obtained by estimating equation (5.9) subject to (n-q) independent linear homogeneous restrictions. This can be shown by solving equation (5.11) for $\alpha$; so that given $\beta = H\alpha$, then $\alpha$ is:

(5.13)

$$\alpha = (H'H)^{-1} \quad H'\beta$$

Then writing equation 5.11 in implicit form as:

(5.14)

$$0 = \beta - H\alpha$$

and substituting for $\alpha$ one obtains:

(5.15)

$$0 = \beta - (H(H'H)^{-1}H')\beta$$

Equation 5.15 simplifies to:

(5.16)

$$0 = (I - (H(H'H)^{-1}H')\beta$$

where $(I-H(H'H)^{-1}H')$ forms a set of (n-q) linear homogeneous restrictions, $R\beta = 0$, on n+1 equations.

While the above outlines a generalized way of developing a restrictions matrix, R, this matrix must be given some specific content. Shirley Almon (1965) proposed that the restrictions matrix be specified by Lagrangian interpolation. In this scheme if the n+1 weights $\beta_i$ are to lie on the graph of a polynominal of degree q, then they could be calculated by linear combinations (formed by Lagrangian constraints) of q+1 points on the polynominal. Clopper Almon (1967) has offered a theoretical exposition of this technique based on maximization of a dynamic structure subject to a chain of Lagrangian constraints.[7] The problem with applying such a technique is that first, the degree of the polynomial must be known from theory. Second the exact length of the lag must be known. Third, the number of Lagrangian coefficients that must be generated form a matrix of size (q+1) x (n+1). While restricted estimation is less subject to error than direct estimation, it is more costly in computations.

A polynomial spline of degree P is a piecewise polynomial function made up of polynomials of degree less than or equal to P. The spline and its derivatives up to and including the (P-1)st, are continuous functions. By analogy an exponential spline of degree P is a piecewise exponential function made up of exponentials with functional arguments of degree less than or equal to P. Again, the spline and its derivatives up to, and including the (P-1)st are continuous functions.

A cubic polynomial spline is a set of cubic polynomials joined together under the restrictions that their first and second derivatives are to be equal at the join points or knots. The function which generates the weight $\beta_i$ is then defined as:

(5.17)

$$\beta = I_{[0,i_1]}(i)g_1(i) + I_{[i_1,i_2]}(i)g_2(i) + I_{[i_1,n]}(i)g_3(i)$$

$I_{[.]}(i)$ is an indicator function whose value is one if the argument, i, is in the interval and zero otherwise. The $i_j$ are the points in the interval [0,n]. The $g_j(i)$ are polynomials of the form:

(5.18)

$$g_j(i) = a_j i^3 + b_j i^2 + c_j i + d_j$$

The restrictions on the derivatives evaluated at the knots require that:

$$\begin{aligned} g_1(i_1) &= g_2(i_1) \\ g'_1(i_1) &= g'_2(i_1) \\ g''_1(i_1) &= g''_2(i_1) \\ g_2(i_2) &= g_3(i_2) \\ g'_2(i_2) &= g'_3(i_2) \\ g''_2(i_2) &= g''_3(i_2) \end{aligned}$$

Rewrite equation (5.12) $y = XH\alpha + e$, as $y = XH^*\alpha^* + e$, where $H^*$ is a block diagonal matrix containing the restrictions set defined by (5.18). The vector $\alpha^*$ is defined as:

$$\alpha^* = (\alpha'_1, \alpha'_2, \alpha'_3)$$

where

$$\alpha_j = (a_j\ b_j\ c_j\ d_j)'$$

for

$$j = 1,\ 2,\ \text{and}\ 3.$$

Hence, for the case of a cubic polynomial spline there will be twelve coefficients $\alpha^*$ estimated subject to six restrictions. The $\beta$'s are then estimated by $b = H^*a^*$. Such spline functions have greater flexibility than Almon polynomial lags, because they are essentially several polynomials spliced together. The cost of this flexibility is an increase in the number of restrictions. One could gain flexibility in the Almon polynomial lag function by increasing the degree of the polynomial, but the use of the spline to gain this flexibility leads to more free parameters.

Several econometric problems arise whether direct or restricted estimation of the $\beta_i$'s is used. Unless the specification of both the degree of the polynomial and the lag length is correct, the distributed lag estimator will be biased. Correct lag length but specification of a polynomial of higher degree than the true polynomial leads to an inefficient estimator. In addition, since the estimators are derived from unknown probability distributions, there are no direct tests to determine if the specification is in fact correct. What is generally done throughout the literature is to attempt to draw conclusions from theory about both the degree of the polynomial and the lag length. This approach has been severely criticized.[8]

# III. THE CUBIC EXPONENTIAL SPLINE MODEL FOR ESTIMATING TERM STRUCTURES

The estimation procedure which we employ is directly related to the distributed lag technique of estimating splines. Our technique, which is in fact a dummy variable technique, also requires the construction of a restriction matrix similar to that discussed above. Each dummy variable can be viewed as permitting the term structure function to take on a different intercept value, and the slope coefficients in interval of the function may take on different values. The independent variable in our term structures is time in months and since we wish to estimate a cubic exponential spline, the estimating form of the independent variable is logarithm of time in months: lnx, $(\ln x)^2$, and $(\ln x)^3$ where x is time in months to maturity.

We have divided the time in months to maturity into five intervals represented by indicator variables $d_1$ through $d_5$ so that:

$d_1$ = time in months less than or equal to ln (6 months)

d2 = time in months greater than ln (6 months) and less than or equal to ln (12 months)

$d_3$ = time in months greater than ln (12 months) and less than or equal to ln (24 months)

$d_4$ = time in months greater than ln (24 months) and less than or equal to ln (48 months) and

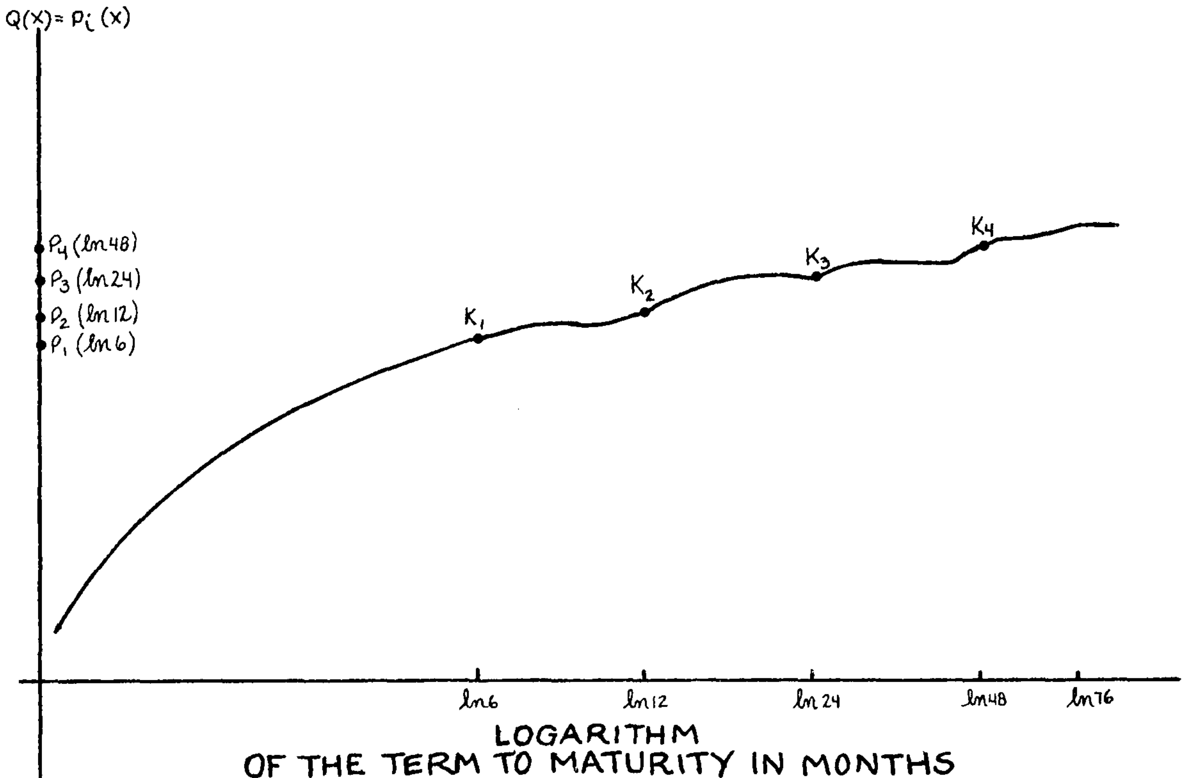
$d_5$ = time in months greater than ln (48 months).

We have concentrated the intervals in the early portion of the term structure given that short term interest rates are more volatile than long term interest rates. The data on inflation also reveal the same pattern. Unequal interval lengths are not commonly employed in spline estimation but this seems to be largely because of the computational burden.

The intervals formed by the indicators $d_1$ through $d_5$ are separated by the spline knot points, and therefore the spline functions and their derivatives must be equal at these knot points. We therefore wish to estimate both the nominal interest rate and the rate of expected inflation as a function of time, subject to these restrictions on the interval functions. These conditions will produce twelve restrictions which will constrain the estimates of twenty spline coefficients. The spline function for both the term structure of inflationary expectations and the term structure of interest rates will be estimated by restricted least squares.

In order to demonstrate the form of the restrictions matrix, consider a function like that depicted in Figure 5.1 below:

FIGURE 5.1
GRAPH OF A GENERALIZED CUBIC
SPLINE FUNCTION




The curve, y = Q(x), may be viewed as a function constructed of pieces on the intervals:

(5.19)

$$Q(x) = \begin{cases} P_1(x) \text{ for } x \le \ln 6 \\ P_2(x) \text{ for } x > \ln 6 \text{ and } x \le \ln 12 \\ P_3(x) \text{ for } x > \ln 12 \text{ and } x \le \ln 24 \\ P_4(x) \text{ for } x > \ln 24 \text{ and } x \le \ln 48 \\ P_5(x) \text{ for } x > \ln 48 \end{cases}$$

where $K_1$ through $K_4$ are our knot points. Knot point $K_1$ has coordinate (ln6, $P_1$(ln6)). $K_2$ has coordinate (ln12, $P_2$(ln12)). $K_3$ and $K_4$ can be written in a similar manner.

Each segment of the function $Q(x)$ in our case is estimated as a cubic exponential of the form:

(5.20)

$$P_i(x) = a_i + b_{i1}\ln x + b_{i2}(\ln x)^2 + b_{i3}(\ln x)^3$$

for $i=1, \ldots, 5$.

At $K_1$, the first knot point, we establish continuity conditions requiring that $P_1(x)$ and $P_2(x)$ when evaluated at $x_1$, are equal, as must be their derivatives:

(5.21)

$$\begin{aligned} P_1(x_1) &= P_2(x_1) \\ P'_{1L}(x_1) &= P'_{2R}(x_1) \\ P''_{1L}(x_1) &= P''_{2R}(x_1) \end{aligned}$$

where the domain of $P_1$ and $P_2$ are respectively:

$$\text{dom } P_1 = (0,x_1] \text{ and } \text{dom } P_2 = [x_1,x_2]$$

and $P'_1$ and $P''_1$ are defined only on the interval $(0,x_1]$ and $P'_2$ and $P''_2$ and are defined only on the interval $[x_1,x_2]$. Hence, we require that the first two segments of the function $Q(x)$ be equal at knot point $K_1$. We also require that the first and second derivatives of these segments be equal when evaluated at $x_1$. By analogy we require that the above conditions must be true of all segments $P_i(x)$ of $Q(x)$ for each knot point.

One can extend the $P_i(x)$ to functions $Q_1(x)$ defined on the entire domain of $Q(x)$ as follows:

(5.22)

$$Q_1(x) = \begin{cases} P_1(x) & \text{for x contained in } (0,x_1] \\ 0 & \text{for x not contained in } (0,x_1] \end{cases}$$

$$Q_2(x) = \begin{cases} P_2(x) & \text{for x contained in } [x_1,x_2] \\ 0 & \text{for x not contained in } [x_1,x_2] \end{cases}$$

$$\vdots$$

$$Q_5(x) = \begin{cases} P_5(x) & \text{for x contained in } [x_4,\text{infinity}] \\ 0 & \text{for x not contained in } [x_4,\text{infinity}] \end{cases}$$

Therefore:

(5.23)

$$Q(x) = Q_1(x) + Q_2(x) + \ldots + Q_5(x)$$

or alternatively,

(5.24)

$$Q(x) = \begin{cases} P_1(x) & \text{for x contained in } (0,x_1] \\ P_2(x) & \text{for x contained in } [x_1,x_2] \\ \vdots & \\ P_5(x) & \text{for x contained in } [x_4,\text{infinity}) \end{cases}$$

The term structures Q(x) are defined at all $x_j$ (time measured in months). The segments $P_1(x)$ through $P_5(x)$ of the term structure Q(x) are defined only on the intervals $(0,x_1]$, $[x_1,x_2]$, . . . , $[x_4,\text{infinity})$ and essentially allow for structural change along the term structure under the restriction that the term structure is to be continuous and twice differentiable for all terms to maturity.

In order to differentiate the segments we establish indicator variables, $d_1$ through $d_5$, defined as follows:

(5.25)

$$d_1(x) = \begin{cases} 1 \text{ for } 0 < x \leq x_1 \\ 0 \text{ otherwise} \end{cases}$$

$$d_2(x) = \begin{cases} 1 \text{ for } x_1 < x \leq x_2 \\ 0 \text{ otherwise} \end{cases}$$

$$\vdots$$

$$d_5(x) = \begin{cases} 1 \text{ for } x > x_4 \\ 0 \text{ otherwise} \end{cases}$$

One may note that these indicators are similar to the usual dummy variable form. Using this notation we may write:

(5.26)

$$Q(x) = d_1(x)Q_1(x) + d_2(x)Q_2(x) + \ . \ . \ . \ d_5(x)Q_5(x)$$

where again the values of x are the logarithm of time in months.

If we substitute the function form of $P_i(x)$ for $Q_i(x)$ in our equation we have

(5.27)

$$\begin{aligned} Q(x) = {} & d_1(x)(a_1 + b_{11}\ln x + b_{12}(\ln x)^2 + b_{13}(\ln x)^3) + \\ & d_2(x)(a_2 + b_{21}\ln x + b_{22}(\ln x)^2 + b_{13}(\ln x)^3) + \\ & . \ . \ . + d_5(x)(a_5 + b_{51}\ln x + b_{52}(\ln x)^2 + b_{53}(\ln x)^3) \end{aligned}$$

By collecting terms, computing the first and second derivatives, and noting the restrictions on evaluation of derivatives at each knot point from the left and right we may derive the restrictions matrix as shown in Table 5.1.

Each of the monthly term structures of both nominal interest rates and inflationary expectations were estimated by restricted least squares. Table 5.1 forms the set of restrictions applied. The result is that each monthly term structure of the observations is summarized by a set of 20 spline coefficients. Using these spline coefficients one can estimate the nominal interest rate of inflation for any maturity. Appendix D contains an example for 2, 3, 6, 12, 24, 36, 48, 60, 72 and 84 months based on the spline coefficients for the sample month presented in Appendix C.

TABLE 5.1

GENERAL FORM FOR RESTRICTIONS MATRIX FOR CUBIC EXPONENTIAL SPLINE: FOUR KNOTS AND FIVE INTERVALS

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $b_{11}$ | $b_{21}$ | $b_{31}$ | $b_{21}$ | $b_{22}$ | $b_{32}$ | $b_{13}$ | $b_{23}$ | $b_{33}$ | $b_{14}$ | $b_{24}$ | $b_{34}$ | $b_{15}$ | $b_{25}$ | $b_{35}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | 0 | 0 | 0 | $x_1$ | $x_1^2$ | $x_1^3$ | $-x_1$ | $-x_1^2$ | $-x_1^3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | knot 1 |
| 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | $x_2$ | $x_2^2$ | $x_2^3$ | $-x_2$ | $-x_2^2$ | $-x_2^3$ | 0 | 0 | 0 | 0 | 0 | 0 | knot 2 |
| 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $x_3$ | $x_3^2$ | $x_3^3$ | $-x_3$ | $-x_3^2$ | $-x_3^3$ | 0 | 0 | 0 | knot 3 |
| 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $x_4$ | $x_4^2$ | $x_4^3$ | $-x_4$ | $-x_4^2$ | $-x_4^3$ | knot 4 |
| 0 | 0 | 0 | 0 | 0 | 1 | $2x_1$ | $3x_1^2$ | -1 | $-2x_1$ | $-3x_1^2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | knot 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $2x_2$ | $3x_2^2$ | -1 | $-2x_2$ | $-3x_2^2$ | 0 | 0 | 0 | 0 | 0 | 0 | knot 2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $2x_3$ | $3x_3^2$ | -1 | $-2x_3$ | $-3x_3^2$ | 0 | 0 | 0 | knot 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $2x_4$ | $3x_4^2$ | -1 | $-2x_4$ | $-3x_4^2$ | knot 4 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | $6x_1$ | 0 | -2 | $-6x_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | knot 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | $6x_2$ | 0 | -2 | $-6x_2$ | 0 | 0 | 0 | 0 | 0 | 0 | knot 2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | $6x_3$ | 0 | -2 | $-6x_3$ | 0 | 0 | 0 | knot 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | $6x_4$ | 0 | -2 | $-6x_4$ | knot 4 |

| Segment One | Segment Two | Segment Three | Segment Four | Segment Five |
|---|---|---|---|---|
| $P_1(x)$ | $P_2(x)$ | $P_3(x)$ | $P_4(x)$ | $P_5(x)$ |
| for all $x \leq x_1$ | for all $x > x_1$ and $x \leq x_2$ | for all $x > x_2$ and $x \leq x_3$ | for all $x > x_3$ and $x \leq x_4$ | for all $x > x_4$ |

$x_1$ = ln (6 months)
$x_2$ = ln (12 months)
$x_3$ = ln (24 months)
$x_4$ = ln (48 months)

# IV. RESULTS OF SPLINE FUNCTION ESTIMATION

Three hundred ten spline functions were estimated so not detailed regression results will be presented. The primary statistical test of interest was the restricted F test. Testing at the 95% confidence level for 12 (restrictions) and 55 (sample size minus twenty coefficients estimated) degrees of freedom, 98% of the restricted regressions were statistically significant.

We openly admit that the empirical work done here has opened a number of issues for future research. For example we did no tests for the appropriateness of our choice of knot placement. We also did not test other functional forms. However, the reader is reminded that our purpose was to obtain estimated spline coefficients which could be used as data in regression. This meant that we could not vary knot placement for each function, nor could we vary the order of the function.

We should make note of two points before proceeding:

1. In interval one, of the spline estimated term structure of nominal interest rates, we usually had an outlier problem. As noted in Chapter IV, our method of estimating nominal interest rates created biased estimates of the short-term nominal interest rate when notes were moved closer to or further from their true date of maturity. In most cases this meant that short-term interest rates (the

15 and 45 day rates) were extremely high compared to the other interest rates estimated. However, one significant advantage of the spline function compared to other flexible form estimation (polynomial functions like the Almon lag function) is that the spline is less sensitive to data concentration.[9] The parameters in each interval are largely estimated from the data in that interval with minimal influence from other segments. The outlier problem should, therefore, have a limited impact on the regression estimates of the Fisher effect.

2. There is still a substantial controversy regarding the relative merits of the cubic polynomial spline and the cubic exponential spline. While the cubic polynomial has an ill-fitting curvature for estimating an exponential function, the exponential spline often tends to drift radically at the long term end of the term structure.[10] However, our choice of a knot placement at a long term maturity seems to have minimized the tendency of our term structure to drift. We have in essence anchored our term structure, but use of the spline function to estimate maturities beyond 76 months should be avoided.

# CHAPTER V

## CHAPTER NOTES

1. As discussed in Chapter IV our data on nominal interest rates are derived from the yields to maturity on securities by removing coupon effects. We did not include any securities with call features. We did not attempt to remove tax effects.

2. The spot rate is the price today for delivery today. The forward rate is the price today for delivery at a specified future date. If the forward rate is above the spot rate, a contract for forward delivery carries a premium. More is paid today for future delivery of a specified quantity than is paid for an equivalent quantity bought at the spot rate. Arbitrage is assumed to take place anytime the spot rate and the forward rate differ. In an efficient market these arbitrage opportunities would quickly disappear.

3. Empirical representations of term structure of yields and spot rates have been developed by numerous researchers. Such empirical term structures are typically constructed by fitting a curve through ordered pairs of maturities (x) and returns (y). The fit of a curve to these (x,y) ordered pairs can be achieved by free hand techniques, ordinary least squares, or restricted least squares. Well known examples may be found in Durand (1942), McCulloch (1971 and 1975a), and Carleton and Cooper (1976).

4. Smoothness and flexibility are not terms that have exact mathematical definitions. The use of these terms follows from the origin of the spline as a tool in architectural drawing. A spline is a thin strip of wood that is sufficiently pliable to bend around attachments to create a smooth curvature. By varying the number and placement of the attachments (knots) the spline can be made to assume a variety of shapes. Spline functions later became a subject of intense study in applied mathematics largely as a technique of approximating functions for practical applications.

   For our purposes flexibility is meant to imply a variety of shapes. This does not, however, mean all possible shapes. Some shapes would not be of interest in modelling term structures. For example, we would not be interested in admitting a set of functions which permitted kinks in the term structure. Such kinks would occur in the case of linear spline functions.

Smoothness is an important characteristic of a term structure. Since we assume arbitrage and an efficient market we expect smoothness or continuity in the forward rate. An excellent discussion can be found in Thies (1982, pp. 46-50) and Smoot (1983, pp. 31-40). (Note that an 'efficient market' in information and 'rational' economic agent behavior should also produce smoothness in forward rates of expected inflation.)

The continuity conditions which we impose to achieve this smoothness does, in a purely mathematical sense, minimize the problem of knot placement. The restrictions that the pieces of the function, the first derivatives and the second derivatives are equal is equivalent to stating that the two functions should be as much alike as possible. If we therefore fit the pieces of the function correctly, the knot point should be invisible. This can be illustrated by considering the relationship between the curvature of the spline function and the curvature of an osculating circle at a knot point.

At a point P(a,b) on y = f(x), the curvature is defined to be

$$k = \frac{|y''|}{[1+(y')^2]^{3/2}} = \frac{|f''(a)|}{[1+(f'(a))^2]^{3/2}}$$

On a circle of radius r, the curvature at any point turns out to be 1/r. Large curvature at a point P means the curve has a sharp bend at P. For a straight line (with equation y = mx+b, we compute y'' = 0 and) the curvature at any point is 0.

At a point Q(x,y) on y = f(x) where the curvature $k \neq 0$, the so-called osculating circle that passes through Q, has the same slope as f(x), and has the same curvature k. Thus, in constructing this osculating circle, one is constructing the circle which is most like y = f(x) at Q. Intuitively, if one considers the graph of y = f(x) as a road, then a very near-sighted driver at point Q could not distinguish the graph (he can see) from the osculating circle. Vasicek and Fong (1982) indicate similar reasoning in deriving their knot point.

Consider a spline function whose pieces y = g(x) and y = h(x) share a knot point P (a,b). The spline conditions are exactly what are required so that the osculating circle constructed at P does not depend on whether construction involved g, g', g'' at P or h, h', h'' at P.

Let us consider the analogy between the graph y = f(x) and a flat road. How fast one can drive along a road (a term structure) without skidding (without an abrupt change in the forward rate) depends on the speed of the vehicle and the curvature of the road. The conditions at the knots of the spline function mean that a driver traveling as fast as possible without skidding along a piece of the spline function will be able to avoid skidding at a knot.

5. Poirier's (1976) technique for modeling structural change in economic relationships is different from rational expectations, but the motivation for examining the method is based on the same theoretical premises as is rational expectations.

6. There is a brief but exceptionally clear discussion of spline modeled structural change in Chapter 10 of Judge et alii, 1980. The discussion there describes the spline model in the context of the general econometric problem of variable parameters models.

7. Also see Theil (1971) on the topics of GLS and restricted regression.

8. See discussion in Griliches (1976), Nerlove (1972), and Sims (1980).

9. See Thies (1982), Kemmsies (1982), and Smoot (1983).

10. See Shea (1983).

# CHAPTER VI

## ESTIMATES OF THE EFFECT OF THE TERM STRUCTURE OF INFLATIONARY EXPECTATIONS ON THE TERM STRUCTURE OF NOMINAL INTEREST RATES

The spline representation of the Fisher effect is:

(6.1)

$$_{t+m}i_Q(x) = {}_{t+m}\alpha_Q(x) + {}_{t+m}\beta({}_{t+m}I^e_Q(X)) + {}_{t+m}v_Q(x)$$

The spline representation of our estimating equation of the Fisher effect is:

(6.2)

$$_{t+m}i_Q(x) = {}_{t+m}A_Q(x) + {}_{t+m}B({}_{t+m}I^e_Q(x)) + {}_{t+m}u_Q(x)$$

Since each of the term structures was fit with a cubic exponential spline over five segments, each spline is represented by 20 coefficients. The five segments are represented by five dummy variables, each with a coefficient. The cubic function, within each segment, is represented by three coefficients. The segments were chosen so that:

1. segment one covers term to maturity less than or equal to six months,
2. segment two covers term to maturity greater than six months but less than or equal to twelve months,
3. segment three covers term to maturity greater than twelve months but less than or equal to twenty-four months,

4. segment four covers term to maturity greater than twenty-four months but less than or equal to forty-eight months,

5. segment five covers term to maturity greater than forty-eight months.

The twenty spline coefficients for each term structure compactly summarize the seventy-six terms to maturity. In order to estimate the Fisher effect regression coefficients, ${}_{t+m}A_{Q(x)}$ and ${}_{t+m}B_{Q(x)}$, the 155 cross-sections (individual term structures) for ${}_{t+m}i_{Q(x)}$ and ${}_{t+m}I^{e}_{Q(x)}$ were pooled by stacking the spline coefficients into time series vectors. The original 11,780 observations per variable were thus reduced to 3,100 observations. The reader is referred to Appendix D for a technical discussion of the derivation of these vectors. The appendix also contains a discussion and demonstration of how one can generate an estimate of the nominal interest rate and the expected rate of inflation for any term to maturity using the spline coefficients.

We estimated the Fisher effect from the pooled cross-section/time series of spline coefficients using two different methods. The first method used was estimation by ordinary least squares (OLS). In addition to the spline coefficients on the expected rate of inflation, the independent variables included a trend variable and a set of monthly seasonal dummy variables. The second method of estimation was estimation using multiple-equation generalized least squares (the Zellner seemingly unrelated regression technique).

In estimating the Fisher effect from the pooled spline coefficients via OLS, we hypothesized that the Fisher effect could be represented by a single cross-section coefficient:

(6.3)

$$_{t+m}B_{Q}(x) = 1$$

This null hypothesis is equivalent to the assumption that there is a complete pass through of expected inflation, on average, across the term structure. We also included a trend variable to estimate drift in nominal rates.

We expected that the set of monthly seasonal dummy variables might affect either the intercept, the coefficent on expected inflation, or both. These monthly seasonal dummy variables were designed so that the t test would not depend on the omitted variable. We chose to omit January. The slope coefficient on the dummy variable for each of the remaining months is an estimate how that month's slope differs from the average change in nominal interest rates, given a change in expected inflation (Pindyck and Rubinfeld, 1981, pp 135-137). The F statistic that all of the monthly dummy variables included to estimate shifts in the intercept had zero coefficients failed to surpass the critical value. However, an F test that all of the monthly dummy variables included to estimate shifts in the slope had zero coefficients led to rejection at the 95 percent level.

Appendix E includes alternative regression models and associated statistics.

The proposed single equation splined Fisher model including trend and seasonal variables is:

(6.4)

$$
\begin{aligned}
{}_{t+m}i_{Q(x)} = \; & {}_{t+m}\alpha_{Q(x)} + {}_{t+m}\beta \, {}_{t+m}I^{e}_{Q(x)} + \phi t + \\
& \tau_1 SDUM_{FE} + \tau_2 SDUM_{MR} + \ldots + \\
& \tau_{11} SDUM_{DC} + {}_{t+m}u_{Q(x)}
\end{aligned}
$$

where:

$${}_{t+m}i_{Q(x)} = \text{spline coefficient of nominal rates}$$

$${}_{t+m}I^{e}_{Q(x)} = \text{spline coefficient of expected inflation}$$

$$t = \text{trend}$$

and:

$SDUM_{FE}$, $SDUM_{MR}$, $SDUM_{AP}$, . . ., $SDUM_{DC}$ are slope dummy variables for the months of February through December. These slope dummies are constructed as the month indicator multiplied by the spline coefficients of expected inflation. The indicator for a month is one for that month, negative one for January, and zero for all other months.[1]

The estimating equation for the splined Fisher effect is:

(6.5)

$$
\begin{aligned}
{}_{t+m}i_{Q(x)} = \; & {}_{t+m}A_{Q(x)} + {}_{t+m}B({}_{t+m}I^{e}_{Q(x)}) + Ct + \\
& D_1 SDUM_{FE} + D_2 SDUM_{MR} + \ldots + \\
& D_{11} SDUM_{DC}
\end{aligned}
$$

where:

A is the estimator for $\alpha$

B is the estimator for $\beta$

C is the estimator for $\phi$

and $D_1, D_2, \ldots, D_{11}$ are the estimators for $\tau_1, \tau_2, \ldots, \tau_{11}$.

The regression results for equation (6.5) are presented in Table 6.1.

TABLE 6.1

ESTIMATE OF THE FISHER EFFECT: OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES WITH TREND AND MONTHLY SLOPE DUMMY VARIABLES

Dependent Variable $_{t+m}i_{Q(x)}$

| Independent Variables | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.2493907E-02 | 0.1181187E-02 | -2.111357 |
| $_{t+m}I^{e}_{Q(x)}$ | 1.249285 | 0.1487487E-01 | 83.98626 |
| $t$ | 0.7600914E-04 | 0.1302535E-04 | 5.835476 |
| $SDUM_{FE}$ | -0.6035958E-01 | 0.4626343E-01 | -1.304693 |
| $SDUM_{MR}$ | -0.2194041 | 0.3872231E-01 | -5.666091 |
| $SDUM_{AP}$ | -0.1676386 | 0.4022767E-01 | -4.167246 |
| $SDUM_{MY}$ | -0.7488156E-01 | 0.4322733E-01 | -1.732274 |
| $SDUM_{JU}$ | -0.1947499 | 0.4013610E-01 | -4.852238 |
| $SDUM_{JL}$ | 0.8810018E-01 | 0.3747746E-01 | 2.350751 |
| $SDUM_{AU}$ | 0.2877342E-01 | 0.4537176E-01 | 0.6341702 |
| $SDUM_{SE}$ | -0.2034053 | 0.4179360E-01 | -4.866902 |
| $SDUM_{OC}$ | 0.6756377E-01 | 0.4126623E-01 | 1.637265 |
| $SDUM_{NO}$ | 0.1820972 | 0.5056912E-01 | 3.600956 |
| $SDUM_{DC}$ | 0.8365628E-02 | 0.4960226E-01 | 0.1686542 |

where:

$_{t+m}i_{Q(x)}$ = nominal interest rates, splined

$_{t+m}I^{e}_{Q(x)}$ = expected rate of inflation, splined

$t$ = trend

$SDUM_{FE}$, $SDUM_{MR}$, $SDUM_{AP}$, . . . , $SDUM_{DC}$

= slope dummy variables for February through December

degrees of freedom = 3086

$\bar{R}_2$ = .71

$F$ = 622.34

Error Sum of Squares (ESS) = .032297

Since, a priori, we did not know how the monthly seasonal dummy variables would be related to nominal interest rates, our regression method involved a strategy. In the case of a strategy Theil (1971) recommends the use of liberal confidence intervals. The critical value for the t test for 80 per cent and 90 per cent confidence intervals are 1.282 and 1.645 respectively. Applying the 90 per cent confidence interval value of t, the change in nominal interest rates in February, August, and December is not significantly different from the average change in nominal interest rates of 1.25, the estimated slope, ${}_{t+m}B_{Q(x)}$.

The change in nominal interest rates is lower than average in March, April, May, June and September. It is higher than average in July, October and November. In order of the magnitude of the difference from the average change in nominal interest rates, March has the largest negative value, followed by September, June, April and May. In order of magnitude above the average change in nominal interest rates, November is largest, followed by July and October.

The change in nominal interest rates for a change in expected inflation is 1.25. This exceeds the hypothesized value of one, in both point and interval form. Nominal interest rates increase (decrease) by more than the increase (decrease) in expected inflation. The strict Fisher hypothesis that ${}_{t+m}B_{Q(x)} = 1$ must be rejected.

There is also a positive and significant trend in nominal interest rates. The coefficient is, however, extremely small.

The constant term is negative and significant. Its coefficient is also quite small.

The above test of the Fisher effect was constrained to estimate a single value for the pass through of inflation expectations to nominal interest rates. We alternatively proposed that the same single equation splined Fisher model including trend and seasonal variable be estimated separately for each spline segment. This can be written as:

(6.6)

$$_{t+m}i_{p_n}(x) = {}_{t+m}\alpha_n P_n(x) = {}_{t+m}\beta_n({}_{t+m}I^e_{p_n}(x)) + \phi_{nt} + \tau_{n1}SDUM_{FE} + \tau_{n2}SDUM_{MR} + \ldots + \tau_{n11}SDUM_{DC} + {}_{t+m}u_{p_n}(x)$$

where:

$_{t+m}i_{p_n}(x)$ = spline coefficients for nominal interest rates for segment n where n = 1 to n = 5

$_{t+m}I^e_{p_n}(x)$ = spline coefficients for expected inflation for segment n, where n = 1 to n = 5

t = trend

and:

$SDUM_{FE}$, $SDUM_{MR}$, $SDUM_{AP}$, . . . $SDUM_{DC}$ are slope dummy variables as defined above.

Referring back to Chapter V, we estimated the Fisher effect across Q(x) in equation (5.24) by estimating equation (6.5) for $P_1(x)$, $P_2(x)$, . . . , $P_5(x)$. Equation (6.5) was rewritten to reflect that the parameters of equation (6.6) will differ for each of the five segments so that:

(6.7)

$$ {}_{t+m}i_{p_n}(x) = {}_{t+m}A_{n}p_n(x) + {}_{t+m}B({}_{t+m}I^{e}_{p_n}(x)) + C_n t + D_{n1}SDUM_{FE} + D_{n2}SDUM_{MR} + \ldots + D_{n11}SDUM_{DC} $$

for all variables as previously defined and for n = 1 to 5. The null hypothesis was altered to represent the joint (OLS) test:

$$ {}_{t+m}B_1P_1(x) = 1 $$

$$ {}_{t+m}B_2P_2(x) = 1 $$

$$ {}_{t+m}B_3P_3(x) = 1 $$

$$ {}_{t+m}B_4P_4(x) = 1 $$

$$ {}_{t+m}B_5P_5(x) = 1 $$

The result of the regression for each of the five segments are shown in Table 6.2 through Table 6.6. The variables for each of the tables are:

${}_{t+m}i_{p_n}(x)$ = nominal interest rates, splined, n = 1 to 5 intervals of the complete spline function ${}_{t+m}i_{Q}(x)$

${}_{t+m}I^{e}_{p_n}(x)$ = expected rate of inflation, splined, n = 1 to n = 5 intervals of the complete spline function ${}_{t+m}I^{e}_{Q}(x)$

t = trend

$SDUM_{FE}$, $SDUM_{MR}$, $SDUM_{AP}$, . . ., $SDUM_{DC}$ = slope dummy variables through December

## TABLE 6.2

### ESTIMATE OF THE FISHER EFFECT FOR SPLINE SEGMENT ONE: OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES WITH TREND AND MONTHLY SLOPE DUMMY VARIABLES

Dependent Variable $_{t+m}i_{p_1(x)}$

| Independent Variables | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.5396067E-02 | 0.3563134E-02 | -1.514416 |
| $_{t+m}I^{e}_{p_1(x)}$ | 1.376102 | 0.4432210E-01 | 31.04777 |
| t | 0.5117649E-04 | 0.3930242E-04 | 1.302121 |
| $SDUM_{FE}$ | -0.1032454 | 0.1368917 | -0.7542121 |
| $SDUM_{MR}$ | -0.3316839 | 0.1097142 | -3.023164 |
| $SDUM_{AP}$ | -0.2081752 | 0.1184043 | -1.758172 |
| $SDUM_{MY}$ | -0.7840005E-01 | 0.1327491 | -0.5905880 |
| $SDUM_{JU}$ | -0.2250461 | 0.1191672 | -1.888491 |
| $SDUM_{JL}$ | 0.2567399 | 0.1115448 | 2.301675 |
| $SDUM_{AU}$ | 0.7799634E-02 | 0.13799890 | 0.5652360E-0 |
| $SDUM_{SE}$ | -0.3548091 | 0.1224161 | -2.898385 |
| $SDUM_{OC}$ | 0.6228110E-01 | 0.1199837 | 0.5190795 |
| $SDUM_{NO}$ | 0.1903143 | 0.1556584 | 1.222641 |
| $SDUM_{DC}$ | -0.2874150E-01 | 0.1499549 | -0.1916677 |

degrees of freedom = 606

$\bar{R}^2$ = .62

Error Sum of Squares = .043568

TABLE 6.3

ESTIMATE OF THE FISHER EFFECT FOR SPLINE SEGMENT TWO: OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES WITH TREND AND MONTHLY SLOPE DUMMY VARIABLES

Dependent Variable $_{t+m}i_{P2}(x)$

| Independent Variables | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.4816283E-03 | 0.2719773E-02 | -0.1770840 |
| $_{t+m}I^{e}_{P2}(x)$ | 1.196756 | 0.3448248E-01 | 34.70619 |
| t | 0.9900240E-04 | 0.2996746E-04 | 3.303664 |
| $SDUM_{FE}$ | -0.4761126E-01 | 0.1083468 | -0.4394339 |
| $SDUM_{MR}$ | -0.2174374 | 0.8983738E-01 | -2.420344 |
| $SDUM_{AP}$ | -0.2187822 | 0.9229999E-01 | -2.370338 |
| $SDUM_{MY}$ | -0.9446262E-01 | 0.9883836E-01 | -0.9557283 |
| $SDUM_{JU}$ | -0.2080906 | 0.9197299E-01 | -2.262519 |
| $SDUM_{JL}$ | 0.4842011E-01 | 0.8539470E-01 | 0.5670153 |
| $SDUM_{AU}$ | 0.4447344E-01 | 0.1050908 | 0.4231905 |
| $SDUM_{SE}$ | -0.2140463 | 0.9538994E-01 | -2.243908 |
| $SDUM_{OC}$ | 0.6107340E-01 | 0.9542906E-01 | 0.6399874 |
| $SDUM_{NO}$ | 0.2264456 | 0.1191654 | 0.900262 |
| $SDUM_{DC}$ | 0.3797894E-01 | 0.1168153 | 0.3251195 |

degrees of freedom = 606

$\bar{R}^2$ = .67

Error Sum of Squares = .033226

TABLE 6.4

ESTIMATE OF THE FISHER EFFECT FOR SPLINE SEGMENT THREE:
OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES
WITH TREND AND MONTHLY SLOPE DUMMY VARIABLES

Dependent Variable $_{t+m}i_{P3}(x)$

| Independent Variables | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.1570508E-02 | 0.2367690E-02 | -0.6633505 |
| $_{t+m}I^{e}_{P3}(x)$ | 1.283547 | 0.2994973E-01 | 42.8567 |
| t | 0.9338162E-04 | 0.2610008E-04 | 3.577829 |
| $SDUM_{FE}$ | -0.8350045E-01 | 0.9332118E-01 | -0.8947642 |
| $SDUM_{MR}$ | -0.2488068 | 0.7737553E-01 | -3.215575 |
| $SDUM_{AP}$ | -0.1609442 | 0.8071930E-01 | -1.993875 |
| $SDUM_{MY}$ | -0.8541848E-01 | 0.878481E-01 | -0.980865 |
| $SDUM_{JU}$ | -0.2240313 | 0.8041994E-01 | -2.785764 |
| $SDUM_{JL}$ | 0.9016909E-01 | 0.7496436E-01 | 1.202826 |
| $SDUM_{AU}$ | 0.1377620E-01 | 0.9162353E-01 | 0.1503566 |
| $SDUM_{SE}$ | -0.2021518 | 0.8526108E-01 | -2.370974 |
| $SDUM_{OC}$ | 0.8647322E-01 | 0.8347459E-01 | 1.035923 |
| $SDUM_{NO}$ | 0.1904368 | 0.1033506 | 1.842628 |
| $SDUM_{DC}$ | 0.7325652E-02 | 0.1009017 | 0.7260184E-01 |

degrees of freedom = 606

$\bar{R}^2$ = .76

Error Sum of Squares = .028933

TABLE 6.5

ESTIMATE OF THE FISHER EFFECT FOR SPLINE SEGMENT FOUR:
OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES
WITH TREND AND MONTHLY SLOPE DUMMY VARIABLES

Dependent Variable $_{t+m}i_{P_4(x)}$

| Independent Variables | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.2527710E-02 | 0.2371845E-02 | -1.065715 |
| $_{t+m}I^{e}_{P_4(x)}$ | 1.174473 | 0.2968873E-01 | 39.55957 |
| t | 0.6781353E-04 | 0.2617940E-04 | 2.590339 |
| $SDUM_{FE}$ | -0.4573340E-01 | 0.9158437E-01 | -0.4993582 |
| $SDUM_{MR}$ | -0.1636994 | 0.7933335E-01 | -2.063438 |
| $SDUM_{AP}$ | -0.1563689 | 0.8051739E-01 | -1.942052 |
| $SDUM_{MY}$ | -0.5989692E-01 | 0.8527020E-01 | -0.7024368 |
| $SDUM_{JU}$ | -0.1650349 | 0.8067508E-01 | -2.045674 |
| $SDUM_{JL}$ | 0.2788719E-01 | 0.7546411E-01 | 0.3695424 |
| $SDUM_{AU}$ | 0.5739257E-01 | 0.9039081E-01 | 0.6349381 |
| $SDUM_{SE}$ | -0.1354322 | 0.8387393E-01 | -1.614712 |
| $SDUM_{OC}$ | 0.6011141E-01 | 0.8303483E-01 | 0.7239301 |
| $SDUM_{NO}$ | 0.1681412 | 0.9869224E-01 | 1.703692 |
| $SDUM_{DC}$ | 0.8355684E-02 | 0.9703559E-01 | 0.8610949E-01 |

degrees of freedom = 606

$\bar{R}^2$ = .72

Error Sum of Squares = .02900

## TABLE 6.6

### ESTIMATE OF THE FISHER EFFECT FOR SPLINE SEGMENT FIVE: OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES WITH TREND AND MONTHLY SLOPE DUMMY VARIABLES

---

Dependent Variable $_{t+m}i_{P5(x)}$

| Independent Variables | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.2564096E-02 | 0.1851697E-02 | -1.384727 |
| $_{t+m}I^{e}_{P5(x)}$ | 1.220513 | 0.2360744E-01 | 51.70034 |
| t | 0.6926900E-04 | 0.2041867E-04 | 3.392434 |
| $SDUM_{FE}$ | -0.2817931E-01 | 0.7336416E-01 | -0.3841018 |
| $SDUM_{MR}$ | -0.1422480 | 0.6364324E-01 | -2.235085 |
| $SDUM_{AP}$ | -0.9868259E-01 | 0.6515494E-01 | -1.514583 |
| $SDUM_{MY}$ | -0.4950415E-01 | 0.6797576E-01 | -0.7282618 |
| $SDUM_{JU}$ | -1.1602007 | 0.6427093E-01 | -2.492584 |
| $SDUM_{JL}$ | -0.3340779E-02 | 0.6022897E-01 | -0.5546797E-01 |
| $SDUM_{AU}$ | 0.2333732E-01 | 0.7021223E-01 | 0.3323825 |
| $SDUM_{SE}$ | -0.1034031 | 0.6708506E-01 | -1.541374 |
| $SDUM_{OC}$ | 0.5041678E-01 | 0.6626328E-01 | 0.7608555 |
| $SDUM_{NO}$ | 0.1526264 | 0.7687979E-01 | 1.985260 |
| $SDUM_{DC}$ | 0.2151264E-01 | 0.7686581E-01 | 0.2798726 |

---

degrees of freedom = 606

$\bar{R}^2$ = .82

Error Sum of Squares = .022617

---

The change in nominal interest rates for a change in expected inflation by spline segments are:

$B_1P_1(x) = 1.376$ $\qquad SE_{B1} = .04$

$B_2P_2(x) = 1.197$ $\qquad SE_{B2} = .04$

$B_3P_3(x) = 1.284$ $\qquad SE_{B3} = .03$

$B_4P_4(x) = 1.174$ $\qquad SE_{B4} = .03$

$B_5P_5(x) = 1.221$ $\qquad SE_{B5} = .02$

where $SE_{Bn}$ is the associated standard error.

These values each exceed the hypothesized value of one. Nominal interest rates increase (decrease) by more than the increase (decrease) in expected inflation in each segment of the term structure. As found previously for the case of the entire spline functions, the Fisher hypothesis of $_{t+m}B_nP_n(x) = 1$ has to be rejected.

The intercepts for the spline segments are all negative. They are statistically significant only in the first and fifth segments. The trend values for the spline segments are all significant. They are also quite small, on the same order of magnitude as reported in Table 6.1.

Table 6.3 contains a quick summary of the significance and sign of the monthly slope dummy variables for each spline segment and for the spline as a whole. March, April, June and September effects are all negative. November is significant and positive in all cases except for segment one. However, even in segment one, the November coefficient is positive, relatively large, and the t value is 1.223 (t critical 1.282). May, February, July, August, October, and

December, like January all take on a value not significantly different from the average slope for the regression.

TABLE 6.7

SIGN AND SIGNIFICANCE OF THE MONTHLY SLOPE DUMMY VARIABLES FOR THE COMPLETE SPLINE FUNCTION AND FOR THE FIVE SPLINE SEGMENTS

| | Spline Function Estimates $Q(x)$ | Segment Estimates $P_1(x)$ | $P_2(x)$ | $P_3(x)$ | $P_4(x)$ | $P_5(x)$ |
|---|---|---|---|---|---|---|
| FE | -(at 80%) | | | | | |
| MR | - | - | - | - | - | - |
| AP | - | -(at 90%) | - | - | - | - |
| MY | - (at 90%) | | | | | |
| JU | - | -(at 90%) | - | - | - | - |
| JL | + | + | | | | |
| AU | | | | | | |
| SE | - | - | - | - | - | - |
| OC | + (at 90%) | | | | | |
| | | | ┌─── | 90% | ───┐ | |
| NO | + | | + | + | + | + |
| DC | | | | | | |

Note: Those signs for which the confidence level is given are significant for that confidence level. Where no sign if given the seasonal dummy is not significant. If the significance level is given it denotes the highest confidence level for which the seasonal dummy is significant. If a sign is given without any note of the significance it is significant at the 95% level of confidence.

We also ran a regression using the joint estimation procedure, outlined by Theil (1971), to:

1. determine the gain in the precision of estimating the Fisher effect across the entire term structure, compared to estimating the effect one maturity at a time, and

2. determine if the slope coefficient measuring the Fisher effect varies across the term structure.

A joint estimation was done allowing the Fisher effect to vary by segment. The null hypotheses, as in the same case examined using OLS regression,

$$_{t+m}B_1P_1(x) = 1$$

$$_{t+m}B_2P_2(x) = 1$$

$$_{t+m}B_3P_3(x) = 1$$

$$_{t+m}B_4P_4(x) = 1$$

$$_{t+m}B_5P_5(x) = 1$$

However, in the Theil joint estimation procedure we did not include the trend and slope dummy variables. Instead, the joint estimation procedure estimates the five regressions as if they were seemingly unrelated regressions. This was done by first estimating each equation by OLS. The residuals from the OLS estimates of the five independent equations were then used to re-estimate each equation. The effect of this re-estimation is that the estimates of the intercept and slope for each of the estimated equations are conditional on all of the other four equations in the system.

Table 6.8 shows the results of the joint estimation procedure for the five spline segment intervals. The five estimated slopes are all smaller than the hypothesized values of one. Unlike the results of the OLS regressions on the spline segments, the intercepts are all positive, all of the intercepts are significant and all are larger than those reported in Tables 6.2 through 6.6.

Theil recommends a chi-square test to evaluate the statistical improvement in the estimates of the jointly estimated equations over the estimation of each equation independently. The null hypothesis is that joint estimation does not improve the regression estimates. Applying the chi-square test, we rejected the null hypothesis. The estimates of the Fisher effect based on joint estimation across the term structure are statistically better estimates of the Fisher effect than the estimation of the Fisher effect based on the same equations estimated for each spline segment independently.[2]

As noted previously variables representing trend and seasonality were not included in the joint (GLS) estimating equation. One should not, therefore, take the chi-square result to imply that estimates of the slopes ${}_{t+m}Bp_{n}(x)$ for n = 1 to 5 by joint (GLS) estimation are statistically better than the estimates shown in Tables 6.2 to 6.6 The trend and the monthly slope dummies were not included in the OLS stage of the joint (GLS) estimation. Since these were statistically significant, the regression in Table 6.8 may exhibit omitted variables bias. However, the chi-square test does confirm improved estimates from mutual segment estimation compared to independent segment estimation.

TABLE 6.8

POOLED CROSS-SECTION TIME SERIES JOINT ESTIMATION OF THE FISHER EFFECT FOR THE FIVE SPLINE SEGMENTS

| | interval one $P_1(x)$ | interval two $P_2(x)$ | interval three $P_3(x)$ | interval four $P_4(x)$ | interval five $P_5(x)$ |
|---|---|---|---|---|---|
| slope | .7770 | .7480 | .7748 | .7332 | .8247 |
| $SE_B$ | .035 | .028 | .023 | .023 | .018 |
| t-statistic | (22.09) | (27.10) | (32.54) | (32.13) | (44.67) |
| intercept | .0102 | .0157 | .0154 | .0115 | .0107 |
| t-statistic | ( 5.29) | (10.64) | (11.84) | ( 9.05) | (10.61) |

Chi-square = 66.89 for four degrees of freedom

Both methods of pooling the data produce results which reject the hypothesis that ${}_{t+m}B = 1$. The results of the OLS regressions suggest that the adjustment of nominal interest rates to expected inflation across the full term structure and within each segment of the term structure exceed a value of one. This result is not inconsistent with the full Fisher effect in a world of taxes. The results of the five segment estimates of the Fisher effect from the joint estimation procedure indicate that we must reject the null hypothesis. Joint (GLS) estimation of the segments indicates that the pass through of expected inflation to nominal interest rates is less than one for each of the five spline segments of the term structure. However, the chi-square test, for the case of joint estimation by segment supports our hypothesis that the associational effects of nominal interest rates and inflationary expectations along the term structure are omitted-variable effects.

## CHAPTER VI

## CHAPTER NOTES

1. This method of constructing a set of dummy variables is demonstrated in Pindyck and Rubinfeld, 1981, pages 135-137. The method is designed to overcome problems in interpretation of the t test which in the usual dummy variable method is dependent on the omitted dummy variable.

2. This result is to be expected, since a benefit of the seemingly unrelated regression procedure is a gain in efficiency over the corresponding OLS estimates.

# CHAPTER VII

## SUMMARY AND CONCLUSIONS

In this study we have examined the effect of the term structure of inflationary expectations on the term structure of nominal interest rates. Our null hypothesis was that a full test of the Fisher effect over the complete term structure would produce interest-on-inflation slope coefficients of one. Our null hypothesis was stated as:

(7.1)

$$_{t+m}B_t = 1.$$

This study began with a review of Irving Fisher's own exposition of the Fisher effect. The review includes Fisher's best known works, "Appreciation and Interest" and the Theory of Interest. Our discussion also included the less widely read Capital and Income, "Our Unstable Dollar and the So-Called Business Cycle," and "The Debt-Deflation Theory of Great Depressions." These less well known works contribute a better understanding of Fisher's empirical models, his evaluation of the Fisher effect, and the conclusions he drew from his empirical research.

Fisher's empirical investigation of the Fisher effect led him to conclude that the nominal rate of interest did not adjust fully to expected inflation. The incomplete adjustment of the nominal interest rate to expected inflation meant that an increase in inflation would cause the real interest rate to fall. Conversely deflation would cause the real interest rate to rise because the nominal rate failed to fully reflect a falling rate of inflation. Fisher suggested that borrowers had better foresight than lenders which resulted in the observed stickiness of the nominal interest rate.

A number of empirical studies of the Fisher effect done during the 1970's found some support for the simple Fisher effect. It has subsequently been found that these results were sensitive to the time period used to estimate the Fisher effect and/or the inclusion of additional variables that determine interest rates. A number of studies of the Fisher effect followed that included a variety of independent variables in addition to expected inflation. These other variables usually include measures of the state of the economy, monetary policy variables, fiscal policy variables, measures of tax effects, measures of the returns on real capital, and supply-side shocks. In general the findings in these studies indicate a substantial or nearly-full adjustment in the nominal interest rate for expected inflation.

While Fisher's own investigation of the Fisher effect included only expected inflation, Fisher concluded that other independent variables influenced interest rates. Among the other variables Fisher discussed were personal income, bank reserves (money supply) and the productivity of capital.

In terms of our empirical work the missing-variables issues which emerged were that:

1. There is significant information about the nominal interest rate for a particular maturity contained in its relationship with associated maturities along the term structure;
2. These associational effects constitute missing variables in studies of the Fisher effect; and
3. to the extent that realizations of the term structure of expected inflation do not contain all of the information about the generating forces of other expectations variables, we would also expect missing variables bias in our estimation of the Fisher effect.

We are convinced that the first two points are valid, and strongly suspect that the third point is also valid.

Our work began by developing a passive learning model of inflationary expectations. This model was used to derive a term structure of inflationary expectations. The data base developed includes one-month to eighty-four months to maturity of inflationary expectations, for a period covering 155 months, January 1970 to November 1982. The passive learning model employed to derive the data base is a general model and could be used to develop other expectations term structures. The estimates of expected inflation produced by this method satisfy the statistical requirements for rationality. The estimation of the passive learning model was simpler to implement than the estimation of an equivalent error learning model. The passive learning model required no prior series of expectations in order to estimate the model's parameters.

We then detailed the estimation procedure used to derive the term structure of spot interest rates from the bank discount on Treasury bills and from the coupon and yield on Treasury notes. Given that yields are biased estimates of spot interest rates, the use of spot interest rates to test our hypothesis permits us to avoid the biases common to studies of the term structure which use yields. The methodology used to estimate spot interest rates is a general method based on a model developed by Carleton and Cooper (1976). The estimation of present values and spot rates for the Treasury securities proved to be most difficult for the period 1970 through 1975. This was because during this early period in our data set there were very few Treasury notes. As a result the maturity horizon of our most limited data series was seventy-six months to maturity. This maturity horizon thus set the limit on the number of months to maturity. The subsequent estimation of the spline-function representations of the term structures for both inflationary expectations and nominal interest rates were based on this seventy-six months to maturity horizon.

The two term structures were modeled using spline functions. Since splines are piecewise-continuous functions, their flexibility has been found to be an excellent method for modeling term structures.

We fit a spline curve to each term structure of our estimated spot interest rates. One result of our use of the splines is that we have obtained data sets on both present values and spot interest rates that are independent of the splines. In most applications of spline functions to the spot rate term structure, the splines are generated more directly from the raw data on bank discounts, coupons and yields, bypassing the step of generating a spot rate data base.

Our spline coefficients were estimated by means of piecewise regression. The knot points, or joint points, can be interpreted as dummy variables, or indicator variables signifying the interval in term to maturity in months. We used a cubic exponential spline with four knot points and five intervals over the seventy-six months to maturity. The joint points were selected to produce smaller intervals at the near end of the term structure than at the far end. This was done because interest rates are usually more variable at the near end of the term structure. They also render those tests of the Fisher effect derived from quarterly and monthly interest rate estimates without seasonal variables particularly suspect.

Finally, we presented the results of our test of the Fisher effect. We found a positive and significant relationship between the term structure of inflationary expectations and the term structure of nominal interest rates. Testing the null hypothesis at the 95 percent confidence level, however, we rejected the hypothesis that ${}_{t+m}B_t = 1$. When we estimated the Fisher effect across the entire term structure the pass through of expected inflation to nominal interest rates was 1.249. When we estimated a separate Fisher effect for each of the five spline segments, the pass through of expected inflation to nominal interest rates ranged from 1.376 to 1.174. A question that this finding poses is: To what extent are the findings in the usual single-maturity estimates of the Fisher effect sensitive to the choice of maturity?

We estimated the Fisher effect by including a trend in the time series of the term structures. The trend was significant, but the coefficient was small in magnitude. We also included monthly dummy variables to check for seasonal variation. February, May, August, October, and December all had coefficients small in magnitude and were all statistically insignificant. November had a large positive coefficient. July also had a positive coefficient. The months of March, April, June, and September all had large, significant, and negative coefficients. These findings suggest that there may be some systematic seasonal influences working on the term structure.

We also estimated our pooled cross-section time series by the joint estimation technique recommended by Theil (1971). We jointly estimated the Fisher effect for the five spline segments. The estimated Fisher effect for each of the five spline segments is positive but less than one. The values range from .73332 to .8247. Unlike the OLS results for the five spline segments the intercepts for the jointly estimated Fisher effects are all positive and statistically significant.

The results of this joint estimation procedure, as applied to estimates of the Fisher effect across the term structure, indicated significantly better estimates of the slope coefficients measuring the Fisher effect than estimates derived one maturity at a time. This indicates that there are important associational effects across maturities and that these near-term associational effects must be considered missing variables in other studies of the Fisher effect.

The two different estimation procedures (OLS and joint) have produced estimates of the Fisher effect which are different from the hypothesized value of one. In the case of OLS estimation the measured Fisher effect is greater than one. In the case of joint estimation the measured Fisher effect is less than one. We tend to favor the OLS findings. Our reason is that the jointly estimated Fisher effects probably have missing variables bias since the joint estimates do not include the trend and seasonal slope dummy variables. The addition of monthly slope dummies which are collectively significant suggest that seasonality in interest rates may significantly affect estimates of the

Fisher effect. We therefore base our remarks regarding the implications of this study on the OLS results.

Our term structures of nominal interest rates are not adjusted to produce after-tax returns. In order to adjust the nominal interest rates for tax effects we would have to develop a model of expected real marginal tax rates. The Darby effect hypothesizes that the pass through of expected inflation to nominal interest rates must exceed one for real after-tax interest rates to reflect the full Fisher effect. In this sense our results are not inconsistent with the idea of a full Fisher effect in a world of taxes.

Our method of estimating the Fisher effect has revealed that information from the term structure of both nominal interest rates and inflationary expectations is important. This term structure information is a missing variable in other studies of the Fisher effect.

However, we cannot dismiss the possibility that we may also have missing variables bias in our study. The missing variables bias may be particularly critical given the time period of our sample, 1970 through 1982. First, the supply side shocks associated with OPEC were a disruption beginning in 1974. There were associated decreased returns to real capital from this interruption, which would be expected to decrease the rate of interest. Second, over our sample period we have several swings in the business cycle. The absence of variables to measure changes in the real rate of interest over the business cycle would create missing variables bias. We would also expect that there would be a significant effect on nominal interest rates associated with changes in income over the business cycle. Measures of the state of the economy are included in most other estimates of the Fisher effect. Third, we have not included a monetary policy variable. To the extent that much of the sample period was characterized by an easy money policy there is likely to be a liquidity effect reducing the rate of interest.

To the extent that our estimates of the Fisher effect contain missing variables bias, that bias probably produced a systematic underestimate of the Fisher effect. The easy monetary policy, the reduced returns to capital resulting from supply side shocks, and the low rate of growth which characterized most of the time period in our sample, would all tend to lower the rate of interest.

The extent to which the missing variables problem can be resolved in the context of our methodology is unclear. On the one hand, given that the model we have developed for generating inflationary expectations is a general model, it is possible to construct other term structures of expectations variables. This may be particularly reasonable in the case of expected income. Other term structures could be summarized by means of spline functions. The primary problem in doing this would be the standardization of spline function form and placement of knots for curves which are likely to be fundamentally different.

Since our objective was to produce spline coefficients which would be compatible for regression it was necessary to standardize knot placement and interval length. Ideally, knot placement may be different for each term structure and attempts to standardize as we have done may serve to force an inappropriate fit over segments of the curve. Additionally, increasing the knots and shortening the intervals at the near end of the term structure carries the potential cost of overfitting the term structure at the near end. There should be some examination of the effect of alternative knot placement on our results.

The choice of the same form and order of the function for both term structures was a necessity. The use of the same functional form to fit both the term structures of nominal interest rates and the term structures of inflationary expectations presumes that these term structures are produced by the same generating forces. While the application of the cubic exponential spline to nominal interest rates has support both from theory and from other applications of the spline method to the term structure of financial returns, there is not a similar foundation for the application of the cubic exponential spline to inflationary expectations. There is no reason to assume that the continuity restrictions of the spline method would not hold, nor is there any reason to assume that inflationary expectations should not have the same tendency to flatten out over the term structure, but other functional forms should be examined.

## APPENDIX A

### TABLE A.1

### MEAN AND VARIANCE FOR EIGHTY-FOUR MATURITIES OF ACTUAL AND EXPECTED ANNUALIZED RATES OF INFLATION

| Term to Maturity | | Actual Inflation | Expected Inflation |
|---|---|---|---|
| MA 1 | Mean | .4536769E-01 | .4562270E-01 |
| | Variance | .2068543E-02 | .2795915E-02 |
| MA2 | Mean | .4528577E-01 | .4554477E-01 |
| | Variance | .1736613E-02 | .2420489E-02 |
| MA3 | Mean | .4527621E-01 | .4542296E-01 |
| | Variance | .1616408E-02 | .1989604E-02 |
| MA4 | Mean | .4528283E-01 | .4534025E-01 |
| | Variance | .1542012E-02 | .1771226E-02 |
| MA5 | Mean | .4528247E-01 | .4533173E-01 |
| | Variance | .1488415E-02 | .1647149E-02 |
| MA6 | Mean | .4529526E-01 | .4532124E-01 |
| | Variance | .1448055E-02 | .1572968E-02 |
| MA7 | Mean | .4530905E-01 | .4533721E-01 |
| | Variance | .1415248E-02 | .1508670E-02 |
| MA8 | Mean | .4535076E-01 | .4535559E-01 |
| | Variance | .1387193E-02 | .1456405E-02 |
| MA9 | Mean | .4539688E-01 | .4540106E-01 |
| | Variance | .1364993E-02 | .1414396E-02 |
| MA10 | Mean | .4542249E-01 | .4543132E-01 |
| | Variance | .1349327E-02 | .1390747E-02 |
| MA11 | Mean | .4547430E-01 | .4547818E-01 |
| | Variance | .1337108E-02 | .1371422E-02 |
| MA12 | Mean | .4553654E-01 | .4552321E-01 |
| | Variance | .1326854E-02 | .1357740E-02 |
| MA13 | Mean | .4561181E-01 | .4559751E-01 |
| | Variance | .1317711E-02 | .1346276E-02 |
| MA14 | Mean | .4569653E-01 | .4570526E-01 |
| | Variance | .1308746E-02 | .1336211E-02 |
| MA15 | Mean | .4578519E-01 | .4577800E-01 |
| | Variance | .1298728E-02 | .1321557E-02 |

| | | | |
|---|---|---|---|
| MA16 | Mean | .4587762E-01 | .4587405E-01 |
| | Variance | .1288766E-02 | .1308368E-02 |
| MA17 | Mean | .4596930E-01 | .4596070E-01 |
| | Variance | .1278634E-02 | .1299779E-02 |
| MA18 | Mean | .4606972E-01 | .4606983E-01 |
| | Variance | .1267040E-02 | .1284906E-02 |
| MA19 | Mean | .4616351E-01 | .4616941E-01 |
| | Variance | .1255497E-02 | .1271761E-02 |
| MA20 | Mean | .4624072E-01 | .4624777E-01 |
| | Variance | .1243597E-02 | .1255385E-02 |
| MA21 | Mean | .4631207E-01 | .4630170E-01 |
| | Variance | .1232268E-02 | .1244986E-02 |
| MA22 | Mean | .4637937E-01 | .4636877E-01 |
| | Variance | .1221170E-02 | .1234272E-02 |
| MA23 | Mean | .4645729E-01 | .4645741E-01 |
| | Variance | .1209331E-02 | .1219744E-02 |
| MA24 | Mean | .4653250E-01 | .4653132E-01 |
| | Variance | .1197909E-02 | .1207693E-02 |
| MA25 | Mean | .4660800E-01 | .4660187E-01 |
| | Variance | .1186294E-02 | .1197419E-02 |
| MA26 | Mean | .4668522E-01 | .4667431E-01 |
| | Variance | .1174882E-02 | .1184328E-02 |
| MA27 | Mean | .4671327E-01 | .4672351E-01 |
| | Variance | .1173926E-02 | .1209804E-02 |
| MA28 | Mean | .4683873E-01 | .4684116E-01 |
| | Variance | .1151994E-02 | .1159169E-02 |
| MA29 | Mean | .4691085E-01 | .4691102E-01 |
| | Variance | .1140737E-02 | .1148091E-02 |
| MA30 | Mean | .4696940E-01 | .4697408E-01 |
| | Variance | .1130282E-02 | .1137321E-02 |
| MA31 | Mean | .4702500E-01 | .4703361E-01 |
| | Variance | .1119537E-02 | .1125718E-02 |
| MA32 | Mean | .4707245E-01 | .4706993E-01 |
| | Variance | .1109106E-02 | .1115197E-02 |
| MA33 | Mean | .4712194E-01 | .4711564E-01 |
| | Variance | .1098372E-02 | .1104372E-02 |
| MA34 | Mean | .4716905E-01 | .4717248E-01 |
| | Variance | .1087859E-02 | .1093417E-02 |

| | | | |
|---|---|---|---|
| MA35 | Mean | .4721499E-01 | .4721344E-01 |
| | Variance | .1077456E-02 | .1082883E-02 |
| MA36 | Mean | .4725580E-01 | .4725060E-01 |
| | Variance | .1067164E-02 | .1072138E-02 |
| MA37 | Mean | .4729540E-01 | .4729446E-01 |
| | Variance | .1057089E-02 | .1061828E-02 |
| MA38 | Mean | .4732785E-01 | .4732854E-01 |
| | Variance | .1047517E-02 | .1051751E-02 |
| MA39 | Mean | .4734855E-01 | .4736254E-01 |
| | Variance | .1038859E-02 | .1042234E-02 |
| MA40 | Mean | .4736325E-01 | .4736619E-01 |
| | Variance | .1030716E-02 | .1034426E-02 |
| MA41 | Mean | .4737379E-01 | .4736830E-01 |
| | Variance | .1022775E-02 | .1027016E-02 |
| MA42 | Mean | .4739142E-01 | .4739017E-01 |
| | Variance | .1014521E-02 | .1018727E-02 |
| MA43 | Mean | .4740247E-01 | .4740901E-01 |
| | Variance | .1007205E-02 | .1010481E-02 |
| MA44 | Mean | .4741510E-01 | .4741062E-01 |
| | Variance | .9993656E-03 | .1002660E-02 |
| MA45 | Mean | .4742024E-01 | .4742472E-01 |
| | Variance | .9921754E-03 | .9956361E-03 |
| MA46 | Mean | .4742696E-01 | .4742809E-01 |
| | Variance | .9846583E-03 | .9875018E-03 |
| MA47 | Mean | .4741990E-01 | .4742986E-01 |
| | Variance | .9779225E-03 | .9800934E-03 |
| MA48 | Mean | .4741145E-01 | .4741473E-01 |
| | Variance | .9708310E-03 | .9730233E-03 |
| MA49 | Mean | .4739981E-01 | .4739878E-01 |
| | Variance | .9639865E-03 | .9663615E-03 |
| MA50 | Mean | .4738013E-01 | .4738037E-01 |
| | Variance | .9575434E-03 | .9600269E-03 |
| MA51 | Mean | .4735523E-01 | .4735402E-01 |
| | Variance | .9515940E-03 | .9538600E-03 |
| MA52 | Mean | .4732974E-01 | .4733528E-01 |
| | Variance | .9456953E-03 | .9477681E-03 |
| MA53 | Mean | .4730129E-01 | .4729775E-01 |
| | Variance | .9398701E-03 | .9418086E-03 |

| | | | |
|---|---|---|---|
| MA54 | Mean | .4727566E-01 | .4727707E-01 |
| | Variance | .9344669E-03 | .9366786E-03 |
| MA55 | Mean | .4724553E-01 | .4724965E-01 |
| | Variance | .9290638E-03 | .9312373E-03 |
| MA56 | Mean | .4721471E-01 | .4721959E-01 |
| | Variance | .9236628E-03 | .9256565E-03 |
| MA57 | Mean | .4718017E-01 | .4718130E-01 |
| | Variance | .9182124E-03 | .9198696E-03 |
| MA58 | Mean | .4714493E-01 | .4715184E-01 |
| | Variance | .9128623E-03 | .9144780E-03 |
| MA59 | Mean | .4710739E-01 | .4711024E-01 |
| | Variance | .9074810E-03 | .9095513E-03 |
| MA60 | Mean | .4706429E-01 | .4706909E-01 |
| | Variance | .9025462E-03 | .9043278E-03 |
| MA61 | Mean | .4701713E-01 | .4702045E-01 |
| | Variance | .8980900E-03 | .8992005E-03 |
| MA62 | Mean | .4698049E-01 | .4697791E-01 |
| | Variance | .8933023E-03 | .8950860E-03 |
| MA63 | Mean | .4694147E-01 | .4694008E-01 |
| | Variance | .8891501E-03 | .8909303E-03 |
| MA64 | Mean | .4690171E-01 | .4689959E-01 |
| | Variance | .8850522E-03 | .8867287E-03 |
| MA65 | Mean | .4686770E-01 | .4686582E-01 |
| | Variance | .8809109E-03 | .8821946E-03 |
| MA66 | Mean | .4683620E-01 | .4684118E-01 |
| | Variance | .8769382E-03 | .8782641E-03 |
| MA67 | Mean | .4680402E-01 | .4680558E-01 |
| | Variance | .8730403E-03 | .8751525E-03 |
| MA68 | Mean | .4677107E-01 | .4677672E-01 |
| | Variance | .8690958E-03 | .8701127E-03 |
| MA69 | Mean | .4674233E-01 | .4674022E-01 |
| | Variance | .8648952E-03 | .8664281E-03 |
| MA70 | Mean | .4670793E-01 | .4670774E-01 |
| | Variance | .8610704E-03 | .8628674E-03 |
| MA71 | Mean | .4667943E-01 | .4667941E-01 |
| | Variance | .8570689E-03 | .8582654E-03 |
| MA72 | Mean | .4665188E-01 | .4665118E-01 |
| | Variance | .8531052E-03 | .8543237E-03 |

| | | | |
|---|---|---|---|
| MA73 | Mean | .4662538E-01 | .4662617E-01 |
| | Variance | .8493921E-03 | .8505655E-03 |
| MA74 | Mean | .4660162E-01 | .4660313E-01 |
| | Variance | .8458570E-03 | .8469905E-03 |
| MA75 | Mean | .4657713E-01 | .4657942E-01 |
| | Variance | .8422021E-03 | .8434417E-03 |
| MA76 | Mean | .4655728E-01 | .4655899E-01 |
| | Variance | .8389998E-03 | .8403442E-03 |
| MA77 | Mean | .4653850E-01 | .4654162E-01 |
| | Variance | .8357285E-03 | .8368906E-03 |
| MA78 | Mean | .4651587E-01 | .4652181E-01 |
| | Variance | .8329626E-03 | .8338481E-03 |
| MA79 | Mean | .4649941E-01 | .4649860E-01 |
| | Variance | .8298504E-03 | .8313609E-03 |
| MA80 | Mean | .4647909E-01 | .4648222E-01 |
| | Variance | .8271430E-03 | .8284612E-03 |
| MA81 | Mean | .4646502E-01 | .4647237E-01 |
| | Variance | .8240884E-03 | .8248288E-03 |
| MA82 | Mean | .4645041E-01 | .4644664E-01 |
| | Variance | .8210570E-03 | .8220905E-03 |
| MA83 | Mean | .4643526E-01 | .4644105E-01 |
| | Variance | .8180715E-03 | .8188437E-03 |
| MA84 | Mean | .4642643E-01 | .4642820E-01 |
| | Variance | .8146669E-03 | .8155464E-03 |

# APPENDIX B

## PROGRAM FOR COMPUTING PRESENT VALUES FOR TREASURY BILLS AND NOTES, PLACING THEM IN A FORM CONSISTENT FOR REGRESSION

This program is designed to read data on Treasury bills and Treasury notes as presented in the Federal Reserve Bulletin. The bill files include the date of maturity and the bankers' discount. The note files include the date of maturity, the coupon, and the yield. The first line of each file contains the closing date. The number of coupon payments is computed for notes, including any fractional coupon payments. The accretion of note prices toward par is also computed. The present value is then computed for all the securities.

In order to place these in a data matrix consistent for regression the remaining portion of the program is concerned with a grid search technique to locate bills and notes within a seven day target of thirty days forward from the last note or bill in the grid. Since there are cases where no actual note or bill may fall within the designated forward range, the program is designed to fill in gaps with what we have referred to as pseudo-zeros. These are computed from the bond equivalent yield for bills and the average yield to maturity for notes.

The data matrix created from this procedure has a number of rows equal to the number of securities and number of columns equal to the number of months to maturity.

Additional information (later referred to as the vector 'key') which is included with the data matrix is the closing date in Julian form, the number of days from closing to the date that the first security is scheduled to mature, the fraction of a month this number of days to maturity represents and the number of months in the data set.

Table B.1 is the listing of the program which computes present values. Table B.2 is the sample output for November 1982.

TABLE B.1

LISTING OF FORTRAN PROGRAM WHICH COMPUTES THE PRESENT VALUE FOR TREASURY BILLS AND NOTES, PLACING THESE IN FORMAT CONSISTENT FOR REGRESSION

---

```
c
c TSMIX program to read bond, bill quotes and place
c       in consistent framework for regression
c
c CFBaum 10.82 mod Jun 84 for Phyllis' data /mod2 dec84/ mod3 nov85
c modified to run on batch unit NU/mod4 apr86
c
      implicit real*8(a-h,o-z)
c
        parameter (nsecur=250,nyrlim=7)
c
      real*8 coup(nsecur),price(nsecur),yield(nsecur),xlbl(nsecur)
      real*8 month/30.4165/,indx(nsecur,nsecur),rindx(nsecur,nsecur)
ca86
        real*8 yieldcol(nsecur),nden(nsecur),rycol(nsecur)
      integer mat(5,nsecur),cdt(5),intdt(nsecur),lbl(nsecur),hyr/182/,tdt
      integer lblord(nsecur),ord(nsecur)
      integer curdt,cpdt
      logical row(nsecur)/nsecur*.false./
c
      integer limit/nsecur/
c
ccc   character*25 billfile,bondfile
c
c zero indx,rindx et.al.
c
      do 70 j=1,limit
      do 70 i=1,limit
      indx(i,j)=0.0
      rindx(i,j)=0.0
        yieldcol(i)=0.0
        nden(i)=0.0
        rycol(i)=0.0
70    continue
c
c get filenames
c
ccc   call chwrsp(6,'$Bond file : ;')
ccc   read(5,80)bondfile
ccc   call chwrsp(6,'$Bill file : ;')
ccc   read(5,80)billfile
80    format(a)
c
c get tightness prior parameter
```

```
c
ccc   call chwrsp(6,'$Tightness prior (0=0.0025) : ;')
ccc   read(5,*)tight
ccc   if(tight.le.0.0)then
c ***** reduce param by one power of ten
cc***** reduce again
            tight=0.00010
ccc   endif
      write(8,81)tight
81    format(/' Tightness prior = ',f15.7/)
c
c read bond data
c
ccc   open(unit=1,file=bondfile,type='old',readonly)
        open(unit=1,type='old',readonly)
c
      read(1,*)(cdt(j),j=1,3)
      write(8,90)(cdt(j),j=1,3)
90    format('0Bond quote date : ',3i3)
c
        write(8,1189)nyrlim,cdt(1),cdt(2),cdt(3)+nyrlim
1189    format('  # yrs = ',i2,'  cutoff = ',3i3)
c
      cdt(4)=juldat(cdt(1),cdt(2),cdt(3))
      nbond=1
      write(8,91)
91    format(/1x,'   #  MM/DD/YY  Jul     Coup   Yld  Days  #Coup  ',
      1 'Price'/)
100   read(1,*,end=110)(mat(j,nbond),j=1,3),coup(nbond),yield(nbond)
      ccp=coup(nbond)
      yyd=yield(nbond)
      coup(nbond)=coup(nbond)/2.0
      yield(nbond)=yield(nbond)/200.0
c
c get (fractional) nr. coupon periods, transform yield to price+a/i
c
        mat(4,nbond)  = juldat(mat(1,nbond),mat(2,nbond),mat(3,nbond))
c1185 only consider "nyrlim" years of data
c
        if(mat(4,nbond).gt.(cdt(4)+nyrlim*1000))goto110
c
        if(mat(4,nbond).lt.cdt(4))then
              write(8,1016)nbond,mat(4,nbond),cdt(4)
1016       format(/' Bond#',i3,' Mat:',i6,' Q dt:',i6,' abort')
              stop 910
              endif
        mat(5,nbond)   = juldif(mat(4,nbond),cdt(4))
        xncoup=  float(mat(5,nbond)) / 182.5
c
            if(yield(nbond).lt.0.001)then
              write(8,1018)nbond,yield(nbond)
1018       format(/' Bond#',i3' inadmit yield = ',e16.8,' abort')
              stop 1018
        endif
c
      pdv=( 1.0- 1.0/(1.0+yield(nbond))**xncoup )*coup(nbond)/yield(nbond)
```

```
      prindv=100.0/(1.0+yield(nbond))**xncoup
101   format(1x,i3,2x,3i3,i6,f8.3,f8.3,i5,f6.2,f8.3)
      price(nbond)=pdv+prindv
      write(8,101)nbond,(mat(j,nbond),j=1,4),ccp,yyd,
      1 mat(5,nbond),xncoup,price(nbond)
ca86    restore yield vector
        yield(nbond)=yyd
      nbond=nbond+1
      if(nbond.gt.limit)goto900
      goto100
110   nbond=nbond-1
c
c read bill data
c
ccc   open(unit=2,file=billfile,type='old',readonly)
        open(unit=2,type='old',readonly)
      read(2,*)(cdt(j),j=1,3)
      write(8,92)(cdt(j),j=1,3)
92    format('0Bill quote date : ',3i3)
      cdt(5)=juldat(cdt(1),cdt(2),cdt(3))
      tdt=cdt(4)
      if(cdt(4).ne.cdt(5))then
                 write(8,921)cdt(4),cdt(5)
921              format('   Bond  Q :',i6,'   <> Bill  Q :',i6'  abort')
                 stop 900
        endif
      nbill=nbond+1
      write(8,93)
93    format(//1x,'   #  MM/DD/YY  Jul   Disct  Days  Price  B-E-Y'/)
120   read(2,*,end=130)(mat(j,nbill),j=1,3),yield(nbill)
c
c translate bankers' discount to price
c
      mat(4,nbill)=juldat(mat(1,nbill),mat(2,nbill),mat(3,nbill))
      mat(5,nbill)=juldif(mat(4,nbill),cdt(4))
      price(nbill)=100.0 - yield(nbill)*float(mat(5,nbill))/360.0
      coup(nbill)=0.0
ca86    place b-e-y in yield vector (from Stigum, money market, p49;
ca86    should be corrected for >6mo
        disct=yield(nbill)
        yield(nbill)=100.0*(365.0*disct/100.0)/(360.0-disct/100.0*
     1  float(mat(5,nbill)))
      write(8,102)nbill,(mat(j,nbill),j=1,4),disct,mat(5,nbill),
     1  price(nbill),yield(nbill)
102   format(1x,i3,2x,3i3,i6,f8.4,i5,f8.3,f8.4)
      nbill=nbill+1
      if(nbill.gt.limit)goto900
      goto120
c
130   nbill=nbill-nbond-1
      n=nbond+nbill
c
c load LBL with mat dates, load INDX with maturity payoff values
c
      do 200 i=1,n
      if(i.eq.1)then
```

```
ca86    start off the process by loading first bond
            ihilbl=1
            lbl(1)=mat(4,i)
            xlbl(1)=mat(5,i)/month
            indx(i,1)=100.0+coup(i)
ca86 load yieldcol,nden
                yieldcol(1)=yield(1)
                nden(1)=1.0
      else
ca86 load additional secs. maturing on sameday into additional rows,
ca86 same column
            do 300 j=1,ihilbl
            if(mat(4,i).eq.lbl(j))then
                  indx(i,j)=100.0+coup(i)
ca86 load yieldcol,nden
                        yieldcol(j)=yieldcol(j)+yield(i)
                        nden(j)=nden(j)+1.0
                  goto301
            endif
c
300         continue
ca86 else place into new column after incrementing ihilbl
            ihilbl=ihilbl+1
            lbl(ihilbl)=mat(4,i)
            xlbl(ihilbl)=mat(5,i)/month
            indx(i,ihilbl)=100.0+coup(i)
ca86 load yieldcol,nden
                yieldcol(ihilbl)=yieldcol(ihilbl)+yield(i)
                nden(ihilbl)=nden(ihilbl)+1.0
301         continue
      endif
200   continue
c
c order label vectors from fifteen days after quote date
c
      iord=1
      nrow=0
c386
        if(cdt(1).eq.12)then
             iem=1
             iey=cdt(3)+1
        else
             iem=cdt(1)+1
             iey=cdt(3)
        endif
      lblord(1)=juldat(iem,15,iey)
ca86 should search over lbl for ihilbl elements only
      nrfbill=iretrv(lblord(1),lbl,ihilbl,mindist)
ccc   type *,lblord(1),mindist
      ord(1)=nrfbill
            do 319 ii=1,n
            if(indx(ii,nrfbill).ne.0.0)then
                  row(ii)=.true.
                  nrow=nrow+1
            endif
319         continue
```

```
c
c move through other maturities by *** 30 *** day intervals
c
c last defined date is last bond in data set
c
      iend=juladd(mat(4,nbond),7)
      ict30=-1
210   ict30=ict30 +1
      lblord(iord+1)=juladd(lblord(iord),i30(ict30))
ca86 should search over lbl for ihilbl defined elements only
      nrlbl=iretrv(lblord(iord+1),lbl,ihilbl,mindist)
ccc   type *,lblord(iord+1),mindist
c
c if security within (386) 7 days of target date, we'll buy it
c
      if(mindist.gt.7)then
            ord(iord+1)=-1
      else
            ord(iord+1)=nrlbl
            do 320 ii=1,n
            if(indx(ii,nrlbl).ne.0.0)then
                  row(ii)=.true.
                  nrow=nrow+1
            endif
320         continue
      endif
      iord=iord+1
      if(iord.gt.limit)stop 210
      if(lblord(iord).le.iend)goto210
c
c all dates loaded, report
c
      write(8,330)nbond,nbill
330   format('0Loaded ',i3,' bond quotes'/'0       ',i3,' bill ',
     1  'quotes'/'0multiple securities maturing on same date appear',
     1  ' in the same column'/)
      call chwr(8,'0Col     Date      Months  Avg Y-T-M  nr.sec.;')
        do 400 i=1,ihilbl
ca86    define yieldcol
        yieldcol(i)=yieldcol(i)/nden(i)
        write(8,401)i,lbl(i),xlbl(i),yieldcol(i),nden(i)
401     format(1x,i3,3x,i6,2x,f10.2,2x,f10.3,f6.0)
400   continue
      write(8,402)ihilbl
402   format('0Number of Distinct Maturities = ',i5)
c
c report reordered columns
c
      iord=iord-1
      call chwr(8,'0Reordered columns;')
      call chwr(8,'0New Old  Load Date  Rept Date   Diff  Avg Y-T-M;')
      do 410 i=1,iord
      if(ord(i).ne.-1)then
            load=lbl(ord(i))
ca86 define rycol
                rycol(i)=yieldcol(ord(i))
```

```
               if(lblord(i).gt.load)then
                      idifff=-juldif(lblord(i),load)
               else
                      idifff=juldif(load,lblord(i))
               endif
      else
            load=0
               idifff=0
      endif
      write(8,420)i,ord(i),load,lblord(i),idifff,rycol(i)
420   format(1x,i3,1x,i3,3x,i6,3x,i6,5x,i3,2x,f10.3)
410   continue
c
      call chwrsp(8,' Old columns not appearing have been dropped.;')
      write(8,422)nrow
422   format(/' Number of usable rows = ',i3/)
c
c shuffle columns of indx into rindx (a86) for the n defined secs. only
c
      do 4501 i=1,iord
      if(ord(i).ne.-1)then
ca86
            do 4502 j=1,n
            rindx(j,i)=indx(j,ord(i))
4502        continue
      endif
4501  continue
c
ca86 locate gaps in ord sequence, fill in with pseudo_zero secs.with
ca86 interpolate Y-T-M implying price
c
        igap=0
        do 470 i=1,iord
        if(ord(i).eq.-1)then
              if(igap.eq.0)istat=i-1
              if(istat.eq.0)stop 470
              igap=1
        elseif(igap.eq.1)then
              iwid=i-istat-1
              write(8,471)iwid,istat,i,rycol(istat),rycol(i)
471           format('0Gap of width  ',i2,'between ',2i4,2f10.4)
              ryc=rycol(istat)
              addon=(rycol(i)-rycol(istat))/float(iwid+1)
              do 475 j=1,iwid
              ryc=ryc+addon
              n=n+1
              row(n)=.true.
              nrow=nrow+1
              if(n.gt.limit)stop 475
              rindx(n,istat+j)=100.0
              xnyrs=(float(istat+j-1)*month+15.0)/365.25
              price(n)=100.0*exp(-(ryc/100.0)*xnyrs)
              write(8,476)istat+j,n,ryc,price(n)
476           format(10x,' Col  ',i3,'  row  ',i3,' Y-T-M = ',f10.4,
     1        ' price = ',f10.4)
475           continue
```

```
                igap=0
         endif
470      continue
c
c crank backward to load coupon dates
c
c
      write(8,451)nrow
451   format('0Rows after augmenting with pseudo-zeros : ',i3)
c
      do 500 i=1,n
c
ca86  if(i.gt.nbond.and.i.le.nreal)goto500
        if(i.gt.nbond)goto501
        if(.not.row(i))goto500
c
      curdt=mat(4,i)
510   cpdt=julsub(curdt,hyr)
      if(cpdt.gt.tdt)then
ca86 should search over lbl for iord defined elements only
ca86
ca86
ca86 WRITTEN 4/30
            nrlbl=iretrv(cpdt,lblord,iord,mindist)
            if(mindist.gt.7)then
                        write(8,511)i,cpdt,mindist
                else
                        rindx(i,nrlbl)=coup(i)
511             format('   No match for # ',i4,2i8)
                endif
cm          rindx(i,nrlbl)=coup(i)
      else
c
c     gone back beyond today's date
c
ccc         intdt(i)=juldif(tdt,cpdt)
ccc         accint(i)=coup(i)/182.5*float(intdt(i))
            goto500
      endif
c
c more coupons to be entered
c
      curdt=cpdt
      goto510
c
500   continue
ca86
501     continue
c
c done with all bonds,
c write price, indx to external file
c
        open(unit=3,type='new',carriagecontrol='list')
        open(unit=4,type='new',carriagecontrol='list')
c
c write header
```

```
c
      iofset=juldif(lblord(1),tdt)
      frac=float(iofset)/month
      write(3,7121)tdt,iofset,frac,iord
7121  format(i7,i7,e16.8,i7)
c
      nchek=0
      do 700 i=1,n
      if(row(i))then
            write(3,701)price(i),(rindx(i,j),j=1,iord)
            nchek=nchek+1
      endif
701   format(5e16.8)
700   continue
      if(nchek.ne.nrow)stop 700
      ncol=iord+1
      write(8,710)nrow,ncol
710   format('0Data matrix ',i3,' x ',i3,' written to TSMIX.DAT...')
      call chwr(8,'0RATS run written to TSMIX.RAT...;')
      call chwr(4,'CAL 1 1 1;')
ccc   call chwr(4,'COLUMNS 80;')
      write(4,7221)nrow
7221  format('ALL 260',1x,i3,' 20 260')
      call chwr(4,'EQV 1 ;')
      call chwr(4,'  priceaci ;')
      call chwr(4,'EQV 200 201 202 203 204 205 206 207 ;')
      call chwr(4,'  key olsbeta mixbeta olserr mixerr intrate/
      xpon matur;')
      CALL CHWR(4,'SUPPRESS LABELING;')
      write(4,722)ncol
722   format('EQV 2 TO ',i3)
      write(4,712)(lblord(i),i=1,iord)
712   format(10(' M',i5),' $')
        write(4,71211)
71211   format('* this is here to slurp the continuation mark')
      write(4,7122)
7122  format('Data(org=var,format=free) 1 4 key')
      write(4,713)nrow,ncol
713   format('Data(org=obs,format=''(5e16.8)'') 1',  i4,' 1 to ',  i3)
      write(4,714)
714   format('procedure mixed ieqn nbeg nend capr lowr v'/
      1 'type rect capr'/'type vect lowr'/'type symm v'/
      2 'local symm xxmixed'/'local vect xymixed'/
      3 'cmoment(equation=ieqn) nbeg nend'/
      4 'regress(equation=ieqn,print) 0 olserr olsbeta'/
      5 'overlay cmom(1,1) with xxmixed(nreg,nreg)'/
      6 'overlay cmom(nreg+1,1) with xymixed(nreg)'/
      7 'mat xxmixed=xxmixed+scale(seesq)*tr(capr)*(inv(v)*capr)'/
      8 'mat xymixed=xymixed+scale(seesq)*tr(capr)*(inv(v)*lowr)'/
      9 'regress(equation=ieqn) 0 mixerr mixbeta'/
      1 'end')
      write(4,715)ncol
715   format('equation(noconst,more) 1 priceaci'/'# 2 to ',i3)
      nrestr=iord-2
      write(4,716)nrestr,iord,nrestr,nrestr,nrestr,tight
716   format('dec rect restr(',i3,',',i3,')'/
```

```
     1       'dec vect zeros(',i3,')'/
     2       'dec symm varcov(',i3,',',i3,')'/
     3       'fmatrix(diff=2) restr 1 1 '/'matrix zeros=const(0.0)'/
     4       'matrix varcov=iden(',f15.7,')')
      write(4,717)nrow,iord,iord
c386 move diffr out of the way
717   format('execute mixed 1 1 ',i3,' restr zeros varcov'/
     1       'eqv 199'/'  diffr'/'set diffr 1 ',i3,' = olsbeta(t) - ',
     1       'mixbeta(t) '/,
     1       'print 1 4 key'/'print 1 ',i3,' olsbeta mixbeta diffr'/
     2       'set matur 1 1 = key(2)')
      write(4,7123)iord,iord,iord,iord,iord
7123  format('set matur 2 ',i3,' = key(2) + (t-1)*30.4165'/
     1       'set xpon 1 ',i3,' = 182.5 / matur(t)'/
     2 'set intrate 1 ',i3,' = (200.0*(1.0/mixbeta(t))**xpon(t))-200.0'/
     3       'copy 1 4 key'/'copy(org=obs) 1 ',i3,' intrate matur'/
     4       'print 1 ',i3,' matur xpon intrate'/'end')
c
      stop
900   call chwr(8,'0Too many rows of data ;')
      stop900
      end
c
c-----------------------------------------------------------
     integer function iretrv(isrch,matdt,nr,mindist)
c
      integer matdt(1)
      mindist=99999
c386
      do 100 i=nr,1,-1
      if(matdt(i).ge.isrch)then
            idist=juldif(matdt(i),isrch)
      else
            idist=juldif(isrch,matdt(i))
      endif
c     idist=iabs(juldif(matdt(i),isrch))
      if(idist.gt.mindist)goto100
      mindist=idist
      iretrv=i
100   continue
      return
      end
c
c-----------------------------------------------------------
      integer function i30(i)
c
c every 12 months, add 2 days
c
      if(mod(i,12).eq.0)then
            i30=32
            return
c
c every three months, add 1 day
c
      else        if(mod(i,3).eq.0)then
                  i30=31
```

```
                return
      else
c
c otherwise just use 30 days
c
      i30=30
      endif
      return
      end
c-----------------------------------------------------------
subroutine avg(vec1,vec2,n,xpay,xprice,price)
c
      real*8 vec1(1),vec2(1),xpay,xprice,price(1)
      xpay=0.0
      xprice=0.0
      ct=0.0
      do 100 i=1,n
      if(vec1(i).gt.0)then
            xpay=xpay+vec1(i)
            xprice=xprice+price(i)
            ct=ct+1.0
      else
      endif
      if(vec2(i).gt.0)then
            xpay=xpay+vec2(i)
            xprice=xprice+price(i)
            ct=ct+1.0
      else
      endif
100   continue
      xpay=xpay/ct
      xprice=xprice/ct
      return
      end
c**********new version of julian calendar routines**********c
C JULFUNCS : JULIAN CALENDAR ROUTINES FOR FINANCIAL MARKET PROCESSING
C
C CFBAUM MAR 1983, REV JUL 1983 AND OCT 1983
c this version is resident in PORT v2.0
C
C-----------------------------------------------------------
C
C JULDAT : CVT GREGORIAN TO JULIAN
C
      INTEGER FUNCTION JULDAT(MM,DD,YY)
C
      INTEGER DA(12)/31,28,31,30,31,30,31,31,30,31,30,31/
      INTEGER IOF(12),YY,DD
C
      IF(MM.GT.12)THEN
            JULDAT=-99999
            RETURN
      ENDIF
      IF(DD.LT.1.OR.DD.GT.DA(MM))THEN
            IF(DD.EQ.29.AND.MM.EQ.2.AND.LEAP(YY).EQ.1)GOTO50
            JULDAT=-99999
```

```
            RETURN
      ENDIF
50      IOF(1)=0
      ILP=LEAP(YY)
      DO 100I=1,11
      IOF(I+1)=IOF(I)+DA(I)
      IF(I.EQ.2.AND.ILP.EQ.1)IOF(I+1)=IOF(I+1)+1
100     CONTINUE
      JULDAT=YY*1000+IOF(MM)+DD
      RETURN
      END
C-----------------------------------------------------------
C
C JULDIF : CALC DIFFERENCE OF JULIAN DATES  [ JA - JB ]
C
      INTEGER FUNCTION JULDIF(JULA,JULB)
C
      IF(JULA.LT.JULB)STOP 'ERROR IN JULDIF'
      JAYR=JULA/1000
      JBYR=JULB/1000
      JADAY=JULA-JAYR*1000
      JBDAY=JULB-JBYR*1000
      JULDIF=(JAYR-JBYR)*365+JADAY-JBDAY+NLEAP(JAYR,JBYR,JADAY,JBDAY)
     1       +LEAPAD(JAYR,JBYR,JADAY,JBDAY)
      RETURN
      END
C-----------------------------------------------------------
C
C JULSUB : SUBTRACT AMT [ < 366 ] FROM JULIAN DATE
C
      INTEGER FUNCTION JULSUB(JUL,JSUB)
C
      IF(JSUB.LT.0.OR.JSUB.GT.366)STOP 'ERROR IN JULSUB'
      JULYR=JUL/1000
      JULDAY=JUL-JULYR*1000
      JULYRO=JULYR
      JULDYO=JULDAY
C
      JULDAY=JULDAY-JSUB
      IF(JULDAY.LE.0)THEN
            JULYR=JULYR-1
            JULDAY=JULDAY+365
      ELSE
      ENDIF
C
      JULSUB=JULYR*1000+JULDAY+NLEAP(JULYRO,JULYR,JULDYO,JULDAY)
      RETURN
      END
C-----------------------------------------------------------
C
C JULADD : ADD AMT [ <366 ] TO JULIAN DATE
C
      INTEGER FUNCTION JULADD(JUL,JADD)
C
      IF(JADD.LT.0.OR.JADD.GT.366)STOP 'ERROR IN JULADD'
      JULYR=JUL/1000
```

```
      JULDAY=JUL-JULYR*1000
      JULYRO=JULYR
      JULDYO=JULDAY
C
      JULDAY=JULDAY+JADD
      IF(JULDAY.GT.365)THEN
            JULYR=JULYR+1
            JULDAY=JULDAY-365
      ELSE
      ENDIF
C
      JULADD=JULYR*1000+JULDAY+NLEAP(JULYR,JULYRO,JULDAY,JULDYO)
      RETURN
      END
C-----------------------------------------------------------------
C
C JULGREG : CVT JULIAN INTO GREGORIAN
C
      SUBROUTINE JULGRG(JUL,MM,DD,YY)
C
      INTEGER DA(12)/31,28,31,30,31,30,31,31,30,31,30,31/
      INTEGER MM,DD,YY,DAYS
C
      YY=JUL/1000
      IF(LEAP(YY).EQ.1)THEN
            DA(2)=29
      ELSE
            DA(2)=28
      ENDIF
      DAYS=JUL-YY*1000
C
      DO 100 I=1,12
      DAYS=DAYS-DA(I)
      IF(DAYS.LE.0)GOTO200
100     CONTINUE
      MM=12
      DD=31
      RETURN
C
200     MM=I
      DD=DAYS+DA(I)
      RETURN
      END
C-----------------------------------------------------------------
C
C GREGDIF : CALC DIFFERENCE OF GREGORIAN DATES
C
      INTEGER FUNCTION IGRGDF(M1,D1,Y1,M2,D2,Y2)
C
      INTEGER M1,M2,D1,D2,Y1,Y2
C
      JUL1=JULDAT(M1,D1,Y1)
      JUL2=JULDAT(M2,D2,Y2)
      IGRGDF=JULDIF(JUL1,JUL2)
      RETURN
      END
```

```
C-----------------------------------------------------
C
C GREGDATE : GET GREGORIAN DATE (VAX SPECIFIC) AS INTEGER
C
      SUBROUTINE GREGDT(MM,DD,YY)
C
      INTEGER*4 IM,ID,IY
      INTEGER MM,DD,YY
      COMMON/GRGDT/IM,ID,IY
C
C VAX SPECIFIC R/T/L CALL
C
CCC       CALL FOR$IDATE(IM,ID,IY)
      MM=IM
      DD=ID
      YY=IY
      RETURN
      END
C
C-----------------------------------------------------
C
C GREGDATEC: GET DATE (VAX SPECIFIC) AS CHARACTER STRING
C
CCC       SUBROUTINE GREGDATEC(CHARDATE)
C
CCC       CHARACTER*9 CHARDATE
C
C VAX SPECIFIC R/T/L CALL
C
CCC       CALLFOR$DATETDS(CHARDATE)
CCC       RETURN
CCC       END
C
C-----------------------------------------------------
C
C LEAP : CALCULATE WHETHER YEAR IS A LEAP YEAR
C
      INTEGER FUNCTION LEAP(YEAR)
C
      INTEGER YEAR
      LEAP=0
      IF(MOD(YEAR,4).EQ.0)LEAP=1
      RETURN
      END
C-----------------------------------------------------
C
C NLEAP : CALCULATE NUMBER OF LEAP YEAR DAYS IN INTERVAL [ JB,JA ]
C         NOT INCLUDING ENDPOINT YEARS
C
      INTEGER FUNCTION NLEAP(JAYR,JBYR,JADAY,JBDAY)
C
      NLEAP=0
      IF(JAYR.LT.JBYR)STOP 'ERROR IN NLEAP'
C
C SAME YEAR
C
```

```
      IF(JAYR.EQ.JBYR)THEN
                  RETURN
C
C DIFFERENT YEARS
C
      ELSE
      JA1=JAYR-1
      JB1=JBYR+1
      IF(JB1.GT.JA1)RETURN
      DO 100 I=JA1,JB1,-1
      IF(LEAP(I).EQ.1)NLEAP=NLEAP+1
100      CONTINUE
      ENDIF
      RETURN
      END
C-----------------------------------------------------------
C
C LEAPADJ : ACCOUNTS FOR LEAP DAYS IN BEGINNING OR ENDING YEARS OF
C           INTERVAL [JB,JA]
C OCT-83
C
      INTEGER FUNCTION LEAPAD(JAYR,JBYR,JADAY,JBDAY)
C
      LEAPAD=0
      IF(JAYR.EQ.JBYR)RETURN
      IF(LEAP(JBYR).EQ.1)THEN
             LEAPAD=1
      ENDIF
      RETURN
      END
c-----------------------------------------------------------
subroutine chwr(iunits,vec)
      logical*1 vec(80),io2
      io2=.false.
      goto8
c
      entry chwr2(iunits,iunit2,vec)
      io2=.true.
      goto8
c
      entry chwrsp(iunits,vec)
      io2=.false.
      write(iunits,3)
3     format(' ')
c
8     i1=1
      call findc(vec,80,';*',2,i1,ifin,ifound,&100,&100)
      ifin=ifin-1
      write(iunits,1)(vec(i),i=1,ifin)
      if(io2)write(iunit2,1)(vec(i),i=1,ifin)
1     format(80a1)
      return
100   write(8,2)
2     format('0error in chwr ')
      return
      end
```

```
C
C-------------------------------------------------------------
C
C     FINDC
C
      SUBROUTINE  FINDC(A,L,AS,LS,IS,IF,IC,*,*)
      LOGICAL*1 A(1),AS(1)
      LOGICAL EQUC
      IF(IS.LE.0.OR.IS.GT.L.OR.LS.LE.0) RETURN 2
      DO 100 I=IS,L
      DO 200 J=1,LS
      IF(EQUC(A(I),AS(J))) GO TO 300
200   CONTINUE
100   CONTINUE
      IF=0
      IC=0
      RETURN 1
300   IF=I
      IC=J
      RETURN
      END
C
C-------------------------------------------------------------
      logical function equc(a1,a2)
      logical*1 a1,a2
      equc=.false.
      if(a1.eq.a2)equc=.true.
      return
      end
```

TABLE B.2

SAMPLE OUTPUT OF THE PRESENT VALUE PROGRAM:
NOVEMBER 1982

Tightness prior = 0.0001000

Bond quote date : 10 29 82
# yrs = 7 cutoff = 10 29 89

| # | MM/DD/YY | Jul | Coup | Yld | Days | #Coup | Price |
|---|---|---|---|---|---|---|---|
| 1 | 11 15 82 | 82319 | 7.130 | 9.290 | 17 | 0.09 | 99.902 |
| 2 | 11 15 82 | 82319 | 7.880 | 9.190 | 17 | 0.09 | 99.940 |
| 3 | 11 30 82 | 82334 | 13.880 | 7.860 | 32 | 0.18 | 100.516 |
| 4 | 12 31 82 | 82365 | 9.380 | 8.150 | 63 | 0.35 | 100.207 |
| 5 | 12 31 82 | 82365 | 15.130 | 7.930 | 63 | 0.35 | 101.211 |
| 6 | 1 31 83 | 83031 | 13.630 | 8.060 | 94 | 0.52 | 101.392 |
| 7 | 2 15 83 | 83046 | 8.000 | 8.630 | 109 | 0.60 | 99.818 |
| 8 | 2 28 83 | 83059 | 13.880 | 8.490 | 122 | 0.67 | 101.740 |
| 9 | 3 31 83 | 83090 | 9.250 | 8.790 | 153 | 0.84 | 100.185 |
| 10 | 3 31 83 | 83090 | 12.630 | 8.760 | 153 | 0.84 | 101.560 |
| 11 | 4 30 83 | 83120 | 14.500 | 8.730 | 183 | 1.00 | 102.772 |
| 12 | 5 15 83 | 83135 | 7.880 | 8.850 | 198 | 1.08 | 99.497 |
| 13 | 5 15 83 | 83135 | 11.630 | 8.830 | 198 | 1.08 | 101.452 |
| 14 | 5 31 83 | 83151 | 15.630 | 9.020 | 214 | 1.17 | 103.694 |
| 15 | 6 30 83 | 83181 | 8.880 | 9.170 | 244 | 1.34 | 99.816 |
| 16 | 6 30 83 | 83181 | 14.630 | 9.180 | 244 | 1.34 | 103.457 |
| 17 | 7 31 83 | 83212 | 15.880 | 9.430 | 275 | 1.51 | 104.587 |
| 18 | 8 15 83 | 83227 | 9.250 | 9.330 | 290 | 1.59 | 99.940 |
| 19 | 8 15 83 | 83227 | 11.880 | 9.440 | 290 | 1.59 | 101.827 |
| 20 | 8 31 83 | 83243 | 16.250 | 9.490 | 306 | 1.68 | 105.327 |
| 21 | 9 30 83 | 83273 | 9.750 | 9.310 | 336 | 1.84 | 100.380 |
| 22 | 9 30 83 | 83273 | 16.000 | 9.500 | 336 | 1.84 | 105.603 |
| 23 | 10 31 83 | 83304 | 15.500 | 9.650 | 367 | 2.01 | 105.481 |
| 24 | 11 15 83 | 83319 | 7.000 | 8.830 | 382 | 2.09 | 98.208 |
| 25 | 11 15 83 | 83319 | 9.880 | 9.550 | 382 | 2.09 | 100.321 |
| 26 | 11 30 83 | 83334 | 12.130 | 9.700 | 397 | 2.18 | 102.452 |
| 27 | 12 31 83 | 83365 | 10.500 | 9.570 | 428 | 2.35 | 101.009 |
| 28 | 12 31 83 | 83365 | 13.000 | 9.750 | 428 | 2.35 | 103.521 |
| 29 | 1 31 84 | 84031 | 15.000 | 10.050 | 459 | 2.52 | 105.714 |
| 30 | 2 15 84 | 84046 | 7.250 | 9.350 | 474 | 2.60 | 97.487 |
| 31 | 2 29 84 | 84060 | 15.130 | 10.120 | 488 | 2.67 | 106.121 |
| 32 | 3 31 84 | 84091 | 14.130 | 10.190 | 519 | 2.84 | 105.096 |
| 33 | 3 31 84 | 84091 | 14.250 | 9.890 | 519 | 2.84 | 105.654 |
| 34 | 4 30 84 | 84121 | 13.880 | 10.150 | 549 | 3.01 | 105.085 |
| 35 | 5 15 84 | 84136 | 9.250 | 9.700 | 564 | 3.09 | 99.368 |
| 36 | 5 15 84 | 84136 | 13.250 | 10.200 | 564 | 3.09 | 104.261 |
| 37 | 5 15 84 | 84136 | 15.750 | 10.180 | 564 | 3.09 | 107.783 |
| 38 | 5 31 84 | 84152 | 13.750 | 10.240 | 580 | 3.18 | 105.030 |

| # | MM/DD/YY | | | Jul | Coup | Yld | Days | #Coup | Price |
|---|---|---|---|---|---|---|---|---|---|
| 39 | 6 | 30 | 84 | 84182 | 8.880 | 9.830 | 610 | 3.34 | 98.568 |
| 40 | 6 | 30 | 84 | 84182 | 14.380 | 10.240 | 610 | 3.34 | 106.215 |
| 41 | 7 | 31 | 84 | 84213 | 13.130 | 10.190 | 641 | 3.51 | 104.621 |
| 42 | 8 | 15 | 84 | 84228 | 7.250 | 9.500 | 656 | 3.59 | 96.361 |
| 43 | 8 | 15 | 84 | 84228 | 13.250 | 10.170 | 656 | 3.59 | 104.945 |
| 44 | 8 | 31 | 84 | 84244 | 11.630 | 10.040 | 672 | 3.68 | 102.613 |
| 45 | 9 | 30 | 84 | 84274 | 12.130 | 10.000 | 702 | 3.85 | 103.645 |
| 46 | 10 | 31 | 84 | 84305 | 9.750 | 9.890 | 733 | 4.02 | 99.751 |
| 47 | 11 | 15 | 84 | 84320 | 14.380 | 10.270 | 748 | 4.10 | 107.426 |
| 48 | 11 | 15 | 84 | 84320 | 16.000 | 10.230 | 748 | 4.10 | 110.430 |
| 49 | 12 | 31 | 84 | 84366 | 14.000 | 10.310 | 794 | 4.35 | 107.030 |
| 50 | 2 | 15 | 85 | 85046 | 8.000 | 9.810 | 840 | 4.60 | 96.350 |
| 51 | 2 | 15 | 85 | 85046 | 14.630 | 10.450 | 840 | 4.60 | 108.359 |
| 52 | 3 | 31 | 85 | 85090 | 13.380 | 10.370 | 884 | 4.84 | 106.304 |
| 53 | 5 | 15 | 85 | 85135 | 10.380 | 9.990 | 929 | 5.09 | 100.858 |
| 54 | 5 | 15 | 85 | 85135 | 14.130 | 10.540 | 929 | 5.09 | 107.836 |
| 55 | 5 | 15 | 85 | 85135 | 14.380 | 10.500 | 929 | 5.09 | 108.474 |
| 56 | 6 | 30 | 85 | 85181 | 14.000 | 10.540 | 975 | 5.34 | 107.877 |
| 57 | 8 | 15 | 85 | 85227 | 8.250 | 9.970 | 1021 | 5.59 | 95.889 |
| 58 | 8 | 15 | 85 | 85227 | 9.630 | 10.020 | 1021 | 5.59 | 99.069 |
| 59 | 8 | 15 | 85 | 85227 | 13.130 | 10.320 | 1021 | 5.59 | 106.680 |
| 60 | 9 | 30 | 85 | 85273 | 15.880 | 10.740 | 1067 | 5.85 | 112.610 |
| 61 | 11 | 15 | 85 | 85319 | 11.750 | 10.280 | 1113 | 6.10 | 103.766 |
| 62 | 12 | 31 | 85 | 85365 | 14.130 | 10.720 | 1159 | 6.35 | 108.977 |
| 63 | 2 | 15 | 86 | 86046 | 13.500 | 10.710 | 1205 | 6.60 | 107.591 |
| 64 | 3 | 31 | 86 | 86090 | 14.000 | 10.890 | 1249 | 6.84 | 108.691 |
| 65 | 5 | 15 | 86 | 86135 | 7.880 | 10.050 | 1294 | 7.09 | 93.660 |
| 66 | 5 | 15 | 86 | 86135 | 13.750 | 10.780 | 1294 | 7.09 | 108.563 |
| 67 | 6 | 30 | 86 | 86181 | 14.880 | 10.980 | 1340 | 7.34 | 111.529 |
| 68 | 8 | 15 | 86 | 86227 | 8.000 | 10.100 | 1386 | 7.59 | 93.510 |
| 69 | 9 | 30 | 86 | 86273 | 12.250 | 10.500 | 1432 | 7.85 | 105.511 |
| 70 | 11 | 15 | 86 | 86319 | 13.880 | 10.950 | 1478 | 8.10 | 109.381 |
| 71 | 11 | 15 | 86 | 86319 | 16.130 | 11.120 | 1478 | 8.10 | 115.986 |
| 72 | 2 | 15 | 87 | 87046 | 9.000 | 10.260 | 1570 | 8.60 | 95.705 |
| 73 | 2 | 15 | 87 | 87046 | 12.750 | 10.690 | 1570 | 8.60 | 106.958 |
| 74 | 5 | 15 | 87 | 87135 | 12.000 | 10.410 | 1659 | 9.09 | 105.644 |
| 75 | 5 | 15 | 87 | 87135 | 14.000 | 11.020 | 1659 | 9.09 | 110.435 |
| 76 | 8 | 15 | 87 | 87227 | 13.750 | 11.000 | 1751 | 9.59 | 110.043 |
| 77 | 11 | 15 | 87 | 87319 | 7.630 | 10.210 | 1843 | 10.10 | 90.014 |
| 78 | 11 | 15 | 87 | 87319 | 12.630 | 10.630 | 1843 | 10.10 | 107.662 |
| 79 | 1 | 15 | 88 | 88015 | 12.380 | 10.690 | 1904 | 10.43 | 106.626 |
| 80 | 4 | 15 | 88 | 88106 | 13.250 | 10.910 | 1995 | 10.93 | 109.447 |
| 81 | 5 | 15 | 88 | 88136 | 8.250 | 10.280 | 2025 | 11.10 | 91.576 |
| 82 | 7 | 15 | 88 | 88197 | 14.000 | 11.150 | 2086 | 11.43 | 111.812 |
| 83 | 10 | 15 | 88 | 88289 | 15.380 | 11.310 | 2178 | 11.93 | 117.321 |
| 84 | 11 | 15 | 88 | 88320 | 8.750 | 10.370 | 2209 | 12.10 | 92.850 |
| 85 | 1 | 15 | 89 | 89015 | 14.630 | 11.280 | 2270 | 12.44 | 114.690 |
| 86 | 4 | 15 | 89 | 89105 | 14.380 | 11.230 | 2360 | 12.93 | 114.210 |
| 87 | 5 | 15 | 89 | 89135 | 9.250 | 10.280 | 2390 | 13.10 | 95.178 |
| 88 | 7 | 15 | 89 | 89196 | 14.500 | 11.230 | 2451 | 13.43 | 115.138 |
| 89 | 10 | 15 | 89 | 89288 | 11.880 | 10.680 | 2543 | 13.93 | 105.794 |

Bill quote date :  10 29 82

| # | MM/DD/YY | Jul | Disct | Days | Price | B-E-Y |
|---|---|---|---|---|---|---|
| 90 | 11 4 82 | 82308 | 7.7300 | 6 | 99.871 | 7.8475 |
| 91 | 11 12 82 | 82316 | 7.7300 | 14 | 99.699 | 7.8610 |
| 92 | 11 18 82 | 82322 | 7.6900 | 20 | 99.573 | 7.8303 |
| 93 | 11 26 82 | 82330 | 7.6600 | 28 | 99.404 | 7.8129 |
| 94 | 12 2 82 | 82336 | 7.6100 | 34 | 99.281 | 7.7716 |
| 95 | 12 9 82 | 82343 | 7.6400 | 41 | 99.130 | 7.8141 |
| 96 | 12 16 82 | 82350 | 7.6400 | 48 | 98.981 | 7.8258 |
| 97 | 12 23 82 | 82357 | 7.7000 | 55 | 98.824 | 7.8999 |
| 98 | 12 30 82 | 82364 | 7.7000 | 62 | 98.674 | 7.9119 |
| 99 | 1 6 83 | 83006 | 7.7600 | 69 | 98.513 | 7.9866 |
| 100 | 1 13 83 | 83013 | 7.8100 | 76 | 98.351 | 8.0512 |
| 101 | 1 20 83 | 83020 | 7.9000 | 83 | 98.179 | 8.1583 |
| 102 | 1 27 83 | 83027 | 7.9000 | 90 | 98.025 | 8.1711 |
| 103 | 2 3 83 | 83034 | 7.9600 | 97 | 97.855 | 8.2474 |
| 104 | 2 10 83 | 83041 | 8.0600 | 104 | 97.672 | 8.3668 |
| 105 | 2 17 83 | 83048 | 8.1000 | 111 | 97.502 | 8.4229 |
| 106 | 2 24 83 | 83055 | 8.1800 | 118 | 97.319 | 8.5221 |
| 107 | 3 3 83 | 83062 | 8.2500 | 125 | 97.135 | 8.6113 |
| 108 | 3 10 83 | 83069 | 8.2600 | 132 | 96.971 | 8.6363 |
| 109 | 3 17 83 | 83076 | 8.2700 | 139 | 96.807 | 8.6614 |
| 110 | 3 24 83 | 83083 | 8.2800 | 146 | 96.642 | 8.6867 |
| 111 | 3 31 83 | 83090 | 8.2700 | 153 | 96.485 | 8.6903 |
| 112 | 4 7 83 | 83097 | 8.3700 | 160 | 96.280 | 8.8141 |
| 113 | 4 14 83 | 83104 | 8.3900 | 167 | 96.108 | 8.8510 |
| 114 | 4 21 83 | 83111 | 8.4200 | 174 | 95.930 | 8.8991 |
| 115 | 4 28 83 | 83118 | 8.3700 | 181 | 95.792 | 8.8591 |
| 116 | 11 4 82 | 82308 | 7.7300 | 6 | 99.871 | 7.8475 |
| 117 | 12 2 82 | 82336 | 7.6100 | 34 | 99.281 | 7.7716 |
| 118 | 12 30 82 | 82364 | 7.7000 | 62 | 98.674 | 7.9119 |
| 119 | 1 27 83 | 83027 | 7.9000 | 90 | 98.025 | 8.1711 |
| 120 | 2 24 83 | 83055 | 8.1800 | 118 | 97.319 | 8.5221 |
| 121 | 3 24 83 | 83083 | 8.2800 | 146 | 96.642 | 8.6867 |
| 122 | 4 21 83 | 83111 | 8.4200 | 174 | 95.930 | 8.8991 |
| 123 | 5 19 83 | 83139 | 8.4800 | 202 | 95.242 | 9.0273 |
| 124 | 6 16 83 | 83167 | 8.5200 | 230 | 94.557 | 9.1356 |
| 125 | 7 14 83 | 83195 | 8.6100 | 258 | 93.829 | 9.3037 |
| 126 | 8 11 83 | 83223 | 8.6100 | 286 | 93.160 | 9.3705 |
| 127 | 9 8 83 | 83251 | 8.6100 | 314 | 92.490 | 9.4384 |
| 128 | 10 6 83 | 83279 | 8.5700 | 342 | 91.858 | 9.4591 |
| 129 | 11 3 83 | 83307 | 8.5400 | 370 | 91.223 | 9.4917 |

Loaded 89 bond quotes
40 bill quotes

multiple securities maturing on same date appear in the same column

| Col | Date | Months | Avg Y-T-M | nr.sec. |
|---|---|---|---|---|
| 1 | 82319 | 0.56 | 9.240 | 2. |
| 2 | 82334 | 1.05 | 7.860 | 1. |
| 3 | 82365 | 2.07 | 8.040 | 2. |
| 4 | 83031 | 3.09 | 8.060 | 1. |
| 5 | 83046 | 3.58 | 8.630 | 1. |

| Col | Date | Months | Avg Y-T-M | nr.sec. |
|---|---|---|---|---|
| 6 | 83059 | 4.01 | 8.490 | 1. |
| 7 | 83090 | 5.03 | 8.747 | 3. |
| 8 | 83120 | 6.02 | 8.730 | 1. |
| 9 | 83135 | 6.51 | 8.840 | 2. |
| 10 | 83151 | 7.04 | 9.020 | 1. |
| 11 | 83181 | 8.02 | 9.175 | 2. |
| 12 | 83212 | 9.04 | 9.430 | 1. |
| 13 | 83227 | 9.53 | 9.385 | 2. |
| 14 | 83243 | 10.06 | 9.490 | 1. |
| 15 | 83273 | 11.05 | 9.405 | 2. |
| 16 | 83304 | 12.07 | 9.650 | 1. |
| 17 | 83319 | 12.56 | 9.190 | 2. |
| 18 | 83334 | 13.05 | 9.700 | 1. |
| 19 | 83365 | 14.07 | 9.660 | 2. |
| 20 | 84031 | 15.09 | 10.050 | 1. |
| 21 | 84046 | 15.58 | 9.350 | 1. |
| 22 | 84060 | 16.04 | 10.120 | 1. |
| 23 | 84091 | 17.06 | 10.040 | 2. |
| 24 | 84121 | 18.05 | 10.150 | 1. |
| 25 | 84136 | 18.54 | 10.027 | 3. |
| 26 | 84152 | 19.07 | 10.240 | 1. |
| 27 | 84182 | 20.05 | 10.035 | 2. |
| 28 | 84213 | 21.07 | 10.190 | 1. |
| 29 | 84228 | 21.57 | 9.835 | 2. |
| 30 | 84244 | 22.09 | 10.040 | 1. |
| 31 | 84274 | 23.08 | 10.000 | 1. |
| 32 | 84305 | 24.10 | 9.890 | 1. |
| 33 | 84320 | 24.59 | 10.250 | 2. |
| 34 | 84366 | 26.10 | 10.310 | 1. |
| 35 | 85046 | 27.62 | 10.130 | 2. |
| 36 | 85090 | 29.06 | 10.370 | 1. |
| 37 | 85135 | 30.54 | 10.343 | 3. |
| 38 | 85181 | 32.05 | 10.540 | 1. |
| 39 | 85227 | 33.57 | 10.103 | 3. |
| 40 | 85273 | 35.08 | 10.740 | 1. |
| 41 | 85319 | 36.59 | 10.280 | 1. |
| 42 | 85365 | 38.10 | 10.720 | 1. |
| 43 | 86046 | 39.62 | 10.710 | 1. |
| 44 | 86090 | 41.06 | 10.890 | 1. |
| 45 | 86135 | 42.54 | 10.415 | 2. |
| 46 | 86181 | 44.06 | 10.980 | 1. |
| 47 | 86227 | 45.57 | 10.100 | 1. |
| 48 | 86273 | 47.08 | 10.500 | 1. |
| 49 | 86319 | 48.59 | 11.035 | 2. |
| 50 | 87046 | 51.62 | 10.475 | 2. |
| 51 | 87135 | 54.54 | 10.715 | 2. |
| 52 | 87227 | 57.57 | 11.000 | 1. |
| 53 | 87319 | 60.59 | 10.420 | 2. |
| 54 | 88015 | 62.60 | 10.690 | 1. |
| 55 | 88106 | 65.59 | 10.910 | 1. |
| 56 | 88136 | 66.58 | 10.280 | 1. |
| 57 | 88197 | 68.58 | 11.150 | 1. |
| 58 | 88289 | 71.61 | 11.310 | 1. |
| 59 | 88320 | 72.63 | 10.370 | 1. |

| Col | Date | Months | Avg Y-T-M | nr.sec. |
|---|---|---|---|---|
| 60 | 89015 | 74.63 | 11.280 | 1. |
| 61 | 89105 | 77.59 | 11.230 | 1. |
| 62 | 89135 | 78.58 | 10.280 | 1. |
| 63 | 89196 | 80.58 | 11.230 | 1. |
| 64 | 89288 | 83.61 | 10.680 | 1. |
| 65 | 82308 | 0.20 | 7.847 | 2. |
| 66 | 82316 | 0.46 | 7.861 | 1. |
| 67 | 82322 | 0.66 | 7.830 | 1. |
| 68 | 82330 | 0.92 | 7.813 | 1. |
| 69 | 82336 | 1.12 | 7.772 | 2. |
| 70 | 82343 | 1.35 | 7.814 | 1. |
| 71 | 82350 | 1.58 | 7.826 | 1. |
| 72 | 82357 | 1.81 | 7.900 | 1. |
| 73 | 82364 | 2.04 | 7.912 | 2. |
| 74 | 83006 | 2.27 | 7.987 | 1. |
| 75 | 83013 | 2.50 | 8.051 | 1. |
| 76 | 83020 | 2.73 | 8.158 | 1. |
| 77 | 83027 | 2.96 | 8.171 | 2. |
| 78 | 83034 | 3.19 | 8.247 | 1. |
| 79 | 83041 | 3.42 | 8.367 | 1. |
| 80 | 83048 | 3.65 | 8.423 | 1. |
| 81 | 83055 | 3.88 | 8.522 | 2. |
| 82 | 83062 | 4.11 | 8.611 | 1. |
| 83 | 83069 | 4.34 | 8.636 | 1. |
| 84 | 83076 | 4.57 | 8.661 | 1. |
| 85 | 83083 | 4.80 | 8.687 | 2. |
| 86 | 83097 | 5.26 | 8.814 | 1. |
| 87 | 83104 | 5.49 | 8.851 | 1. |
| 88 | 83111 | 5.72 | 8.899 | 2. |
| 89 | 83118 | 5.95 | 8.859 | 1. |
| 90 | 83139 | 6.64 | 9.027 | 1. |
| 91 | 83167 | 7.56 | 9.136 | 1. |
| 92 | 83195 | 8.48 | 9.304 | 1. |
| 93 | 83223 | 9.40 | 9.371 | 1. |
| 94 | 83251 | 10.32 | 9.438 | 1. |
| 95 | 83279 | 11.24 | 9.459 | 1. |
| 96 | 83307 | 12.16 | 9.492 | 1. |

Number of Distinct Maturities = 96

Reordered columns

| New | Old | Load Date | Rept Date | Diff | Avg Y-T-M |
|---|---|---|---|---|---|
| 1 | 1 | 82319 | 82319 | 0 | 9.240 |
| 2 | 71 | 82350 | 82351 | -1 | 7.826 |
| 3 | 75 | 83013 | 83016 | -3 | 8.051 |
| 4 | 5 | 83046 | 83046 | 0 | 8.630 |
| 5 | 84 | 83076 | 83077 | -1 | 8.661 |
| 6 | 87 | 83104 | 83107 | -3 | 8.851 |
| 7 | 9 | 83135 | 83137 | -2 | 8.840 |
| 8 | 91 | 83167 | 83168 | -1 | 9.136 |
| 9 | 92 | 83195 | 83198 | -3 | 9.304 |
| 10 | 13 | 83227 | 83228 | -1 | 9.385 |

| New | Old | Load Date | Rept Date | Diff | Avg Y-T-M |
|---|---|---|---|---|---|
| 11 | -1 | 0 | 83259 | 0 | 0.000 |
| 12 | -1 | 0 | 83289 | 0 | 0.000 |
| 13 | 17 | 83319 | 83319 | 0 | 9.190 |
| 14 | -1 | 0 | 83351 | 0 | 0.000 |
| 15 | -1 | 0 | 84016 | 0 | 0.000 |
| 16 | 21 | 84046 | 84046 | 0 | 9.350 |
| 17 | -1 | 0 | 84077 | 0 | 0.000 |
| 18 | -1 | 0 | 84107 | 0 | 0.000 |
| 19 | 25 | 84136 | 84137 | -1 | 10.027 |
| 20 | -1 | 0 | 84168 | 0 | 0.000 |
| 21 | -1 | 0 | 84198 | 0 | 0.000 |
| 22 | 29 | 84228 | 84228 | 0 | 9.835 |
| 23 | -1 | 0 | 84259 | 0 | 0.000 |
| 24 | -1 | 0 | 84289 | 0 | 0.000 |
| 25 | 33 | 84320 | 84319 | 1 | 10.250 |
| 26 | -1 | 0 | 84351 | 0 | 0.000 |
| 27 | -1 | 0 | 85016 | 0 | 0.000 |
| 28 | 35 | 85046 | 85046 | 0 | 10.130 |
| 29 | -1 | 0 | 85077 | 0 | 0.000 |
| 30 | -1 | 0 | 85107 | 0 | 0.000 |
| 31 | 37 | 85135 | 85137 | -2 | 10.343 |
| 32 | -1 | 0 | 85168 | 0 | 0.000 |
| 33 | -1 | 0 | 85198 | 0 | 0.000 |
| 34 | 39 | 85227 | 85228 | -1 | 10.103 |
| 35 | -1 | 0 | 85259 | 0 | 0.000 |
| 36 | -1 | 0 | 85289 | 0 | 0.000 |
| 37 | 41 | 85319 | 85319 | 0 | 10.280 |
| 38 | -1 | 0 | 85351 | 0 | 0.000 |
| 39 | -1 | 0 | 86016 | 0 | 0.000 |
| 40 | 43 | 86046 | 86046 | 0 | 10.710 |
| 41 | -1 | 0 | 86077 | 0 | 0.000 |
| 42 | -1 | 0 | 86107 | 0 | 0.000 |
| 43 | 45 | 86135 | 86137 | -2 | 10.415 |
| 44 | -1 | 0 | 86168 | 0 | 0.000 |
| 45 | -1 | 0 | 86198 | 0 | 0.000 |
| 46 | 47 | 86227 | 86228 | -1 | 10.100 |
| 47 | -1 | 0 | 86259 | 0 | 0.000 |
| 48 | -1 | 0 | 86289 | 0 | 0.000 |
| 49 | 49 | 86319 | 86319 | 0 | 11.035 |
| 50 | -1 | 0 | 86351 | 0 | 0.000 |
| 51 | -1 | 0 | 87016 | 0 | 0.000 |
| 52 | 50 | 87046 | 87046 | 0 | 10.475 |
| 53 | -1 | 0 | 87077 | 0 | 0.000 |
| 54 | -1 | 0 | 87107 | 0 | 0.000 |
| 55 | 51 | 87135 | 87137 | -2 | 10.715 |
| 56 | -1 | 0 | 87168 | 0 | 0.000 |
| 57 | -1 | 0 | 87198 | 0 | 0.000 |
| 58 | 52 | 87227 | 87228 | -1 | 11.000 |
| 59 | -1 | 0 | 87259 | 0 | 0.000 |
| 60 | -1 | 0 | 87289 | 0 | 0.000 |
| 61 | 53 | 87319 | 87319 | 0 | 10.420 |
| 62 | -1 | 0 | 87351 | 0 | 0.000 |
| 63 | 54 | 88015 | 88016 | -1 | 10.690 |
| 64 | -1 | 0 | 88046 | 0 | 0.000 |

| New | Old | Load Date | Rept Date | Diff | Avg Y-T-M |
|---|---|---|---|---|---|
| 65 | -1 | 0 | 88077 | 0 | 0.000 |
| 66 | 55 | 88106 | 88107 | -1 | 10.910 |
| 67 | 56 | 88136 | 88137 | -1 | 10.280 |
| 68 | -1 | 0 | 88168 | 0 | 0.000 |
| 69 | 57 | 88197 | 88198 | -1 | 11.150 |
| 70 | -1 | 0 | 88228 | 0 | 0.000 |
| 71 | -1 | 0 | 88259 | 0 | 0.000 |
| 72 | 58 | 88289 | 88289 | 0 | 11.310 |
| 73 | 59 | 88320 | 88319 | 1 | 10.370 |
| 74 | -1 | 0 | 88351 | 0 | 0.000 |
| 75 | 60 | 89015 | 89016 | -1 | 11.280 |
| 76 | -1 | 0 | 89046 | 0 | 0.000 |
| 77 | -1 | 0 | 89077 | 0 | 0.000 |
| 78 | 61 | 89105 | 89107 | -2 | 11.230 |
| 79 | 62 | 89135 | 89137 | -2 | 10.280 |
| 80 | -1 | 0 | 89168 | 0 | 0.000 |
| 81 | 63 | 89196 | 89198 | -2 | 11.230 |
| 82 | -1 | 0 | 89228 | 0 | 0.000 |
| 83 | -1 | 0 | 89259 | 0 | 0.000 |
| 84 | 64 | 89288 | 89289 | -1 | 10.680 |

Old columns not appearing have been dropped.

Number of usable rows = 56

```
Gap of width   2between   10  13    9.3850    9.1900
Col    11   row   130 Y-T-M =     9.3200 price =    92.1787
Col    12   row   131 Y-T-M =     9.2550 price =    91.5230

Gap of width   2between   13  16    9.1900    9.3500
Col    14   row   132 Y-T-M =     9.2433 price =    90.1349
Col    15   row   133 Y-T-M =     9.2967 price =    89.3862

Gap of width   2between   16  19    9.3500   10.0267
Col    17   row   134 Y-T-M =     9.5756 price =    87.6763
Col    18   row   135 Y-T-M =     9.8011 price =    86.6946

Gap of width   2between   19  22   10.0267    9.8350
Col    20   row   136 Y-T-M =     9.9628 price =    85.0674
Col    21   row   137 Y-T-M =     9.8989 price =    84.4566

Gap of width   2between   22  25    9.8350   10.2500
Col    23   row   138 Y-T-M =     9.9733 price =    82.9598
Col    24   row   139 Y-T-M =    10.1117 price =    82.0513

Gap of width   2between   25  28   10.2500   10.1300
Col    26   row   140 Y-T-M =    10.2100 price =    80.5128
Col    27   row   141 Y-T-M =    10.1700 price =    79.9016

Gap of width   2between   28  31   10.1300   10.3433
Col    29   row   142 Y-T-M =    10.2011 price =    78.5016
Col    30   row   143 Y-T-M =    10.2722 price =    77.7017
```

```
Gap of width   2between   31  34   10.3433   10.1033
Col    32   row   144 Y-T-M =    10.2633 price =     76.4015
Col    33   row   145 Y-T-M =    10.1833 price =     75.9154

Gap of width   2between   34  37   10.1033   10.2800
Col    35   row   146 Y-T-M =    10.1622 price =     74.6840
Col    36   row   147 Y-T-M =    10.2211 price =     73.9258

Gap of width   2between   37  40   10.2800   10.7100
Col    38   row   148 Y-T-M =    10.4233 price =     72.2206
Col    39   row   149 Y-T-M =    10.5667 price =     71.2682

Gap of width   2between   40  43   10.7100   10.4150
Col    41   row   150 Y-T-M =    10.6117 price =     69.9188
Col    42   row   151 Y-T-M =    10.5133 price =     69.5395

Gap of width   2between   43  46   10.4150   10.1000
Col    44   row   152 Y-T-M =    10.3100 price =     68.8375
Col    45   row   153 Y-T-M =    10.2050 price =     68.5151

Gap of width   2between   46  49   10.1000   11.0350
Col    47   row   154 Y-T-M =    10.4117 price =     66.8236
Col    48   row   155 Y-T-M =    10.7233 price =     65.4351

Gap of width   2between   49  52   11.0350   10.4750
Col    50   row   156 Y-T-M =    10.8483 price =     63.9465
Col    51   row   157 Y-T-M =    10.6617 price =     63.8708

Gap of width   2between   52  55   10.4750   10.7150
Col    53   row   158 Y-T-M =    10.5550 price =     63.0399
Col    54   row   159 Y-T-M =    10.6350 price =     62.2659

Gap of width   2between   55  58   10.7150   11.0000
Col    56   row   160 Y-T-M =    10.8100 price =     60.6800
Col    57   row   161 Y-T-M =    10.9050 price =     59.8680

Gap of width   2between   58  61   11.0000   10.4200
Col    59   row   162 Y-T-M =    10.8067 price =     59.0728
Col    60   row   163 Y-T-M =    10.6133 price =     59.1070

Gap of width   1between   61  63   10.4200   10.6900
Col    62   row   164 Y-T-M =    10.5550 price =     58.2451

Gap of width   2between   63  66   10.6900   10.9100
Col    64   row   165 Y-T-M =    10.7633 price =     56.6032
Col    65   row   166 Y-T-M =    10.8367 price =     55.8776

Gap of width   1between   67  69   10.2800   11.1500
Col    68   row   167 Y-T-M =    10.7150 price =     54.7583

Gap of width   2between   69  72   11.1500   11.3100
Col    70   row   168 Y-T-M =    11.2033 price =     52.2909
Col    71   row   169 Y-T-M =    11.2567 price =     51.6434

Gap of width   1between   73  75   10.3700   11.2800
Col    74   row   170 Y-T-M =    10.8250 price =     51.5555
```

```
Gap of width   2between   75  78   11.2800   11.2300
Col    76   row  171 Y-T-M =    11.2633 price =    49.2580
Col    77   row  172 Y-T-M =    11.2467 price =    48.8500

Gap of width   1between   79  81   10.2800   11.2300
Col    80   row  173 Y-T-M =    10.7550 price =    49.0679

Gap of width   2between   81  84   11.2300   10.6800
Col    82   row  174 Y-T-M =    11.0467 price =    47.2521
Col    83   row  175 Y-T-M =    10.8633 price =    47.4128

Rows after augmenting with pseudo-zeros : 102

Data matrix 102 x  85 written to TSMIX.DAT...

RATS run written to TSMIX.RAT...
```

# APPENDIX C

## COMPUTATION OF SPOT INTEREST RATES FROM REGRESSION COEFFICIENTS

Following the logic of Carleton and Cooper (1976) as presented in Chapter IV, the present value of the Treasury securities is regressed against their terms to maturity, via the mixed estimation procedure outlined in the RATS User's Manual, Version 4.1, pages 13-14. The spot rate $i_{mat(t)}$ of interest for term to maturity t is computed as:

(C.1)

$$i_{mat(t)} = (200.0 * (100.0/(b_{mat(t)})^{t}) - 200.0$$

The value of $i_{mat(t)}$ is labeled INTRATE in Table C.1 and is the annualized interest rate for each term to maturity. The value $b_{mat(t)}$, also shown in Table C.1, is the mixed beta of the regression of present value as a function of term to maturity t. The exponent (t) is labeled XPON in Table C.1.

A final file shown in Table C.2 is written with the values of the vector 'key' as the first line, are carried forward from the program described in Appendix B. Two column vectors are then written: the annualized spot interest rate and the number of days to maturity. These form the data from which the spline functions are estimated.

TABLE C.1

OUTPUT FROM MIXED ESTIMATION PROCEDURE DEMONSTRATING THE COMPUTATION OF SPOT RATES OF INTEREST: NOVEMBER 1982

```
CHANGE INPUT 2
CAL 1 1 1
ALL 260 102 20 260
EQV 1
  PRICEACI
EQV 200 201 202 203 204 205 206 207
  KEY OLSBETA MIXBETA OLSERR MIXERR INTRATE XPON MATUR
SUPPRESS LABELING
EQV 2 TO  85
 M82319 M82351 M83016 M83046 M83077 M83107 M83137 M83168
M83198 M83228 $
 M83259 M83289 M83319 M83351 M84016 M84046 M84077 M84107
M84137 M84168 $
 M84198 M84228 M84259 M84289 M84319 M84351 M85016 M85046
M85077 M85107 $
 M85137 M85168 M85198 M85228 M85259 M85289 M85319 M85351
M86016 M86046 $
 M86077 M86107 M86137 M86168 M86198 M86228 M86259 M86289
M86319 M86351 $
 M87016 M87046 M87077 M87107 M87137 M87168 M87198 M87228
M87259 M87289 $
 M87319 M87351 M88016 M88046 M88077 M88107 M88137 M88168
M88198 M88228 $
 M88259 M88289 M88319 M88351 M89016 M89046 M89077 M89107
M89137 M89168 $
 M89198 M89228 M89259 M89289 M
* THIS IS HERE TO SLURP THE CONTINUATION MARK
DATA(ORG=VAR,FORMAT=FREE) 1 4 KEY
DATA(ORG=OBS,FORMAT='(5E16.8)') 1 102 1 TO  85
 78715 PROCEDURE MIXED IEQN NBEG NEND CAPR LOWR V
 78723 TYPE RECT CAPR
 78723 TYPE VECT LOWR
 78723 TYPE SYMM V
 78723 LOCAL SYMM XXMIXED
 78726 LOCAL VECT XYMIXED
 78729 CMOMENT(EQUATION=IEQN) NBEG NEND
 78743 REGRESS(EQUATION=IEQN,PRINT) 0 OLSERR OLSBETA
 78762 OVERLAY CMOM(1,1) WITH XXMIXED(NREG,NREG)
 78782 OVERLAY CMOM(NREG+1,1) WITH XYMIXED(NREG)
 78803 MAT
XXMIXED=XXMIXED+SCALE(SEESQ)*TR(CAPR)*(INV(V)*CAPR)
 78828 MAT
XYMIXED=XYMIXED+SCALE(SEESQ)*TR(CAPR)*(INV(V)*LOWR)
 78853 REGRESS(EQUATION=IEQN) 0 MIXERR MIXBETA
 78870 END
EQUATION(NOCONST,MORE) 1 PRICEACI
```

# 2 TO 85

| | | |
|---|---|---|
| VAR | 2 | M82319 |
| VAR | 3 | M82351 |
| VAR | 4 | M83016 |
| VAR | 5 | M83046 |
| VAR | 6 | M83077 |
| VAR | 7 | M83107 |
| VAR | 8 | M83137 |
| VAR | 9 | M83168 |
| VAR | 10 | M83198 |
| VAR | 11 | M83228 |
| VAR | 12 | M83259 |
| VAR | 13 | M83289 |
| VAR | 14 | M83319 |
| VAR | 15 | M83351 |
| VAR | 16 | M84016 |
| VAR | 17 | M84046 |
| VAR | 18 | M84077 |
| VAR | 19 | M84107 |
| VAR | 20 | M84137 |
| VAR | 21 | M84168 |
| VAR | 22 | M84198 |
| VAR | 23 | M84228 |
| VAR | 24 | M84259 |
| VAR | 25 | M84289 |
| VAR | 26 | M84319 |
| VAR | 27 | M84351 |
| VAR | 28 | M85016 |
| VAR | 29 | M85046 |
| VAR | 30 | M85077 |
| VAR | 31 | M85107 |
| VAR | 32 | M85137 |
| VAR | 33 | M85168 |
| VAR | 34 | M85198 |
| VAR | 35 | M85228 |
| VAR | 36 | M85259 |
| VAR | 37 | M85289 |
| VAR | 38 | M85319 |
| VAR | 39 | M85351 |
| VAR | 40 | M86016 |
| VAR | 41 | M86046 |
| VAR | 42 | M86077 |
| VAR | 43 | M86107 |
| VAR | 44 | M86137 |
| VAR | 45 | M86168 |
| VAR | 46 | M86198 |
| VAR | 47 | M86228 |
| VAR | 48 | M86259 |
| VAR | 49 | M86289 |
| VAR | 50 | M86319 |
| VAR | 51 | M86351 |
| VAR | 52 | M87016 |
| VAR | 53 | M87046 |
| VAR | 54 | M87077 |
| VAR | 55 | M87107 |
| VAR | 56 | M87137 |

```
VAR    57     M87168
VAR    58     M87198
VAR    59     M87228
VAR    60     M87259
VAR    61     M87289
VAR    62     M87319
VAR    63     M87351
VAR    64     M88016
VAR    65     M88046
VAR    66     M88077
VAR    67     M88107
VAR    68     M88137
VAR    69     M88168
VAR    70     M88198
VAR    71     M88228
VAR    72     M88259
VAR    73     M88289
VAR    74     M88319
VAR    75     M88351
VAR    76     M89016
VAR    77     M89046
VAR    78     M89077
VAR    79     M89107
VAR    80     M89137
VAR    81     M89168
VAR    82     M89198
VAR    83     M89228
VAR    84     M89259
VAR    85     M89289
DEC RECT RESTR( 82, 84)
DEC VECT ZEROS( 82)
DEC SYMM VARCOV( 82, 82)
FMATRIX(DIFF=2) RESTR 1 1
MATRIX ZEROS=CONST(0.0)
MATRIX VARCOV=IDEN(       0.0001000)
EXECUTE MIXED 1 1 102 RESTR ZEROS VARCOV

VARIABLES IN CROSS-MOMENT MATRIX
FROM     1- 1 UNTIL  102- 1
VAR     2     M82319
VAR     3     M82351
VAR     4     M83016
VAR     5     M83046
VAR     6     M83077
VAR     7     M83107
VAR     8     M83137
VAR     9     M83168
VAR    10     M83198
VAR    11     M83228
VAR    12     M83259
VAR    13     M83289
VAR    14     M83319
VAR    15     M83351
VAR    16     M84016
VAR    17     M84046
VAR    18     M84077
```

| | | |
|---|---|---|
| VAR | 19 | M84107 |
| VAR | 20 | M84137 |
| VAR | 21 | M84168 |
| VAR | 22 | M84198 |
| VAR | 23 | M84228 |
| VAR | 24 | M84259 |
| VAR | 25 | M84289 |
| VAR | 26 | M84319 |
| VAR | 27 | M84351 |
| VAR | 28 | M85016 |
| VAR | 29 | M85046 |
| VAR | 30 | M85077 |
| VAR | 31 | M85107 |
| VAR | 32 | M85137 |
| VAR | 33 | M85168 |
| VAR | 34 | M85198 |
| VAR | 35 | M85228 |
| VAR | 36 | M85259 |
| VAR | 37 | M85289 |
| VAR | 38 | M85319 |
| VAR | 39 | M85351 |
| VAR | 40 | M86016 |
| VAR | 41 | M86046 |
| VAR | 42 | M86077 |
| VAR | 43 | M86107 |
| VAR | 44 | M86137 |
| VAR | 45 | M86168 |
| VAR | 46 | M86198 |
| VAR | 47 | M86228 |
| VAR | 48 | M86259 |
| VAR | 49 | M86289 |
| VAR | 50 | M86319 |
| VAR | 51 | M86351 |
| VAR | 52 | M87016 |
| VAR | 53 | M87046 |
| VAR | 54 | M87077 |
| VAR | 55 | M87107 |
| VAR | 56 | M87137 |
| VAR | 57 | M87168 |
| VAR | 58 | M87198 |
| VAR | 59 | M87228 |
| VAR | 60 | M87259 |
| VAR | 61 | M87289 |
| VAR | 62 | M87319 |
| VAR | 63 | M87351 |
| VAR | 64 | M88016 |
| VAR | 65 | M88046 |
| VAR | 66 | M88077 |
| VAR | 67 | M88107 |
| VAR | 68 | M88137 |
| VAR | 69 | M88168 |
| VAR | 70 | M88198 |
| VAR | 71 | M88228 |
| VAR | 72 | M88259 |
| VAR | 73 | M88289 |
| VAR | 74 | M88319 |

```
VAR    75     M88351
VAR    76     M89016
VAR    77     M89046
VAR    78     M89077
VAR    79     M89107
VAR    80     M89137
VAR    81     M89168
VAR    82     M89198
VAR    83     M89228
VAR    84     M89259
VAR    85     M89289
VAR     1     PRICEACI
```

(Actual Regressions Not Shown)

```
EQV 199
  DIFFR
SET DIFFR 1  84 = OLSBETA(T) - MIXBETA(T)
PRINT 1 4 KEY
```

| ENTRY | KEY 200 |
|---|---|
| 1 | 82302.9 |
| 2 | 17.0000 |
| 3 | 0.558907 |
| 4 | 84.0000 |

```
PRINT 1  84 OLSBETA MIXBETA DIFFR
```

| ENTRY | OLSBETA 201 | MIXBETA 202 | DIFFR 199 |
|---|---|---|---|
| 1 | 0.962288 | 0.959865 | 0.242241E-02 |
| 2 | 0.989813 | 0.970145 | 0.196683E-01 |
| 3 | 0.983512 | 0.972019 | 0.114932E-01 |
| 4 | 0.958926 | 0.966469 | -0.754315E-02 |
| 5 | 0.968069 | 0.957139 | 0.109296E-01 |
| 6 | 0.961080 | 0.942820 | 0.182594E-01 |
| 7 | 0.912462 | 0.927520 | -0.150573E-01 |
| 8 | 0.945567 | 0.924493 | 0.210734E-01 |
| 9 | 0.938295 | 0.921140 | 0.171551E-01 |
| 10 | 0.909658 | 0.914917 | -0.525898E-02 |
| 11 | 0.921787 | 0.908647 | 0.131405E-01 |
| 12 | 0.915230 | 0.898527 | 0.167027E-01 |
| 13 | 0.875766 | 0.887027 | -0.112608E-01 |
| 14 | 0.901349 | 0.885119 | 0.162292E-01 |
| 15 | 0.893862 | 0.881904 | 0.119579E-01 |
| 16 | 0.874485 | 0.874224 | 0.261128E-03 |
| 17 | 0.876763 | 0.861810 | 0.149534E-01 |
| 18 | 0.866946 | 0.843637 | 0.233089E-01 |
| 19 | 0.810506 | 0.825820 | -0.153149E-01 |
| 20 | 0.850674 | 0.826132 | 0.245419E-01 |
| 21 | 0.844566 | 0.827167 | 0.173988E-01 |
| 22 | 0.823268 | 0.823234 | 0.338255E-04 |
| 23 | 0.829598 | 0.814126 | 0.154719E-01 |
| 24 | 0.820513 | 0.798396 | 0.221168E-01 |
| 25 | 0.760494 | 0.781983 | -0.214891E-01 |

| ENTRY | OLSBETA 201 | MIXBETA 202 | DIFFR 199 |
|---|---|---|---|
| 26 | 0.805128 | 0.781916 | 0.232111E-01 |
| 27 | 0.799016 | 0.782278 | 0.167377E-01 |
| 28 | 0.777440 | 0.778229 | -0.788752E-03 |
| 29 | 0.785016 | 0.770096 | 0.149197E-01 |
| 30 | 0.777017 | 0.756112 | 0.209057E-01 |
| 31 | 0.729441 | 0.741627 | -0.121863E-01 |
| 32 | 0.764015 | 0.742508 | 0.215071E-01 |
| 33 | 0.759154 | 0.744264 | 0.148903E-01 |
| 34 | 0.742467 | 0.742672 | -0.204711E-03 |
| 35 | 0.746840 | 0.737793 | 0.904680E-02 |
| 36 | 0.739258 | 0.727858 | 0.114007E-01 |
| 37 | 0.698912 | 0.715415 | -0.165029E-01 |
| 38 | 0.722206 | 0.708987 | 0.132185E-01 |
| 39 | 0.712682 | 0.702155 | 0.105269E-01 |
| 40 | 0.686110 | 0.694807 | -0.869654E-02 |
| 41 | 0.699188 | 0.689032 | 0.101560E-01 |
| 42 | 0.695395 | 0.681586 | 0.138092E-01 |
| 43 | 0.663794 | 0.674072 | -0.102775E-01 |
| 44 | 0.688375 | 0.675216 | 0.131591E-01 |
| 45 | 0.685151 | 0.674841 | 0.103101E-01 |
| 46 | 0.676957 | 0.669046 | 0.791069E-02 |
| 47 | 0.668236 | 0.656033 | 0.122028E-01 |
| 48 | 0.654351 | 0.637363 | 0.169885E-01 |
| 49 | 0.600366 | 0.620422 | -0.200562E-01 |
| 50 | 0.639465 | 0.621240 | 0.182252E-01 |
| 51 | 0.638708 | 0.625003 | 0.137048E-01 |
| 52 | 0.628375 | 0.625597 | 0.277786E-02 |
| 53 | 0.630399 | 0.620627 | 0.977154E-02 |
| 54 | 0.622659 | 0.609471 | 0.131880E-01 |
| 55 | 0.586160 | 0.596171 | -0.100112E-01 |
| 56 | 0.606800 | 0.591596 | 0.152038E-01 |
| 57 | 0.598680 | 0.587973 | 0.107076E-01 |
| 58 | 0.574353 | 0.584782 | -0.104296E-01 |
| 59 | 0.590728 | 0.583794 | 0.693346E-02 |
| 60 | 0.591070 | 0.580690 | 0.103797E-01 |
| 61 | 0.574184 | 0.574460 | -0.275622E-03 |
| 62 | 0.582451 | 0.569581 | 0.128703E-01 |
| 63 | 0.546307 | 0.564090 | -0.177837E-01 |
| 64 | 0.566032 | 0.562169 | 0.386227E-02 |
| 65 | 0.558776 | 0.560147 | -0.137147E-02 |
| 66 | 0.551545 | 0.556197 | -0.465193E-02 |
| 67 | 0.555648 | 0.547837 | 0.781013E-02 |
| 68 | 0.547583 | 0.536036 | 0.115469E-01 |
| 69 | 0.495442 | 0.523541 | -0.280993E-01 |
| 70 | 0.522909 | 0.518613 | 0.429565E-02 |
| 71 | 0.516434 | 0.516564 | -0.130116E-03 |
| 72 | 0.504235 | 0.514754 | -0.105188E-01 |
| 73 | 0.523656 | 0.510483 | 0.131726E-01 |
| 74 | 0.515555 | 0.501945 | 0.136092E-01 |
| 75 | 0.462377 | 0.491751 | -0.293743E-01 |
| 76 | 0.492580 | 0.489006 | 0.357402E-02 |
| 77 | 0.488500 | 0.489097 | -0.596919E-03 |
| 78 | 0.481970 | 0.489116 | -0.714653E-02 |
| 79 | 0.500641 | 0.485872 | 0.147688E-01 |

| ENTRY | OLSBETA 201 | MIXBETA 202 | DIFFR 199 |
|---|---|---|---|
| 80 | 0.490679 | 0.478027 | 0.126521E-01 |
| 81 | 0.440970 | 0.469440 | -0.284701E-01 |
| 82 | 0.472521 | 0.470011 | 0.251077E-02 |
| 83 | 0.474128 | 0.476221 | -0.209203E-02 |
| 84 | 0.483829 | 0.485749 | -0.192021E-02 |

```
SET MATUR 1 1 = KEY(2)
SET MATUR 2  84 = KEY(2) + (T-1)*30.4165
SET XPON 1  84 = 182.5 / MATUR(T)
SET INTRATE 1  84 = (200.0*(1.0/MIXBETA(T))**XPON(T))-200.0
COPY 1 4 KEY
COPY(ORG=OBS) 1  84 INTRATE MATUR
PRINT 1  84 MATUR XPON INTRATE
```

| ENTRY | MATUR 207 | XPON 206 | INTRATE 205 |
|---|---|---|---|
| 1 | 17.0000 | 10.7353 | 110.462 |
| 2 | 47.4165 | 3.84887 | 24.7470 |
| 3 | 77.8330 | 2.34476 | 13.7616 |
| 4 | 108.249 | 1.68592 | 11.8372 |
| 5 | 138.666 | 1.31611 | 11.8698 |
| 6 | 169.082 | 1.07935 | 13.1230 |
| 7 | 199.499 | 0.914792 | 14.2508 |
| 8 | 229.915 | 0.793770 | 12.8603 |
| 9 | 260.332 | 0.701028 | 11.8550 |
| 10 | 290.748 | 0.627690 | 11.4805 |
| 11 | 321.165 | 0.568244 | 11.1892 |
| 12 | 351.581 | 0.519083 | 11.4224 |
| 13 | 381.998 | 0.477751 | 11.7889 |
| 14 | 412.414 | 0.442516 | 11.0972 |
| 15 | 442.831 | 0.412121 | 10.6314 |
| 16 | 473.247 | 0.385633 | 10.6407 |
| 17 | 503.664 | 0.362345 | 11.0733 |
| 18 | 534.081 | 0.341709 | 11.9645 |
| 19 | 564.497 | 0.323297 | 12.7652 |
| 20 | 594.913 | 0.306767 | 12.0687 |
| 21 | 625.330 | 0.291846 | 11.3879 |
| 22 | 655.746 | 0.278309 | 11.1254 |
| 23 | 686.163 | 0.265972 | 11.2436 |
| 24 | 716.579 | 0.254682 | 11.8036 |
| 25 | 746.996 | 0.244312 | 12.3847 |
| 26 | 777.412 | 0.234753 | 11.8902 |
| 27 | 807.829 | 0.225914 | 11.4079 |
| 28 | 838.245 | 0.217717 | 11.2213 |
| 29 | 868.662 | 0.210093 | 11.2838 |
| 30 | 899.078 | 0.202986 | 11.6778 |
| 31 | 929.495 | 0.196343 | 12.0890 |
| 32 | 959.911 | 0.190122 | 11.6472 |
| 33 | 990.328 | 0.184282 | 11.1876 |
| 34 | 1020.74 | 0.178791 | 10.9261 |
| 35 | 1051.16 | 0.173618 | 10.8428 |
| 36 | 1081.58 | 0.168735 | 11.0122 |

| ENTRY | MATUR 207 | XPON 206 | INTRATE 205 |
|---|---|---|---|
| 37 | 1111.99 | 0.164120 | 11.3002 |
| 38 | 1142.41 | 0.159750 | 11.2956 |
| 39 | 1172.83 | 0.155607 | 11.3129 |
| 40 | 1203.24 | 0.151673 | 11.3562 |
| 41 | 1233.66 | 0.147934 | 11.3294 |
| 42 | 1264.08 | 0.144374 | 11.3807 |
| 43 | 1294.49 | 0.140982 | 11.4362 |
| 44 | 1324.91 | 0.137745 | 11.1171 |
| 45 | 1355.33 | 0.134654 | 10.8768 |
| 46 | 1385.74 | 0.131698 | 10.8711 |
| 47 | 1416.16 | 0.128870 | 11.1654 |
| 48 | 1446.58 | 0.126160 | 11.6940 |
| 49 | 1476.99 | 0.123562 | 12.1514 |
| 50 | 1507.41 | 0.121069 | 11.8653 |
| 51 | 1537.82 | 0.118674 | 11.4723 |
| 52 | 1568.24 | 0.116372 | 11.2203 |
| 53 | 1598.66 | 0.114158 | 11.1933 |
| 54 | 1629.07 | 0.112027 | 11.4078 |
| 55 | 1659.49 | 0.109973 | 11.7060 |
| 56 | 1689.91 | 0.107994 | 11.6654 |
| 57 | 1720.32 | 0.106085 | 11.5912 |
| 58 | 1750.74 | 0.104242 | 11.5042 |
| 59 | 1781.16 | 0.102461 | 11.3389 |
| 60 | 1811.57 | 0.100741 | 11.2567 |
| 61 | 1841.99 | 0.990776E-01 | 11.2915 |
| 62 | 1872.41 | 0.974682E-01 | 11.2786 |
| 63 | 1902.82 | 0.959101E-01 | 11.2896 |
| 64 | 1933.24 | 0.944011E-01 | 11.1752 |
| 65 | 1963.66 | 0.929389E-01 | 11.0681 |
| 66 | 1994.07 | 0.915212E-01 | 11.0314 |
| 67 | 2024.49 | 0.901462E-01 | 11.1493 |
| 68 | 2054.91 | 0.888119E-01 | 11.3882 |
| 69 | 2085.32 | 0.875165E-01 | 11.6540 |
| 70 | 2115.74 | 0.862583E-01 | 11.6543 |
| 71 | 2146.15 | 0.850358E-01 | 11.5557 |
| 72 | 2176.57 | 0.838475E-01 | 11.4519 |
| 73 | 2206.99 | 0.826919E-01 | 11.4353 |
| 74 | 2237.40 | 0.815677E-01 | 11.5664 |
| 75 | 2267.82 | 0.804737E-01 | 11.7563 |
| 76 | 2298.24 | 0.794087E-01 | 11.6904 |
| 77 | 2328.65 | 0.783715E-01 | 11.5303 |
| 78 | 2359.07 | 0.773610E-01 | 11.3768 |
| 79 | 2389.49 | 0.763762E-01 | 11.3354 |
| 80 | 2419.90 | 0.754162E-01 | 11.4485 |
| 81 | 2450.32 | 0.744801E-01 | 11.5879 |
| 82 | 2480.74 | 0.735669E-01 | 11.4229 |
| 83 | 2511.15 | 0.726758E-01 | 11.0793 |
| 84 | 2541.57 | 0.718060E-01 | 10.6432 |

TABLE C.2

OUTPUT FILE PRODUCED FROM MIXED ESTIMATION: THE DATA FILE FROM WHICH THE SPLINE FUNCTION IS ESTIMATED NOVEMBER 1982

| | | | |
|---|---|---|---|
| 82302.000 | 17.000000 | 0.55890717 | 84.000000 (Key) |

| Interest Rate | Days to Maturity |
|---|---|
| 110.46215 | 17.000000 |
| 24.747020 | 47.416500 |
| 13.761613 | 77.833000 |
| 11.837173 | 108.24950 |
| 11.869786 | 138.66600 |
| 13.122973 | 169.08250 |
| 14.250819 | 199.49900 |
| 12.860253 | 229.91550 |
| 11.855020 | 260.33200 |
| 11.480530 | 290.74850 |
| 11.189178 | 321.16500 |
| 11.422445 | 351.58150 |
| 11.788946 | 381.99800 |
| 11.097214 | 412.41450 |
| 10.631374 | 442.83100 |
| 10.640659 | 473.24750 |
| 11.073320 | 503.66400 |
| 11.964543 | 534.08050 |
| 12.765197 | 564.49700 |
| 12.068685 | 594.91350 |
| 11.387898 | 625.33000 |
| 11.125426 | 655.74650 |
| 11.243571 | 686.16300 |
| 11.803556 | 716.57950 |
| 12.384663 | 746.99600 |
| 11.890235 | 777.41250 |
| 11.407900 | 807.82900 |
| 11.221305 | 838.24550 |
| 11.283754 | 868.66200 |
| 11.677797 | 899.07850 |
| 12.089000 | 929.49500 |
| 11.647208 | 959.91150 |
| 11.187620 | 990.32800 |
| 10.926100 | 1020.7445 |
| 10.842843 | 1051.1610 |
| 11.012204 | 1081.5775 |
| 11.300199 | 1111.9940 |
| 11.295615 | 1142.4105 |
| 11.312924 | 1172.8270 |
| 11.356221 | 1203.2435 |
| 11.329387 | 1233.6600 |

| Interest Rate | Days to Maturity |
|---|---|
| 11.380702 | 1264.0765 |
| 11.436196 | 1294.4930 |
| 11.117103 | 1324.9095 |
| 10.876754 | 1355.3260 |
| 10.871132 | 1385.7425 |
| 11.165393 | 1416.1590 |
| 11.694020 | 1446.5755 |
| 12.151430 | 1476.9920 |
| 11.865294 | 1507.4085 |
| 11.472304 | 1537.8250 |
| 11.220295 | 1568.2415 |
| 11.193261 | 1598.6580 |
| 11.407797 | 1629.0745 |
| 11.706046 | 1659.4910 |
| 11.665412 | 1689.9075 |
| 11.591227 | 1720.3240 |
| 11.504158 | 1750.7405 |
| 11.338857 | 1781.1570 |
| 11.256701 | 1811.5735 |
| 11.291478 | 1841.9900 |
| 11.278632 | 1872.4065 |
| 11.289626 | 1902.8230 |
| 11.175155 | 1933.2395 |
| 11.068058 | 1963.6560 |
| 11.031355 | 1994.0725 |
| 11.149255 | 2024.4890 |
| 11.388232 | 2054.9055 |
| 11.653981 | 2085.3220 |
| 11.654305 | 2115.7385 |
| 11.555704 | 2146.1550 |
| 11.451924 | 2176.5715 |
| 11.435339 | 2206.9880 |
| 11.566433 | 2237.4045 |
| 11.756322 | 2267.8210 |
| 11.690389 | 2298.2375 |
| 11.530298 | 2328.6540 |
| 11.376835 | 2359.0705 |
| 11.335410 | 2389.4870 |
| 11.448455 | 2419.9035 |
| 11.587860 | 2450.3200 |
| 11.422888 | 2480.7365 |
| 11.079251 | 2511.1530 |
| 10.643229 | 2541.5695 |

APPENDIX D

# DERIVATION OF DATA VECTORS FROM SPLINE FUNCTION COEFFICIENTS AND ESTIMATION OF TERM STRUCTURES USING SPLINE COEFFICIENTS

The spline representation of the estimating equation of the Fisher effect is:

(D.1)

$$_{t+m}i_{Q(x)} = {}_{t+m}A_{Q(x)} + {}_{t+m}B({}_{t+m}I^{e}_{Q(x)}) + {}_{t+m}u_{Q(x)}$$

However, in Chapter V, we have shown that the spline function, Q(x), for a cubic exponential spline with four knots and five segments is written as:

(D.2)

$$\begin{aligned} Q(x) = {} & d_1(x)(a_1 + b_{11}\ln x + b_{12}\ln x^2 + b_{13}\ln x^3) \\ & + d_2(x)(a_2 + b_{21}\ln x + b_{22}\ln x^2 + b_{23}\ln x^3) \\ & + \ldots + d_5(x)(a_5 + b_{51}\ln x + b_{52}\ln x^2 + b_{53}\ln x^3) \end{aligned}$$

where x is the time in months to maturity.

The $d_1(x), d_2(x), \ldots d_5(x)$ represent the dummy indicator values for being in a particular segment and the cubic function, $(a_m + b_{mn}\ln x + b_{mn}\ln x^2 + b_{mn}\ln x^3)$, for m=1 to 5 segments and n=1 to 3 terms, is the function estimated in each interval.

By substitution then, D.1 can be rewritten as:

(D.3)

$$i_{d_1}(x)(a_1 + b_{11}\ln x + b_{12}\ln x^2 + b_{13}\ln x^3)$$
$$+ i_{d_2}(x)(a_2 + b_{21}\ln x + b_{22}\ln x^2 + b_{23}\ln x^3)$$
$$+ \ldots + i_{d_5}(x)(a_5 + b_{51}\ln x + b_{52}\ln x^2 + b_{53}\ln x^3)$$
$$= {}_{t+m}A + {}_{t+m}B[I^e l_1(x)(m_1 + c_{11}\ln x + c_{12}\ln x^2 + c_{13}\ln x^3)$$
$$+ l_2(x)(m_2 + c_{21}\ln x + c_{22}\ln x^2 + c_{13}\ln x^3)$$
$$+ \ldots + l_5(x)(m_5 + c_{51}\ln x + c_{52}\ln x^2 + c_{53}\ln x^3)$$

The $d_1(x), d_2(x), \ldots, d_5(x)$ represent the dummy indicator values for the five spline segments for nominal interest rates and $l_1(x), l_2(x), \ldots, l_5(x)$ represent the dummy indicator values for the five spline segments for expected inflation. A cubic exponential function is estimated for each spline segment p(x) for both nominal interest rates and expected inflation. The sum of these spline segments, p(x), as demonstrated in Chapter V, is the spline function Q(x). The entire spline functions Q(x) representing the term structure of nominal interest rates and expected inflation is estimated by twenty spline coefficients each. The term structure of nominal interest rates is represented by the twenty spline coefficients:

$d_1a_1$, $d_2a_2$, . . . , $d_5a_5$; segment indicators.

$b_{11}$, $b_{21}$, . . . , $b_{51}$; term one of the cubic function.

$b_{12}$, $b_{22}$, . . . ,$b_{52}$; term two of the cubic function.

$b_{13}$, $b_{23}$, . . . ,$b_{53}$; term three of the cubic function.

The term structure of expected inflation is represented by the twenty spline coefficients:

$l_1m_1$, $l_2m_2$, . . . , $l_5m_5$; segment indicators.

$c_{11}$, $c_{21}$, . . . , $c_{51}$; term one of the cubic function.

$c_{12}$, $c_{22}$, . . . , $c_{52}$; term two of the cubic function.

$c_{13}$, $c_{12}$, . . . , $c_{53}$; term three of the cubic function.

Like coefficients are stacked, pairwise, into two cross-section time series vectors. Representations of these data vectors are shown in Figure D.1.

FIGURE D.1

STACKED CROSS-SECTION TIME SERIES
COEFFICIENTS VECTORS

| | $_{t+m}{}^{i}Q(x)$ | | $_{t+m}I^{e}Q(x)$ |
|---|---|---|---|
| Spline Segment One | $b_{11}$JA70 | first order terms | $c_{11}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{11}$NO82 | | $c_{11}$NO82 |
| | $b_{12}$JA70 | second order terms | $c_{12}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{12}$NO82 | | $c_{12}$NO82 |
| | $b_{13}$JA70 | third order terms | $c_{13}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{13}$NO82 | | $c_{13}$NO82 |
| | $d_{1}a_{1}$JA70 | indicator terms | $l_{1}m_{1}$JA70 |
| | ⋮ | | ⋮ |
| | $d_{1}a_{1}$NO82 | | $l_{1}m_{1}$NO82 |
| Spline Segment Two | $b_{21}$JA70 | first order terms | $c_{21}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{21}$NO82 | | $c_{21}$NO82 |
| | ⋮ | | ⋮ |
| | $b_{22}$JA70 | second order terms | $c_{22}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{22}$NO82 | | $c_{22}$NO82 |
| | ⋮ | | ⋮ |
| | $b_{23}$JA70 | third order terms | $c_{23}$JA70 |
| | ⋮ | | ⋮ |

| | | | |
|---|---|---|---|
| | $b_{23}$NO82 | | $c_{23}$NO82 |
| | ⋮ | | ⋮ |
| | $d_2a_2$JA70 | indicator terms | $l_2m_2$JA70 |
| | ⋮ | | ⋮ |
| | $d_2a_2$NO82 | | $l_2m_2$NO82 |
| | (⋮) | | (⋮) |
| Spline Segment Five | $b_{51}$JA70 | first order terms | $c_{51}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{51}$NO82 | | $c_{51}$NO82 |
| | ⋮ | | ⋮ |
| | $b_{52}$JA70 | second order terms | $c_{52}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{52}$NO82 | | $c_{52}$NO82 |
| | $b_{53}$JA70 | third order terms | $c_{53}$JA70 |
| | ⋮ | | ⋮ |
| | $b_{53}$NO82 | | $c_{53}$NO82 |
| | ⋮ | | ⋮ |
| | $d_5a_5$JA70 | indicator terms | $l_5m_5$JA70 |
| | ⋮ | | ⋮ |
| | $d_5a_5$NO82 | | $l_5m_5$NO82 |

Notes: JA70 . . . NO82 = January 1970 through November 1982

155 time series observations

To demonstrate how the coefficients compactly describe the entire term structure we use the spline coefficients for November 1982 to estimate the nominal interest rate and expected inflation for 2, 3, 6, 12, 24, 36, 48, 60, 72, and 84 months to maturity.

The values for the term to maturity in months, for the cubic exponential spline are shown in Table D.2.

TABLE D.2

VALUES FOR THE TERM OF MATURITY IN MONTHS, BY INTERVAL, FOR THE CUBIC EXPONENTIAL SPLINE

| Term to Maturity in Months | | lnx | $lnx^2$ | $lnx^3$ |
|---|---|---|---|---|
| interval one | 2 | .693147 | .480453 | .333025 |
| | 3 | 1.09861 | 1.20695 | 1.32597 |
| | 6 | 1.79176 | 3.21040 | 5.75227 |
| interval two | 12 | 2.48491 | 6.17476 | 15.3437 |
| interval three | 24 | 3.17805 | 10.1000 | 32.0984 |
| interval four | 36 | 3.58352 | 12.8416 | 46.0181 |
| | 48 | 3.87120 | 14.9862 | 58.0146 |
| interval five | 60 | 4.09434 | 16.7637 | 68.6362 |
| | 72 | 4.27667 | 18.2899 | 78.2197 |
| | 84 | 4.43082 | 19.6321 | 86.9864 |

The spline coefficients for nominal interest rates for November 1982 are shown in Table D.3.

TABLE D.3

SPLINE COEFFICIENTS FOR THE TERM STRUCTURE FOR NOMINAL INTEREST RATES FOR NOVEMBER 1982

| | interval one | interval two | interval three | interval four | interval five |
|---|---|---|---|---|---|
| segment indicator | .194232 | .167324 | .175571 | .157651 | .153178 |
| first order term | -.059691 | .027380 | -.003599 | -.014637 | .002235 |
| second order term | .001079 | -.024594 | -.006242 | -.000541 | -.007678 |
| third order term | .000920 | .001100 | -.004211 | .000024 | .001177 |

Similarly, the spline coefficient for expected inflation for November 1982 are shown in D.4.

## TABLE D.4

### SPLINE COEFFICIENTS FOR THE TERM STRUCTURE OF EXPECTED INFLATION FOR NOVEMBER 1982

| | interval one | interval two | interval three | interval four | interval five |
|---|---|---|---|---|---|
| segment indicator | .0281937 | .0330412 | .0283737 | .0272801 | .0402237 |
| first order term | .0133497 | -.0030730 | -.0014851 | .0052333 | .0014569 |
| second order term | -.0023279 | .0108684 | -.0008109 | -.0020236 | .0119007 |
| third order term | -.0011357 | -.0019896 | .0018700 | .0014554 | -.0022127 |

Referring back to equation (D.2), in order to compute the nominal interest rate for a term to maturity of two months, only the first interval, or piece of the spline function of nominal interest rates is relevant so, we would employ the equation,

(D.4)

$$_{t+2}i_{p1}(x) = d_1(x)(a_1 + b_{11}\ln x + b_{12}\ln x^2 + b_{13}\ln x^3).$$

(Note: The value for month one is not computed, since in month one the value is simply $d_1 a_1$)

Using the values for interval one in Table D.2 and Table D.3 the two month ahead nominal interest rate is computed as:

(D.5)

$$_{t+2}i_{p1}(x) = \underset{(a_1)}{.194232}\underset{(d_1)}{(1.0000)} - \underset{(b_{11})}{.059691}\underset{(\ln x)}{(.69315)}$$

$$+ \underset{(b_{12})}{.001079}\underset{(\ln x^2)}{(.48045)} + \underset{(b_{13})}{.000920}\underset{(\ln x^3)}{(.33303)}$$

Similarily, in order to compute the expected rate of inflation for term to maturity of two months, only the first piece of the spline function of expected inflation is relevant so, we would employ the equation

(D.6)

$$_{t+m}I^{e}{}_{p1}(x) = l_1(x)(m_1 + c_{11}\ln x + c_{12}\ln x^2 + c_{13}\ln x^3).$$

Using the values for interval one in Table D.2 and D.4 the two month ahead expected rate of inflation is computed as:

(D.7)

$$_{t+2}I^{e}{}_{p1}(x) = \underset{(m_1)}{.0281937}\underset{(l_1)}{(1.00000)} + \underset{(c_{11})}{.0133497}\underset{(\ln x)}{(.69315)}$$

$$-\underset{(c_{12})}{.0023279}\underset{(\ln x^2)}{(.48045)} - \underset{(c_{13})}{.0011357}\underset{(\ln x^3)}{(.333025)}$$

For a term to maturity of thirty-six months only the fourth segment of a spline function is relevant. The nominal interest rate estimated for spline segment four is estimated by:

(D.8)

$$_{t+36}i_{p4}(x) = d_4(x)(a_4 + b_{41}\ln x + b_{42}\ln x^2 + b_{43}\ln x^3).$$

The expected rate of inflation estimated for spline segment four is:

(D.9)

$$_{t+36}I^{e}_{p4}(x) = l_4(x)(m_4 + c_{41}\ln x + c_{42}\ln x^2 + c_{43}\ln x^3).$$

Using the values for interval four from Tables D.2 and D.3 the thirty-six month ahead nominal interest rate is computed as:

(D.10)

$$_{t+36}i_{p4}(x) = \underset{(a_4)}{.157651}\underset{(d_4)}{(1.00000)} - \underset{(b_{41})}{.014637}\underset{(\ln x)}{(3.5835)} + \underset{(b_{42})}{.000541}\underset{(\ln x^2)}{(12.8416)} + \underset{(b_{43})}{.000024}\underset{(\ln x^3)}{(46.0181)}$$

Similarily, using the values for interval four from Table D.2 and D.4 the thirty-six month ahead expected rate of inflation is computed as:

(D.11)

$$_{t+36}I^{e}_{p4}(x) = \underset{(m_4)}{.027280}\underset{(l_4)}{(1.00000)} + \underset{(c_{41})}{.0052333}\underset{(\ln x)}{(3.5835)} - \underset{(c_{42})}{.0020236}\underset{(\ln x^2)}{(12.8416)} + \underset{(c_{43})}{.0014554}\underset{(\ln x^3)}{(46.0181)}$$

Similar computation can be made for any term to maturity for nominal interest rates and expected inflation. Tables D.5 and D.6 below show the values for selected terms to maturity for nominal interest and expect inflation computed from the spline coefficients for November 1982. Table D.5 also contains the nominal interest rate for the term to maturity for the closest date estimated from the Treasury securities data. Table D.6 ontains expected inflation for the term to maturity estimated by our model of the term structure of expected inflation for November 1982.

TABLE D.5
SPLINE ESTIMATED NOMINAL INTEREST RATES AND TREASURY SECURITY INTEREST RATES FOR SELECTED TERMS TO MATURITY: NOVEMBER 1982

| Term to Maturity in Months | Spline Estimated Nominal Interest Rates | Treasury Security Interest Rates |
|---|---|---|
| 2 | .15368 | .1376 |
| 3 | .13118 | .1184 |
| 6 | .09603 | .1425 |
| 12 | .10037 | .1179 |
| 24 | .09138 | .1238 |
| 36 | .11326 | .1130 |
| 48 | .11050 | .1215 |
| 60 | .11440 | .1126 |
| 72 | .11436 | .1144 |
| 84 | .11472 | Not Available |

TABLE D.6

SPLINE ESTIMATED EXPECTED INFLATION AND MODEL ESTIMATED EXPECTED INFLATION FOR SELECTED TERMS TO MATURITY: NOVEMBER 1982

| Term to Maturity in Months | Spline Estimated Expected Inflation | Model Estimated Expected Inflation |
|---|---|---|
| 2 | .035950 | .0269 |
| 3 | .038544 | .0379 |
| 6 | .038107 | .0620 |
| 12 | .061987 | .0552 |
| 24 | .075568 | .0708 |
| 36 | .087022 | .0886 |
| 48 | .101650 | .0961 |
| 60 | .093819 | .0956 |
| 72 | .091042 | .0925 |
| 84 | .087842 | Not available |

# APPENDIX E

## RESULTS OF ALTERNATIVE REGRESSION MODELS AND ASSOCIATED STATISTICS

Four tables of regression results are presented in this appendix. The primary purpose for presenting these alternative regression results is so that the reader may examine the associated test statistics. The error sum of squares from the regression results shown here and from the regressions shown in Table 6.1 of Chapter VI are used to compute the F statistic for improvements in $R^2$ resulting from the joint significance of a subset of regression coefficients. (Pindyck and Rubinfeld, 1981, pp 117-119).

## TABLE E.1

### OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES, NOMINAL INTEREST RATES AND EXPECTED INFLATION

Dependent Variable $_{t+m}i_{Q(x)}$

| Independent Variable | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | 0.3690702E-02 | 0.6703736E-03 | 5.505442 |
| $_{t+m}I^{e}_{Q(x)}$ | 1.227559 | 0.1497072E-01 | 81.99734 |

$_{t+m}i_{Q(x)}$ = nominal interest rates, splined

$_{t+m}I^{e}_{Q(x)}$ = expected rate of inflation, splined

degrees of freedom = 3098

$\bar{R}^2$ = .68

Error Sum of Squares (ESS) = .03349

TABLE E.2

OLS REGRESSION, POOLED CROSS-SECTION
TIME SERIES, NOMINAL INTEREST RATES,
EXPECTED INFLATION, AND TREND

Dependent Variable $_{t+m}i_{Q(x)}$

| Independent Variable | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.2118257E-02 | 0.1218167E-02 | -1.738889 |
| $_{t+m}I^{e}_{Q(x)}$ | 1.219278 | 0.1496597E-01 | 81.46997 |
| t | 0.7657423E-04 | 0.1343701E-04 | 5.698756 |

$_{t+m}i_{Q(x)}$ = nominal interest rates, splined

$_{t+m}I^{e}_{Q(x)}$ = expected rate of inflation, splined

t = trend

degree of freedom = 3097

$\bar{R}^2$ = .69

Error Sum of Squares (ESS) = .03332

TABLE E.3

OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES, NOMINAL INTEREST RATES, EXPECTED INFLATION, TREND, AND MONTHLY INTERCEPT DUMMY VARIABLES

Dependent Variable $_{t+m}i_{Q(x)}$

| Independent Variable | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -.2173547E-02 | 0.1218339E-02 | -1.784024 |
| $_{t+m}I^{e}_{Q(x)}$ | 1.221273 | 0.1495802E-01 | 81.64670 |
| t | 0.7676999E-04 | 0.1344622E-04 | 5.709410 |
| $DUM_{FE}$ | 0.9978346E-04 | 0.1976554E-02 | 0.5048355E-01 |
| $DUM_{MR}$ | -0.3013530E-02 | 0.1976241E-02 | -1.524880 |
| $DUM_{AP}$ | 0.1199570E-02 | 0.1975856E-02 | 0.6071141 |
| $DUM_{MY}$ | -0.6810335E-03 | 0.1975716E-02 | -0.3447022 |
| $DUM_{JU}$ | -0.5280195E-02 | 0.1975935E-02 | -2.672251 |
| $DUM_{JL}$ | -0.6349749E-03 | 0.1976579E-02 | -0.3212494 |
| $DUM_{AU}$ | -0.1477989E-03 | 0.1975861E-02 | -0.7480229E-01 |
| $DUM_{SE}$ | -0.1882871E-02 | 0.1976073E-02 | -0.9528346 |
| $DUM_{OC}$ | 0.1508850E-02 | 0.1976394E-02 | 0.7634359 |
| $DUM_{NO}$ | 0.2938053E-02 | 0.1977247E-02 | 1.485932 |
| $DUM_{DE}$ | -0.8489417E-04 | 0.2049260E-02 | -0.4142674E-01 |

$_{t+m}i_{Q(x)}$ = nominal interest rates, splined
$_{t+m}I^{e}_{Q(x)}$ = expected rate of inflation, splined
t = trend
$DUM_{FE}$, $DUM_{MR}$, . . . $DUM_{DC}$ = intercept dummy variables February through December

degree of freedom = 3086
$\bar{R}^2$ = .69
Error Sum of Squares (ESS) = .033226

TABLE E.4

OLS REGRESSION, POOLED CROSS-SECTION TIME SERIES, NOMINAL INTEREST RATES, EXPECTED INFLATION TREND, MONTHLY SLOPE AND INTERCEPT DUMMY VARIABLES

Dependent Variable $_{t+m}i_{Q(x)}$

| Independent Variable | Coefficient | Stand.Error | T-Statistic |
|---|---|---|---|
| Constant | -0.2510733E-02 | 0.1182350E-02 | -2.123510 |
| $_{t+m}I^{e}_{Q(x)}$ | 1.251397 | 0.1489782E-01 | 83.99868 |
| t | 0.7566855E-04 | 0.1304392E-04 | 5.801060 |
| $DUM_{FE}$ | 0.1328099E-02 | 0.2124183E-02 | 0.6252282 |
| $DUM_{MR}$ | 0.1779614E-02 | 0.2116467E-02 | 0.8408421 |
| $DUM_{AP}$ | 0.5329888E-02 | 0.2109647E-02 | 2.526436 |
| $DUM_{MY}$ | 0.8401523E-03 | 0.2145402E-02 | 0.3916060 |
| $DUM_{JU}$ | -0.1520527E-02 | 0.2150448E-02 | -0.7070746 |
| $DUM_{JL}$ | -0.3763617E-02 | 0.2151277E-02 | -1.749481 |
| $DUM_{AU}$ | -0.1061035E-02 | 0.2157339E-02 | -0.4918258 |
| $DUM_{SE}$ | 0.2491123E-02 | 0.2126026E-02 | 1.171727 |
| $DUM_{OC}$ | -0.2715754E-04 | 0.2130485E-02 | -0.1274712E-01 |
| $DUM_{NO}$ | -0.3781942E-03 | 0.2161960E-02 | -0.1749312 |
| $DUM_{DC}$ | -0.4096506E-03 | 0.2226986E-02 | -0.1839484 |
| $SDUM_{FE}$ | -0.7582423E-01 | 0.5123943E-01 | -1.479802 |
| $SDUM_{MR}$ | -0.2360865 | 0.4273719E-01 | -5.524147 |
| $SDUM_{AP}$ | -0.2153970 | 0.4426468E-01 | -4.866114 |
| $SDUM_{MY}$ | -0.854680E-01 | 0.4837421E-01 | -1.758102 |
| $SDUM_{JU}$ | -0.1820736 | 0.4501554E-01 | -4.044683 |
| $SDUM_{JL}$ | 0.1192979 | 0.4203664E-01 | 2.837950 |
| $SDUM_{AU}$ | 0.3879062E-01 | 0.5105189E-01 | 0.7598273 |
| $SDUM_{SE}$ | -0.2284069 | 0.4633958E-01 | -4.928980 |
| $SDUM_{OC}$ | 0.6614906E-01 | 0.4584329E-01 | 1.442939 |
| $SDUM_{NO}$ | 0.1852485 | 0.5698021E-01 | 3.251103 |
| $SDUM_{DC}$ | 0.1142995E-01 | 0.5555031E-01 | 0.2057585 |

$_{t+m}i_{Q(x)}$ = nominal interest rates, splined

$_{t+m}I^{e}_{Q(x)}$ = expected rate of inflation, splined

$t$ = trend

$DUM_{FE}$, $DUM_{MR}$, . . . , $DUM_{DC}$ = intercept dummy variables February through December

$SDUM_{FE}$, $SDUM_{MR}$, . . . , $SDUM_{DC}$ = slope dummy variables February through December

Degrees of freedom = 3075

Error Sum of Squares (ESS) = .032254

We used an F test to determine the joint significance of the sets of dummy variables. (Pindyck and Rubinfeld, 1981, pp 117-119) Testing the null hypothesis that the set of intercept dummy variables all have zero coefficients (Table E.3) when added to the simple regression of the Fisher effect and trend (Table E.2), the value of F was .4486. Since this is less than the critical value of 1.75 we accept the null hypothesis that the set of intercept dummy variables all have coefficients of zero. This is equivalent to accepting the null hypothesis that there was no significant improvement in the $R^2$ for the regression shown in Table E.2 when the intercept dummies are added to the regression shown in Table E.3. Testing the null hypothesis that the set of intercept dummy variables all have zero coefficients (Table E.4) when added to the regression of nominal interest rates, splined, on expected inflation, trend and slope dummy variables (Table 6.2), the value of F was 0.9. Since this is less than the critical value of 1.46 we accept the null hypothesis that the set of intercept dummy variables all have coefficients of zero. There is no significant improvement in $R^2$ from the addition of the intercept dummy variables to the regression model which includes slope dummy variables.

Testing the null hypothesis that the set of slope dummy variables (Table 6.1) all have zero coefficients when added to the simple regression of the Fisher effect and trend (Table E.2), the value of F was 9.05. Since this is greater than the critical value of 1.75 we reject the null hypothesis that the set of slope dummy variables all have zero coefficients. We accept the alternative hypothesis which is equivalent to accepting the hypothesis that there is a significant improvement in $R^2$ resulting from the addition of slope dummy variables.

## APPENDIX F

## USING THE REGRESSION COEFFICIENTS TO ESTIMATE NOMINAL INTEREST RATES FOR TERMS TO MATURITY FROM EXPECTED INFLATION FOR TERMS TO MATURITY

In this appendix we will briefly demonstrate how one can use the various estimates of the regression parameters to compute the nominal interest rate for a specific term to maturity given the expected rate of inflation for the same term to maturity. The reader will find tables of values for selected points on the term structure for the cubic function of term to maturity in Appendix D. Appendix D also contains tables of spline coefficients for November to maturity 1982 for nominal interest rates and expected inflation.

Regression model one, Table 6.1, has a single intercept value, A, and a single slope value, B, estimating the Fisher effect across the entire term structure, -.0025 and 1.25 respectively. There are additional coefficients for trend, .000076, and a slope dummy for the month. Following the example in Appendix D we will use the month of November 1982 which has the spline coefficient value of .1821 for the slope dummy variable in regression model one. To estimate the nominal interest rates for two and thirty-six months ahead, as an example, we use the two and thirty-six month ahead estimates of expected inflation, ${}_{t+2}I^{e}_{Q(x)}$ = .03595, and ${}_{t+36}I^{e}_{Q(x)}$ = .087022. These can also be found in Appendix D. The nominal interest rate for two months ahead:

(F.1)

$$_{t+2}i_{Q(x)} = A + {}_{t+2}B({}_{t+2}I^{e}_{Q(x)}) + C(t) + D_{10}SDUM_{NO}$$

$$_{t+2}i_{Q(x)} = \underset{(A)}{.0025} + \underset{(B)}{1.25}({}_{t+2}I^{e}_{Q(x)}) + \underset{(C)}{.000076}(t) + \underset{(D_{10})}{.1825}\ SDUM_{NO}$$

$$_{t+2}i_{Q(x)} = .061$$

The actual Treasury security rate for $_{t+2}I^{e}_{Q(x)}$ was .1376.

The nominal interest rate for thirty-six months ahead,

(F.2)

$$_{t+36}i_{Q(x)} = A + {}_{t+36}B({}_{t+36}I^{e}_{Q(x)}) + C(t) + D_{10}SDUM_{NO}$$

$$_{t+36}i_{Q(x)} = \underset{(A)}{.0025} + \underset{(B)}{1.25}({}_{t+36}I^{e}_{Q(x)}) + \underset{(C)}{.000076}(t) + \underset{(D_{10})}{.1825}\ SDUM_{NO}$$

$$_{t+36}i_{Q(x)} = .135$$

The actual Treasury security rate for $_{t+36}i_{Q(x)}$ was .1130.

Regression model two, shown in Tables 6.2 through 6.6, has a different intercept and slope for the Fisher effect for each interval. Therefore, in order to compute the nominal interest rate for two months ahead we use the values for interval one. To compute the nominal interest rate for thirty-six months ahead we use the values for interval four. The values for trend and the monthly slope dummies also differ as do the values for expected inflation. Equations (F.3) and (F.4) show the estimation of $_{t+2}i_{P_1(x)}$ and $_{t+36}i_{P_4(x)}$.

(F.3)

$$_{t+2}i_{p_1(x)} = A + {}_{t+2}B_1({}_{t+m}I^{e}_{p_1(x)}) + C_1(t) + D_{10}SDUM_{NO}$$

$$_{t+2}i_{p_1(x)} = \underset{(A)}{-.0054} + \underset{(B_1)}{1.37}({}_{t+2}I^{e}_{p_1(x)}) + \underset{(C_1)}{.00005}(t)$$

$$+ \underset{(D_{10})}{.1903}\ SDUM_{NO}$$

$$_{t+2}i_{p_1(x)} = .0594$$

given:

$$_{t+2}I^{e}_{p_1(x)} = .03595$$

$$t = 155$$

$$SDUM_{NO} = .03595$$

(F.4)

$$_{t+36}i_{p_4(x)} = A + ({}_{t+36}B_4(I^{e}_{p_4(x)}) + C(t) + D_{10}\ SDUM_{NO}$$

$$_{t+36}i_{p_4(x)} = \underset{(A)}{-.0025} + \underset{(B)}{1.17}({}_{t+36}I^{e}_{p_4(x)}) + \underset{(C_4)}{.000069}(t)$$

$$+ \underset{(D_{10})}{.1681}\ SDUM_{NO}$$

$$_{t+36}i_{p_4(x)} = .116$$

given:

$$_{t+36}I^{e}_{p_4(x)} = .087$$

$$t = 155$$

$$SDUM_{NO} = .087$$

Comparing the results of regression model one and two, there is little difference in the estimates for $_{t+2}i$. Both are approximately 6% when the actual nominal rate for t+2 was approximately 14%. In both cases the short term nominal interest rate is underestimated by the model. However, the estimate for $_{t+36}i$ for model two is .116 when the actual was .113. The estimate for $_{t+36}i$ for model one is .135.

Joint estimation of the five segments, like the OLS regressions on the five segments, results in a different intercept and slope for each interval. Therefore, as in the example above in equations (F.3) and (F.4), the nominal interest rate for two months ahead has to be estimated with the values for spline segment one and the nominal interest rate for thirty-six months ahead has to be estimated with the values for spline segment four. These can be found in Table 6.8. The results for $_{t+2}i_{P_1(x)}$ and $_{t+36}i_{P_4(x)}$ are shown in equations (F.5) and (F.6), respectively.

(F.5)

$$_{t+2}i_{P_1(x)} = A + {}_{t+2}B_1(I^e_{P_1(x)})$$

$$_{t+2}i_{P_1(x)} = \underset{(A)}{.0102} + \underset{(B_1)}{.7770}(I^e_{P_1(x)})$$

$$_{t+2}i_{P_1(x)} = .038$$

(F.6)

$$_{t+36}i_{P_4(x)} = A + {}_{t+36}B_4(I^e_{P_4(x)})$$

$$_{t+36}i_{P_4(x)} = .0115 + .7332(I^e_{P_4(x)})$$

$$_{t+36}i_{P_4(x)} = .075$$

Both of the estimates are below the actual Treasury security nominal interest rate for the designated term to maturity.

We also applied the Theil joint estimation procedure to estimate the Fisher effect for each spline coefficient. Since each term structure is represented by twenty spline coefficients, there were twenty equations in the entire system. Five of these represent the five segment indicator dummy variables, and 15 represent the linear, quadratic, and cubic terms of each of the five segments. These results are presented in Table F.1.

TABLE F.1

POOLED CROSS-SECTION TIME SERIES JOINT ESTIMATION OF THE FISHER EFFECT FOR THE TWENTY SPLINE COEFFICIENTS

| | interval one | interval two | interval three | interval four | interval five |
|---|---|---|---|---|---|
| Segment Dummy: | d1 | d2 | d3 | d4 | d5 |
| slope | .2016 | .1942 | .2334 | .2353 | .3134 |
| intercept | .1173 | .1041 | .1044 | .0950 | .0858 |
| First Order Term: | | | | | |
| slope | -.0925 | -.02896 | .1603 | -.0312 | -.01373 |
| intercept | -.0318 | .014568 | -.0018 | -.0086 | .00156 |
| Second Order Term: | | | | | |
| slope | .01828 | -.09265 | -.1026 | .04136 | -.01626 |
| intercept | .00053 | -.012968 | -.0033 | .00031 | -.00427 |
| Third Order Term: | | | | | |
| slope | .00365* | .00764* | -.0694 | .0360 | 0 |
| intercept | .000578 | .0058 | -.0025 | .00007 | 0 |
| Chi square = 2305.25 for eighteen degrees of freedom | | | | | |

Note: The third-order cubic spline equation in segment five had to be dropped from the system because of multicolinearity. A * denotes the coefficients which were not statistically significant.

Again, the chi-square test resulted in rejecting the null hypotheses. Estimates of the Fisher effect based on joint estimation across the term structure are statistically better than the estimates of the Fisher effect based on the twenty independent OLS regressions. These coefficients cannot be easily interpreted to directly test the Fisher hypothesis. The reason for this difficulty is that the coefficients depend indirectly on the term to maturity and must be scaled by the cubic exponential function of term to maturity. However, we demonstrate below how these coefficients can be used to estimate the nominal interest rate from expected inflation.

The computation of the nominal interest rate using the coefficients from joint estimation of the 20 spline coefficients is less direct than computing the nominal interest rate in any of the other cases shown here. The reason is that the regression coefficients on the 20 spline coefficients are specifically about both spline coefficients and their respective terms to maturity. In order to compute the nominal interest rate for a specific term to maturity we must use the cubic values for term to maturity in months (see Appendix D) and the spline coefficient unique to each term in the relevant spline segment. Equations (F.7) and (F.8) show these computations for two and 36 months ahead nominal interest rates.

(F.7)

$$
\begin{aligned}
{}_{t+2}i_{p_1(x)} = {} & [A_1 + B_1 I^e_{l_1(x)}) * m_1)] \\
& + [A_2 + B_2 I^e_{l_1(x)}) \ln x] \\
& \qquad (C_{11}) \\
& + [A_3 + B_3 I^e_{l_1(x)}) \ln x^2] \\
& \qquad (C_{12}) \\
& + [A_4 + B_4 I^e_{l_1(x)}) \ln x^2] \\
& \qquad (C_{13})
\end{aligned}
$$

$$
\begin{aligned}
{}_{t+2}i p_1(x) &= [\underset{(A_1)}{.1173} + \underset{(B_1)}{.2016}(\underset{(l_1 m_1)}{.0281937})] \\
&+ [\underset{(A_2)}{-.0318} - \underset{(B_2)}{.0925}(\underset{(C_{11})}{.01335})\underset{(\ln x)}{1.693147}] \\
&+ [\underset{(A_3)}{.00053} + \underset{(B_3)}{.01828}(\underset{(C_{12})}{-.00233})(\underset{(\ln x^2)}{.48045})] \\
&+ [\underset{(A_4)}{.000578} + \underset{(B_4)}{.00366}(\underset{(C_{13})}{-.0011})(\underset{(\ln x^3)}{.333025})] \\
{}_{t+2}i p_1(x) &= .0904
\end{aligned}
$$

where $A_1$ through $A_4$ are the four joint regression intercepts, $B_1$ through $B_4$ are the four joint regression slopes, and $I^e{}_{l1(x)}$ are the three spline coefficients on expected inflation: $C_{11}$, $C_{12}$, and $C_{13}$, respectively. The reader may be helped with the notation by referring to Table D.1 in Appendix D.

(F.8)

$$
\begin{aligned}
{}_{t+36}i p_4(x) &= [A_1 + B_1(I^e{}_{l4(x)}) * m_4)] \\
&+ [A_2 + B_2\underset{(C_{41})}{(I^e{}_{l4(x)})} \ln x] \\
&+ [A_3 + B_3\underset{(C_{42})}{(I^e{}_{l4(x)})} \ln x^2] \\
&+ [A_4 + B_4\underset{(C_{43})}{(I^e{}_{l4(x)})} \ln x^2]
\end{aligned}
$$

$$
\begin{aligned}
{}_{t+36}i p_4(x) &= [\underset{(A_1)}{.0950} + \underset{(B_1)}{.2353}(\underset{(l_4 m_4)}{.02738})] \\
&+ [\underset{(A_2)}{.0086} - \underset{(B_2)}{.0312}(\underset{(C_{41})}{-.01464})(\underset{(\ln x)}{3.584})] \\
&+ [\underset{(A_3)}{.00031} + \underset{(B_3)}{.04135}(\underset{(C_{42})}{.00541}(\underset{(\ln x^2)}{12.8416})]
\end{aligned}
$$

$$+ [.00007 + .03601(.000025)(46.0181)]$$

$$(A_4) \quad (B_4) \quad (C_{43}) \quad (\ln x^3)$$

$${}_{t+36}i_{p_4}(x) = .0973$$

where the notation is the same as for equation (F.7), except that the spline coefficients are for interval four.

Both of these are underestimates of the actual treasury security rate for the specified term to maturity. However, both are better estimates than those produced by regression coefficients joint estimation for the segments.

No extensive comment can be made here regarding the overall relative performance of each of the estimating techniques. The above serves only as a simple illustration of how each of the estimating equations may be applied. However, this simple set of examples does suggest that there should be further study of the possible advantages of allowing the Fisher effect to vary across the term structure.

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