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Estimation of the Parameters of the Generalized Exponential Distribution in the Presence of One Outlier Generated from Uniform Distribution

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Abstract

Gupta and Kundu (1999) defined the cumulative distribution function of the generalized exponential (GE). It has many properties that are quite similar to those of the gamma distribution. This paper deals with the estimation of the parameters of the generalized exponential distribution in the presence of one outlier generated from uniform distribution. The maximum likelihood, moment and mixture of the estimators are derived. These estimators are compared empirically when all of the parameters are unknown. Their bias and mean square error (MSE) are investigated with help of numerical techniques.

Keywords: Generalized exponential distribution, uniform distribution, outlier, maximum likelihood, moment and mixture estimators

1 Introduction

Recently the two-parameter generalized exponential (GE) distribution has been proposed by the authors. It has been studied extensively by Gupta and Kundu (1999, 2001a, 2001b, 2002, 2003a,2003b, 2004), Raqab (2002), Raqab and Ahsanullah (2001) and Zheng (2002). Note that the generalized exponential distribution is a sub-model of the exponentiated weibull distribution introduced by Mudholkar and Srivastava (1993) and later studied by Mudholkar, Srivastara and Freimer (1995) and Mudholkar and Huston (1996). Dixit, Moore and Barnett (1996), assumed that a set of random variables $(X_1, X_2, ..., X_n)$ represent the distance of an infected sampled plant from a plant from a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and are distributed

according to the exponential distribution. The other observations out of n random variables (say k) are present because aphids which are know to be carriers of barley yellow mosaic dwarf virus (BYMDV) have passed the virus into the plants when the aphids feed on the sap. Dixit and Nasiri (2001) considered estimation of the parameters of the exponential distribution in the presence of outliers generated from uniform distribution. In this paper, we consider generalized exponential distribution in presence of one outlier generated from uniform distribution.
The two-parameter GE distribution has the following density function

$$f(x;\alpha) = \alpha e^{-x}(1-e^{-x})^{\alpha-1}, \quad x>0, \alpha>0, \tag{1}$$

and the distribution function

$$F(x;\alpha) = (1-e^{-x})^{\alpha}, \quad x>0, \alpha>0. \tag{2}$$

Let the random variables $(X_1, X_2, ..., X_n)$ are such that $n-1$ of them are distributed with pdf $f(x,\alpha)$

$$f(x;\alpha) = \alpha e^{-x}(1-e^{-x})^{\alpha-1}, \quad x>0, \alpha>0, \tag{3}$$

and one random variable are distributed with pdf $g(x;\alpha,\theta)$

$$g(x;\alpha,\theta) = \frac{1}{\alpha\theta}\mathbf{I}_{(0,\alpha\theta)}(x), \quad \alpha>0, \theta>0, \tag{4}$$

where $\mathbf{I}$ is an indicator function.

Therefore the joint distribution of $(X_1, X_2, ..., X_n)$ is

$$f(x_1,x_2,...,x_n;\alpha,\theta) = \frac{(n-1)!}{n!}\prod_{i=1}^{n} f(x_i;\alpha)\sum_{A_1=1}^{n}\frac{g(x_{A_1})}{f(x_{A_1})}, \tag{5}$$

For $f(x;\alpha)$ and $g(x;\alpha,\theta)$ are given in (3) and (4), $f(x_1, x_2, ..., x_n)$ is

$$\begin{aligned} f(x_1,x_2,...,x_n;\alpha,\theta) &= \frac{(n-1)!}{n!}\alpha^n e^{-\sum_{i=1}^n x_i}\prod_{i=1}^{n}(1-e^{-x_i})^{\alpha-1}\sum_{A_1=1}^{n}\frac{\frac{1}{\alpha\theta}\mathbf{I}_{(0,\alpha\theta)}(x_{A_1})}{\alpha e^{-x_{A_1}}(1-e^{-x_{A_1}})^{\alpha-1}} \\ &= \frac{(n-1)!}{n!}\frac{\alpha^{n-1}e^{-n\bar{x}}}{(\alpha\theta)}\prod_{i=1}^{n}(1-e^{-x_i})^{\alpha-1}\sum_{A_1=1}^{n}\frac{\mathbf{I}_{(0,\alpha\theta)}(x_{A_1})}{e^{-x_{A_1}}(1-e^{-x_{A_1}})^{\alpha-1}}. \end{aligned} \tag{6}$$

It is easy to show that the marginal distribution of X is

$$f(x;\alpha,\theta)=\frac{1}{n}\frac{1}{\alpha\theta}\mathbf{I}_{(0,\alpha\theta)}(x)+\frac{n-1}{n}\alpha e^{-x}(1-e^{-x})^{\alpha-1},\quad x>0,\alpha>0,\theta>0. \quad (7)$$

The paper is organized as follows:
Section 2, 3 and 4 discusses the methods of moment (MM), maximum likelihood (ML) and mixture estimators (Mix). The bias and MSE will be investigated with help of numerical technique in section 5.

2 Method of moment

The r^{th} moments of X may be determined direct or using the moment of generating function. Here we consider the moment generating function.

$$\begin{aligned}
M(t) &= E(e^{tX})=\int_0^\infty e^{tx}f(x;\alpha,\theta)dx\\
&= \int_0^\infty e^{tx}[\frac{1}{n}\frac{1}{\alpha\theta}\mathbf{I}_{(0,\alpha\theta)}(x)+\frac{n-1}{n}\alpha e^{-x}(1-e^{-x})\alpha^{\alpha-1}]dx\\
&= \frac{1}{n}\int_0^{\alpha\theta}\frac{e^{tx}}{\alpha\theta}dx+\frac{n-1}{n}\int_0^\infty \alpha e^{(t-1)x}(1-e^{-x})^{\alpha-1}dx\\
&= \frac{1}{n}\frac{e^{\alpha\theta t}-1}{\alpha\theta t}+\frac{n-1}{n}\int_0^\infty \alpha e^{(t-1)x}(1-e^{-x})^{\alpha-1}dx.
\end{aligned}$$

Let $y=e^{-x}$ then $x=-\ln(y)$, $dx=\frac{-dy}{y}$, and $M(t)$ is given by

$$\begin{aligned}
M(t) &= \frac{1}{n}\frac{e^{\alpha\theta t}-1}{\alpha\theta t}+\frac{n-1}{n}\int_0^\infty \alpha y^{-t}(1-y)^{\alpha-1}dx\\
&= \frac{1}{n}\frac{e^{\alpha\theta t}-1}{\alpha\theta t}+\frac{n-1}{n}\frac{\Gamma(\alpha+1)\Gamma(1-t)}{\Gamma(\alpha-t+1)}, \quad (8)
\end{aligned}$$

where $\Gamma(.)$ is the gamma function. Differentiating $M(t)$ and evaluating at $t=0$, we get the $E(X)$ and $E(X^2)$ as

$$\begin{aligned}
M'(t) &= \frac{1}{n\alpha\theta}[\frac{\alpha\theta e^{\alpha\theta t}}{t}-\frac{e^{\alpha\theta t}-1}{t^2}]\\
&+ \frac{n-1}{n}[\frac{\Gamma(\alpha+1)\Gamma(1-t)\Gamma'(\alpha-t)}{\Gamma^2(\alpha-t+1)}-\frac{\Gamma(\alpha+1)\Gamma'(1-t)}{\Gamma(\alpha-t+1)}], \quad (9)
\end{aligned}$$

where $\Gamma'(.)$ is the derivative of the gamma function.

$$\begin{aligned}
M(t)'' &= \frac{1}{n\alpha\theta}[\frac{(\alpha\theta)^2e^{\alpha\theta t}}{t}-\frac{2\alpha\theta e^{\alpha\theta t}}{t^2}+\frac{2(e^{\alpha\theta t}-1)}{t^3}]+\frac{n-1}{n}[\frac{\Gamma(1-t)\Gamma'(\alpha-t+1)}{\Gamma^2(\alpha-t+1)}\\
&- \frac{\Gamma(\alpha+1)(\Gamma'(1-t)\Gamma'(\alpha-t+1)+\Gamma(1-t)\Gamma''(\alpha-t+1))}{\Gamma^2(\alpha-t+1)}]. \quad (10)
\end{aligned}$$

where $\Gamma''(.)$ is derivative of $\Gamma'(.)$.

From the equations (9) and (10), $E(X)$ and $E(X^2)$ are given by

$$E(X) = M'(0) = \frac{1}{n}\frac{\alpha\theta}{2} + \frac{n-1}{n}[\frac{\Gamma'(\alpha+1)}{\Gamma(\alpha+1)} - \frac{\Gamma'(1)}{\Gamma(1)}], \tag{11}$$

and

$$E(X^2) = M''(0) = \frac{1}{n}\frac{(\alpha\theta)^2}{3} + \frac{n-1}{n}[\frac{2\Gamma'(\alpha+1)}{\Gamma^2(\alpha+1)} - \frac{\Gamma'(1)\Gamma'(\alpha+1)+\Gamma''(\alpha+1)}{\Gamma(\alpha+1)}]. \tag{12}$$

Let A and B be a function of α as

$$A = \frac{n-1}{n}[\frac{\Gamma'(\alpha+1)}{\Gamma(\alpha+1)} - \frac{\Gamma'(1)}{\Gamma(1)}], \tag{13}$$

and

$$B = \frac{n-1}{n}[\frac{2\Gamma'(\alpha+1)}{\Gamma^2(\alpha+1)} - \frac{\Gamma'(1)\Gamma'(\alpha+1)+\Gamma''(\alpha+1)}{\Gamma(\alpha+1)}]. \tag{14}$$

Then

$$m_1 = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\frac{\hat{\alpha}\hat{\theta}}{2} + A, \tag{15}$$

and

$$m_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{1}{n}\frac{(\hat{\alpha}\hat{\theta})^2}{3} + B. \tag{16}$$

Equations (15) and (16) imply that

$$m_2 - B - \frac{4n}{3}(m_1 - A)^2 = 0 \tag{17}$$

To estimate of α we can solve (17) by Newton - Raphson method. Hence solution of the equation is

$$\alpha_{i+1} = \alpha_i - \frac{g(\alpha_i)}{g'(\alpha_i)} \tag{18}$$

where

$$\begin{aligned} g(\alpha_i) &= m_2 - B(\alpha_i) - \frac{4n}{3}(m_1 - A(\alpha_i))^2 \\ g'(\alpha_i) &= -B'(\alpha_i) + \frac{8n}{3}A'(\alpha_i)(m_1 - A(\alpha_i)) \end{aligned}$$

where , $A'(\alpha_i)$ and $B'(\alpha_i)$ are

$$A'(\alpha_i) = \frac{n-1}{n}[\frac{\Gamma''(\alpha_i+1)}{\Gamma(\alpha_i+1)} - \frac{\Gamma'^2(\alpha_i+1)}{\Gamma^2(\alpha_i+1)}]$$

$$\begin{aligned} B'(\alpha_i) &= \frac{n-1}{n}[\frac{2\Gamma''(\alpha_i+1)}{\Gamma^2(\alpha_i+1)} - \frac{4\Gamma'^2(\alpha_i+1)}{\Gamma^3(\alpha_i+1)}] \\ &- \frac{n-1}{n}[\frac{\Gamma'(1)\Gamma''(\alpha_i+1)+\Gamma'''(\alpha_i+1)}{\Gamma(\alpha_i+1)} - \frac{\Gamma'(1)\Gamma'^2(\alpha_i+1)+\Gamma'(\alpha_i+1)\Gamma''(\alpha_i+1)}{\Gamma^2(\alpha_i+1)}] \end{aligned}$$

Hence $\hat{\theta}$ can be obtained as

$$\hat{\theta} = \frac{1}{\hat{\alpha}}\frac{3(m_2-\hat{B})}{2(m_1-\hat{A})}. \tag{19}$$

3 Method of maximum likelihood

Proceeding with the method of maximum likelihood, the likelihood function for a sample of size n, $(X_1, X_2, ..., X_n)$, is given by

$$\begin{aligned} L(\alpha,\theta) &= \frac{1!(n-1)!}{n!}\alpha^n e^{-\sum_{i=1}^n x_i}\prod_{i=1}^n(1-e^{-x_i})^{\alpha-1}\sum_{A_1=1}^n\prod_{j=1}^1\frac{\frac{1}{\alpha\theta}\mathbf{I}_{(0,\alpha\theta)}(x)}{\alpha e^{-x_{A_1}}(1-e^{-x_{A_1}})^{\alpha-1}} \\ &= \frac{1!(n-1)!}{n!}\frac{\alpha^{n-1}e^{-n\bar{x}}}{(\alpha\theta)}\prod_{i=1}^n(1-e^{-x_i})^{\alpha-1}\sum_{A_1=1}^n\frac{\mathbf{I}_{(0,\alpha\theta)}(x_{A_1})}{e^{-x_{A_1}}(1-e^{-x_{A_1}})^{\alpha-1}}. \end{aligned} \tag{20}$$

Then

$$L(\alpha,\theta) \simeq \frac{\alpha^{n-1}e^{-n\bar{x}}}{x_{(n)}}(1-e^{-x_i})^{\alpha-1}\sum_{A=1}^n\frac{1}{e^{-x_{A_1}}(1-e^{-x_{A_1}})^{\alpha-1}}, \tag{21}$$

where
$\hat{\alpha\theta} = x_{(n)} = max(X_1, X_2, ..., X_n)$.
Hence

$$\hat{\theta} = \frac{x_n}{\alpha} \tag{22}$$

To estimate α, we consider $\ln(L(\alpha,\theta))$ as

$$\begin{aligned} \ln(L(\alpha,\theta)) &\simeq (n-1)\ln(\alpha) - n\bar{x} - \ln(x_{(n)}) + (\alpha-1)\sum_{i=1}^n\ln(1-e^{-x_i}) \\ &+ \ln(\sum_{A_1=1}^n\frac{1}{e^{-x_{A_1}}(1-e^{-x_{A_1}})^{\alpha-1}}). \end{aligned} \tag{23}$$

Taking the derivative with respect to α and equating to 0, we obtain the normal equation as

$$\begin{aligned} \frac{dlnl(\alpha)}{d\alpha} &\simeq \frac{n-1}{\alpha}+\sum_{i=1}^{n}\ln(1-e^{-x_i}) \\ &- \frac{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}\ln(1-e^{-x_{A_1}})}{\sum_{A=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}} = 0, \end{aligned} \tag{24}$$

Since $\frac{dlnl(\alpha)}{d\alpha}=0$, hence

$$\frac{n-1}{\alpha}+\sum_{i=1}^{n}\ln(1-e^{-x_i}) \simeq \frac{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}\ln(1-e^{-x_{A_1}})}{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}}. \tag{25}$$

Here, we need to use either the scoring algorithm or the Newton Raphson method to solve the non-linear equation.

Here, We solve (25) by Newton - Raphson method. Hence solution of the equation is

$$\alpha_{i+1}=\alpha_i-\frac{g(\alpha_i)}{g'(\alpha_i)} \tag{26}$$

where,

$$g(\alpha) \simeq \frac{n-1}{\alpha}+\sum_{i=1}^{n}\ln(1-e^{-x_i}) \tag{27}$$

$$- \frac{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}\ln(1-e^{-x_{A_1}})}{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}}. \tag{28}$$

$$g'(\alpha) \simeq -\frac{n-1}{\alpha^2} \tag{29}$$

$$+ \frac{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}(\ln(1-e^{-x_{A_1}}))^2}{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}} \tag{30}$$

$$- [\frac{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}\ln(1-e^{-x_{A_1}})}{\sum_{A_1=1}^{n} e^{x_{A_1}}(1-e^{-x_{A_1}})^{1-\alpha}}]^2 \tag{31}$$

4 Mixture of method of moment and maximum likelihood

Read (1981) proposed the methods, which avoid the difficulty of complicated equations. According to Read (1981), replacement of some, but not all of the equations in the system of likelihood may make it more manageable.

One sees from (24) the ML estimator for the parameter α of the GE distribution with presence of outlier can not be obtained in closed forms and therefore that is little point in considering the method any further. So from (19)

$$\hat{\theta} = \frac{1}{\hat{\alpha}} \frac{3(m_2 - B)}{2(m_1 - A)}, \tag{32}$$

and

$$\hat{\alpha} = \frac{x_{(n)}}{\hat{\theta}}. \tag{33}$$

So we can easily find the mixture estimators of α and θ.

5 Numerical experiments and discussions

In order to have some idea about Bias and Mean Square Error (MSE) of methods of moment, MLE and mixture, we perform sampling experiments using a R software. The result of bias and MSE of the estimators are given in Tables 1 to 12, for $\alpha = 0.5(0.5)2$, and $\theta = 1, 3, 5, 7$ respectively.

According to Tables 1 , 3 , 5, 7 , 9 and 11, the bias of mixture are less than the others. Table 2 , 4 , 6, 8 and 12 are shown that the MSE of mixture estimators are less than the MSE of the other estimators for all values of n . So the mixture estimators are more efficient than the others.

Table 1

Bias for $\alpha = 0.5$ and $\theta = 5$.

	MME	MLE	Mix	MME	MLE	Mix
n	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$
3	0.1730	1.0905	0.3162	-3.8681	-4.7750	-3.8681
4	-0.1303	0.0102	0.0125	-4.1626	-4.7946	-4.1626
5	-0.1713	0.0361	0.0300	-3.8274	-4.9343	-3.8274
6	-0.1415	-0.0240	0.0124	-3.9011	-4.7122	-3.9011
7	0.2095	0.3820	0.1077	-4.4908	-4.9378	-4.4908
8	-0.0800	0.1259	0.0221	-4.3359	-4.9765	-4.3359
9	-0.3624	-0.2787	0.0251	-4.1475	-4.9035	-4.1475
10	-0.2934	-0.2196	0.0324	-4.2438	-4.9126	-4.2438
15	0.1159	-0.1312	0.0436	-4.3907	-4.9280	-4.3907
20	-0.0579	-0.1928	0.0264	-4.2572	-4.9362	-4.2572
25	-0.1534	-0.2070	0.0196	-4.1124	-4.9405	-4.1124
30	-0.0432	-0.1595	0.0366	-4.1456	-4.9082	-4.1456

Table 2
MSE for $\alpha = 0.5$ and $\theta = 5$.

	MME	MLE	Mix	MME	MLE	Mix
n	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$
3	1.3887	8.7781	0.3337	18.8058	22.9525	18.8058
4	0.7004	1.3014	0.2188	20.8334	23.1991	20.8334
5	0.5695	1.4385	0.2254	21.5238	24.3689	21.5238
6	0.6625	1.1337	0.2186	21.2566	22.6192	21.2566
7	3.5681	5.5920	0.2351	21.9822	24.4093	21.9822
8	1.2414	2.7580	0.2318	21.8872	24.7697	21.8872
9	0.3018	0.5185	0.2312	23.7426	24.1279	23.7426
10	0.4702	0.7559	0.2281	23.1561	24.2027	23.1561
15	3.4269	1.2414	0.2254	22.6195	24.3316	22.6195
20	1.7628	0.8866	0.2306	23.0897	24.4026	23.0897
25	1.1045	0.8157	0.2342	24.0026	24.4407	24.0026
30	1.8795	1.0689	0.2268	23.7563	24.1662	23.7563

Table 3
Bias for $\alpha = 0.5$ and $\theta = 1$.

	MME	MLE	Mix	MME	MLE	Mix
n	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$
3	-1.6305	-0.2219	0.2064	-0.8083	-0.8577	-0.8083
4	0.0585	-0.0298	0.1396	0.4382	-0.5731	0.4382
5	0.0215	-0.2420	0.1906	0.8288	0.3508	0.8288
6	1.7109	0.3011	0.1516	0.1000	-0.7918	0.1000
7	-0.3357	-0.3333	0.0639	2.1033	0.1891	2.1033
8	0.0339	-0.2361	0.1631	0.7568	0.0858	0.7568
9	-0.0153	-0.2069	0.2169	1.6530	0.9633	1.6530
10	1.9082	0.4266	0.1481	-0.1027	-0.8566	-0.1027
15	-0.0895	-0.2590	0.1105	1.2620	0.0371	1.2620
20	0.5572	-0.0118	0.2213	0.7883	-0.1894	0.7883
25	0.2704	-0.1868	0.1989	1.6314	0.6710	1.6314
30	-0.0038	-0.1350	0.1398	2.4663	0.3273	2.4663

Table 4
MSE for $\alpha = 0.5$ and $\theta = 1$.

	MME	MLE	Mix	MME	MLE	Mix
n	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$
3	7.7712	0.3586	0.0513	0.0800	0.8166	0.0800
4	1.2512	0.8853	0.0416	0.8466	1.0574	0.8466
5	1.0883	0.3248	0.0482	1.4066	7.4212	1.4066
6	22.4790	2.6578	0.0427	0.4850	0.8003	0.4850
7	0.2207	0.2223	0.0413	4.2945	5.6914	4.2945
8	1.1412	0.3343	0.0440	1.2918	4.7232	1.2918
9	0.9401	0.3864	0.0537	3.0886	16.3467	3.0886
10	26.8395	3.6162	0.0423	0.3231	0.8160	0.3231
15	0.6820	0.2994	0.0401	2.2060	4.3035	2.2060
20	4.7815	0.9534	0.0547	1.3413	2.6639	1.3413
25	2.4473	0.4274	0.0498	3.0358	11.6198	3.0358
30	0.9848	0.5511	0.0416	5.4144	7.1543	5.4144

Table 5
Bias for $\alpha = 1$ and $\theta = 3$.

	MME	MLE	Mix	MME	MLE	Mix
n	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$
3	-0.0991	2.3296	0.2380	-2.3608	-2.9543	-2.3608
4	-1.3166	-0.8673	0.0352	-0.0893	-2.2282	-0.0893
5	-0.5331	0.2911	0.1452	-2.0239	-2.8902	-2.0239
6	-0.7465	-0.5526	0.0351	-1.0059	-2.8438	-1.0059
7	-0.7189	-0.4852	0.1166	-1.4169	-2.6415	-1.4169
8	-0.8247	-0.6895	0.0137	0.9241	-2.8269	0.9241
9	-0.7206	-0.7518	0.0847	-1.0241	-2.3262	-1.0241
10	-0.7550	-0.6729	0.0609	-0.6427	-2.5614	-0.6427
15	-0.8318	-0.7201	0.0425	0.6673	-2.4427	0.6673
20	-0.7369	-0.6894	0.0631	-0.2141	-2.4337	-0.2141
25	-0.7609	-0.5432	0.0288	-0.2231	-2.8249	-0.2231
30	-0.6954	-0.6883	0.0473	0.1276	-2.5256	0.1276

Table 6
MSE for $\alpha = 1$ and $\theta = 3$.

	MME	MLE	Mix	MME	MLE	Mix
n	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$
3	3.2563	49.7731	0.1614	0.7208	8.7362	0.7208
4	2.1344	0.8227	0.1872	3.3897	7.3476	3.3897
5	1.1562	6.7528	0.1630	0.7907	8.4016	0.7907
6	0.8143	1.1060	0.1872	1.6917	8.1846	1.6917
7	0.8329	1.2955	0.1670	1.2033	7.4915	1.2033
8	0.8030	0.8610	0.1947	6.2450	8.1113	6.2450
9	0.8315	0.8116	0.1733	1.6666	7.2272	1.6666
10	-0.7550	-0.6729	0.0609	-0.6427	-2.5614	-0.6427
15	0.8050	0.8319	0.1848	5.4241	7.2091	5.4241
20	0.8199	0.8612	0.1787	3.1090	7.2057	3.1090
25	0.8077	1.1298	0.1893	3.0895	8.1026	3.0895
30	0.8547	0.8623	0.1833	3.9145	7.2788	3.9145

Table 7
Bias for $\alpha = 1.5$ and $\theta = 5$.

	MME	MLE	Mix	MME	MLE	Mix
n	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$
3	-1.2494	-1.1758	0.1437	-2.6520	-3.9593	-2.6520
4	-1.2859	-1.2262	0.1220	-1.3864	-3.3892	-1.3864
5	-1.3550	-1.1622	0.0615	-2.2685	-4.5023	-2.2685
6	-1.2813	-1.1918	0.1158	-2.3080	-3.9884	-2.3080
7	-1.3802	-1.2280	0.0383	-1.0941	-4.4506	-1.0941
8	-1.2941	-1.2735	0.1014	-1.1625	-3.2815	-1.1625
9	-1.2623	-1.2555	0.1117	-2.2201	-3.7295	-2.2201
10	-1.3329	-1.2039	0.0887	0.0376	-3.4912	0.0376
15	-1.3840	-1.2941	0.0190	0.7983	-4.4661	0.7983
20	-1.2986	-1.2960	0.0831	-0.7680	-3.2769	-0.7680
25	-1.3566	-1.2801	0.0650	2.1179	-2.8956	2.1179
30	-1.3004	-1.2599	0.0868	0.3885	-3.0516	0.3885

Table 8
MSE for $\alpha = 1.5$ and $\theta = 5$.

	MME	MLE	Mix	MME	MLE	Mix
n	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$
3	1.8122	1.8029	0.3844	2.9085	20.0083	2.9085
4	1.8369	1.8034	0.3917	5.4153	21.8654	5.4153
5	1.9202	1.8071	0.4169	3.4991	21.2617	3.4991
6	1.8331	1.8003	0.3939	3.4314	20.0007	3.4314
7	1.9624	1.8039	0.4285	6.2220	21.0151	6.2220
8	1.8443	1.8270	0.3994	6.0257	22.5814	6.0257
9	1.8194	1.8154	0.3954	3.5840	20.3657	3.5840
10	1.8884	1.8001	0.4047	10.1511	21.2946	10.1511
15	1.9693	1.8443	0.4390	13.5116	21.0863	13.5116
20	1.8487	1.8461	0.4071	7.2230	22.6147	7.2230
25	1.9226	1.8320	0.4152	20.7142	26.0985	20.7142
30	1.8504	1.8179	0.4054	11.6295	24.4969	11.6295

Table 9
Bias for $\alpha = 2$ and $\theta = 7$.

	MME	MLE	Mix	MME	MLE	Mix
n	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$
3	-1.4771	-1.6688	0.3014	-4.2300	-4.4792	-4.2300
4	-1.7753	-1.8354	0.1297	-0.6504	-1.9985	-0.6504
5	-1.7689	-1.8059	0.1269	-3.5825	-4.7661	-3.5825
6	-1.7285	-1.8422	0.1532	-2.6995	-2.8249	-2.6995
7	-1.8893	-1.8619	0.0606	2.0756	-3.0177	2.0756
8	-1.7866	-1.8126	0.1203	-1.1046	-3.2169	-1.1046
9	-1.8672	-1.7993	0.0725	0.8204	-4.1765	0.8204
10	-1.8362	-1.8569	0.0790	-1.0935	-3.7379	-1.0935
15	-1.8537	-1.7932	0.0771	1.0710	-3.9924	1.0710
20	-1.8528	-1.7634	0.0437	-2.0408	-6.0834	-2.0408
25	-1.8673	-1.8158	0.0682	3.4879	-3.1200	3.4879
30	-1.8984	-1.7905	0.0343	2.4710	-5.4500	2.4710

Table 10
MSE for $\alpha = 2$ and $\theta = 7$.

	MME	MLE	Mix	MME	MLE	Mix
n	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$
3	3.2756	3.2237	0.6497	4.8584	45.4806	4.8584
4	3.3537	3.4770	0.7131	16.1690	104.0550	16.1690
5	3.3426	3.4119	0.7146	5.9552	42.6773	5.9552
6	3.2826	3.4933	0.7009	8.1265	77.7060	8.1265
7	3.6184	3.5431	0.7552	33.3778	72.5402	33.3778
8	3.3740	3.4260	0.7183	14.0243	67.5957	14.0243
9	3.5570	3.3985	0.7473	24.5309	49.3315	24.5309
10	3.4790	3.5301	0.7430	14.0742	56.5375	14.0742
15	3.5219	3.3867	0.7443	26.1713	52.1218	26.1713
20	3.5195	3.3335	0.7669	10.2539	40.3682	10.2539
25	3.5573	3.4328	0.7501	45.2154	69.9508	45.2154
30	3.6452	3.3814	0.7737	36.4908	39.3125	36.4908

Table 11
Bias for $\alpha = 1.5$ and $\theta = 7$.

	MME	MLE	Mix	MME	MLE	Mix
n	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\alpha}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$	Bias of $\hat{\theta}$
3	-0.9268	-1.0619	0.3275	-5.0123	-5.5138	-5.0123
4	-1.4412	-1.4194	0.0319	6.1865	-1.7810	6.1865
5	-1.4370	-1.3338	0.0161	-0.0074	-6.3217	-0.0074
6	-1.3605	-1.3173	0.0784	0.4892	-3.7843	0.4892
7	-1.2166	-1.2580	0.1595	-2.9191	-4.3098	-2.9191
8	-1.3907	-1.3577	0.0484	-1.5270	-5.1402	-1.5270
9	-1.2219	-1.2884	0.1456	-2.9162	-4.1898	-2.9162
10	-1.3082	-1.1656	0.0999	-3.1880	-5.8612	-3.1880
15	-1.3421	-1.3047	0.0783	-0.8224	-4.5229	-0.8224
20	-1.3920	-1.2786	0.0257	-0.3468	-6.2276	-0.3468
25	-1.3588	-1.2408	0.0626	-0.1916	-5.3564	-0.1916
30	-1.3694	-1.2589	0.0029	0.9402	-6.9049	0.9402

Table 12
MSE for $\alpha = 1.5$ and $\theta = 7$.

	MME	MLE	Mix	MME	MLE	Mix
n	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\alpha}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$	MSE of $\hat{\theta}$
3	2.1731	1.8953	0.3608	4.0927	39.2372	4.0927
4	2.0908	2.0407	0.4319	73.3804	112.1241	73.3804
5	2.0808	1.8896	0.4406	19.5587	41.8041	19.5587
6	1.9288	1.8688	0.4091	22.4590	55.6836	22.4590
7	1.8014	1.8168	0.3797	7.5136	47.5230	7.5136
8	1.9818	1.9243	0.4233	12.2148	40.2571	12.2148
9	1.8024	1.8391	0.3838	7.5213	49.1437	7.5213
10	1.8585	1.8059	0.4000	6.8288	39.5411	6.8288
15	1.9010	1.8548	0.4091	15.3327	45.0005	15.3327
20	1.9843	1.8309	0.4352	17.7183	41.1695	17.7183
25	1.9261	1.8083	0.4164	18.5453	39.4967	18.5453
30	1.9434	1.8174	0.4483	25.3071	47.7143	25.3071

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