Estimation of the spontaneous emission factor for microdisk lasers via the approximation of whispering gallery modes

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(Received 27 May 1993; accepted for publication 18 November 1993)

Microdisk structures have been used to achieve low-threshold lasing. For these microcavity lasers, the spontaneous emission coupling factor β is an important parameter since it determines the threshold current of the laser. Theoretical calculation of β based on the exact solution of the modes in a microdisk is complicated. A simple, approximate method for solving the waveguide modes and the density of states is developed here, using conformal transformation and the Wentzel–Kramers–Brillouin approximation and taking into account the effect of the disk thickness. We find that the β value for a microdisk laser is smaller than that of an ideal laser that has a cylindrical waveguide structure with a strong index guiding. A considerably high value of β can still be achieved, however, in a microdisk laser.

I. INTRODUCTION

Recently, McCall *et al.* demonstrated a two-dimensional guided microdisk structure to achieve low-threshold lasing.[1] This structure is an example of photonic confined microcavities, in which the photon density of states and the spontaneous emission characteristics are significantly modified.[2] The threshold current of a microcavity laser with low transparency current is dependent mainly on the spontaneous emission coupling factor β, which is the spontaneous emission rate into the lasing modes divided by the total spontaneous emission rate. We have previously calculated the β value of a microring laser.[3] The cross section of a microring laser is a one-dimensional dielectric cylindrical waveguide with a high index guiding region of diameter d_D. We showed that, when d_D is less than the cutoff diameter for the second-order guided mode, the β value can be as high as 0.96. In our decay rate calculations, the spontaneous decay is modeled as stimulated decay due to stochastic vacuum field fluctuations.[2-4] The spontaneous emission rate γ due to the vacuum field in either the guided or the radiation modes is proportional to the modal density of states and the vacuum mode field intensity at the location of the dipoles. More precisely, the contribution to γ from the guided mode n is[3]

$$\gamma_{gn}=\frac{2\pi}{\hbar^2}\frac{\hbar\omega}{2L_zA}\,|\bar{E}_n\cdot\bar{\mu}|^2\,\frac{L_z}{2\pi}\frac{dk_{nz}}{d\omega}, \tag{1}$$

where A is basically the spatial mode area (i.e., the transverse area of the mode) times the dielectric constant, $\bar{E}_n$ is the mode function for the guided mode, and $\bar{\mu}$ is the usual dipole matrix element. In this case, the field has been quantized in the z direction via the traveling-wave modes. Thus the number of states per unit angular frequency is $d\rho_n=(L_z/2\pi)(dk_{nz}/d\omega)$ for each waveguide mode. Usually, an exact solution of $\bar{E}_n$ and the density of states are very complicated. To extend our theory to a microdisk laser for the purpose of estimating the β value, a simple, approximate method for solving the waveguide modes and the density of states is developed here. Knowledge of the mode size and the density of states allows us to obtain the relative value of γ_{gn} according to Eq. (1) and, thus, estimate the β value of the microdisk laser. A microdisk is characterized by two size parameters, d and R, where d is the thickness and R the radius of the disk. The waveguide modes travel around the edge of the disk by repeated total internal reflections, in a way similar to the "whispering gallery modes."[5] In this article the whispering gallery modes are solved approximately by using conformal transformation and the Wentzel–Kramers–Brillouin (WKB) approximation. The modal density of states and the mode field sizes are then evaluated as a function of d and R. Finally, an estimation for the β values of microdisk lasers is given. We would like to note that there are also other methods for investigating the properties of disk waveguides.[6,7] However the conformal transformation method gives us an estimation of the spatial widths of the guided modes that is needed for our purpose here.

II. METHOD OF CALCULATION

A microdisk structure can be seen as part of a cylindrical waveguide, as shown in Fig. 1. To facilitate future calculations, let us first consider the mode solutions of the cylindrical waveguide. The wave equation for the z component ψ in cylindrical coordinates is[8]

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}+\frac{\partial^2\psi}{\partial z^2}+k^2\psi=0. \tag{2}$$

The z components of the electric and magnetic fields have the form $\psi=F(r)\exp(\pm im\phi)\exp(ik_zz)$, representing a traveling wave in the z direction, where m is the azimuthal number given by $m=0, 1, 2,\ldots$, and k_z is the k vector in the z direction (see Ref. 8). In this case Eq. (2) can be reduced to the wave equation for the radial mode function $F(r)$:[8]

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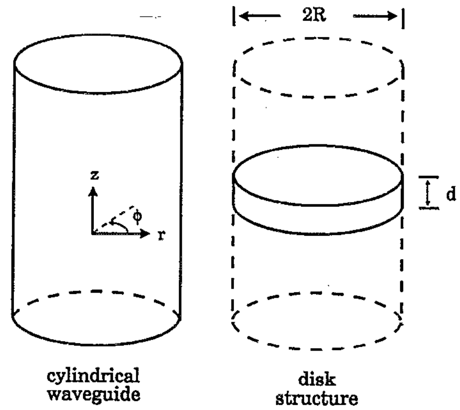

FIG. 1. Schematic diagram of a cylindrical waveguide and a disk structure.

$$\frac{\partial^2 F}{\partial r^2}+\frac{1}{r}\frac{\partial F}{\partial r}+\left[q^2-\left(\frac{m^2}{r^2}\right)\right]F=0, \tag{3}$$

where $q^2=k^2-k_z^2$. For each m, given waveguide frequency ν, and radius R, one can solve for q by matching the boundary conditions of the electric and magnetic fields at $r=R$. The propagation vector k_z can then be determined.

Unlike a cylindrical waveguide where the mode propagates along the z axis of the cylinder, the waveguide modes in a microdisk are stationary in the z direction and propagate around the disk. Hence, the eigenmodes of a microdisk structure can be constructed from the cylindrical waveguide modes by summing both $+z$ and $-z$ propagating modes to form stationary modes in the z direction. It is important to point that out because of the boundary conditions imposed by the top and bottom boundaries of the disk; the k_z vector in the microdisk case is determined solely by the disk thickness. For a given disk thickness, there can be more than one allowed k_z value, corresponding to resonant modes with different numbers of nodes along the z direction and which will be referred to as the planar mode number $p=0, 1, 2, 3,\ldots$. These modes are simply the planar waveguide modes. Thus, in the case of the microdisk k_z is determined by the structure, and we want to solve the resonance frequency ν for each azimuthal mode number m. The azimuthal mode number m determines the number of nodes along the ϕ direction, which is obvious from the function of ψ. It turns out that for each m, there are many resonance frequencies ν, corresponding to resonance modes with different numbers of nodes in the radial direction [and given by different solutions to $F(r)$], which will be referred to as the radial mode number. The radial mode number will be denoted by $l=0, 1, 2,\ldots$. Thus the resonance mode frequency ν is parameterized by three mode numbers: m,l,p. We are interested mainly in the case where the disk is so thin that only the $p=0$ mode exists and we shall drop the p parameter below. We will denote the frequency for mode m, l, $(p=0)$ as ν_{ml}.

Thus the modes in the microdisk can, in principle, be solved approximately using the cylindrical waveguide modes.[9] However, the solution is complicated. Here, we

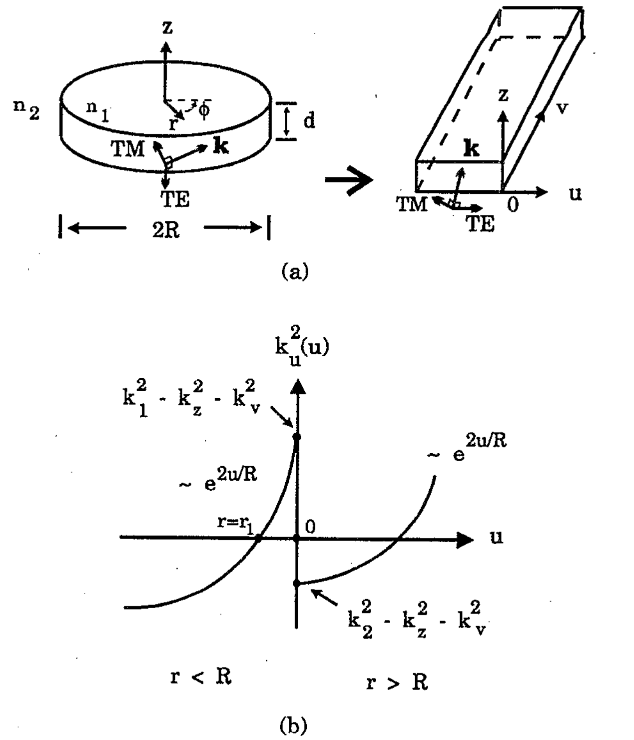

FIG. 2. (a) Schematic structure of a disk waveguide with radius R and thickness d and its transformation. n_1 and n_2 are the refractive indices inside and outside the disk. (b) Illustration of $k_u^2(u)$ as a function of u.

adopt an approximation method to solve for the modes based on the method of conformal transformation developed in Ref. 10. There, the conformal transformation is applied to find the bending loss of a waveguide. Here, we apply the conformal transformation to treat the cavity of the disk structure. Besides the difference in applications, our approach differs from Ref. 10 in that we have taken the finite thickness of the microdisk into consideration.

Figure 2(a) shows the geometry of the microdisk characterized by a radius R and a thickness d. It is assumed that $R>d$ for a microdisk. The indices in the figure are assumed to be that given by a semiconductor microdisk with $n_1=3.4$ and $n_2=1$. Equation (2) can be transformed into the Cartesian form

$$[\nabla_{u,v}^2+p^2(u,v)]\psi=0 \tag{4}$$

in a coordinate system (u,v) that is related to (r,ϕ) by a conformal transformation as follows:[10]

$$u=R\ln\left(\frac{r}{R}\right),\quad v=R\phi. \tag{5}$$

The resulting wave equation is

$$\frac{\partial^2\psi}{\partial u^2}+\frac{\partial^2\psi}{\partial v^2}+q^2\exp\left(\frac{2u}{R}\right)\psi=0, \tag{6}$$

where $q^2=(k_1^2-k_z^2)$ for $r<R$ and $q^2=(k_2^2-k_z^2)$ for $r>R$, $k_1=kn_1$, $k_2=kn_2$, and $k=2\pi\nu/c$. The disk waveguide is thus equivalent to a straight waveguide in the (u,v) space shown in Fig. 2(a), but the index distribution has become a function of u. Note that in Fig. 2(a), $u=0$ corresponds to $r=R$, $u=-\infty$ corresponds to $r=0$, $v=0$ corresponds to $\phi=0$, and $v=2\pi R$ corresponds to $\phi=2\pi$.

Under the transformation, the form of ψ is transformed to $\psi=\widetilde{F}(u)\exp(ik_v v)\cos(k_z z)$, where $k_v v=m\phi$. The cosinusoidal variation of ψ in the z direction accounts for the waveguiding in the plane of the disk. Let us define an effective index n_v as $k_v=2\pi n_v/\lambda$ (λ is the free-space wavelength). Since $k_v v=m\phi$, using $v=R\phi$ from Eq. (5) to eliminate ϕ we have

$$\lambda=\frac{2\pi n_v R}{m}. \tag{7}$$

Later we will see that n_v in Eq. (7) is dependent on the radial mode number l. As a result, the allowed values of λ are discrete and are dependent on mode numbers m and l.

Note that in the disk waveguide, for each k vector there are two modes corresponding to two polarizations. We shall define them as transverse electric (TE) and transverse magnetic (TM) modes with the electric field and magnetic field, respectively, parallel to the r direction, as shown in Fig. 2(a) (in the figure the direction of the electric field is indicated). This corresponds to having the electric field and magnetic field, respectively, parallel to the u direction in the transformed coordinate. Under our definition of the TE and TM modes, ψ is the z component of the magnetic field for the case of the TE mode (i.e., the electric field has no z component).[8] Likewise, ψ is the z component of the electric field for the case of the TM mode. The equation for the radial function $\widetilde{F}(u)$ can be obtained by substituting the function of ψ previously given into Eq. (6), giving

$$\frac{d^2\widetilde{F}}{du^2}=-k_u^2(u)\widetilde{F}, \tag{8}$$

where $k_u^2(u)=(k_1^2-k_z^2)\exp(2u/R)-k_v^2$ for $r<R$, $k_u^2(u)=(k_2^2-k_z^2)\exp(2u/R)-k_v^2$ for $r>R$, and k_v must satisfy the condition for guided mode $(k_2^2-k_z^2)<k_v^2<(k_1^2-k_z^2)$. The variable k_u can be interpreted as an effective k vector component in the u direction. The form of $k_u^2(u)$ is illustrated in Fig. 2(b). Let us define an effective index n_u so that $k_u=2\pi n_u/\lambda$. From Fig. 2(b) we see that n_u is a strong function of u. The form of the index distribution $n_u(u)$ inside the waveguide entails that the radial distribution of the mode is skewed towards the edge (i.e., $r=R$ or $u=0$) since the index has a maximum there. In the case where $(k_2^2-k_z^2)>0$, the effective refractive index outside the disk increases radially as $e^{u/R}$ so at some point the index will be large enough where the oscillatory mode can again exist and resulting in radiation loss. On the other hand, if $(k_2^2-k_z^2)$ is negative, as when k_z is large, then the field is completely evanescent outside the guide. Since $n_2=1$, from Fig. 3 we see that $n_z<n_2$ or $k_z^2<k_2^2$ for a thick disk, while $k_z^2>k_2^2$ for a thin disk. Hence for a thick disk, we expect the wave to be completely evanescent outside the disk. For the thin disk of interest here where d is small, there could be propagating wave outside the disk according to the theory here. However, it must be pointed out that the wave will experience diffraction once it leaves the disk and k_z can become large there due to diffraction [diffraction has been totally neglected in Eq. (8)]. It is possible that when diffraction is properly included, the wave outside the disk may be completely evanescent also for a thin disk. Hence, the question of radiation loss cannot be appropriately addressed by the theory here.

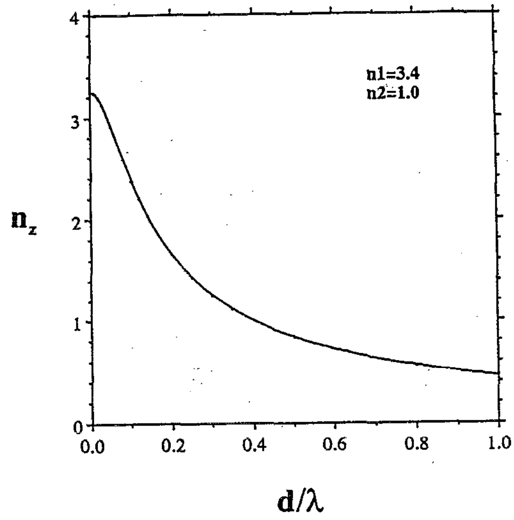

FIG. 3. Calculated n_z, defined by $k_z=(2\pi/\lambda)n_z$, for the lowest TE mode as a function of d/λ.

As mentioned earlier, k_z can be determined by matching the boundary conditions at the interface of the planar waveguide. For the lowest-order TE modes, this gives

$$\tan\left(\frac{k_z d}{2}\right)=\frac{h}{k_z}, \tag{9}$$

where d is the thickness of the disk, and h is a variable defined by $k_z^2+h^2\equiv k^2(n_1^2-n_2^2)$. Writing $k_z=kn_z$, we plot n_z as a function of d/λ in Fig. 3. Note that $n_z>n_2$ for most values of d/λ of interest. The most interesting region of d/λ is just below $0.5/n_1$ where only the lowest-order TE mode is guided and the confinement of the spontaneous emission in the plane of the disk is at its peak.[4]

Next, k_v is determined in the WKB approximation[11] by requiring the quantization condition

$$\int k_u(u)du=l\pi+\phi_1+\phi_2, \quad l=0,1,2\ldots, \tag{10}$$

where the integral is taken over the classical region from the lower limit defined by $k_u(u)=0$ to the upper limit $u=0$ (or $r=R$), l is the radial mode number, and ϕ_1 and ϕ_2 are phases determined by the shape of the potential function at the turning points. Doing so yields the implicit equation[10]

$$2\sqrt{n_1^2-n_z^2}\,\frac{R}{\lambda}\,(\sqrt{1-a^2}-a\cos^{-1}a)$$

$$=l+\frac{1}{4}+\frac{1}{\pi}\tan^{-1}\sqrt{1-a^2/a^2+b^2}, \quad l=0,1,2,\ldots, \tag{11}$$

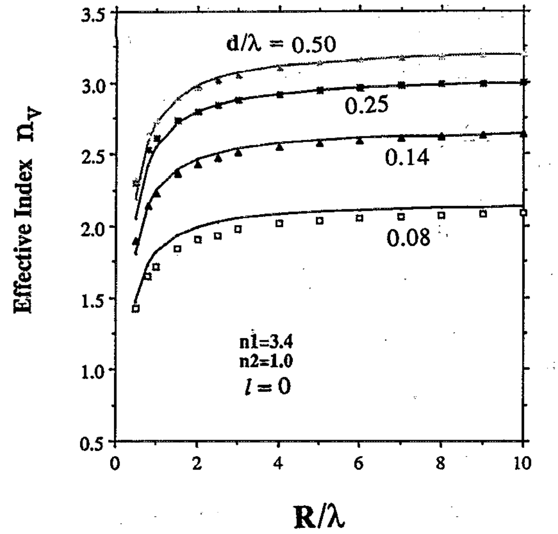

FIG. 4. The calculated effective index n_v for the $l=0$ radial mode as a function of R/λ for various values of d/λ. The solid lines are the curve fitting for the calculated points.

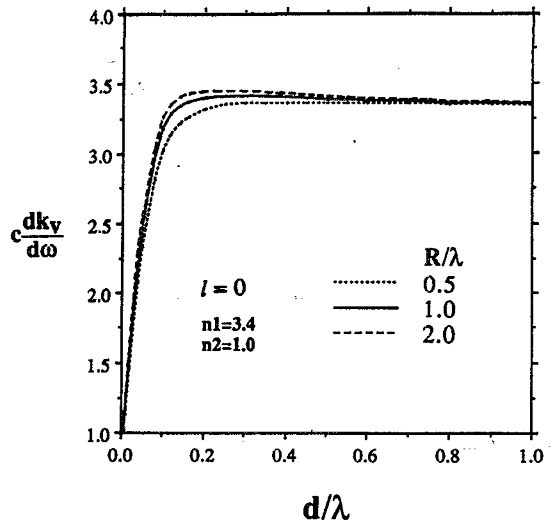

FIG. 6. The dependence of the mode density $dk_v/d\omega$ for the lowest-order TE mode on d/λ_{ml} and R/λ_{ml}.

where a and b are defined by $a \equiv n_v/\sqrt{n_1^2-n_z^2}$, and $b \equiv \sqrt{n_z^2-n_2^2}/\sqrt{n_1^2-n_z^2}$. Thus n_v is a function of R/λ, d/λ (from n_z), and l. For $l=0$, the calculated n_v as a function of R/λ and d/λ is shown in Fig. 4. Note that (i) n_v is relatively insensitive to R/λ for $R/\lambda>2$, and (ii) n_v decreases with d/λ.

To facilitate later calculations for the case of $l=0$, we fit the calculated n_v with simple curves, shown by the solid lines in Fig. 4, which have the following simple expression

$$n_v=\alpha\sqrt{n_1^2-n_z^2}, \quad \alpha=f-\frac{g}{(R/\lambda)}, \tag{12}$$

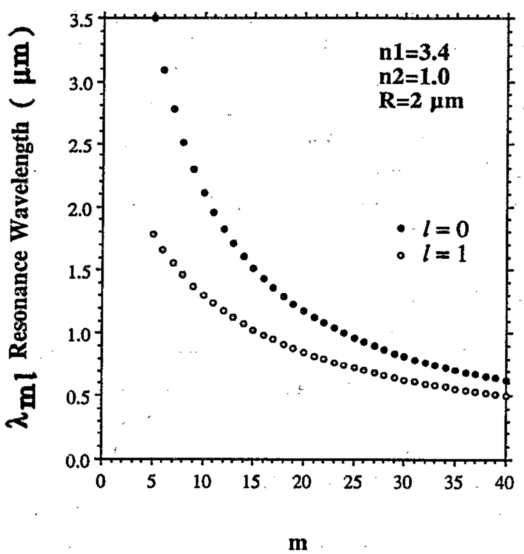

FIG. 5. Cavity resonance wavelength vs the azimuthal mode number m for the TE and $l=0$ and $l=1$ modes of a 2 μm radius disk waveguide.

where $f=0.984$ and $g=0.163$ for $l=0$. Using Eq. (7) to eliminate n_v in Eq. (12), it follows that, for a fixed d/λ and, hence, a fixed $\sqrt{n_1^2-n_z^2}$, there is a linear relationship between R/λ and m (i.e., $m=2\pi\alpha\sqrt{n_1^2-n_z^2}R/\lambda$). Since m is an integer, this determines, for a given R, the resonance wavelength λ corresponding to each value of m and l. We shall denote λ for the mode number m and l as λ_{ml}. An example of λ_{ml} for $R=2$ μm, $n_z=2.7$, and $l=0$ and 1, is shown in Fig. 5. The cavity resonance frequency ν_{ml} is then given by $\nu_{ml}=c/\lambda_{ml}$.

In order to evaluate the spontaneous emission coupling factor, we need to determine the mode area A as well as the modal density of states. The modal density of states $dk_v/d\omega$ for any guided mode can, in principle, be derived analytically from Eq. (11). However, a simpler approach is by making use of Eq. (12), noting that $k_v=\omega n_v/c$ and, if n_v is a function of only R/λ_{ml} and d/λ_{ml}, then a general and complete expression for $dk_v/d\omega$ is

$$c\frac{dk_v}{d\omega}=n_v+\frac{R}{\lambda_{ml}}\frac{dn_v}{d(R/\lambda_{ml})}+\frac{d}{\lambda_{ml}}\frac{dn_v}{d(d/\lambda_{ml})}. \tag{13}$$

With Eq. (12), this gives

$$c\frac{dk_v}{d\omega}=n_v\left[\frac{f}{f-\frac{g}{R/\lambda_{ml}}}-\frac{n_z}{n_1^2-n_2^2}\frac{d}{\lambda_{ml}}\frac{dn_z}{d(d/\lambda_{ml})}\right], \tag{14}$$

where $dn_z/d(d/\lambda_{ml})$ may be obtained from Fig. 3 or derived analytically from Eq. (9). In Fig. 6, $cdk_v/d\omega$ for the lowest-order radial mode ($l=0$) is plotted as a function of d/λ_{ml} for various values of R/λ_{ml}. It should be noted that $dk_v/d\omega$ is not sensitive to R/λ_{ml}, and that its dependence on d/λ_{ml} is significant only if d/λ_{ml} is smaller than 0.2. We can expect that this is also the region where the spontaneous emission rate is modified most. This behavior can also be inferred from Fig. 4.

For the lowest-order radial mode, we obtained A by approximating the two-dimensional mode profiles with a

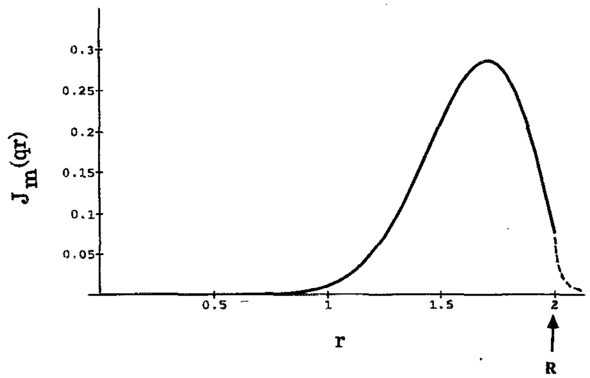

FIG. 7. Plot of the Bessel function $J_m(qr)$ for $m=12$ and $l=0$.

product of cosine functions of the form $\cos(\pi x/w_r)\cos(\pi y/d_z)$, where w_r is the effective width of the radial mode function, and d_z is the effective mode thickness in the direction perpendicular to the disk. The effective mode thickness d_z is given by the width of the planar waveguide mode. We are interested mainly in the case where the lasing mode is the lowest-order planar waveguide mode. The value of w_r is determined by the radial functions for the (m,l) modes, which are Bessel functions of the order m, $J_m(qr)$, where $q = (2\pi/\lambda_{ml})\sqrt{n_1^2-n_z^2}$ is dependent on l through λ_{ml}. An example of the radial mode function is shown for $m=12$ and $l=0$ in Fig. 7. The outer limit of w_r is simply given by $r_{\text{out}}=R$ (or $u=0$) in Fig. 2(b). The inner limit of w_r, denoted by r_{in}, is given approximately by the classical turning point at $r=r_1$, shown in Fig. 2(b) where $k_u(u)=0$. Setting $k_u(u)=0$ gives $\sqrt{n_1^2-n_z^2}\exp(u/R) = n_v$. Using Eq. (12), we then obtain $r_1 = R\alpha = Rn_v/\sqrt{n_1^2-n_z^2}$. Using r_1 as an estimate for r_{in} tends to underestimate w_r because of the weak mode confinement; a better estimate is given by $r_{\text{in}}\simeq Rn_v/n_1$ which is slightly smaller than r_1. Using this estimation for r_{in}, we obtain the following equation for w_r/λ_{ml}:

$$\frac{w_r}{\lambda_{ml}}=\left(\frac{R}{\lambda_{ml}}\right)\left(1-\frac{n_v}{n_1}\right). \tag{15}$$

In this form, w_r/λ_{ml} is not only a function of R/λ_{ml}, but also of d/λ_{ml}. Figure 8 shows w_r/λ_{ml} for the $l=0$ modes as a function of R/λ_{ml} and d/λ_{ml}. Note that the radial mode widths are actually larger for the disks with smaller d/λ_{ml} since they have larger n_z and, hence, smaller n_v [see Eq. (12)].

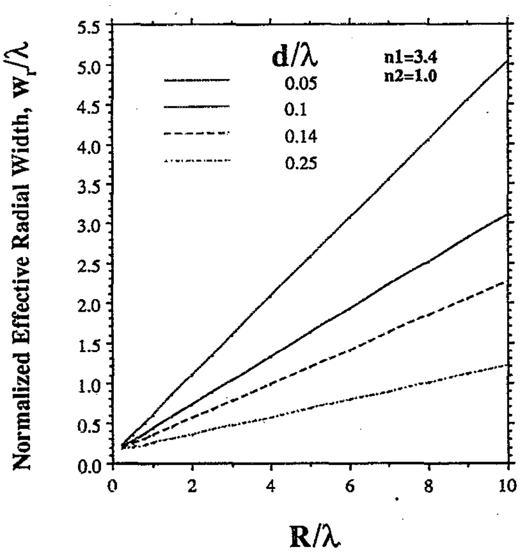

FIG. 8. The normalized effective radial width of the lowest-order TE mode as a function of d/λ_{ml} and R/λ_{ml}.

III. RESULTS

The modal density of states and the effective mode size allow us to determine the spontaneous emission coupling factor β for a microdisk laser. It is given by

$$\beta=\frac{R_L}{R_T}, \tag{16}$$

where R_L is the emission rate into the lasing mode, and R_T is the total emission rate. We shall take R_L and R_T as normalized rates normalized by the spontaneous emission rate in a bulk medium of uniform index n_1 given by

$$\gamma_{\text{bulk}}=\frac{1}{3\pi}\frac{n_1}{\hbar}\left(\frac{\omega}{c}\right)^3|\mu|^2. \tag{17}$$

We assume that the spontaneous emission into the lasing mode only emits into one guided mode spatially and into one cavity resonance spectrally. The condition for the spontaneous emission to go into one single resonance mode spectrally is $\Delta\nu_c>\Delta\nu_{\text{sp}}$, where $\Delta\nu_{\text{sp}}$ is the spontaneous emission width, and $\Delta\nu_c$ is the intermode frequency spacing given by $\Delta\nu_c\simeq c/(2\pi Rn_v)$.[3] If $\Delta\nu_c<\Delta\nu_{\text{sp}}$, then the spontaneous emission will emit into other nonlasing modes spectrally and the β value will decrease. This condition, therefore, determines the maximum size of the disk before the β value decreases. For example, the spontaneous emission width of a quantum well is typically 1% of the optical frequency so, at $\lambda_{ml}=1.5$ μm, the largest diameter of the disk that satisfies the condition $\Delta\nu_c>\Delta\nu_{\text{sp}}$ is 9.5 μm. If several radial modes are allowed, the dominant mode is one with the largest cavity Q. The evaluation of Q is complicated, as it depends on the particular dominant loss mechanism. In the case of radiation loss due to tunneling from the disk, it can be shown that the Q is larger for the modes with the larger values of m.[12] Note in Fig. 5 that the values of m for the $l=0$ modes are significantly larger than those for the $l=1$ modes. Hence, the $l=0$ modes will have larger Q values than the $l>0$ modes and will be the most likely to lase. The cavity resonance width $\Delta\nu_{\text{cav}}$ is also related to Q according to $\Delta\nu_{\text{cav}}=\Delta\nu_c/Q$. If $\Delta\nu_{\text{sp}}\ll\Delta\nu_{\text{cav}}$, then the spontaneous emission rate into the guided modes will be enhanced by a factor of Q. On the other hand, if $\Delta\nu_{\text{sp}}>\Delta\nu_{\text{cav}}$, then the spontaneous emission rate into the guided mode will not be strongly affected by the cavity. The cavity enhancement factor will be averaged to around

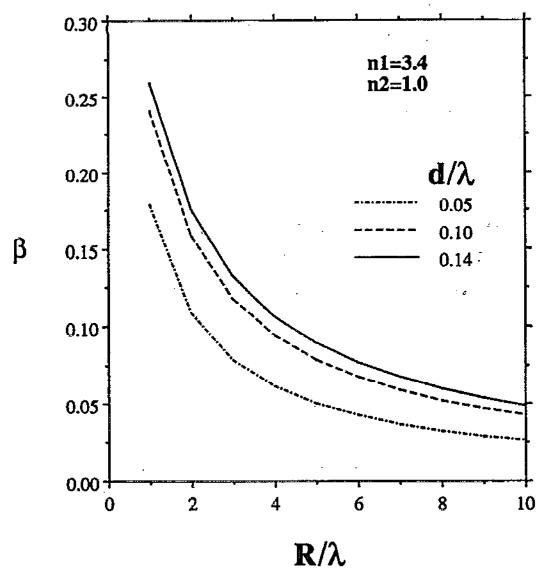

FIG. 9. The calculated spontaneous emission coupling factor β for microdisk lasers with varying size parameters $(R/\lambda_{ml}, d/\lambda_{ml})$, assuming only one guided (lasing) mode is supported.

unity if the spontaneous emission width approaches the intermode spacing. We will assume for our estimation of β that this is the case.[3]

It turns out that for the case of the microdisk, R_T is close to the bulk emission rate because of the high probability of spontaneous emission from the center and then from the side of the disk. Thus we can take $R_T \simeq 1$. In fact, even for the strongly confined waveguide of a microring cavity, R_T does not vary from 1 by more than 20%.[3] By knowing $dk_\nu/d\omega$ and A are functions of R/λ_{ml} and d/λ_{ml}, one can determine R_L for specific combinations of R/λ_{ml} and d/λ_{ml}. We will restrict d/λ_{ml} to $0<d/\lambda_{ml}<0.5/n_1$, which is the region of interest for microdisk lasers where we want all the emissions to be in a single lasing mode. The calculated β is plotted in Fig. 9 with respect to R/λ_{ml} and d/λ_{ml}. Note that the β value for a particular R/λ is the highest at around $d/\lambda \simeq 0.14$, right at the disk thickness that cut off the second-order planar guided mode. The β value reduces with reduced disk thickness d because of a broadening of the guided mode at small d, leading to a larger mode width w_r. On the other hand, the β value increases with decreasing disk radius R due to the reduction of the mode width at a smaller disk radius. Physically, a smaller mode width corresponds to a larger mode guiding angle, which allows the lasing mode to capture more spontaneous emission.

The calculation of the β value is done with the assumption that only the radial dipole is excited by the pump (i.e., the nonisotropic pumping). In the case of isotropic excitation (i.e., all three dipoles are equally excited, which is normally the case in practice), the β value will vary. However, we note that it has been shown[4] that the radiation is nonisotropic even when the excitation is isotropic. In particular, the z-dipole emission is suppressed by the thin disk due to the reduction of the z component of the vacuum field. In fact the z-dipole emission rate will be further reduced if quantum wells are used as the active medium inside the disk.[13] As a result, there will only be r and ϕ dipole emissions, even under isotropic excitation. Since only r dipole emits into the lasing mode, the β value will be approximately half of that considered here.

The β value obtained here can be compared to the case of the microring laser mentioned earlier.[3] The β value of a microdisk laser is generally smaller than that of a microring laser; this is primarily because of the weaker mode confinement in a microdisk structure, except when the radius is of the order of a wavelength. For the microdisk case, if the laser is multimode, with several radial modes (l=0, 1,...), then the β value will be even smaller.

IV. CONCLUSIONS

We have developed an approximate method for solving the whispering gallery modes in a microdisk laser. Conformal transformation of the wave equation for the circular disk is used to show the effective radial index distribution that provides a guiding action to confine the mode to near the edge of the disk. The effect of disk thickness is also evident in this approach. We showed that the spontaneous emission coupling factor of a microdisk laser is smaller than that of a laser with an ideal cylindrical waveguide structure with strong index guiding. Nevertheless, a considerably high value of β, 0.1–0.2, can still be achieved in a microdisk laser with a cavity Q value of unity. In practice, the microring laser geometry is not easy to achieve.

ACKNOWLEDGMENT

This work is supported by the National Science Foundation under Contract No. ECS-9210434.

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