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Estimation of Uncertainty Bounds on Unmeasured Variables via Nonlinear Finite Element Model Updating

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Summary

The uncertainty associated with simulation input parameters for a nonlinear finite element structural dynamics model is estimated using a previously developed model updating algorithm. The utility of a model updating approach to estimate unmeasurable or unmeasured experimental parameters is demonstrated.

Introduction

Finite element model validation is a topic of current interest to many researchers in the field of linear and nonlinear structural dynamics. Model validation refers to "substantiation that a model, within its domain of applicability, possesses a satisfactory range of accuracy consistent with the intended application of the model." [1]. Validation is accomplished primarily by comparison of simulation results to experimental results to confirm the accuracy of the mechanics models in the simulation and the values of the parameters employed in the simulation, and to explore how the simulation might be improved.

The assessment of uncertainties in the simulation mechanics models and their associated parameters plays a critical role in the credible validation of nonlinear structural dynamics models. The study of the effects that these uncertainties produce is termed uncertainty quantification (UQ). A major issue in UQ is the determination of how the distributions of the model parameters (which essentially form a set of inputs to the simulation) should be represented in order to accurately reflect the real-world response of the structure.

In the case of repeated experiments, it is sometimes adequate to monitor the values of the input variables (e.g. forces, temperatures, velocities, etc.) and estimate a distribution from these observations. However, in many structural dynamics experiments, there can be significant input variables that are either unmeasurable (such as the actual orientation of parts during an impact event) or unmeasured (such as the level of torque applied to an interface during assembly). In these cases, it is necessary to estimate the distributions of the key input variables by indirect means.

In this paper, a previously developed model updating technique for nonlinear structural dynamics models is applied to data from repeated experimental trials to estimate the distributions of four key input parameters for a transient impact event. The model updating technique itself, along with the selection of the key simulation parameters, is not the focus of this paper, and so these issues are only addressed in summary form.

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Nonlinear Finite Element Model Updating Using Metamodels

Researchers at Los Alamos National Laboratory (LANL) have been exploring the issue of model updating as applied to nonlinear finite element structural dynamics simulations since about 1997. The approach focuses first on the selection of the response features of interest, which will be used to form the metric for the model updating optimization. The term “features” applies to any characteristic of the model output that is of interest, is of relatively low dimension, and is experimentally observable. In the case of nonlinear structural dynamics, often the feature of interest is a peak acceleration, stress, or displacement magnitude. For linear structural dynamics, the feature of interest could be a modal frequency.

The next step is the selection of the key parameters in the finite element model that exhibit significant influence on the response features. The parameter selection process begins with engineering judgment about which parameters will influence the features of interest the most, and have a significant chance of changing or varying over a range of experimental tests or physical units. Once a set of candidate parameters has been established, quantitative techniques are employed to downselect from this set of candidate parameters to a lower dimension set of parameters. This low dimension set of parameters will be better suited for model validation, sensitivity analysis, model updating, and uncertainty quantification.

The selection of the smaller set of parameters is accomplished using metamodel (or response surface) analysis. A metamodel in this context refers to a relatively low-order function that relates several inputs to a single output. A common approach is to use a low-order metamodel to reduce the dimensionality of the parameter set, then run a larger set of levels for fewer parameters to form a higher dimension metamodel. This higher dimension metamodel can be used to understand the relationship between the parameters and the feature of interest. Typically one metamodel must be constructed for each response feature of interest.

Model updating is then accomplished in the parameter space of the metamodel by searching for the combination of parameters that minimizes the error between the metamodel prediction and the experimental observation. Further details on the specific technique have been previously presented. [2]

Experimental Apparatus & Computational Model

In the summer of 1999 a series of impact experiments was performed LANL to provide a database of experimental results for the development of methodologies for nonlinear finite element model validation. A schematic of the experimental assembly is shown in Figure 1, and the details of the experiment have been previously presented. [2] The carriage is dropped and brought to a sudden stop resulting in the compression of the foam pad by the steel cylinder. The foam pad exhibits hyperelastic behavior and is similar in consistency to a common mousepad.

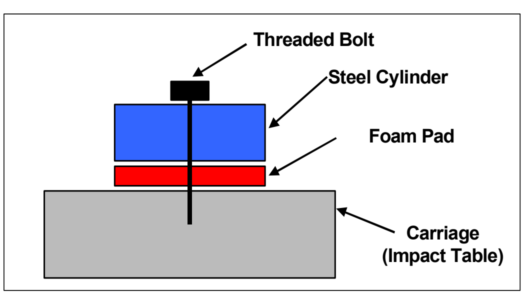

Figure 1. Impact Test Assembly Schematic

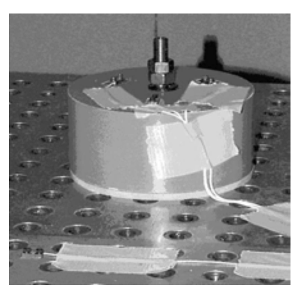

Figure 2: Impact Test Assembly Photo

Four acceleration measurements were collected during each test. The input acceleration was measured on the top surface of the carriage and three output accelerations were measured on top of the steel cylinder. The tightening of the threaded bolt that hold the assembly together results in a preload in the foam pad that turns out to be of critical importance to the simulated system response, but was not measured during the experiment. The carriage is dropped from an initial height of 13 inches (0.33 meters) and the hyper-foam pad used in this configuration is 0.25 inch thick (6.3 mm). The impact tests were repeated 10 times to provide multiple sets of feature measurements for the purposes of validation.

A three-dimensional finite element model of this assembly was developed, as shown in Figure 3. An initial analysis of the system indicated that eight parameters were expected to exhibit significant uncertainty or variability and have potentially significant influence on the features of interest. These parameters are: angle of impact of the cylinder (two values), bolt preload, stiffening of the stress-strain model of the foam pad (two values), input acceleration magnitude, friction coefficient between the steel cylinder and bolt, and linear bulk viscosity (a numerical convergence parameter).

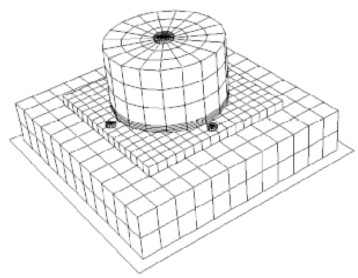

Figure 3: 3D Model of Drop Test Assembly

Feature Extraction and Construction of Metamodel

The feature of interest is typically driven by the engineering considerations that motivate the modeling in the first place. In this experiment, the features of interest are the peak acceleration response of each channel and the arrival time of the impulse (the delay between the acceleration input from the carriage and the acceleration response of the cylinder.) Acceleration signals measured during the ten experimental trials at sensor #1 are shown in Figures 4. A close-up of the peak acceleration signals collected during these ten 'identical' trials is shown in Figure 5.

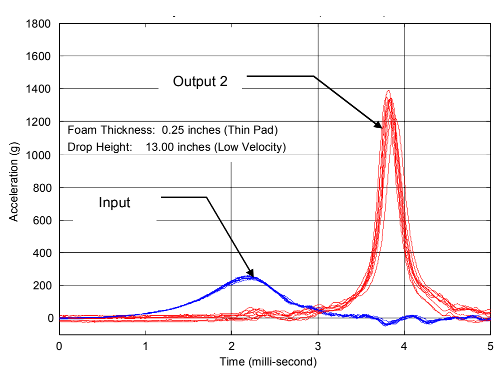

Figure 4: Measured Accelerations

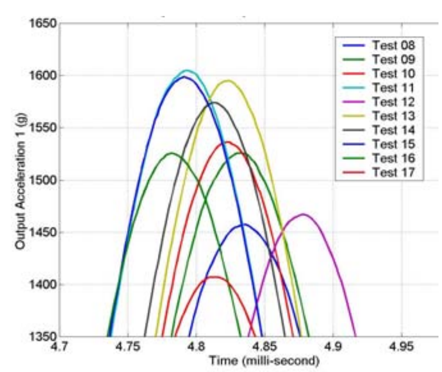

Figure 5: Variability of the Peak Acceleration

A linear metamodel was constructed over the eight simulation parameters mentioned above. Two values were selected for each parameter, and a full factorial set of simulations was executed. The linear metamodel was used to perform hypothesis testing on each of the coefficient values. [3] Based on these screening results, the number of parameters was reduced from eight to four. A full factorial simulation was then performed on these four parameters using four levels for each parameter. A quadratic model with interactions was fit to these results. (It is acknowledged that in practice with large simulation models full factorial designs are not practical. The issues associated with using partial factorial designs will be addressed in future publications. However, the full factorial results are used in this study.) These four parameters, along with their ranges of interest, are shown in Table 1. In the model updating process, the variables are encoded with reference letters and scaled to the range [-1,1] to eliminate numerical scaling incompatibilities. (Uniform distributions were assumed for all parameters at this point.) All four of these parameters were unmeasured in the experiments.

Table 1: Four Parameters for Model Updating

Parameter Name	Parameter Code	Min Value	Max Value
Angle #1	A	0 deg	1 deg
Angle #2	B	0 deg	1 deg
Bolt Preload	C	0 lb	500 lb
Input Scaling	D	0.9	1.1

Parameter Updating using the Metamodel

The four parameters in the metamodel will be updated to establish estimates of their true values during the experiment. Using these results, an uncertainty bound can be established on these parameters for future replications of the experiment. The model updating process begins with the definition of an objective function, J, which in this case is the 2-norm of a vector formed using the error in peak accelerations ΔP_i between measured and simulated values, along with the perturbation values of the simulation parameters. (The parameter values are included in the objective function to avoid non-uniqueness in the solution, as demonstrated below.)

$$J = \left\| \Delta P_1 \quad \Delta P_2 \quad \Delta P_3 \quad A \quad B \quad C \quad D \right\|_2 \tag{2}$$

For each update result described below, the parameters are each assigned an initial value of 0, and the optimization is run until J converges with an error tolerance of 1e-12. The measured peak accelerations at each sensor i for each trial j are denoted $\hat{P}_{ij}$, and the mean values over all 10 trials are denoted $\overline{P}_i$, where $\overline{P}_i = \sum_j \hat{P}_{ij}$.

The first update result is obtained by using the mean of the measured values $\overline{P}_i$. This update yields "mean" updated encoded values for the parameters of

$$\lfloor A \quad B \quad C \quad D \rfloor = \lfloor -0.61 \quad 1.06 \quad -0.10 \quad -0.04 \rfloor \tag{2}$$

The surface of the metamodel over C and D for P_1 at these values of A and is shown in Figure 6. To illustrate the range of combinations of C and D that produce P_i values equal to the measurements, a plane is plotted at $P_1 = \overline{P}_1$. The line of intersection between these two planes represents a continuum of C, D combinations that satisfy $P_1 = \overline{P}_1$. The inclusion of the $\lfloor A \quad B \quad C \quad D \rfloor$ values in the objective function of Eqn. (1) ensures that the point on this line closest to the origin is selected. (The rationale being that the point with minimum perturbation from the assumed nominal values of $\lfloor A \quad B \quad C \quad D \rfloor$ deviates the least from the best judgment-based estimate of the parameter values, and so is the "most likely" solution.) A contour plot of the line of intersection, along with the nominal parameter values (the origin) and the updated parameter point is shown in Figure 7.

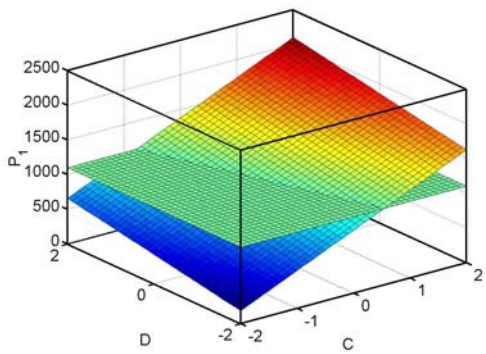

Figure 6: Metamodel of P_1 vs. C and D with Plane at $\overline{P}_1$

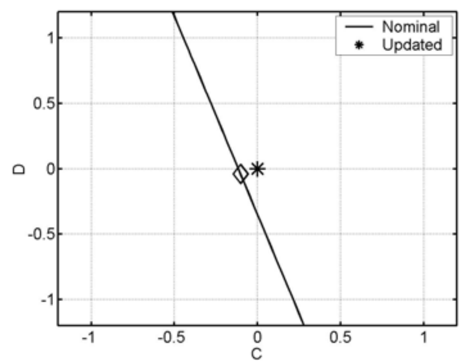

Figure 7: Line of Intersection from Figure 1 with Nominal and Updated Parameter Values

To understand the variability in A,B,C,D from test-to-test, the update is performed on each of the 10 experimental measurement results. Overlaying these results with their corresponding experimental-metamodel contours yields the results shown in Figure 8. All of the updated C,D combinations lie precisely on their corresponding line contours, except for trial #9. Examining Figure 9 indicates that the optimized point for Trial #9 does not produce P_1 predictions that are as accurate as the other 9 trials. This is possibly a result of influences of other variables that have not been included in the update set.

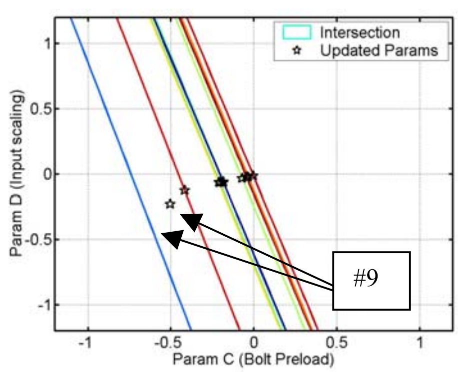

Figure 8: Lines of Intersection for Each Experimental Trial With Corresponding Updated Parameter Values

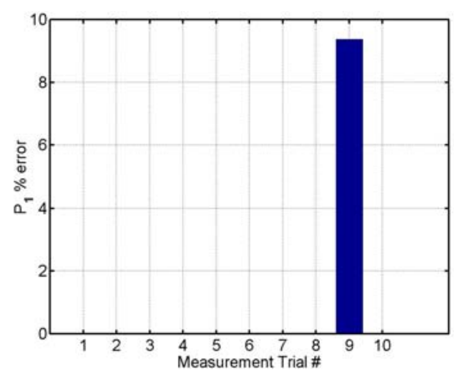

Figure 9: Error Between Updated P_1 Predictions and P_1 measurements

Finally, the updated values of A,B,C and D can be analyzed to assess the variability from test-to-test. Figure 10 shows a scatter plot of the A, B, C, D updates for each experimental trial. The spread of these values gives a range of variability that can be expected for each parameter for a repetition of this experiment. The sample mean and sample standard deviation for each parameter is shown in Table 2. Thus the uncertainty associated with each of these 4 parameters has been estimated.

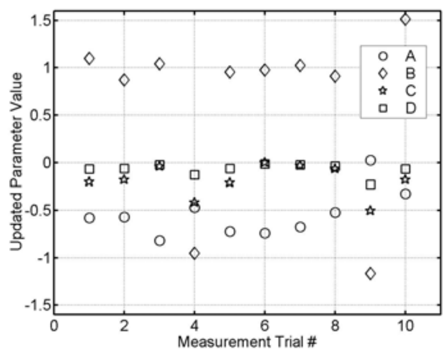

Figure 10: Updated Parameter Values for Each Experimental Trial

Table 2: Encoded and Decoded Mean and STD Values for Updated Parameters

Parameter	Encoded Mean	Encoded STD	Mean	STD
Impact Angle 1	-0.54	0.24	0.23 deg	0.12 deg
Impact Angle 2	0.63	0.91	0.81 deg	0.45 deg
Bolt Preload	-0.18	0.17	203.90 lb	41.3 lb
Input Scaling	-0.071	0.065	0.99	0.0064

References

1 Schlesinger, S., et al., 1979, “Terminology for Model Credibility,” *Simulation* **32**(3), 103-104.

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3 Myers, R.H. and Montgomery, D.C., 1995, *Response Surface Methodology*, Wiley & Sons.