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RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

UNIVERSITY OF VIRGINIA

Charlottesville, Virginia 22901

Annual Report
on NASA Grant NSG-1509

EVALUATING AND MINIMIZING NOISE IMPACT DUE TO AIRCRAFT FLYOVER

Submitted to:

NASA Scientific and Technical Information Facility
P.O. Box 8757
Baltimore/Washington International Airport
Baltimore, MD 21240

Submitted by:

Ira D. Jacobson
Associate Professor

Gerald Cook
Professor

Report No. UVA/528166/MAE79/101

May 1979

RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

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I. INTRODUCTION

This report presents the results of a study on the evaluation and reduction of noise impact to a community due to aircraft operation. Existing techniques have been used to assess the noise impact and to optimize the flight paths of an approaching aircraft with respect to the annoyance produced. Major achievements have been: (1) the development of a population model suitable for determining the noise impact, (2) generation of a numerical computer code which uses this population model along with the steepest descent algorithm to optimize approach/landing trajectories, (3) implementation of this optimization code in several fictitious cases as well as for the community surrounding Patrick Henry International Airport.

Previous work has centered on developing noise annoyance criteria for flyover (i.e. NEF, NNI, CNR, etc.) and ground noise signatures for aircraft. Some of these criteria are discussed in References 1-5 with a review of many of the noise effect measures being summarized in Ref. 6. Typical of the noise footprint work is Ref. 7. The annoyance criterion used in the study is the noise impact index (NII).

The details of the models used, their advantages and disadvantages and the results obtained are outlined in the following sections.

II. PROBLEM FORMULATION

A. Overview

Analysis of the problem consists of six parts: (1) aircraft noise signatures, (2) population models, (3) cost (annoyance) function, (4) aircraft flight-path model, (5) aircraft constraints, and (6) approach/landing path optimization. A modular concept has been employed so that modification of any of these segments may be effected with relative ease. The sections below describe each of these parts in detail.

B. A/C Noise Signature

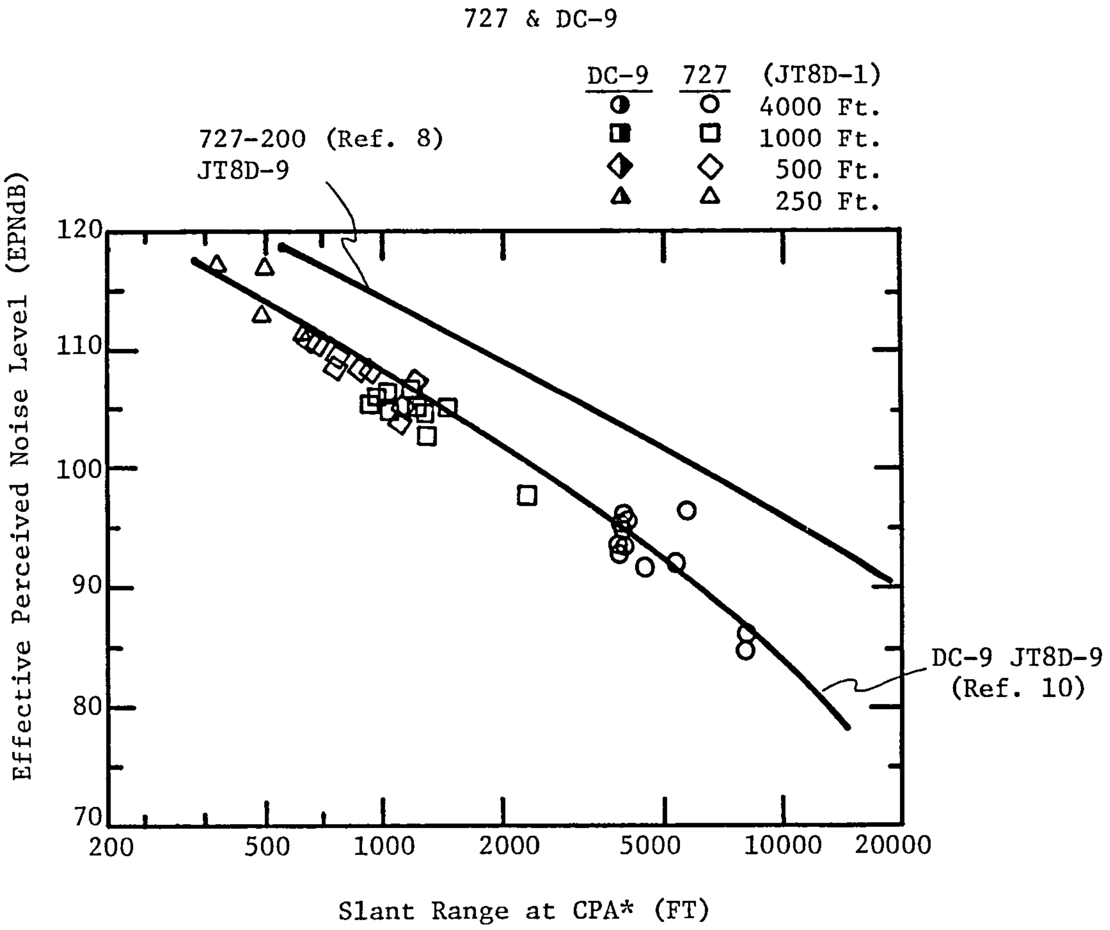
The aircraft noise signature is obtained using data from Ref. 3. Here the effective perceived noise level (EPNdB) is given as a function of slant range to the closest point of approach for a variety of aircraft. A typical plot of the slant range variation is shown in Figure 1. These data were fit using standard least squares techniques to yield an expression for EPNdB given by

$$\text{EPNdB} = 115-22.5 \log_{10} x \text{ (Slant Range)}. \tag{1}$$

This equation is used for calculation of the maximum noise level at each location for a flyover. A typical footprint for a straight in approach along a 3-degree glide slope is shown in Figure 2.

C. Population Model

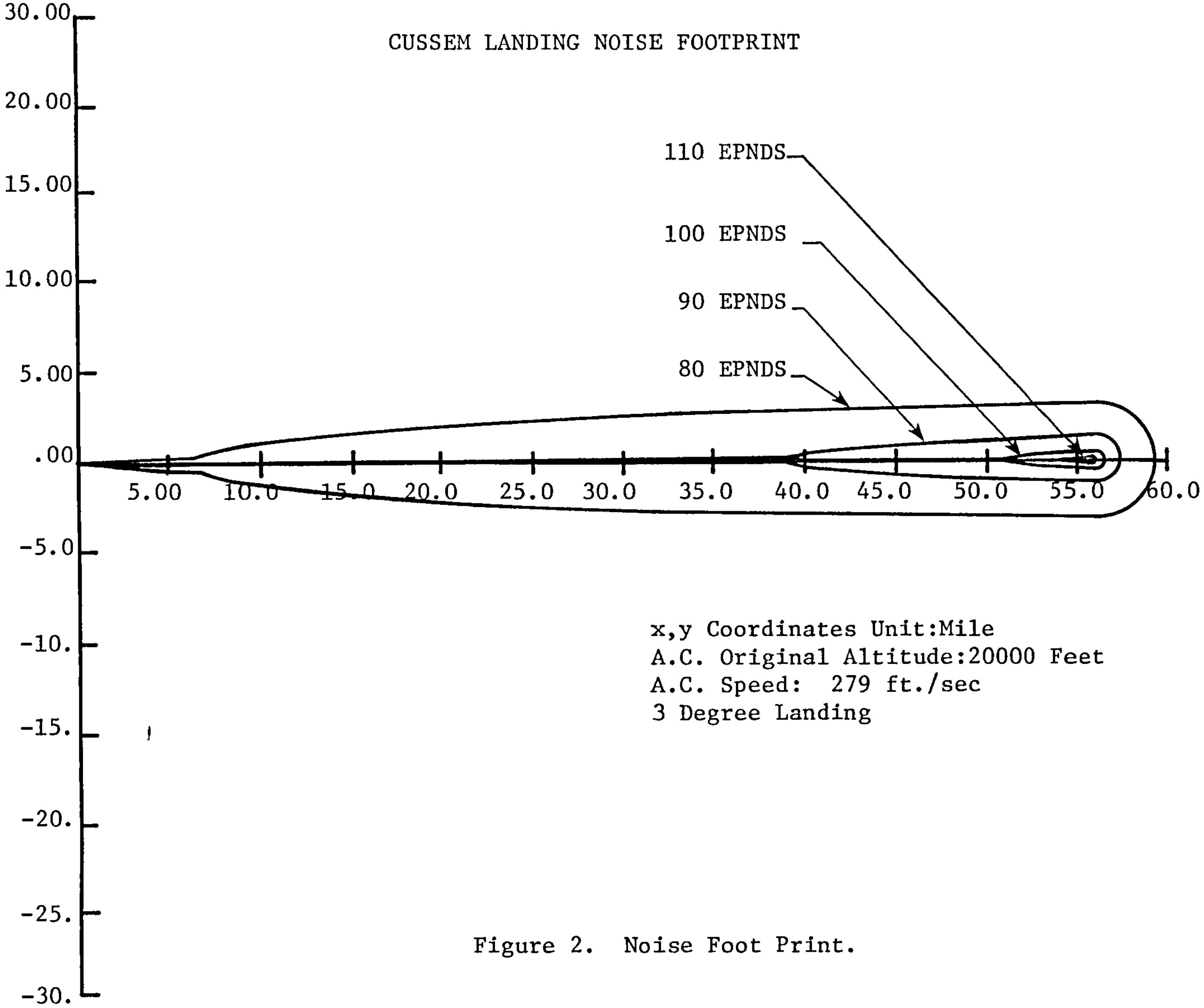
To model the population, a map of the community is overlaid with a grid and the population in each section of the grid determined. The population distribution within each section is assumed to be uniform. Several grid geometries were examined (see Figure 3). These geometries



<u>FLYBY NOISE LEVEL</u>
(1.93 - 1.95 EPR 727 Aircraft) FIG. A-1**
(1.94 EPR DC-9 Aircraft) FIG. D-1**

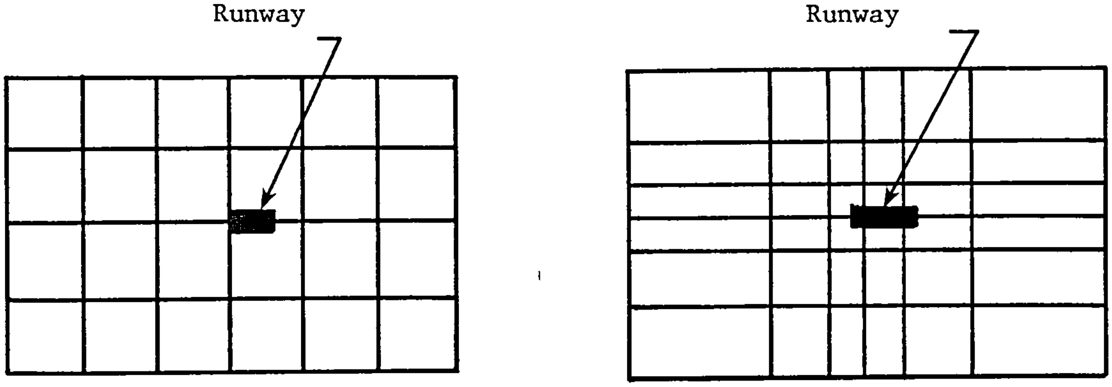
*Closest Point of Approach
**FAA-RD-71-83 (Ref. 6)

Figure 1. EPNL vs. Slant Range

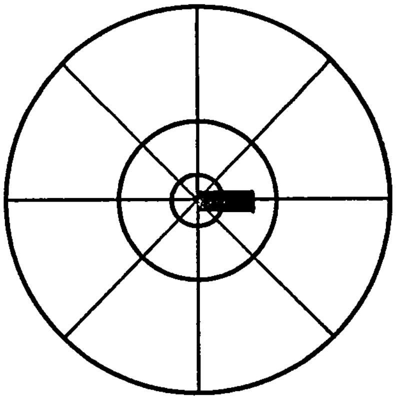

CUSSEM LANDING NOISE FOOTPRINT

x,y Coordinates Unit:Mile
A.C. Original Altitude:20000 Feet
A.C. Speed: 279 ft./sec
3 Degree Landing

Figure 2. Noise Foot Print.



1. Equal Size Blocks 2. Variable Size Blocks



3. Concentric Circles

Figure 3. Grid Geometries.

included: (1) rectangular sections of equal size, (2) rectangular sections whose dimensions increased with distance from the airport runway, and (3) concentric circles divided by several radial lines. The second scheme was chosen since it requires fewer rectangular sections than the first and is easier to implement than the third. Computer time required for determining the optimum trajectory varies directly with the number of grid sections. Furthermore, in light of the dependence of noise levels on distance and the fact that the aircraft has higher altitude when further from the runway, the need for high resolution of the population density diminishes with distance from the airport. While the third population scheme could offer this same advantage, it is somewhat more difficult to determine the population and noise impact for each grid section with such a geometry.

Within a grid section, the population is determined by use of the SITE II system, (Ref. 8), available on the CDC 7600 computer at the NASA-Langley facility. This system requires as input the latitude and longitude of a reference point and the coordinates of the corners of each rectangular section. Although SITE II allows for simple retrieval of 1970 census data, there is some question about its resolution capabilities for small grid sections. In addition, in rapidly growing areas the population data may lag actual population. The SITE II program is capable of producing detailed census information as shown in Figure 4. However for the present analysis only population information is used.

```
                                   DEMOGRAPHIC PROFILE REPORT          PAGE  1
SEVEN CORNERS
SALES        TERRITORY
SITE TOTAL                           * * * * * * * * * * * * * * * * * * * * *
                                     *                             1970-1975 *
       DEG  MIN  SEC                 *                        1975   CHANGE  *
LATITUDE   38  52  10                * POPULATION           369003   -18006  *
LONGITUDE  77   9  20                * HOUSEHOLDS           138552     1076  *
                                     * PER CAPITA INCOME    $ 7464   $ 2384  *
                                     * *
                                     *   ANNUAL COMPOUND GROWTH  -0.9%       *
                                     * * * * * * * * * * * * * * * * * * * * *

                                  1970 CENSUS DATA

POPULATION                       AGE AND SEX
TOTAL     387009  100.0%                     MALE          FEMALE         TOTAL
WHITE     367224   94.9%  0-5      19328  10.4%     18646   9.2%          9.8%
NEGRO      15414    4.0%  6-13     26757  14.5%     25269  12.5%         13.4%
OTHER       4371    1.1%  14-17    13645   7.4%     13194   6.5%          6.9%
                          18-20     7536   4.1%     10413   5.2%          4.6%
SPAN       13839    3.6%  21-29    35499  19.2%     39587  19.6%         19.4%
                          30-39    23840  12.9%     22964  11.4%         12.1%
                          40-49    23476  12.7%     27719  13.7%         13.2%
FAMILY INCOME (000)       50-64    27112  14.7%     30045  14.9%         14.8%
$0-5        7945    7.8%  65 +      7859   4.2%     14113   7.0%          5.7%
$5-7        6942    6.8%  TOTAL  185052            201950
$7-10      14752   14.4%  MEDIAN(AGE)      27.4               28.6       28.0
$10-15     25949   25.4%
$15-25     32623   31.9% HOME VALUE (000)          OCCUPATION
$25-50     12867   12.6% $0-10        339   0.7%   MGR/PROF    68537  41.8%
$50 +       1109    1.1% $10-15      1084   2.1%   SALES       12291   7.5%
TOTAL     102187         $15-20      4450   8.6%   CLERICAL    48735  29.8%
                         $20-25      8491  16.3%   CRAFT       12810   7.8%
AVERAGE  $15763          $25-35     17183  33.1%   OPERTIVS     6010   3.7%
MEDIAN   $14134          $35-50     14380  27.7%   LABORER      2144   1.3%
                         $50 +       6012  11.6%   FARM          114   0.1%
                          TOTAL     51939          SERVICE     11469   7.0%
RENT                                               PRIVATE      1663   1.0%
$0-100      8737  10.5%  AVERAGE $34161
$100-150   35292  42.5%  MEDIAN  $31754
$150-200   28662  34.5%  % OWNER   38.5            EDUCATION  ADULTS > 25
$200-250    6645   8.0%                            0-8         20729   9.6%
$250 +      3792   4.6%                            9-11        24297  11.3%
TOTAL      83128         AUTOMOBILES               12          69170  32.0%
                         NONE       13451   9.8%   13-15       37764  17.5%
AVERAGE    $ 150         ONE        71744  52.2%   16 +        64003  29.6%
MEDIAN     $ 147         TWO        44475  32.3%
% RENTER    61.5         THREE+      7872   5.7%
                                                   HOUSEHOLD PARAMETERS
                                                   FAM POP    335153  86.6%
UNITS IN STRUCTURE       HOUSEHOLDS WITH:          INDIVIDS    45881  11.9%
1          66945  48.7%  TV        126239  91.8%   GRP QTRS     5975   1.5%
2           1304   0.9%  WASHER     71594  52.1%   TOT POP    387009
3-4         5510   4.0%  DRYER      54258  39.5%
5-9        11809   8.6%  DISHWSH    56277  40.9%  NO OF HH'S     137476
10-49      31569  23.0%  AIRCOND    79438  57.8%  NO OF FAM'S    101961
50 +       20288  14.7%  FREEZER    28600  20.8%  AVG HH SIZE       2.8
MOBILE       125   0.1%  2 HOMES     2856   2.1%  AVG FAM SIZE      3.3
                                                                 CACI,INC
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Figure 4. Demographic Profile Report

D. Flight Path Model

There are two ways in which the trajectory of the aircraft can be determined. In one, a discrete time integration of the equations of motion with control deflections yields point by point spatial coordinates and orientation. Although this allows the flexibility of building in control constraints as well as dynamical constraints (e.g. max roll angle) it requires a considerable number of states to be stored in the optimization routine. In a multi-aircraft, multi-runway problem, as is anticipated, these storage requirements become prohibitive.

Thus, another method was adopted which utilizes only the functional form of the trajectory to describe the flight path. First a starting path was assumed which went from the initial point to the desired runway and ended up with the proper heading, i.e., velocity vector aligned with the runway. The following equation was used to generate this starting trajectory. (See Figure 5).

$$y_s(x) = \left[\left(\frac{y_f - y_p}{x_f - x_p}\right) (x-x_p) + (y_p-y_o)\right] \text{EXP} \left[-C(x-x_f)/(x_o-x_f)\right] + y_o \tag{2}$$

For the vertical motion a simple three degree descent path was assumed.

Next the first five Fourier sine harmonics were used to introduce deviation from this starting path. One advantage in using this type representation is the fact that each of the forms contributes zero at the end points. Therefore if the starting path satisfies the boundary conditions the curve with the deviations will also. An exponential decay at the final point was used to eliminate heading deviations.

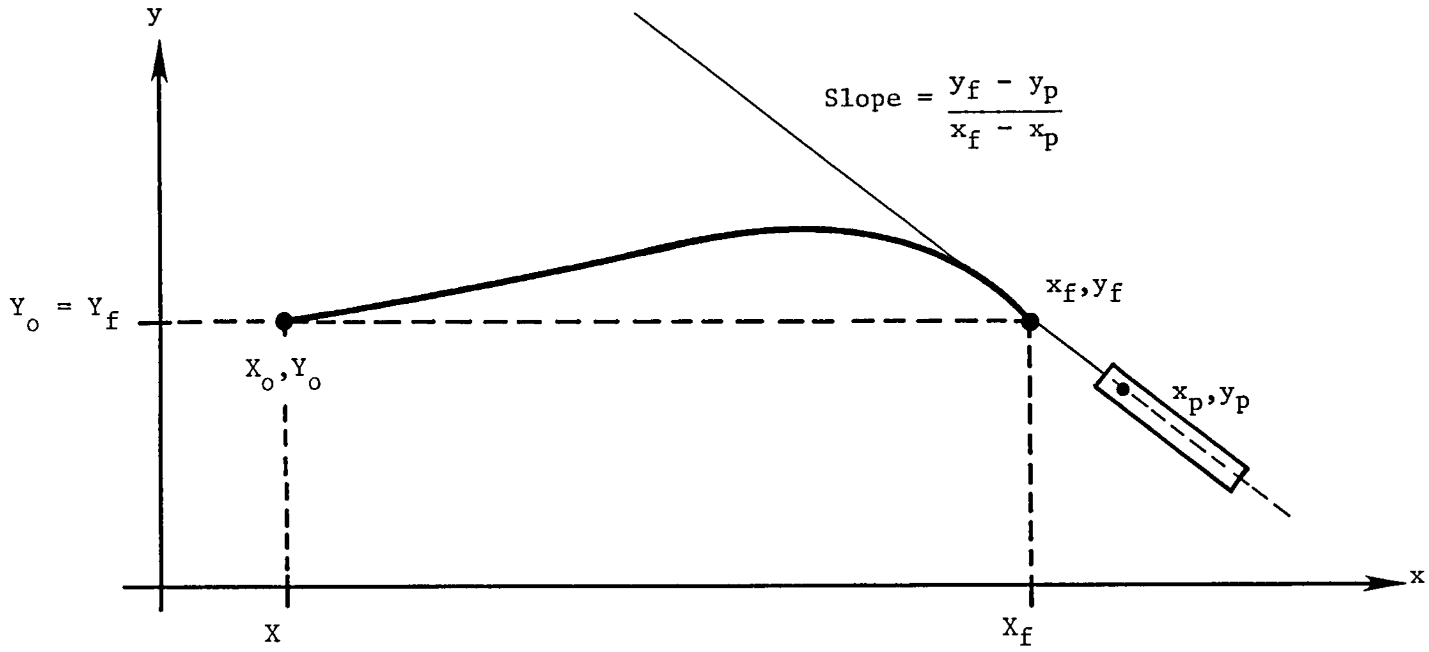

Figure 5. Rotated Coordinate System for Establishing Nominal Flight Trajectory from Initial Point to Runway Approach.

The equations with the deviations thus become

$$y(x) = \left\{ \sum_{i=1}^{5} \alpha_i \sin[\pi i(x-x_o)/(x_f-x_o)] \right\} \left\{ 1-\mathrm{EXP}[(x-x_f)/C_i] \right\} + y_s(x) \tag{3a}$$

$$Z(x) = \left\{ \sum_{i=1}^{5} \beta_i \sin[\pi i(x-x_o)/(x_f-x_o)] \right\} \left\{ 1-\mathrm{EXP}[(x-x_f)/C_i] \right\} + Z_s(x) \tag{3b}$$

A second advantage to using a Fourier Series representation for the curve is that it provides a means of representing a function with a finite number of parameters. This reduces the optimization problem from a variational one to an ordinary one.

E. Constraints

The use of a functional form of the flight path for the trajectory requires the reformulation of constraints into parameters which can be used in the optimization. This is accomplished by translating the steady state solutions of the lateral and longitudinal perturbation equations into geometric constraints. An exact derivation is given in Appendix A. The constraints are incorporated by determining maximum curvature and slope parameters as a function of aerodynamics and physical constraints. For example the constraint of a maximum roll angle, ϕ_{max}, yields

$$\frac{\frac{d^2y}{dx^2}}{1+\left(\frac{dy}{dx}\right)^2} \leq \frac{C_2+C_1C_3}{C_4+C_1C_5} \; \frac{\phi_{max}}{V_{avg}} \tag{4}$$

where C_1 through C_5 depend upon aircraft stability and control derivations (see Appendix A for details) and V_{avg} is the average velocity. Similar expressions are given in Appendix A for constraints on aileron rudder and elevator deflection, flight path angle and pitch rate limits.

F. Cost Function

A large number of criteria have been proposed by evaluating noise annoyance (e.g., EPNdB, NII, sleep interference index, speech interference index, etc.). The recent trend in noise assessment work is toward a universal measure--the noise impact index (NII). This measure is a weighted day-night model which accounts for population density. It is described in detail in Ref. 9. Briefly, the total population exposed to each incremental average day-night model sound level is multiplied by the weighting function for the level. The weighting function used in shown in Figure 6. This weighting factor $W(L_{dn})$ multiplied by the population exposed to that L_{dn} is summed and normalized by the total population giving the Noise Impact Index for the area.

$$\mathrm{NII} = \frac{\sum_{L_{dn}} P(L_{dn})W(L_{dn})}{\sum_{L_{dn}} P(L_{dn})} \tag{5}$$

The cost function or payoff for the optimization procedure is taken to be the NII plus penalties for violating constraints. Basically the optimization procedure is set up to "drive" the aircraft trajectory to the path which will minimize the NII and at the same time not violate any constraints. As an example of the constraint of flight path angle not exceeding a maximum descent angle, γ_d, nor a maximum climb angle, γ_c, is written as

$$\tan\gamma_c < \frac{dZ}{dx} < \tan\gamma_d \tag{6}$$

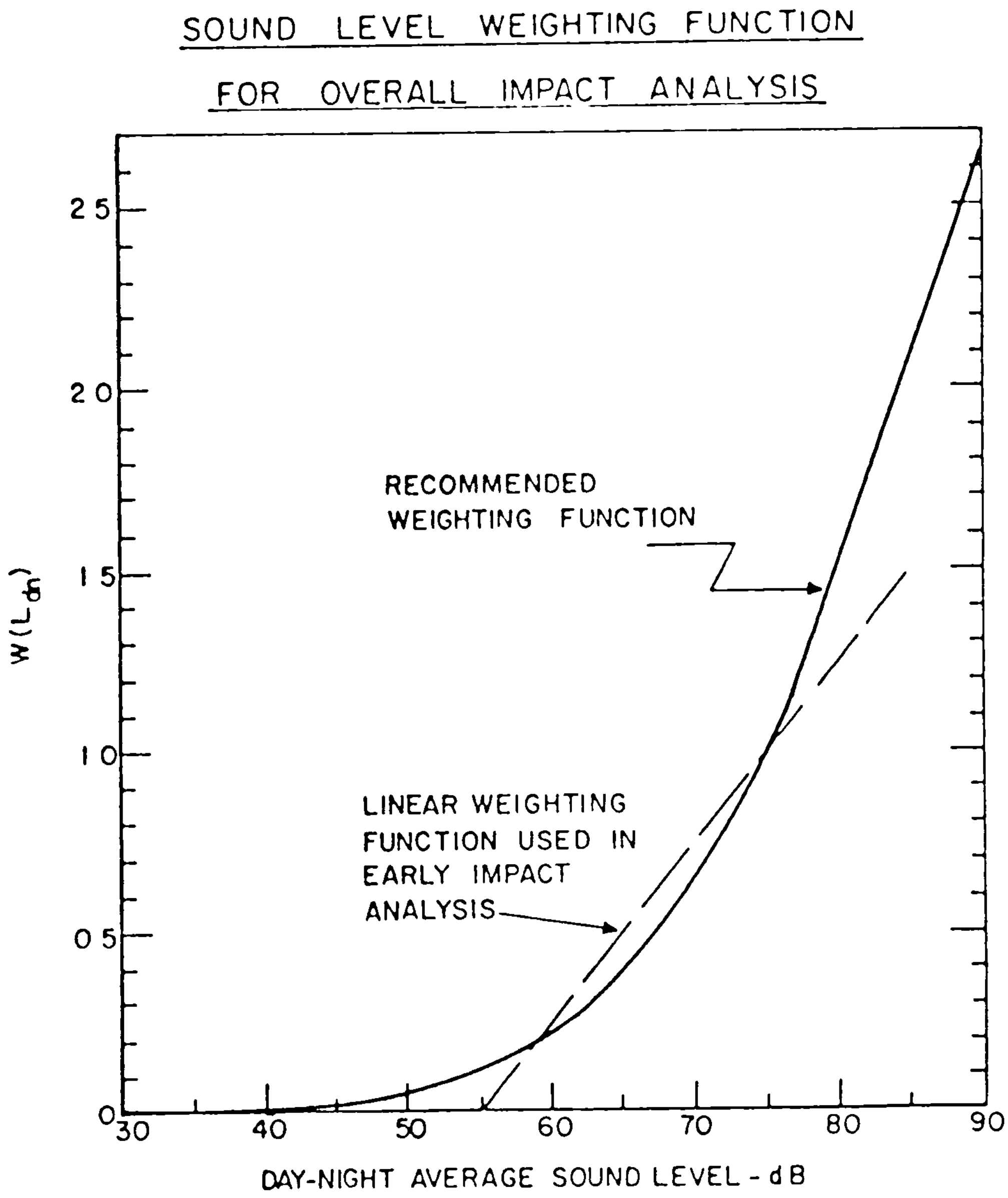

Figure 6. Sound Level Weighting Function for Overall Impact Analysis.

Each is converted to a penalty which is added to the NII in the form

$$\text{Cost} = \text{NII}+(\frac{dZ}{dx}/\tan\gamma_d)^{20} + (\tan\gamma_c/\frac{dZ}{dx})^{20} \tag{7}$$

As is seen for values of the flight path angle within the allowable range the penalty is negligible; however for values outside this range the penalty and thus the increase in cost is great. Other terms are added in a like manner.

III. OPTIMIZATION

The optimum trajectory is determined by calculating values of the α_i's and β_i's (Eq. 2) which minimize the total cost (NII plus penalties). A steepest descent algorithm is employed here. Basically, this method computes the gradient of the cost function, C, with respect to the α_i's and β_i's, then searches along the negative gradient direction for values of α_i's and β_i's which reduce the cost. The change in cost is given by

$$\Delta C = \sum_{i=1}^{5} \left(\frac{\partial C}{\partial \alpha_i} \Delta\alpha_i + \frac{\partial C}{\partial \beta_i}\Delta\beta_i\right) \tag{8}$$

The process continues iteratively until the cost converges to within a specified tolerance. While implementation of the algorithm is fairly straightforward, convergence near the optimal set of α_i's and β_i's is inherently slow. Most of the cost reduction, however, occurs in the first few iterations.

A. The Optimization Algorithm

A computer code has been developed which implements the functions described above. Figure 7 shows a flow chart for this code. Initial data (population map, aircraft constraints, initial and final aircraft positions, etc.) are required for each airport/airplane configuration to be evaluated. To facilitate calculation of the Fourier coefficients, the coordinate axes are rotated such that a line joining the initial and final aircraft positions is made to be parallel to the x axis. A nominal trajectory is generated which constrains the heading of the aircraft to asymptotically approach the runway. The steepest descent search then begins and continues until the stopping criterion is met. This criterion

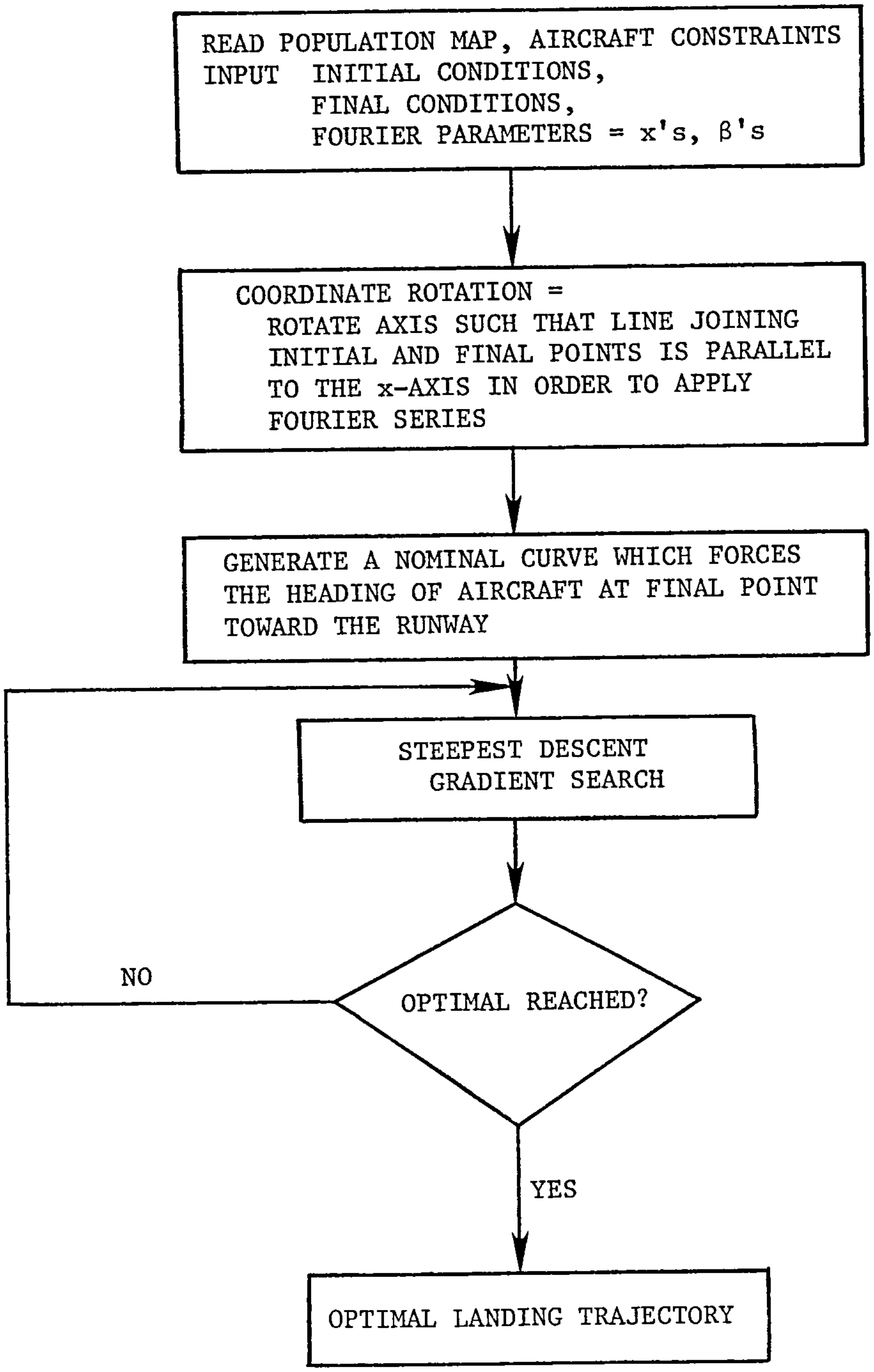

Figure 7. Flow Chart.

is met if successive improvements become negligible.

In order to provide a more accurate noise impact in each population section the impact is integrated using quadratures. This procedure can be found in Ref. 9.

The various functions such as the population model, the cost function, and the aircraft signature are incorporated as subroutines. This will allow ease of upgrading or modification if different models are desired.

Appendix B contains the Fortran code as written for a CDC Cyber 172 machine.

B. Results

Several cases have been run to test the benefits that can be obtained by this approach. First, a fictitious set of data incorporating a population valley is used. As can be seen in Figure 8 the optimization algorithm moves the aircraft (a Convair 880) towards the valley (i.e. fewer people impacted) with a corresponding improvement in the NII of 32%.

The second case models the Patrick Henry Airport in Hampton, Virginia. Here the SITE II program was used to generate the census data for each block as shown in Figure 9. Two initial trajectories were flown. One entering the area from the northwest over the Swing VOR station and the other from the southwest over the Franklin VOR station. Both of these paths are specified IFR trajectories (Figure 10). The aircraft enters the area approximately 30,000 meters from the runway. In addition several straight-in paths were evaluated. Figure 9 shows each of the trajectories. The associated

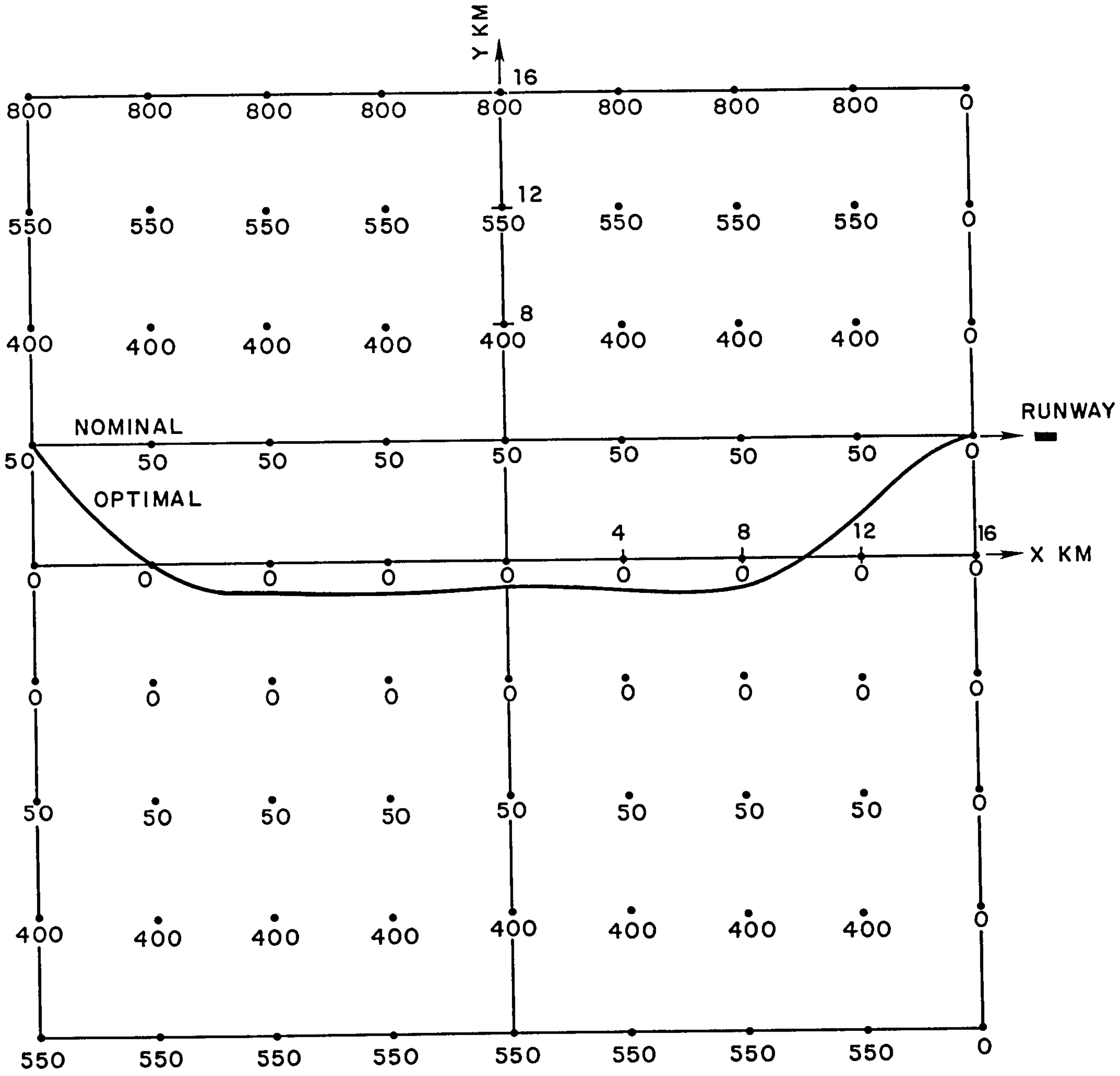

Figure 8. Optimization Results Using Fictitious Population Data.

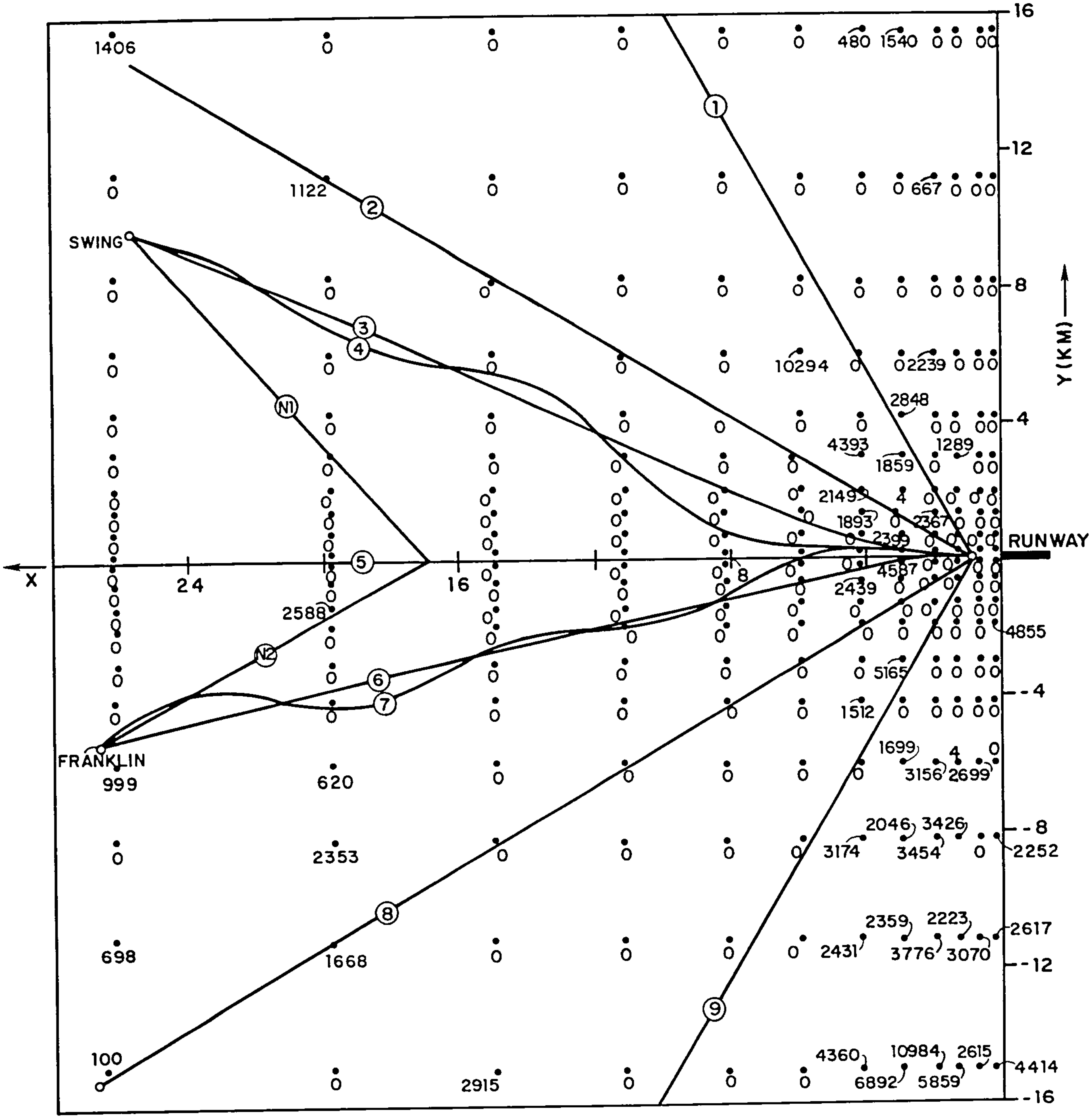

Figure 9. Population Model and Optimization Results for Patrick Henry Airport.

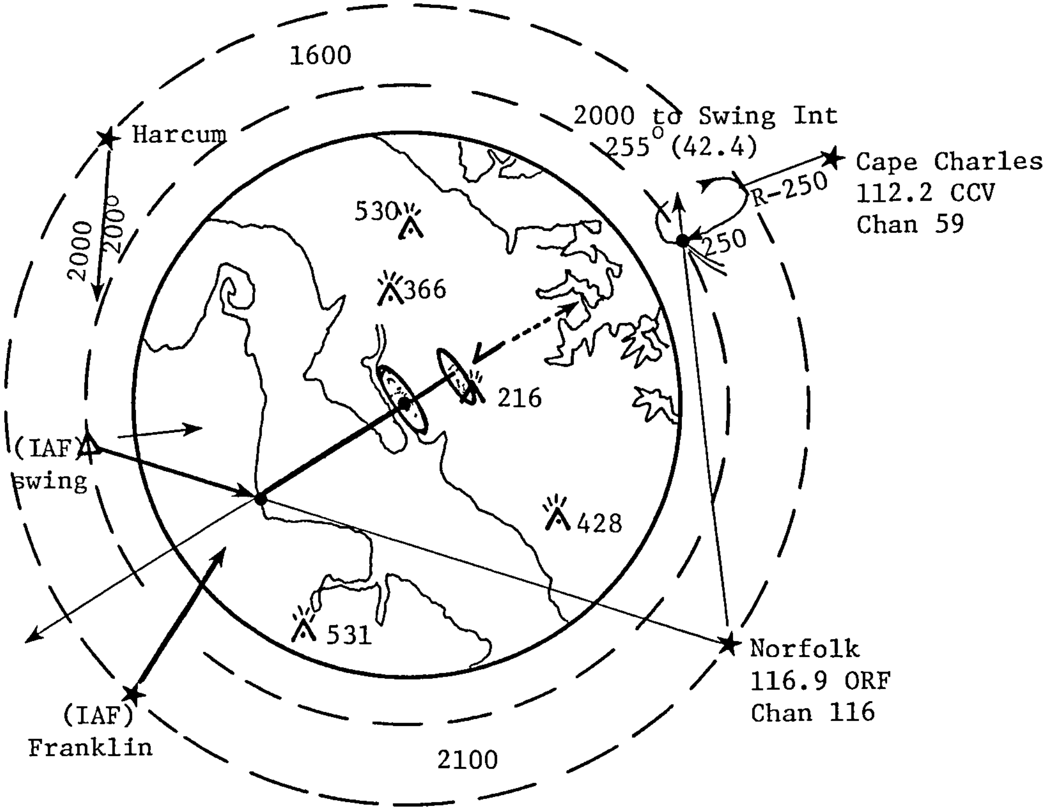

Figure 10. Conventional Approach Pattern

Table I

Northwest Approach

Entry Point: Swing

Traj. No.	Description	Cost (NII x 10^{-2})	% Change from Present
1	60 deg wrt runway	2.373	+3.2%
2	30 deg wrt runway	2.438	+6.0%
3	Initial iteration	2.27	-1.3%
4	Optimal	2.213	-3.8%
5	Straight in	2.316	+.1%
N1	Presently used	2.300	0%

Table II

Southwest Approach

Entry Point: Franklin

Traj. No.	Description	Cost (NII x 10^{-2})	% Change From Present
5	Straight in	2.316	-1.3%
6	Initial iteration	2.408	+2.6%
7	Optimal	2.241	-4.5%
8	30 deg wrt runway	2.598	+10.7%
9	60 deg wrt runway	2.687	+14.5%
N2	Presently used	2.346	0%

NII's are summarized in Tables I and II.

As is seen that even for this case, where the population is sparse away from the runway and congested near the end of it, an improvement of 3 to 5% is achieved using the optimization algorithm. It should also be noted that this technique allows not only the optimization of the path but also can be used to evaluate existing or proposed paths, such as the nominal and straight-in paths indicated.

Conclusion

A method has been formulated to optimize the path of an aircraft during approach or take-off from any airport. Models have been developed using available data where possible for population, aircraft signature, noise impact, constraints and flight path. An algorithm using steepest descent has been implemented and tested. This approach allows

1) The evaluation of the noise impact of existing flight paths,
2) The evaluation of the noise impact of proposed flight paths, and
3) The optimization of the flight path to minimize the noise impact under constraints.

This method has been applied to the Patrick Henry International Airport. Both nominal and other straight paths were evaluated. Also an optimal path was determined for each of two terminal area entry points. Performance ranged from a 15% degradation of the NII to 4.5% improvement compared to the presently used approaches. It is significant that as much as 3 to 5% improvement could be achieved in light of the fact that most of the population is concentrated at the end of the runway.

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APPENDIX A

Derivation of Parameterized Trajectory Constraints

Lateral perturbation equations

$$\text{Y eq'n:}\quad -\frac{b}{2Vr}\,C_{y_p}\dot{\phi} - \frac{mg}{q_\infty S}\cos\phi_o\phi + \left(\frac{mV_T}{q_\infty S} - \frac{b}{2V_T}\,C_{y_r}\right)\dot{\Psi} - \frac{mg}{q_\infty S}\sin\theta_o\Psi$$

$$+\frac{mV_T}{q_\infty S}\,\dot{\beta} - C_{y_\beta}\beta = C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r$$

$$\text{L eq'n:}\quad \frac{I_{xx}}{q_\infty Sb}\ddot{\phi} - \frac{b}{2V_T}\,C_{\ell_p}\phi - \frac{I_{xz}}{q_\infty Sb}\,\ddot{\Psi} - \frac{b}{2V_T}\,C_\ell\,\dot{\Psi} - C_{\ell_\beta}\beta = C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r$$

$$\text{N eq'n:}\quad -\frac{I_{xz}}{q_\infty Sb}\,\ddot{\phi} - \frac{b}{2V_T}\,C_{n_p}\dot{\phi} + \frac{I_{zz}}{q\ Sb}\ddot{\Psi} - \frac{b}{2V_T}\,C_{n_r}\Psi - C_{n_\beta}\beta = C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \qquad (1)$$

If we assume all turns to be coordinated (no sideslip)

Then letting $-\frac{b}{2V_T}\,C_{y_p} = \overline{C}_{y_p}$, etc.

$$\frac{mg}{q_\infty S}\cos\theta_o = \overline{g}_1 \quad \frac{mg}{q_\infty S}\sin\theta_o = \overline{g}_2$$

$$\frac{I_{xx}}{q_\infty Sb} = i_x,\ \text{etc.}\ \frac{mV_T}{q_\infty S} = \overline{m}$$

$$\text{L eq'n:}\quad i_x\ddot{\phi} - C_{\ell_p}\dot{\phi} - i_x Z\ddot{\Psi} - \overline{C}_{\ell_r}\dot{\Psi} = C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r$$

$$\text{N eq'n:}\quad -ixZ\ddot{\phi} - \overline{C}_{n_p}\dot{\phi} + iZ\ddot{\Psi} - \overline{C}_{n_r}\dot{\Psi} = C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$$

$$\text{Y eq'n:}\quad -\overline{C}_{y_p}\dot{\phi} - \overline{g}_1 + (\overline{m} - \overline{C}_{y_r})\dot{\Psi} - \overline{g}_2\Psi = C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r \qquad (2)$$

Taking the Laplace transform (I.C.'s = 0)

$$\text{L eq'n:}\quad (i_x s^2 - \overline{C}_{\ell_p}s)\ \phi\ (s) + (-i_x Z s^2 - \overline{C}_{\ell_r}s)\ \Psi\ (s) = C_{\ell_{\delta_a}}\delta_a(s) + C_{\ell_{\delta_r}}\delta_r(s)$$

$$\text{N eq'n:}\quad (-i_{xZ}s^2-\overline{C}_{n_p})\,\phi\,(s) + (i_Z s^2-C_{n_r}s)\,\Psi\,(s) = C_{n_{\delta_a}}\delta_a(s)+C_{n_{\delta_r}}\delta_r\,(s)$$

$$\text{Y eq'n:}\quad (-\overline{C}_{y_p}s-\overline{g}_i)\,\phi(s) + [(\overline{m}-\overline{C}_{y_p})s-\overline{g}_2]\Psi\,(s) = C_{y_{\delta_a}}\delta_a(s)+C_{y_{\delta_r}}\delta_r(s) \tag{3}$$

To determine the required δ_a for a given δ_r we consider δ_a an unknown along with $\phi(s)$ and $\Psi(s)$ [i.e. move δ_a to the left hand side of the equations] and solve for δ_a/δ_r using Cramer's rule

$$\frac{\delta_a}{\delta_r} = \left| \frac{\begin{matrix} i_x s^2-\overline{C}_{\ell_p}s & -i_{xZ}s^2-\overline{C}_{\ell_r}s & +C_{\ell_{\delta_r}} \\ -i_xZs^2-\overline{C}_{n_p}s & i_Z s^2-C_{n_r}s & +C_{n_{\delta_r}} \\ -\overline{C}_{y_p}s-\overline{g}_1 & (\overline{m}-\overline{C}_{y_p})s-\overline{g}_2 & +C_{y_{\delta_r}} \end{matrix}}{\begin{matrix} i_x s^2-\overline{C}_{\ell p}s & -i_xZs^2-\overline{C}_{\ell_r}s & -C_{\ell_{\delta_a}} \\ -i_xZs^2-\overline{C}_{np}s & +i_Z s^2-\overline{C}_{n_r}s & -C_{n_{\delta_a}} \\ -\overline{C}_{y_p}s-\overline{g}_1 & (\overline{m}-\overline{C}_{y_p})s-\overline{g}_2 & -C_{y_{\delta_a}} \end{matrix}} \right| = \frac{N(s)}{\Delta(s)} \tag{4}$$

The denominator (characteristic eqn.) is given by:

$$\begin{aligned} \Delta(s) = {} & s^4\{-C_{y_{\delta_a}}(i_x i_Z-i_x^2Z)\}+s^3\{C_{y_{\delta_a}}[i_Z\overline{C}_{\ell_p}+i_x\overline{C}_{n_r}+i_{xZ}(\overline{C}_{\ell_r}+\overline{C}_{n_p})] \\ & +C_{n_{\delta_a}}[-i_xZ\overline{C}_{y_p}+(\overline{m}-\overline{C}_{y_r})i_x]+\overline{C}_{\ell_{\delta_a}}[i_x\overline{C}_{y_p}-(\overline{m}-\overline{C}_{y_r})i_xZ]\} \\ & +s^2\{C_{y_{\delta_a}}(\overline{C}_{n_p}\overline{C}_{\ell_r}-\overline{C}_{\ell_p}\overline{C}_{n_r})+C_{n_{\delta_a}}[-i_xZ\overline{g}_1-i_x\overline{g}_2-\overline{C}_{y_p}\overline{C}_{\ell_r}-(\overline{m}-\overline{C}_{y_r})\overline{C}_{\ell_p}] \\ & +C_{\ell_{\delta_a}}[\overline{g}_1 i_Z-\overline{g}_2 i_xZ-\overline{C}_{y_p}\overline{C}_{n_r}+(\overline{m}-\overline{C}_{y_r})\overline{C}_{n_r}]\} \\ & +s\{C_{n_{\delta_a}}(\overline{g}_2\overline{C}_{\ell p}-\overline{g}_1\overline{C}_{\ell r})+C_{\ell_{\delta_a}}(\overline{g}_2\overline{C}_{np}-\overline{g}_1\overline{C}_{n_r})\} \end{aligned} \tag{5}$$

The numerator is:

$$N(s) = s^4\{C_{y_{\delta_r}}(i_x i_Z - i_x^2 Z\} + s^3\{-C_{y_{\delta_r}}[i_Z\bar{C}_{\ell_p} + i_x\bar{C}_{n_r} + i_{x_Z}(\bar{C}_{\ell_r} + \bar{C}_{n_p})]$$

$$-C_{n_{\delta_r}}[-i_x Z\bar{C}_{y_p} + (\bar{m} - \bar{C}_{y_r})i_x] - C_{\ell_{\delta_r}}\ i_x\bar{C}_{y_p} - (\bar{m} - \bar{C}_{y_r})i_x Z]\}$$

$$+s^2\{-C_{y_{\delta_r}}(\bar{C}_{n_p}\bar{C}_{\ell_r} - \bar{C}_{\ell_p}\bar{C}_{n_r}) - C_{n_{\delta_a}}[-i_x\ \bar{g}_1 - i_x\bar{g}_2 - \bar{C}_{y_p}\bar{C}_{\ell_r} - (\bar{m} - \bar{C}_{y_r})\bar{C}_{\ell_p}]$$

$$-C_{\ell_{\delta_r}}[\bar{g}_1 i_Z - \bar{g}_2 i_x Z - \bar{C}_{y_p}\bar{C}_{n_r} + (\bar{m} - \bar{C}_{y_r})\bar{C}_{n_r}]\}$$

$$+s\{-C_{n_{\delta_r}}(\bar{g}_2\bar{C}_{\ell_p} - \bar{g}_1\bar{C}_{\ell_r}) - C_{\ell_{\delta_r}}(\bar{g}_2\bar{C}_{n_p} - \bar{g}_1\bar{C}_{n_r})\} \qquad (6)$$

Now assuming that only the steady state (st. st.) condition is of interest,

$$\lim_{s\to 0}\frac{N(s)}{\Delta(s)} = \left(\frac{\delta_a}{\delta_r}\right)\text{ st. st.}$$

we get

$$\left(\frac{\delta_a}{\delta_r}\right)\text{ st. st.} = \frac{-C_{n_{\delta_r}}(\bar{g}_2\bar{C}_{\ell_p} - \bar{g}_1\bar{C}_{\ell_r}) - C_{\ell_{\delta_r}}(\bar{g}_2\bar{C}_{n_p} - \bar{g}_1\bar{C}_{n_r})}{C_{n_{\delta_a}}(\bar{g}_2\bar{C}_{\ell_p} - \bar{g}_1\bar{C}_{\ell_r}) + C_{\ell_{\delta_a}}(\bar{g}_2\bar{C}_{n_p} - \bar{g}_1\bar{C}_{n_r})} \qquad (7)$$

$$\left(\frac{\delta_a}{\delta_r}\right)\text{ st. st.} = \frac{\cos\theta_o(C_{n_{\delta_r}}C_{\ell_r} + C_{\ell_{\delta_r}}C_{n_r}) - \sin\theta_o(C_{n_{\delta_r}}C_{\ell_p} + C_{\ell_{\delta_r}}C_{n_p})}{-\cos\theta_o(C_{n_{\delta_a}}C_{\ell_r} + C_{\ell_{\delta_a}}C_{n_r}) + \sin\theta_o(C_{n_{\delta_a}}C_{\ell_p} + C_{\ell_{\delta_a}}C_{n_p})} \qquad (8)$$

For small initial flight path angle (i.e. $\theta_o \simeq 0$)

$$\left(\frac{\delta_a}{\delta_r}\right)\text{ st. st.} = -\frac{C_{n_{\delta_r}}C_{\ell_r} + C_{\ell_{\delta_r}}C_{n_r}}{C_{n_{\delta_r}}C_{\ell_r} + C_{\ell_{\delta_r}}C_{n_r}} = C_1 \qquad (9)$$

Assuming $\theta_o = 0$ to simplify we can write the transfer functions for ϕ and $\dot{\Psi}$ as (in the st. st.)

$$\frac{\dot{\Psi}}{\delta_r} = \frac{C_{\ell_{\delta_r}} C_{n_\beta} - C_{n_{\delta_r}} C_{\ell_\beta}}{C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r}} = C_2 \tag{10}$$

$$\frac{\dot{\Psi}}{\delta_a} = \frac{C_{\ell_{\delta_a}} C_{n_\beta} - C_{n_{\delta_a}} C_{\ell_\beta}}{C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r}} = C_3 \tag{11}$$

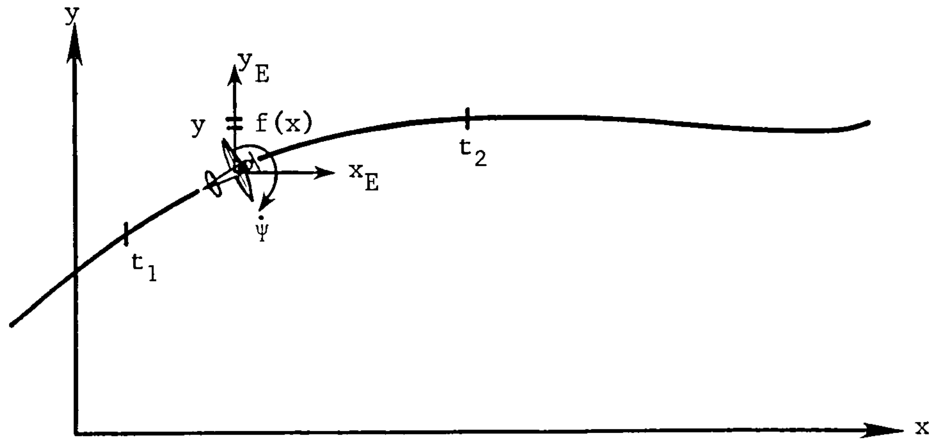
$$\frac{\phi}{\delta_r} = \frac{C_{y_{\delta_r}} (\overline{C}_{\ell_r} C_{n_\beta} - C_{\ell_\beta} \overline{C}_{n_r}) + C_{\ell_{\delta_r}} (C_{y_p} \overline{C}_{n_r} + C_{n_\beta} (\overline{m} - \overline{C}_{y_r})) + C_{n_{\delta_r}} (C_{\ell_\beta} (\overline{m} - \overline{C}_{y_r}) + C_{y_\beta} \overline{C}_{\ell_r})}{\frac{mg}{q_\infty S} (C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r})}$$

$$= C_4 \tag{12}$$

$$\frac{\phi}{\delta_r} = \frac{C_{y_{\delta_a}} (\overline{C}_{\ell_r} C_{n_\beta} - C_{\ell_\beta} \overline{C}_{n_r}) + C_{\ell_{\delta_a}} (C_{y_\beta} \overline{C}_{n_r} + C_{n_\beta} (\overline{m} - \overline{C}_{y_r})) + C_{n_{\delta_a}} (C_{\ell_\beta} (\overline{m} - \overline{C}_{y_r}) + C_{y_\beta} \overline{C}_{\ell_r})}{\frac{mg}{q_\infty S} (C_{\ell_\beta} \overline{C}_{n_r} - C_{n_\beta} \overline{C}_{\ell_r})}$$

$$= C_5 \tag{13}$$

Consider the aircraft trajectory shown



The slope at any point is $\frac{dy}{dx}$ and the angle the slope makes with the x axis is $\tan^{-1}\left(\frac{dy}{dx}\right)$.

The angular rate $\dot{\Psi}$ is then $\frac{d}{dt}\ \tan^{-1}\left(\frac{dy}{dx}\right)$

or $\frac{\partial}{\partial x}\left\{\tan^{-1}\frac{dy}{dx}\right\}\ \frac{dx}{dt} = V_{avg}\ \frac{\partial}{\partial x}\left\{\tan^{-1}\left(\frac{dy}{dx}\right)\right\}$

$$\text{Then } \Psi = V_{avg}\ \frac{\frac{d^2y}{dx^2}}{1+\left(\frac{dy}{dx}\right)^2} = V_{avg}\ \left\{\frac{f''(x)}{1+\ f''[(x)]^2}\right\} \qquad (14)$$

If we know δ_r we can determine δ_a from $\delta_a = C_1\delta_r$

$$\text{Also } \dot{\Psi} = C_2\delta_r+C_3\delta_a = (C_2+C_1+C_3)\delta_r \quad . \qquad (15)$$

We can also write

$$\phi = C_4\delta_r+C_5\delta_a = (C_4+C_1C_5)\delta_r \quad , \qquad (16)$$

Constraining δ_a to be $\leq \delta_{a_{max}}$, (17)

δ_r to be $\leq \delta_{r_{max}}$, (18)

and ϕ to be $\leq \phi_{\ max}$ ($\simeq$ max bank angle) (19)

we get the following expressions

$$\delta_{r_1} \leq \frac{\phi max}{C_4+C_1C_5} \qquad (20)$$

$$\delta_{r_2} \leq \delta_{r_{max}} \qquad (21)$$

$$\delta_{r_3} \leq \frac{\delta_{a_{max}}}{C_1} \qquad (22)$$

The constraining value is given by

$$\delta_{r_{max}} = \min(\delta_{r_1},\delta_{r_2},\delta_{r_3}) \tag{23}$$

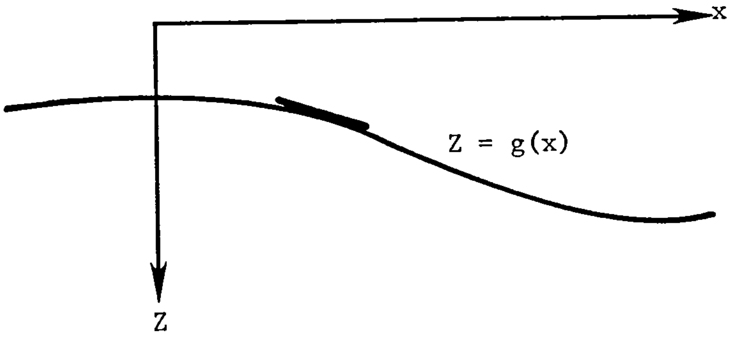
which yields

$$\dot{\Psi}_{max} = (C_2+C_1C_3) \min(\delta_{r_1},\delta_{r_2},\delta_{r_3}) \tag{24}$$

This condition incorporates all three constraints ((17)-(19)) as

$$\frac{\frac{d^2y}{dx^2}}{1+(\frac{dy}{dx})^2} = \frac{f''(x)}{1+ f'(x)^2} \le \frac{(C_2+C_1C_3)}{V_{avg}} \min (\delta_{r_1},\delta_{r_2},\delta_{r_3})$$

Longitudinally we wish to constrain the behavior of the trajectory so that we restrict γ (the flight path angle) and θ (the pitching rate). The trajectory is given by



Then, assuming the aircraft center of mass follows this trajectory γ is given by

$$\gamma = \tan^{-1} \frac{dz}{dx}$$

or

$$\frac{dz}{dx} = \tan \gamma$$

We wish to constrain γ to a maximum descent angle, $\gamma_{d_{max}}$ and a maximum angle, $\gamma_{c_{max}}$.

Thus

$$\tan\gamma_{c_{max}} \le \frac{dz}{dx} \le \tan \gamma_{d_{max}}$$

APPENDIX B

```
      PROGRAM NOISE (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE97,TAPE98
     1,TAPE99)
      COMMON ALFA(5),BETA(5),POSIT(53,3),ARRAY(578,9),NMAP
      COMMON /CURVE/ YCURVE(51),ADY(51),ADDY(51)
      COMMON /LABEL/ LINFO(4),LLOC(3)
      COMMON /AIRPORT/ XPORT,YPORT,ZPORT
      COMMON /SCALE/ XMIN,XINC,YMIN,YINC
      INTEGER COUNT,HALF
      DIMENSION ALFAOD(5), BETAOD(5), GY(5), GZ(5), DALFA(5), DBETA(5)
      DIMENSION AGY(5), BGY(5), AGZ(5), BGZ(5)
C
C ....................................................................
C .                                                                  .
C .   READ MAP FROM DISC INTO MEMORY                                 .
C .                                                                  .
C ....................................................................
C
      READ (5,*) A11,A12
      READ (5,*) NMAP,XPORTO,YPORTO
      READ (5,*) ((ARRAY(I,J),J=1,9),I=1,NMAP)
C
C ....................................................................
C .                                                                  .
C .   INPUT INITIAL CONDITIONS                                       .
C .                                                                  .
C ....................................................................
C
      READ (5,*) MAXIT,YALLOW,ZALLOW,(ALFA(I),I=1,5),(BETA(I),I=1,5),XO,
     1YO,ZO,XF,YF,ZF
      READ (5,9100) (LLOC(I),I=1,3),(LINFO(I),I=1,4)
      WRITE (6,9110) (LLOC(I),I=1,3),(LINFO(I),I=1,4)
      WRITE (6,9120) MAXIT
      WRITE (6,9010) XO,YO,ZO,XF,YF,ZF,XPORTO,YPORTO
      WRITE (6,9130) YALLOW,ZALLOW
      WRITE (6,9020) A11,A12
      WRITE (6,9140) (I,ALFA(I),BETA(I),I=1,5)
C
C ....................................................................
C .                                                                  .
C .   COORDINATE ROTATION                                            .
C .                                                                  .
C ....................................................................
C
      THETA = ATAN2((YF-YO),(XF-XO))
      A = (XF*COS(THETA)+YF*SIN(THETA))-(XO*COS(THETA)+YO*SIN(THETA))
      PHI = ATAN2((ZF-ZO),A)
      XOCAP = XO*COS(THETA)*COS(PHI)+YO*COS(PHI)*SIN(THETA)+ZO*SIN(PHI)
      XFCAP = XF*COS(THETA)*COS(PHI)+YF*SIN(THETA)*COS(PHI)+ZF*SIN(PHI)
      XPORT = XPORTO*COS(THETA)*COS(PHI)+YPORTO*SIN(THETA)*COS(PHI)
      YOCAP = -XO*SIN(THETA)+YO*COS(THETA)
      YFCAP = -XF*SIN(THETA)+YF*COS(THETA)
      YPORT = -XPORTO*SIN(THETA)+YPORTO*COS(THETA)
      ZOCAP = -XO*COS(THETA)*SIN(PHI)-YO*SIN(THETA)*SIN(PHI)+ZO*COS(PHI)
      ZFCAP = -XF*COS(THETA)*SIN(PHI)-YF*SIN(THETA)*SIN(PHI)+ZF*COS(PHI)
      ZPORT = -XPORTO*COS(THETA)*SIN(PHI)-YPORTO*SIN(THETA)*SIN(PHI)
C
C ....................................................................
```

```
C .                                                                  .  A  580
C .   START OPTIMIZATION                                             .  A  590
C .                                                                  .  A  600
C ...................................................................   A  610
C                                                                       A  620
      INDEX = 0                                                         A  630
      DLXCAP = (XFCAP-XOCAP)/50.                                        A  640
C                                                                       A  650
C ...................................................................   A  660
C .                                                                  .  A  670
C .   FIRST FIND A CURVE WHICH FORCES THE HEADING OF THE             .  A  680
C .   AIRCRAFT TOWARD THE RUNWAY AT THE FINAL POINT                  .  A  690
C .                                                                  .  A  700
C ...................................................................   A  710
C                                                                       A  720
      SLOPE = (YFCAP-YPORT)/(XFCAP-XPORT)                               A  730
      YCURVE(1) = YOCAP                                                 A  740
      ADY(1) = 0.                                                       A  750
      ADDY(1) = 0.                                                      A  760
      XCAP = XOCAP                                                      A  770
      DO 10 I = 1,50                                                    A  780
        XCAP = XCAP+DLXCAP                                              A  790
        EXPO = -5.*(XCAP-XFCAP)/(XOCAP-XFCAP)                           A  800
        YCURVE(I+1) = (SLOPE*(XCAP-XPORT)+(YPORT-YOCAP))*EXP(EXPO)+YOCAP A  810
        ADY(I+1) = -5./(XOCAP-XFCAP)*(YCURVE(I+1)-YOCAP)+(SLOPE)*EXP(EXP A  820
     1  O)                                                              A  830
        ADDY(I+1) = ((-5./(XOCAP-XFCAP))**2)*(YCURVE(I+1)-YOCAP)+(-5./(X A  840
     1  OCAP-XFCAP))*SLOPE*EXP(EXPO)*2.                                 A  850
   10   CONTINUE                                                        A  860
      COUNT = 0.                                                        A  870
C                                                                       A  880
C ...................................................................   A  890
C .                                                                  .  A  900
C .   INITIAL COST                                                   .  A  910
C .                                                                  .  A  920
C ...................................................................   A  930
C                                                                       A  940
      XMIN = -40000                                                     A  950
      XINC = 2500                                                       A  960
      YMIN = -40000                                                     A  970
      YINC = 2500                                                       A  980
      H = 0.07                                                          A  990
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL A 1000
     1TY)                                                               A 1010
      COST1 = TOTAL                                                     A 1020
      A = COST1-PNALTY                                                  A 1030
      WRITE (6,9150) COUNT,COST1,A,PNALTY                               A 1040
      WRITE (6,9220)                                                    A 1050
      WRITE (6,9230) ((POSIT(I,J),J=1,3),I=1,51)                        A 1060
      WRITE (6,9030)                                                    A 1070
      DO 20 I = 1,5                                                     A 1080
        WRITE (6,9040) I,ALFA(I),BETA(I)                                A 1090
   20   CONTINUE                                                        A 1100
      WRITE (6,9050)                                                    A 1110
   30 DO 40 I = 1,5                                                     A 1120
        DALFA(I) = A11                                                  A 1130
   40   DBETA(I) = A12                                                  A 1140
```

```
C                                                                       A 1150
C ...................................................................... A 1160
C .                                                                   . A 1170
C .   CALCULATE GRADIENT                                              . A 1180
C .                                                                   . A 1190
C ...................................................................... A 1200
C                                                                       A 1210
   50 DO 60 I = 1,5                                                     A 1220
        ALFA(I) = ALFA(I)+DALFA(I)                                      A 1230
        CALL COST (1,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PN A 1240
     1  ALTY)                                                           A 1250
        COST2 = TOTAL                                                   A 1260
        GY(I) = (COST2-COST1)/ABS(DALFA(I))                             A 1270
        IF (INDEX.EQ.0) AGY(I) = GY(I)                                  A 1280
        IF (INDEX.EQ.1) BGY(I) = GY(I)                                  A 1290
        WRITE (6,9160) I,GY(I)                                          A 1300
   60   ALFA(I) = ALFA(I)-DALFA(I)                                      A 1310
      DO 70 I = 1,5                                                     A 1320
        BETA(I) = BETA(I)+DBETA(I)                                      A 1330
        CALL COST (1,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PN A 1340
     1  ALTY)                                                           A 1350
        COST2 = TOTAL                                                   A 1360
        GZ(I) = (COST2-COST1)/ABS(DBETA(I))                             A 1370
        GZ(I) = 0.                                                      A 1380
        IF (INDEX.EQ.0) AGZ(I) = GZ(I)                                  A 1390
        IF (INDEX.EQ.1) BGZ(I) = GZ(I)                                  A 1400
        WRITE (6,9170) I,GZ(I)                                          A 1410
   70   BETA(I) = BETA(I)-DBETA(I)                                      A 1420
      IF (INDEX.EQ.1) GO TO 190                                         A 1430
      GYMAX = ABS(GY(1))                                                A 1440
      GZMAX = ABS(GZ(1))                                                A 1450
      DO 80 I = 2,5                                                     A 1460
        IF (GYMAX.LT.ABS(GY(I))) GYMAX = ABS(GY(I))                     A 1470
   80   IF (GZMAX.LT.ABS(GZ(I))) GZMAX = ABS(GZ(I))                     A 1480
C                                                                       A 1490
C ...................................................................... A 1500
C .                                                                   . A 1510
C .   DETERMINE SIZE OF STEP CHANGE                                   . A 1520
C .                                                                   . A 1530
C ...................................................................... A 1540
C                                                                       A 1550
      YALLOW = (YALLOW-A11)*0.95+A11                                    A 1560
      ZALLOW = (ZALLOW-A12)*0.95+A12                                    A 1570
      IF (GYMAX.EQ.0.) YRATIO = 0.                                      A 1580
      IF (GYMAX.NE.0.) YRATIO = YALLOW/GYMAX                            A 1590
      IF (GZMAX.EQ.0.) ZRATIO = 0.                                      A 1600
      IF (GZMAX.NE.0.) ZRATIO = ZALLOW/GZMAX                            A 1610
      DO 90 I = 1,5                                                     A 1620
        ALFAOD(I) = ALFA(I)                                             A 1630
        ALFA(I) = ALFA(I)-YRATIO*GY(I)                                  A 1640
        BETAOD(I) = BETA(I)                                             A 1650
   90   BETA(I) = BETA(I)-ZRATIO*GZ(I)                                  A 1660
      CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL A 1670
     1TY)                                                               A 1680
      COST2 = TOTAL                                                     A 1690
      IF (COST2.GE.COST1) GO TO 150                                     A 1700
  100 PRCENT = ABS(COST2-COST1)/COST1                                   A 1710
```

```
C                                                                       A 1720
C .....................................................................  A 1730
C .                                                                   .  A 1740
C .   STOP CRITERION -- PERCENTAGE CHANGE IN COST INSIGNIFICANT       .  A 1750
C .                                                                   .  A 1760
C .....................................................................  A 1770
C                                                                       A 1780
      IF (PRCENT.GE.1.E-5) GO TO 110                                    A 1790
      COUNT = COUNT+1                                                   A 1800
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 1810
     1TY)                                                               A 1820
      WRITE (6,9180) COUNT                                              A 1830
      CALL MONIT (COUNT,COST2,PNALTY)                                   A 1840
      STOP                                                              A 1850
  110 CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 1860
     1TY)                                                               A 1870
      COST1 = TOTAL                                                     A 1880
      COUNT = COUNT+1                                                   A 1890
      A = COST1-PNALTY                                                  A 1900
      WRITE (6,9150) COUNT,COST1,A,PNALTY                               A 1910
      WRITE (6,9220)                                                    A 1920
      WRITE (6,9230) ((POSIT(I,J),J=1,3),I=1,51)                        A 1930
      DO 120 I = 1,5                                                    A 1940
C                                                                       A 1950
C .....................................................................  A 1960
C .                                                                   .  A 1970
C .   STOP CRITERION -- ALL GRADIENT COMPONENTS EQUAL TO ZERO         .  A 1980
C .                                                                   .  A 1990
C .....................................................................  A 2000
C                                                                       A 2010
        IF (GY(I).NE.0.) GO TO 130                                      A 2020
        IF (GZ(I).NE.0.) GO TO 130                                      A 2030
  120   CONTINUE                                                        A 2040
      WRITE (6,9060) COUNT                                              A 2050
      CALL MONIT (COUNT,COST1,PNALTY)                                   A 2060
      STOP                                                              A 2070
  130 WRITE (6,9070)                                                    A 2080
      DO 140 I = 1,5                                                    A 2090
        WRITE (6,9190) I,ALFA(I),BETA(I)                                A 2100
  140   CONTINUE                                                        A 2110
      COST2 = TOTAL                                                     A 2120
C                                                                       A 2130
C .....................................................................  A 2140
C .                                                                   .  A 2150
C .   STOP CRITERION -- MAXIMUM NUMBER OF ITERATIONS REACHED          .  A 2160
C .                                                                   .  A 2170
C .....................................................................  A 2180
C                                                                       A 2190
      IF (COUNT.LT.MAXIT) GO TO 30                                      A 2200
      WRITE (6,9200)                                                    A 2210
      CALL MONIT (COUNT,COST1,PNALTY)                                   A 2220
      STOP                                                              A 2230
  150 HALF = 1                                                          A 2240
C                                                                       A 2250
C .....................................................................  A 2260
C .                                                                   .  A 2270
C .   REDUCE SIZE OF STEP CHANGE BY HALF                              .  A 2280
```

```
C .   IF COST HAS NOT DECREASED                                       .   A 2290
C .                                                                   .   A 2300
C .....................................................................   A 2310
C                                                                         A 2320
      DO 170 J = 1,3                                                      A 2330
        DO 160 I = 1,5                                                    A 2340
          ALFA(I) = (ALFA(I)+ALFAOD(I))/2.                                A 2350
  160     BETA(I) = (BETA(I)+BETAOD(I))/2.                                A 2360
        HALF = J                                                          A 2370
        WRITE (6,9210) HALF                                               A 2380
        CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PN  A 2390
     1  ALTY)                                                             A 2400
        COST2 = TOTAL                                                     A 2410
        IF (COST2.LT.COST1) GO TO 100                                     A 2420
  170   CONTINUE                                                          A 2430
      HALF = 4                                                            A 2440
      INDEX = 1                                                           A 2450
      DO 180 I = 1,5                                                      A 2460
        DALFA(I) = -DALFA(I)                                              A 2470
  180   DBETA(I) = -DBETA(I)                                              A 2480
C                                                                         A 2490
C .....................................................................   A 2500
C .                                                                   .   A 2510
C .   PERTURB CURVE IN THE OPPOSITE DIRECTION                         .   A 2520
C .                                                                   .   A 2530
C .....................................................................   A 2540
C                                                                         A 2550
      GO TO 50                                                            A 2560
  190 DO 200 I = 1,5                                                      A 2570
        IF (AGY(I).LT.0.) GO TO 220                                       A 2580
        IF (BGY(I).LT.0.) GO TO 220                                       A 2590
        IF (AGZ(I).LT.0.) GO TO 220                                       A 2600
        IF (BGZ(I).LT.0.) GO TO 220                                       A 2610
  200   CONTINUE                                                          A 2620
      WRITE (6,9080)                                                      A 2630
      DO 210 I = 1,5                                                      A 2640
        ALFA(I) = ALFAOD(I)                                               A 2650
  210   BETA(I) = BETAOD(I)                                               A 2660
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 2670
     1TY)                                                                 A 2680
      CALL MONIT (COUNT,COST1,PNALTY)                                     A 2690
      STOP                                                                A 2700
  220 BGYMAX = ABS(BGY(1))                                                A 2710
      BGZMAX = ABS(BGZ(1))                                                A 2720
      DO 230 I = 2,5                                                      A 2730
        IF (BGYMAX.LT.ABS(BGY(I))) BGYMAX = ABS(BGY(I))                   A 2740
  230   IF (BGZMAX.LT.ABS(BGZ(I))) BGZMAX = ABS(BGZ(I))                   A 2750
  240 WRITE (6,9210) HALF                                                 A 2760
C                                                                         A 2770
C .....................................................................   A 2780
C .                                                                   .   A 2790
C .   CHECK EACH GRADIENT COMPONENT TO DETERMINE SIZE                 .   A 2800
C .   OF STEP CHANGE                                                  .   A 2810
C .                                                                   .   A 2820
C .....................................................................   A 2830
C                                                                         A 2840
      DO 320 I = 1,5                                                      A 2850
```

```
            IF (HALF.EQ.7) GO TO 250                                          A 2860
            IF (GYMAX.NE.0.) AY = YALLOW/GYMAX/FLOAT(HALF-3)*AGY(I)           A 2870
            IF (BGYMAX.NE.0.) BY = YALLOW/BGYMAX/FLOAT(HALF-3)*BGY(I)         A 2880
            IF (GZMAX.EQ.0.) AZ = 0.                                          A 2890
            IF (BGZMAX.EQ.0.) BZ = 0.                                         A 2900
            IF (GYMAX.EQ.0.) AY = 0.                                          A 2910
            IF (BGYMAX.EQ.0.) BY = 0.                                         A 2920
            IF (GZMAX.NE.0.) AZ = ZALLOW/GZMAX/FLOAT(HALF-3)*AGZ(I)           A 2930
            IF (BGZMAX.NE.0.) BZ = ZALLOW/BGZMAX/FLOAT(HALF-3)*BGZ(I)         A 2940
            GO TO 260                                                         A 2950
  250       AY = -DALFA(I)                                                    A 2960
            BY = DALFA(I)                                                     A 2970
            AZ = -DBETA(I)                                                    A 2980
            BZ = DBETA(I)                                                     A 2990
  260       IF (AGY(I).LE.0.) GO TO 270                                       A 3000
            IF (BGY(I).GE.0.) ALFA(I) = ALFAOD(I)                             A 3010
            IF (BGY(I).LT.0.) ALFA(I) = ALFAOD(I)+BY                          A 3020
            GO TO 290                                                         A 3030
  270       IF (AGY(I).LT.0.) GO TO 280                                       A 3040
            IF (BGY(I).GE.0.) ALFA(I) = ALFAOD(I)                             A 3050
            IF (BGY(I).LT.0.) ALFA(I) = ALFAOD(I)+BY                          A 3060
            GO TO 290                                                         A 3070
  280       IF (AGY(I).LT.BGY(I)) ALFA(I) = ALFAOD(I)-AY                      A 3080
            IF (AGY(I).GE.BGY(I)) ALFA(I) = ALFAOD(I)+BY                      A 3090
  290       IF (AGZ(I).LE.0.) GO TO 300                                       A 3100
            IF (BGZ(I).GE.0.) BETA(I) = BETAOD(I)                             A 3110
            IF (BGZ(I).LT.0.) BETA(I) = BETAOD(I)+BZ                          A 3120
            GO TO 320                                                         A 3130
  300       IF (AGZ(I).LT.0.) GO TO 310                                       A 3140
            IF (BGZ(I).GE.0.) BETA(I) = BETAOD(I)                             A 3150
            IF (BGZ(I).LT.0.) BETA(I) = BETAOD(I)+BZ                          A 3160
            GO TO 320                                                         A 3170
  310       BETA(I) = BETA(I)-AZ                                              A 3180
  320       CONTINUE                                                          A 3190
          CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 3200
         1TY)                                                                 A 3210
          COST2 = TOTAL                                                       A 3220
          IF (COST2.LT.COST1) GO TO 420                                       A 3230
          HALF = HALF+1                                                       A 3240
          IF (HALF.LT.7) GO TO 240                                            A 3250
          WRITE (6,9210) HALF                                                 A 3260
          GYMIN = AGY(1)                                                      A 3270
          J = 1                                                               A 3280
          GZMIN = AGZ(1)                                                      A 3290
          K = 1                                                               A 3300
          DO 340 I = 2,5                                                      A 3310
            IF (GYMIN.LE.AGY(I)) GO TO 330                                    A 3320
            GYMIN = AGY(I)                                                    A 3330
            J = I                                                             A 3340
  330       IF (GZMIN.LE.AGZ(I)) GO TO 340                                    A 3350
            GZMIN = AGZ(I)                                                    A 3360
            K = I                                                             A 3370
  340     CONTINUE                                                            A 3380
          DO 360 I = 1,5                                                      A 3390
            IF (GYMIN.LE.BGY(I)) GO TO 350                                    A 3400
            GYMIN = BGY(I)                                                    A 3410
            J = I+5                                                           A 3420
```

```
  350   IF (GZMIN.LE.BGZ(I)) GO TO 360                                    A 3430
        GZMIN = BGZ(I)                                                    A 3440
        K = I+5                                                           A 3450
  360   CONTINUE                                                          A 3460
      IF ((GYMIN.LT.0.0).OR.(GZMIN.LT.0.0)) GO TO 370                     A 3470
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 3480
     1TY)                                                                 A 3490
      COUNT = COUNT+1                                                     A 3500
      WRITE (6,9090) COUNT                                                A 3510
      CALL MONIT (COUNT,COST1,PNALTY)                                     A 3520
      STOP                                                                A 3530
  370 DO 380 I = 1,5                                                      A 3540
        ALFA(I) = ALFAOD(I)                                               A 3550
  380   BETA(I) = BETAOD(I)                                               A 3560
      IF ((GYMIN.LT.0.0).AND.(GZMIN.GE.0.0)) GO TO 390                    A 3570
      IF ((GYMIN.LT.0.).AND.(GZMIN.LT.0.)) GO TO 400                      A 3580
      IF (K.LE.5) BETA(K, = BETA(K)-DBETA(K)                              A 3590
      IF (K.GT.5) BETA(K-5) = BETA(K-5)+DBETA(K-5)                        A 3600
      GO TO 420                                                           A 3610
  390 IF (J.LE.5) ALFA(J) = ALFA(J)-DALFA(J)                              A 3620
      IF (J.GT.5) ALFA(J-5) = ALFA(J-5)+DALFA(J-5)                        A 3630
      GO TO 420                                                           A 3640
  400 IF (J.LE.5) ALFA(J) = ALFA(J)-DALFA(J)                              A 3650
      IF (J.GT.5) ALFA(J-5) = ALFA(J-5)+DALFA(J-5)                        A 3660
      IF (K.LE.5) BETA(K) = BETA(K)-DBETA(K)                              A 3670
      IF (K.GT.5) BETA(K-5) = BETA(K-5)+DBETA(K-5)                        A 3680
      CALL COST (0,0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 3690
     1TY)                                                                 A 3700
      COST2 = TOTAL                                                       A 3710
      IF (COST2.LT.COST1) GO TO 420                                       A 3720
      DO 410 I = 1,5                                                      A 3730
  410   BETA(I) = BETAOD,I)                                               A 3740
  420 INDEX = 0                                                           A 3750
      CALL COST (0,1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL  A 3760
     1TY)                                                                 A 3770
      GO TO 100                                                           A 3780
C                                                                         A 3790
C                                                                         A 3800
 9010 FORMAT (5X,14HINITAL X,Y,Z: ,3(F12.2,3X),7H METERS,/,5X,13HFINAL X  A 3810
     1,Y,Z: ,3(F12.2,3X),7H METERS,/,5X,23HAIRPORT LOCATION, X,Y: ,2(F12  A 3820
     2.2,3X),7H METERS)                                                   A 3830
 9020 FORMAT (5X,43HPERTURB TRAJECTORY IN Y AND Z DIRECTONS BY ,F6.2,5H   A 3840
     1AND ,F6.2,42H METERS, RESPECTIVELY FOR CALCULATING GRAD,5HIENTS)    A 3850
 9030 FORMAT (13X,4HALFA,16X,4HBETA)                                      A 3860
 9040 FORMAT (10X,I1,1PE16.9,4X,1PE16.9)                                  A 3870
 9050 FORMAT (////)                                                       A 3880
 9060 FORMAT (////,1X,13HAT ITERATION ,I2,49H ALL GRADIENTS EQUAL TO      A 3890
     1   ZERO, PROGRAM STOPS)                                             A 3900
 9070 FORMAT (10X,2HNO,1X,4HALFA,16X,4HBETA)                              A 3910
 9080 FORMAT (5X,43HALL GRADIENTS PERTURBED BOTH DIRECTIONS > 0)          A 3920
 9090 FORMAT (1X,13HAT ITERATION ,I2,16H OPTIMUM REACHED)                 A 3930
 9100 FORMAT (3A10,/,4A10)                                                A 3940
 9110 FORMAT (1H1,20X,3A10,//,4A10,////)                                  A 3950
 9120 FORMAT (1X,19HINFORMATION INPUT: ,//,5X,21HMAXIMUM ITERATION SET,1  A 3960
     1H:,I3)                                                              A 3970
 9130 FORMAT (5X,47HMAXIMUM ALLOWED CHANGES PER ITERATON IN Y AND Z,27H   A 3980
     1DIRECTIONS, RESPECTIVELY: ,1PE10.3,5H AND ,1PE10.3,7H METERS)       A 3990
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```
      9140 FORMAT (5X,22HINITIAL ALFA AND BETA:,/,13X,4HALFA,16X,4HBETA,5(/,1  A 4000
          10X,I1,1X,1PE16.9,4X,1PE16.9))                                       A 4010
      9150 FORMAT (////,1X,10HITERATION ,I3,//,5X,14HTOTAL COST IS ,1PE16.9,/  A 4020
          1,5X,22HTRUE ANNOYACE(NII) IS ,1PE16.9,/,5X,42HPENALTY DUE TO AI     A 4030
          2RCRAFT CONSTRAINTS IS ,1PE16.9,//)                                  A 4040
      9160 FORMAT (10X,I2,17HTH Y-GRADIENT IS ,1PE16.9)                        A 4050
      9170 FORMAT (10X,I2,17HTH Z-GRADIENT IS ,1PE16.9)                        A 4060
      9180 FORMAT (////,1X,13HAT ITERATION ,I2,24H PERCENTAGE CHANGE IN CO,33  A 4070
          1HST LESS THAN .001%, PROGRAM STOPS)                                 A 4080
      9190 FORMAT (10X,I1,2X,1PE16.9,4X,1PE16.9)                               A 4090
      9200 FORMAT (10X,41HREACH MAXIMUM ITERATION SET, PROGRAM STOP)           A 4100
      9210 FORMAT (10X,7HHALF = ,I2)                                           A 4110
      9220 FORMAT (10X,10HTRAJECTORY,/,10X,12HX COORDINATE,8X,12HY COORDINATE  A 4120
          1,8X,12HZ COORDINATE,/,12X,7H(METER),13X,7H(METER),13X,7H(METER))    A 4130
      9230 FORMAT (10X,3(1PE16.9,4X))                                          A 4140
           END                                                                 A 4150

45000B CM STORAGE USED        7.828 SECONDS
```

```
      SUBROUTINE COST (IGRAD,IWRITE,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA B   10
     1,PHI,TOTAL,PNALTY)                                                 B   20
      COMMON ALFA(5),BETA(5),POSIT(53,3),ARRAY(578,9),NMAP               B   30
      COMMON /CURVE/ YCURVE(51),ADY(51),ADDY(51)                         B   40
      COMMON /AIRPORT/ XPORT,YPORT,ZPORT                                 B   50
      COMMON /AC/ X,Y,Z                                                  B   60
      EXTERNAL FCN                                                       B   70
      PNALTY = 0.                                                        B   80
      XCAP = XOCAP                                                       B   90
      PI = ATAN(1.)*4.                                                   B  100
      C2 = PI/ABS(XFCAP-XOCAP)                                           B  110
      C3 = ABS(XFCAP-XOCAP)/4.                                           B  120
      DO 10 I = 1,NMAP                                                   B  130
        ARRAY(I,4) = 0.                                                  B  140
   10   ARRAY(I,5) = 0.                                                  B  150
C                                                                        B  160
C .................................................................... B  170
C .                                                                  . B  180
C .   MULTIPLY BY EXPONENTIAL TERM SUCH THAT THE FINAL                 . B  190
C .   HEADING OF AIRCRAFT IS TOWARD THE RUNWAY                         . B  200
C .                                                                  . B  210
C .................................................................... B  220
C                                                                        B  230
      DO 50 I = 1,51                                                     B  240
        Y2 = 1.-EXP(-(XFCAP-XCAP)/C3)                                    B  250
        Y5 = (Y2-1.)/C3                                                  B  260
        Y9 = 0.0                                                         B  270
        Y8 = Y9                                                          B  280
        Y7 = Y8                                                          B  290
        Y6 = Y7                                                          B  300
        Y3 = Y6                                                          B  310
C                                                                        B  320
C .................................................................... B  330
C .                                                                  . B  340
C .   GENERATE SINE HARMnNICS                                          . B  350
C .                                                                  . B  360
C .................................................................... B  370
C                                                                        B  380
        DO 20 J = 1,5                                                    B  390
          TRIGOX = FLOAT(J)*(XCAP-XOCAP)*C2                              B  400
          Y3 = Y3+ALFA(J)*SIN(TRIGOX)                                    B  410
          Y8 = Y8+BETA(J)*SIN(TRIGOX)                                    B  420
          Y6 = Y6+FLOAT(J)*C2*ALFA(J)*COS(TRIGOX)                        B  430
          Y7 = Y7-FLOAT(J**2)*(C2**2)*ALFA(J)*SIN(TRIGOX)                B  440
   20     Y9 = Y9+FLOAT(J)*C2*BETA(J)*COS(TRIGOX)                        B  450
        DLYCAP = Y2*Y3                                                   B  460
        DLZCAP = Y2*Y8                                                   B  470
        ZCAP = ZOCAP+DLZCAP                                              B  480
        YCAP = DLYCAP+YCURVE(I)                                          B  490
C                                                                        B  500
C .................................................................... B  510
C .                                                                  . B  520
C .   AIRCRAFT CONSTRA'NTS                                             . B  530
C .                                                                  . B  540
C .................................................................... B  550
C                                                                        B  560
        DY = Y2*Y6+Y3*Y5                                                 B  570
```

```
        DY = DY+ADY(I)
        DDY = Y2*Y7+2.*Y5*Y6+Y3*Y5/C3
        DDY = DDY+ADDY(I)
        DDY = DDY/(1+DY**2)
        DZ = Y2*Y9+Y5*Y8
        DZ = DZ+TAN(PHI)
        DZ = 0.
        PNALTY = PNALTY+(DDY/.001)**(20)+(DZ/.14)**(20)
        X = XCAP*COS(THETA)*COS(PHI)-YCAP*SIN(THETA)-ZCAP*COS(THETA)*SIN
     1  (PHI)
        Y = XCAP*SIN(THETA)*COS(PHI)+YCAP*COS(THETA)-ZCAP*SIN(THETA)*SIN
     1  (PHI)
        Z = XCAP*SIN(PHI)+ZCAP*COS(PHI)
        DO 40 K = 1,NMAP
          RANGE = ((X-ARRAY(K,1))**2+(Y-ARRAY(K,2))**2+Z**2)**.5
          DB = 115.-22.5*ALOG10(3.281*RANGE/500.)
          IF (DB.LE.ARRAY(K,4)) GO TO 40
          ARRAY(K,4) =  A
          IF (ARRAY(K,4).LT.55.) GO TO 40
          IF (ARRAY(K,3).EQ.0.) GO TO 30
C
C ...................................................................
C .                                                                 .
C .   ANNOYANCE INTEGRATION OVER A SINGE BLOCK                      .
C .                                                                 .
C ...................................................................
C
          SMALLP = ARRAY(K,3)/(ARRAY(K,7)-ARRAY(K,6))/(ARRAY(K,9)-ARRAY(
     1    K,8))
          CALL GAUSS (ARRAY(K,6),ARRAY(K,7),ARRAY(K,8),ARRAY(K,9),FCN,TE
     1    MP)
          ARRAY(K,5) = TEMP*SMALLP
          GO TO 40
   30     ARRAY(K,5) = 0.
   40     CONTINUE
        IF (IWRITE.EQ.0) GO TO 50
        II = I
        POSIT(II,1) = X
        POSIT(II,2) = Y
        POSIT(II,3) = Z
   50   XCAP = XCAP+DLXCAP
C
C ...................................................................
C .                                                                 .
C .   TOTAL POPULATON EXPOSED TO NOISE ABOVE 55 EPNDB               .
C .                                                                 .
C ...................................................................
C
       PEOPLE = 0.
       DO 60 K = 1,NMAP
         IF (ARRAY(K,5).EQ.0.0) GO TO 60
         PEOPLE = ARRAY(K,3)+PEOPLE
   60    CONTINUE
       FX = 0.
       DO 70 K = 1,NMAP
         ARRAY(K,5) = ARRAY(K,5)/PEOPLE
         FX = FX+ARRAY(K,5)
```

```
 115          70   CONTINUE                                                      B 1150
                  TOTAL = FX+PNALTY                                              B 1160
                  RETURN                                                         B 1170
                  END                                                            B 1180

41000B CM STORAGE USED          .874 SECONDS
```

```
SUBROUTINE MONIT      73/172  TS                            FTN 4.6+452        04/27/79  11.45.47         PAGE      1

                    SUBROUTINE MONIT (IA,AA,BB)
                    COMMON ALFA(5),BETA(5),POSIT(53,3),ARRAY(578,9),NMAP
                    COMMON /LABEL/ LINFO(4),LLOC(3)
                    COMMON /SCALE/ XMIN,XINC,YMIN,YINC
                    DIMENSION PCRIT(10)
                    DIMENSION XM(1026), YM(1026)
                    DIMENSION XP(53), YP(53), ZP(53), NA(5), NB(3)
                    EQUIVALENCE (XM(1),ARRAY(1,1)), (YM(1),ARRAY(1,2))
                    EQUIVALENCE (XP(1),POSIT(1,1)), (YP(1),POSIT(1,2)), (ZP(1),POSIT(1
                   1,3))
                    DATA NB/10HTOTAL POPU,10HLATION ANN,9HOYANCE = /
              C
              C ...................................................................
              C .                                                                 .
              C .   DOCUMENTATION                                                 .
              C .                                                                 .
              C ...................................................................
              C
                    CC = AA-BB
                    WRITE (6,9010)
                    WRITE (6,9020) IA,AA,CC,BB
                    DO 10 I = 1,51
                      WRITE (6,9030) (POSIT(I,J),J=1,3)
                 10   CONTINUE
                    WRITE (6,9040)
                    DO 20 I = 1,NMAP
                      WRITE (6,9050) (ARRAY(I,J),J=1,5)
                 20   CONTINUE
                    WRITE (97,9060) ((POSIT(I,J),J=1,3),I=1,51)
                    WRITE (97,9070) ((ARRAY(I,J),J=1,5),I=1,NMAP)
                    RETURN
              C
               9010 FORMAT (10X,55HOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIR
                   1PORT,//,10X,59HNOISE BELOW 55 EPNDB IS CONSIDERED NOT NOISY, ANNOY
                   2ANCE = 0,/,10X,23HUNIT FOR NOISE IS EPNDB,/,10X,50HUNIT FOR COORDI
                   3NATES, AIRCRAFT TRAJECTORY IS METER,//)
               9020 FORMAT (10X,13HAT ITERATION ,I2,/,15X,14HTOTAL COST IS ,1PE16.9,/,
                   115X,23HTRUE ANNOYANCE(NII) IS ,1PE16.9,/,15X,11HPENALTY IS ,1PE16.
                   29,//,10X,18HOPTIMUM TRAJECTORY,/,10X,12HX COORDINATE,8X,12HY COORD
                   3INATE,8X,12HZ COORDINATE,/,12X,7H(METER),13X,7H(METER),13X,7H(METE
                   4R))
               9030 FORMAT (10X,3(1PE16.9,4X))
               9040 FORMAT (//,10X,32HPOPULATION-NOISE-ANNOYANCE CHART,/,10X,10HX-POSI
                   1TION,5X,10HY-POSITION,5X,10HPOP. INDEX,5X,11HNOISE LEVEL,4X,9HANNO
                   2YANCE)
               9050 FORMAT (10X,3(F10.3,5X),2(1PE10.3,5X))
               9060 FORMAT (3E12.6)
               9070 FORMAT (5E12.6)
                    END

41000B CM STORAGE USED          .284 SECONDS
```

```
    SUBROUTINE GAUSS      73/172  TS                            FTN 4.6+452        04/27/79  11.45.47        PAGE      1

                          SUBROUTINE GAUSS (XN,XX,YN,YX,FCN,FINT)
                          COMMON /AC/ XA,YA,ZA
                          DIMENSION X(5), Y(5), F(5), XI(5), W(5)
                          DATA XI,W,N/-0.577350269,0.577350269,0,0,0,1.,1.,0,0,0,2/
                   C
                   C ......................................................................
                   C .                                                                    .
                   C .   GAUSSIAN QUADRATURE INTEGRATION WITH FOUR POINTS                 .
                   C .                                                                    .
                   C ......................................................................
                   C
                          DO 10 I = 1,N
                            Y(I) = (YX-YN)/2.*XI(I)+(YX+YN)/2.
                       10   X(I) = (XX-XN)/2.*XI(I)+(XX+XN)/2.
                          FINT = 0.
                          DO 30 J = 1,N
                            F(J) = 0.
                            DO 20 I = 1,N
                       20     F(J) = F(J)+W(I)*FCN(X(I),Y(J))
                            F(J) = F(J)*(XX-XN)/2.
                       30   FINT = FINT+W(J)*F(J)
                          FINT = FINT*(YX-YN)/2
                          RETURN
                          END

  41000B CM STORAGE USED          .186 SECONDS
```

```
FUNCTION FCN        73/172  TS                            FTN 4.6+452       04/27/79  11.45.47        PAGE      1

                  FUNCTION FCN (X,Y)                                                        E   10
                  COMMON /AC/ XA,YA,ZA                                                      E   20
                  RANGE = SQRT((X-XA)**2+(Y-YA)**2+ZA**2)                                   E   30
                  ARG = 129.12-22.5*ALOG10(RANGE)                                           E   40
                  FCN = (3.36E-6*10.**(.103*ARG))/(.2*10.**(.03*ARG)+1.43E-4*10.**(.  E   50
                 108*ARG))                                                                  E   60
                  RETURN                                                                    E   70
                  END                                                                       E   80

41000B CM STORAGE USED          .100 SECONDS
```

```
                         >>> COST REPORT FOR LISTOAF <<<

                    04/27/79                           11.45.59

       RESOURCE                  BILLING RATE                  UNITS USED           COST

CENTRAL PROCESSOR     -      $105.00  /HOUR                 9.314 CP SECONDS      $    .27
PERIPHERAL PROCESSOR           20.00  /HOUR                 9.737 PP SECONDS           .05
I/O                            80.00  /HOUR                 2.926 IO SECONDS           .07
FIELD LENGTH                    3.00  /KILO-WRD-HOUR      205.576 KILO-WRD-SECS.       .17
                                                                                  --------
(BASIC COST EXCLUDES LINES PRINTED, CARDS PUNCHED                     BASIC COST       .56
  AND PLOTTER TIME CHARGES)

JOB PRIORITY  3         PRIORITY COST FACTOR  1.00         APPROXIMATE ADJUSTED COST      .56

       AS OF LAST ACCOUNT UPDATE, ACCOUNT EXPIRES 04/30/79, FUNDS LEFT $  6037.31


  04/27/79    UVA NOS/BE 1.2   LEVEL 454-03/11/78
11.45.47.LISTOAF  FROM *GD/AB
11.45.47.LIST,M3117A,T100.
11.45.47.ATTACH,Q,NEWTIDY.
11.45.47.PF CYCLE NO. = 002
11.45.47.FTN(I=Q)
11.45.59.  45000B   CM STORAGE USED
11.45.59.     9.292 CP SECONDS COMPILATION TIME
11.45.59.    STOP
11.46.00.EJ  END OF JOB, AB

PRINT COST $000.88 LISTOAF //// END OF LIST ////   0000803 LINES
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UNIVERSITY OF VIRGINIA

School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,000 students with a graduate enrollment of 350 There are approximately 120 faculty members, a majority of whom conduct research in addition to teaching

Research is an integral part of the educational program and interests parallel academic specialties These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering, and Applied Mathematics and Computer Science In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics All departments offer the doctorate, the Biomedical and Materials Science Departments grant only graduate degrees

The School of Engineering and Applied Science is an integral part of the University (approximately 1,400 full-time faculty with a total enrollment of about 14,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, and Business Administration In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service

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