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James A. Chalfant, Richard S. Gray, and Kenneth J. White

An almost ideal demand system for meats is estimated using Canadian data. A Bayesian approach is used to impose inequality restrictions on substitution elasticities, via Monte Carlo integration and importance sampling, in order to conform with prior beliefs about curvature and monotonicity restrictions and substitution relationships. Results are more consistent with the concavity and monotonicity restrictions from demand theory than with the added restriction that all meats are substitutes.

Key words: almost ideal demand system, Bayesian inference, inequality constraints, meats demand.

The widespread use of flexible functional forms in demand analysis has given researchers the ability to model consumer preferences with no restrictions on the nature of substitution or complementarity relationships between pairs of goods. Unfortunately, theoretical restrictions automatically met by simpler forms need not hold with flexible forms, so estimated demand systems often conflict with prior beliefs. Symmetry and homogeneity restrictions may be violated when tested, but they are generally imposed using equality restrictions on the parameters of the model.

More difficult are restrictions represented as inequality constraints. It is common to observe predicted budget shares outside the 0–1 interval, violating monotonicity, or violation of curvature restrictions, reflected in a matrix of elasticities of substitution between goods that is not negative semidefinite. Thus, the researcher who desires to approximate arbitrary preferences often ends up with an approximation that suggests that they are badly behaved.

In this paper we show how to impose the inequality constraints of monotonicity and concavity of the consumer's expenditure function, using an almost ideal demand system for per capita consumption of meats and fish in Canada. While one might expect that many of the factors affecting U.S. consumption have also been present in Canada, relative prices have not always been similar. In particular, while beef and pork are, to a large extent, freely traded between the two countries, live poultry and poultry meat are not. During the early 1970s, Canada introduced a supply management scheme under Article 11(c) of GATT. Under this scheme, domestic prices have been supported above U.S. prices through production quotas for producers within Canada and import quotas to restrict trade. As a result, some of the relative decline in relative poultry prices that has occurred in the United States since 1970 was not observed in Canada.

In addition to the inequality restrictions from consumer theory, we suggest a new set of inequality restrictions not generally imposed on a demand system, but which have a compelling motivation. It seems reasonable to expect that no pair of foods that play essentially the same role in the diet should be complements. One thinks of coffee and cream or beef and gravy, but not beef and fish as complementary items. Yet, complementarity is common when flexible forms are estimated. In many applications, an intermediate case between the rigidity of Cobb-Douglas or CES preferences and the flexible form would be desirable.

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While inequality restrictions on the signs of elasticities of substitution are suggested by empirical observation, rather than the underlying theory, they seem just as important as compatibility with theory in judging the degree to which an estimated demand system conforms to prior beliefs about consumer behavior. For applications such as this one, then, the constraint that all meats are substitutes can be viewed as another requirement that any well-behaved demand system must satisfy.

We make use of a Bayesian approach based on Geweke's (1986, 1988, 1989) work. He shows how to make inferences about or impose inequality restrictions in regression models. Chalfant and White used his method to impose curvature and monotonicity restrictions on the translog cost function but did not address restrictions on individual elasticities of substitution.

There are familiar alternatives to imposing prior beliefs with the Bayesian approach. One can always search over flexible forms to find one that is consistent with the desired restrictions but with the obvious consequences for inferences once a well-behaved demand system has been obtained (Judge and Bock). Alternatively, inequality restrictions can be imposed through constraints on a maximum-likelihood estimation procedure. Unfortunately, such constraints are difficult to interpret statistically (e.g., Gallant and Golub, Hazilla and Kopp, Wolak); the usual likelihood ratio test does not apply, for instance. Even if testing the restrictions is not the goal, imposing inequality constraints in this manner is likely to yield parameter values that lie on one of the constraints. For instance, constraining a demand elasticity to be nonpositive may well produce a vertical demand curve if the constraint is binding. Thus, a constrained maximum-likelihood approach is neither intuitively satisfying nor statistically attractive.

The next section discusses the almost ideal demand system. The Bayesian approach to inequality restrictions then is described in detail, followed by the application to meats demand. The data are described and demand system estimates are obtained for curvature and monotonicity restrictions alone and then with the added restriction that all meats are substitutes.

The Almost Ideal Demand System

Demand theory suggests that the demand for a good should be a function of its price, the prices of other goods, and income. In order to estimate demand relationships in a system of a reasonable size, it is common to invoke weak separability; choices concerning the allocation of expenditures among a subset of goods consumed are assumed to be made independently from the prices of goods outside that group. For example, the quantity of beef consumed is likely to be a function of the prices of beef, pork, chicken, fish, and total expenditure on meat but is not a function of the price of bananas.[1]

Whether or not it is appropriate to assume separability for a particular demand system is an empirical question. Theory suggests that any partial demand system representing the separable parts of larger systems should satisfy the conditions of symmetry, homogeneity, monotonicity, and concavity. Indeed, one interpretation of these conditions not holding in an estimated system is that the goods included in the demand system do not make up a separable group—a relevant price has presumably been omitted. Violation of those conditions also may indicate the presence of structural change, aggregation bias, or some other specification error.

Deaton and Muellbauer suggested the almost ideal demand system as a particular representation of price-independent, generalized logarithmic (PIGLOG) preferences. Such preferences are consistent with the aggregation of individual preferences. In addition, the functional form they chose is locally flexible in the sense used by Diewert and Barnett—it can attain arbitrary values for substitution elasticities at a given set of prices.

The equations for budget shares take the following form:

$$S_i = \alpha_i + \sum_{j=1}^{M} \gamma_{ij} \ln P_j + \beta_i \ln (x/P),$$

where P_j is the jth good's price, x denotes total expenditure on the M goods, and P is a price index. The specification of the underlying expenditure function introduced by Deaton and Muellbauer leads to the expression

$$\ln P = \alpha_0 + \sum_{i=1}^{M} \alpha_i \ln P_i + 1/2 \sum_{i=1}^{M}\sum_{j=1}^{M} \gamma_{ij} \ln P_i \ln P_j,$$

[1] Except, of course, to the extent that the prices of goods outside the group under study affect the total group expenditure, perhaps in a preliminary stage of allocating expenditures to aggregates such as meats, other foods, shelter, etc.

but this model, which is nonlinear in the parameters, is usually not estimated (for an exception, see Georgantelis, Phillips, and Zhang). More common is to follow the advice of Deaton and Muellbauer and replace this expression for ln P with Stone's geometric price index

$$\ln P = \sum_{k=1}^{M} S_k \log P_k$$

giving rise to an approximate almost ideal demand system that Blanciforti and Green (1983a, b) termed the linear approximate model because it is linear in parameters.

For either model, a system of these share equations can be estimated to obtain parameter estimates, and simple formulas convert the parameter estimates to elasticities. The system is easily restricted to satisfy symmetry ($\gamma_{ij} = \gamma_{ji} \forall i, j$) and homogeneity ($\Sigma_{j=1}^{M} \gamma_{ij} = 0 \forall i$). The adding-up property holds, given these restrictions, provided that $\Sigma_{i=1}^{M} \alpha_i = 1$ and $\Sigma_{i=1}^{M} \beta_i = 0$. Concavity or monotonicity restrictions are more difficult because they involve multiple inequality restrictions on the parameters. For concavity, the matrix of second derivatives of the expenditure function, or equivalently, of elasticities of substitution, must be negative semidefinite. For monotonicity, predicted budget shares must all be between 0 and 1 to ensure that predicted quantities consumed are positive.[2] Such restrictions are difficult to impose using most econometric packages and even harder to interpret statistically.

As with other flexible functional forms, elasticities are not constant with respect to prices and expenditures. The income elasticity of demand for good i is

$$\eta_i = 1 + \frac{\beta_i}{S_i}$$

for either specification of the price index. The Marshallian elasticity of demand for good i with respect to price j in the nonlinear model is

$$\eta_{ij} = -\delta_{ij} + \frac{1}{S_i} \cdot \left[\gamma_{ij} - \beta_i \alpha_j - \beta_i \sum_{k=1}^{M} \gamma_{jk} \ln P_k \right]$$
$$= -\delta_{ij} + \frac{1}{S_i} \cdot \left[\gamma_{ij} - \beta_i \left(\alpha_j + \sum_{k=1}^{M} \gamma_{jk} \ln P_k \right) \right],$$

where δ_{ij} equals 1 when $i = j$ and 0 otherwise.

The implied compensated price elasticity (η_{ij}^*) or elasticity of substitution (σ_{ij}) can then be obtained by manipulating the Slutsky equation in elasticity form:

$$\eta_{ij}^* = S_j \sigma_{ij} = \eta_{ij} + S_j \eta_i,$$

which yields, for the elasticity of substitution,

$$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{S_i S_j} + \frac{-\delta_{ij}}{S_i} + \frac{\beta_i}{S_i} \left[1 - \frac{\alpha_j + \sum_{k=1}^{M} \gamma_{jk} \ln P_k}{S_j} \right].$$

The term in square brackets disappears if Stone's price index is used, leaving

$$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{S_i S_j} + \frac{-\delta_{ij}}{S_i}.$$

Green and Alston noted that the latter formula is obtained for the linear-approximate model only if the budget shares appearing in Stone's price index are treated as exogenous, and they offered alternative formulas to accommodate this problem. However, the Monte Carlo results in Foster, Green, and Alston show that, if the nonlinear model is viewed as the underlying demand system and the linear-approximate model is indeed viewed as an approximation, the simpler formula provides a good approximation. This can be seen by noting that for homothetic models (where each $\beta_i = 0$), the two formulas are identical, and for nonhomothetic models, the discrepancy is likely small.

A final practical advantage of the simpler formulas is that they remain consistent with the income elasticity η_i above, whereas treating the budget shares in the price index as endogenous means that the model no longer will be characterized by these income elasticities. Deaton and Muellbauer noted that these elasticities were an advantage of the almost ideal model. In the application below, the simpler formulas are used, but the method would also apply with the "true" nonlinear model or the Green and Alston formulas.

The Bayesian Approach to Testing Inequality Restrictions

An alternative approach to imposing inequality restrictions in a demand system is made possible using a Bayesian approach, which permits the

[2] Chapter 3 of Varian, or a comparable text, provides further details concerning the restrictions from consumer theory.

formal inclusion of such prior information. Often, prior information can be imposed by choice of functional form. An extreme case is the Cobb-Douglas utility function, which would impose all of the restrictions from consumer theory plus some less desirable ones, such as additivity of preferences and that elasticities of substitution are each one. The latter restriction might lead one to question the stability of a demand system, for instance, if substitution patterns in response to relative price changes were not consistent with the assumed elasticities. Thus, flexible functional forms are a preferred alternative.

Symmetry and homogeneity restrictions from demand theory represent prior information that is often imposed on flexible forms through equality restrictions on the parameters. Such restrictions reduce the dimensionality of the parameter space when demand systems based on these forms are estimated; the symmetry and homogeneity restrictions, for instance, provide considerable gains in degrees of freedom. Prior information taking the form of an inequality restriction is less informative than such equality restrictions, in the sense that this information serves to truncate the parameter space, rather than reduce the number of free parameters. For instance, a particular parameter θ may be restricted to be positive. Conventional approaches to estimation do not permit the formal inclusion of such information (e.g., Judge et al.), and most econometric packages do not permit such restrictions to be imposed.

The problem of prior beliefs that take the form of inequality constraints is easily handled in the context of Bayesian inference. The Bayesian approach begins with a prior density function, defined over the vector of parameters, θ, call it $p(\theta)$.[3] This prior density summarizes all of the information the researcher has about θ prior to estimation. Specifying $p(\theta)$ permits the formal inclusion of information about the parameters. For instance, if a particular parameter is considered equally likely to be positive or negative, a zero median characterizes the marginal probability density function used to describe prior beliefs about that parameter. If there is no prior information about θ, $p(\theta)$ is simply defined to be proportional to a constant over all real numbers, thus making it an improper density and a so-called "diffuse prior." Alternatively, $p(\theta)$ could be a proper density function that reflects various beliefs about θ in the form of probability statements. A very simple case is the prior p.d.f. which says that θ is contained in some region D with probability one:

$$p(\theta) \propto c \quad \forall\ \theta \in D,$$

where $\propto$ denotes "is proportional to"; D may be an open or closed interval, depending on the application. We consider below how such a prior density can be used to represent prior information about the parameters of a demand system.

Bayes' theorem shows how to combine prior and sample information to obtain a posterior distribution for the parameters in θ given a data set y:

$$f(\theta|y) \propto p(\theta)\, L(\theta|y),$$

where $L(\cdot)$ is the likelihood function based on the observed data. Unlike the sampling-theoretic approach to estimation, the Bayesian approach recognizes that posterior beliefs are conditional on the observed data set rather than emphasizing the performance of estimators in repeated samples.

The posterior distribution $f(\theta|y)$ summarizes all information available about θ, both prior and sample information. It can serve as the end result of an investigation, or it can be used to calculate confidence intervals and probabilities related to hypotheses about θ or to obtain a point estimate of θ or some related quantity such as a demand elasticity. The optimal point estimate for θ depends on the investigator's objective function. Constrained maximum-likelihood estimation, which yields the mode of the posterior distribution as a point estimate, corresponds to a "zero-one" loss function (e.g., Zellner 1988). This is an implausible loss function, in that it places the same weight on being wrong for estimates arbitrarily close to the true θ and for choices very far from that value. More reasonable loss functions can be imagined, and different point estimates will result. For instance, if the investigator's loss function is quadratic, the mean of the posterior distribution for θ minimizes expected loss (e.g., Judge et al.). All that is needed, then, to find Bayesian point estimates of the parameters of a demand system is a means to describe prior beliefs in the form of inequality restrictions using $p(\theta)$, a way to obtain the posterior density function, and then a way to find its mean.

Below we illustrate this approach using the quadratic loss function. With the inequality re-

[3] See Zellner (1971) or Judge et al. for much more detailed descriptions of the properties of the Bayesian approach. An excellent introduction to inequality restrictions in the Bayesian framework is provided by Griffiths.

strictions imposed, it is straightforward to obtain the mean of the posterior distribution, call it $\bar{\theta}$. This serves as the optimal point estimate of the parameters of the demand system.

Also of interest is some measure of the plausibility of the restrictions, given the data. Suppose prior beliefs are completely uninformative; that is, all parameter values are considered equally likely. In this case, the sample information dominates the posterior density function and an optimal point estimate is the mean of the (unrestricted) posterior distribution. The probability that the restrictions are correct can be calculated using the unconstrained posterior density. This probability is interpreted as the degree of belief that the restrictions are true, based on observed data, found by obtaining the probability that θ lies in D.

Both restrictions on the signs of substitution elasticities and restrictions on the entire matrix of substitution elasticities, to satisfy curvature restrictions, can be examined by calculating substitution elasticities. To evaluate these inequality restrictions, then, the behavior of elasticities of substitution must be examined everywhere in the parameter space where the researcher wishes to impose them. Similarly, the monotonicity restriction can be evaluated using predicted budget shares. Each set of restrictions is then imposed by truncating the parameter space so that each restriction holds. To obtain a Bayesian point estimate (with a quadratic loss function), the researcher must find the mean of the truncated posterior distribution for the parameter vector.

While these calculations are in principle straightforward, requiring that integrals over the posterior density function be evaluated, the analytic solutions cannot be obtained in practice, except for fairly simple models. The dimension of the posterior density is likely to be too great, even if the density function and the region of the parameter space of interest can be described easily. Instead, it is necessary to evaluate the integrals using Monte Carlo integration. This permits estimating the solutions to integrals by random sampling.[4]

To describe the method, we begin by specifying a data-generating process. We assume that prices and expenditures may be treated as exogenous, so that the parameters of the system of $M - 1$ equations for budget shares could be estimated using seemingly unrelated regressions (SUR). As is well known, the equation for the Mth budget share cannot be included without implying a singular contemporaneous covariance matrix for the error terms in the M share equations (Barten), but deleting the Mth share and using restrictions on the parameters allows the complete set of parameter estimates to be obtained. Use of iterated SUR was shown by Barten to lead to maximum-likelihood estimates that are invariant to the equation chosen for deletion.

We assume that each time period's $M - 1$ vector of errors, and therefore the vector of budget shares, follows the multivariate normal distribution. Strictly speaking, one might prefer a distribution more compatible with the fact that observed shares are bounded by 0 and 1 (e.g., Woodland, Rossi), in which case imposing monotonicity is not an issue, but we prefer to stick with the more widely used distribution to illustrate the method. Our approach could easily be adjusted for nonnormal errors.

To illustrate Monte Carlo integration, suppose that Σ, the variance-covariance matrix of the errors, is known. Suppose also that $p(\theta)$, the prior information about the α_i's, γ_{ij}'s, and β_i's, indicates that some region D, a proper subset of R^J, contains the true parameter vector, where J denotes the number of free parameters in the model and R^J denotes the J-dimensional real numbers. Finally, suppose that the investigator has a quadratic loss function, and desires a point estimate of θ; as noted earlier, the mean of the posterior density for θ minimizes expected loss.

The steps involved in finding an estimate of the mean, $\bar{\theta}$, are straightforward. With no prior information about θ and a known Σ, the posterior distribution for θ estimator would be the multivariate normal, centered at $\hat{\theta}$ with variance-covariance matrix $V(\hat{\theta})$, where $\hat{\theta}$ and $V(\hat{\theta})$ are obtained using Σ-restricted SUR. Given the prior information, the posterior distribution for θ then becomes the truncated multivariate normal, since θ is known to lie in D. The task becomes finding the mean of a truncated, J-variate normal:

$$E(\theta) = \int_{\theta_1 \in D} \cdots \int_{\theta_J \in D} \theta N_J[\theta | \hat{\theta}, V(\hat{\theta})] \, d\theta_1 \ldots d\theta_J.$$

Needless to say, such a calculation is infeasible for all but trivial examples.

Monte Carlo integration is based on the idea that an expectation such as the one above can

[4] Kloek and van Dijk, van Dijk and Kloek, and Geweke (1986, 1988, 1989) provide the foundations for the Monte Carlo integration and importance sampling, described below. The application to demand systems follows the discussion in Chalfant and White. Gallant and Monahan; and Barnett, Geweke, and Yue have also followed a Bayesian approach to demand system estimation.

be estimated (arbitrarily accurately, given the Law of Large Numbers) using random sampling. One way of estimating the mean of a random variable with p.d.f. $f(\theta)$ is to generate a large number of replications in a random sample from that distribution, and calculate

$$\bar{\theta} = \frac{\sum_{i=1}^{N} \theta_i}{N},$$

where N is the number of replications. $\bar{\theta}$ serves as an estimate of $E(\theta)$, of course. Since N is determined by the investigator, $E(\theta)$ can be estimated with an arbitrarily high degree of accuracy. To apply that approach using the multivariate normal would require five steps:

(*a*) Estimate the parameters of the share equations, obtaining $\hat{\theta}$ and $V(\hat{\theta})$.

(*b*) Treat these as parameters of the posterior distribution for θ that would be consistent with no restrictions on θ's range, the J-variate normal density.

(*c*) Use $N_J[\hat{\theta}, V(\hat{\theta})]$ and a random number generator to obtain a random sample from this multivariate normal. Omit those draws θ_i which are not contained in D, leaving a random sample of size n from the truncated multivariate normal.

(*d*) Estimate $E(\theta)$ using the average of the n replications in D:

$$\bar{\theta} = \frac{\sum_{i=1}^{n} \theta_i}{n}.$$

(*e*) A byproduct of the procedure is that $\hat{p}_D = n/N$ estimates the area under the multivariate normal density contained in D, i.e., the probability that the restrictions hold, given no prior information. If either p_D or $E(\theta)$ are estimated with less than the desired precision, increase N and repeat the process.

While somewhat computer-intensive, these steps are certainly feasible. They can be performed using the commonly available statistical packages for any desired posterior distribution. All that is required is a random number generator and some simple calculations.

Importance Sampling for Exact Results

The procedure outlined above relies on the asymptotic properties of the estimation procedure by making use of a normal approximation, since the covariance matrix Σ will invariably be unknown. While this is comparable to what is done using non-Bayesian approaches, it will not yield results consistent with the exact posterior density function. Unfortunately, the posterior distribution for θ is no longer of the multivariate normal form when the variance matrix Σ is unknown. Effectively, the procedure outlined above substituted a conditional distribution $f(\theta|y, \hat{\Sigma})$ for the marginal distribution $f(\theta|y)$.

Such a marginal distribution for θ can be obtained from a joint posterior density for θ and the parameters of Σ by integrating over a posterior density for Σ. Following Zellner (1971, p. 242), Judge et al. (p. 478), or Srivastava and Giles, with a diffuse prior density for both Σ and θ, the resulting posterior density for θ is given by

$$f(\theta|y) \propto |A|^{-T/2},$$

where T denotes the number of sample observations. A typical element a_{ij} of the $M - 1$ by $M - 1$ matrix A is given by

$$a_{ij} = [(e_i(\theta))'(e_j(\theta))],$$

where $e_i(\theta)$ is the vector of residuals for share equation i evaluated using any value of θ where the posterior density is defined. This density function corresponds to the posterior density for θ with no prior information about likely values. Should one wish to impose the restriction that θ could take on only values consistent with the inequality restrictions, a truncated version of this posterior density must be used. In this framework, imposing the Bayesian restrictions and finding a posterior mean $\bar{\theta}$ requires sampling from the truncated density $f^R(\theta|y)$, but this is not a familiar density, so it is difficult to obtain an appropriate random sample. The procedure outlined above, whereby the untruncated density $f(\theta|y)$ could be used, also cannot be applied, for the same reason.

Instead, the steps outlined above must be modified, correcting for the fact that the multivariate normal is at best only approximately the correct posterior density. The technique for doing so is called importance sampling (e.g., Kloek and van Dijk, van Dijk and Kloek, Geweke 1986, 1988, 1989). The concept which underlies importance sampling is relatively straightforward. Before returning to the problem at hand, its use is illustrated with a simple example.

Consider estimating the mean of Z, a beta random variable with $\alpha = 9$ and $\beta = 1$ and density function $f(z)$. Such a random variable has

$$E(Z) = \int_0^1 z f(z)\, dz = \frac{\alpha}{\alpha + \beta} = .9.$$

Of course, this is an example where the integral could be evaluated and where random draws can also be obtained from the correct distribution, but suppose that we had available only a uniform random number generator, whose density function we denote $g(z)$. How might the mean of the beta distribution be calculated, using random draws from $U(0, 1)$?

The sample mean of replications drawn from the $U(0, 1)$ will underestimate $E(Z)$ because it will tend toward one-half. The reason, of course, is that values close to zero for Z occur more often under $U(0, 1)$ and most values above one-half will occur less often than under the beta distribution we have chosen. Importance sampling corrects for this by adjusting the "importance" given to each replication. The appropriate weight for each replication z_i is the ratio of the probability density function of the beta distribution at z_i, $f(z_i)$, and the density of the uniform at z_i, $g(z_i)$. In this way, those values drawn which are closer to one will receive a large weight while those closer to zero will receive a smaller weight.

To see why this works, note that the expected value of Z using the density function given by $f(z)$ can be found by integrating over $g(z)$ instead:

$$E(Z) = \int z f(z)\, dz = \int z \frac{f(z)}{g(z)} g(z)\, dz.$$

In the first instance, $E(Z)$ is taken with respect to f; and in the second, $E[Zf(Z)/g(Z)]$ is taken with respect to g. Just as $E(Z)$ could be estimated using a sample mean of replications from $f(z)$, then so could it be estimated by drawing from $g(z)$ and calculating

$$\frac{\sum_{i=1}^{N} z_i \frac{f(z_i)}{g(z_i)}}{N}.$$

One surprising aspect of this procedure is that any density function can be used as $g(z)$, provided it is strictly positive over the range of Z determined from $f(Z)$. Otherwise, division by $g(z)$ within the integral is not allowed, and the implication would be that some values of Z, which do occur when sampling from $f(z)$, would never be drawn using $g(z)$. Naturally, if the weights applied to each z_i are close to one, so that $f(z)$ and $g(z)$ are similar, fewer draws will be required to obtain good estimates of the values of these integrals (Kloek and van Dijk; van Dijk and Kloek; Geweke 1986, 1988, 1989).

This procedure can be applied for estimating the mean of the exact posterior density for θ. The posterior distribution is analogous to the beta distribution in the case above, in that it is the correct density but difficult to work with. Meanwhile, the multivariate t will be used as was the uniform distribution, to generate replications for θ. In the case of uninformative priors, draws from the multivariate t can be adjusted by the ratio of the two density functions to obtain an estimate of the probability that the inequality restrictions hold. Alternatively, the inequality restrictions can be imposed, so that the posterior density is truncated or restricted.

To find the posterior mean, find the solution to

$$E(\theta|y) = \int_{\forall\theta\in D} \theta f^R(\theta|y)\, d\theta.$$

At the same time, to calculate the probability that the restrictions hold, find

$$p = \int_{\forall\theta\in D} I(\theta) f(\theta|y)\, d\theta,$$

where $I(\theta) = 1$ if the restrictions hold and $I(\theta) = 0$ otherwise. Each of these could be accomplished by sampling from the exact posterior density $f(\theta|y)$, if it were of a known form, and the steps outlined earlier could be used. Because these integrands are too complicated to permit analytic solutions, importance sampling must be used. To reiterate, notice that the posterior mean can also be found by

$$E(\theta|y) = \int_{\forall\theta\in D} \theta \frac{f^R(\theta|y)}{g^R(\theta|y)} g^R(\theta|y)\, d\theta,$$

where $g^R(\theta|y)$ is the truncated multivariate t p.d.f. and D is the region of the parameter space consistent with the concavity and monotonicity restrictions. Again, the density $g^R(\theta|y)$ must be positive over the entire range (D) of the posterior density $f^R(\theta|y)$.

The modified steps now required for the calculations, taken from Chalfant and White, are given below:

(*a*) Estimate the parameters of the demand system using iterated seemingly unrelated regressions to obtain maximum likelihood estimates $\hat{\theta}$ and the estimated covariance matrix $V(\hat{\theta})$.

(*b*) Calculate a matrix H such that $HH' = V(\hat{\theta})$. Draw a random vector of the same length as $\hat{\theta}$ from the standard normal distribution

$$w \sim N(0, I),$$

where I is the identity matrix of order J. Replications of θ that follow a multivariate t-distribution with λ degrees of freedom can be generated using the steps in van Dijk and Kloek. We obtain

$$\theta^A = \hat{\theta} + Hw/v$$

and its "antithetic replication"

$$\theta^B = \hat{\theta} - Hw/v,$$

where v is obtained by taking a vector z of λ draws from $N(0, 1)$ and calculating

$$v = [(z'z)/\lambda]^{1/2}.$$

The inclusion of antithetic replications was suggested by Geweke (1988) to improve convergence.

Using the multivariate t,

$$g(\theta|y) \propto \left[\lambda + (\theta - \hat{\theta})' {\sum}^{-1} (\theta - \hat{\theta})\right]^{-(\lambda+J)/2}$$

(c) Check each replication to see if it violates concavity, monotonicity, or substitutability. To do so, we calculated elasticities of substitution using each replication and the means of observed budget shares for the four meats. Concavity and substitutability were checked using these elasticities. (To check these restrictions for all 29 observations would have involved simply repeating the same check for each observed share vector.) To check monotonicity, we used each replication to obtain new predicted shares for all twenty-nine data points.

(d) Estimate the mean of the posterior distribution using the n draws of θ^A or θ^B that satisfy the restrictions

$$\bar{\theta} = \frac{\sum_{k=1}^{n} \theta_k \dfrac{f^R(\theta_k|y)}{g^R(\theta_k|y)}}{\sum_{k=1}^{n} \dfrac{f^R(\theta_k|y)}{g^R(\theta_k|y)}}.$$

As noted by Chalfant and White, if $f(\theta|y)$ and $g(\theta|y)$ were proper density functions, a denominator of N would suffice. Otherwise, the denominator serves as a normalizing constant to correct for the fact that we use only the kernels of proper densities.

(e) To estimate the probability that the restrictions hold, use all replications, letting the first n be those consistent with the restrictions, and calculate

$$\hat{p}_D = \frac{\sum_{k=1}^{N} I(\theta_k) \dfrac{f(\theta_k|y)}{g(\theta_k|y)}}{\sum_{k=1}^{N} \dfrac{f(\theta_k|y)}{g(\theta_k|y)}} = \frac{\sum_{k=1}^{n} \dfrac{f^R(\theta_k|y)}{g^R(\theta_k|y)}}{\sum_{k=1}^{N} \dfrac{f(\theta_k|y)}{g(\theta_k|y)}}.$$

(f) Check to see if the number of replications is large enough to arrive at stable estimates of $\bar{\theta}$ or $\hat{p}$ and of their standard deviations, if desired. If not, increase N.

A measure of the numerical accuracy of each estimated expectation, analogous to the usual standard error of the estimate of a population mean, was suggested by Geweke (1989). These can be used in step (f). The numerical standard error (nse) of the estimate of the mean of the posterior is given by

$$nse(\theta) = \frac{\sum_{k=1}^{N} (\theta_k - \bar{\theta}_k)^2 \left(\dfrac{f(\theta_k|y)}{g(\theta_k|y)}\right)^2}{\left(\sum_{k=1}^{N} \dfrac{f(\theta_k|y)}{g(\theta_k|y)}\right)^2}.$$

As also pointed out by Barnett, Geweke, and Yue, these measures are reliable estimates of the true nse's if and only if the tails of the distribution being sampled from are at least as "fat" as the tails of the posterior distribution. This characteristic of the importance sampling procedure is easily checked by plotting the replications of each θ_k against the ratio $f^R(\theta|y)/g^R(\theta|y)$. The weights should approach zero in the tails of each posterior distribution.

The standard deviation of the posterior distribution can be estimated by taking the square root of the estimated variance of the posterior distribution, obtained by using importance sampling to estimate $E[(\theta_k - \bar{\theta}_k)^2]$:

$$s.d.(\theta) = \left[\frac{\sum_{k=1}^{N} (\theta_k - \bar{\theta}_k)^2 \left(\dfrac{f^R(\theta_k|y)}{g^R(\theta_k|y)}\right)}{\sum_{k=1}^{n} \dfrac{f^R(\theta_k|y)}{g^R(\theta_k|y)}}\right]^{1/2}.$$

Application to Aggregate Meat Consumption in Canada

In this section, an almost ideal demand system for meat and fish products is estimated using aggregate Canadian data for the years 1960 to

1988.[5] The data are from Agriculture Canada (Robbins) and were also used by Alston and Chalfant. The demands for four goods were examined—beef, pork, poultry, and fish (henceforth, the meats group). Consumer preferences for the meats group were assumed weakly separable from all other goods, and the demand system was treated as the second-stage allocation model, conditional on prices and a predetermined level of expenditures.[6] Using the Agriculture Canada data on price indices and per capita quantities consumed for each meat, along with 1981 nominal prices reported in Van Kooten, we converted indices into current prices and calculated total expenditures on the group.

One remaining variable was included, a "trend" term taking the value 1 in 1975 and increasing by 1 in every subsequent period. Alston and Chalfant found that, although this data set is consistent with the hypothesis of stable preferences, the almost ideal demand system cannot be fit to these data without some allowance for dynamic influences; we chose the trend term as being more interpretable, albeit conditional on this specification, than an autocorrelation correction. A trend that began in the middle of the sample, 1975, seemed to correspond more to a hypothesized structural change or other type of model failure than did one beginning in 1960.

The share equations of the linear approximate form, obtained by deflating nominal expenditures by Stone's geometric price index, were then augmented by including this trend coefficient in each share equation. Thus the expression for the share of the budget allocated to the *i*th meat is

$$S_i = \alpha_i + \sum_{j=1}^{4} \gamma_{ij} \ln(P_j) + \beta_i \ln(x/P) + \tau_i t.$$

A system of three such equations was estimated using the nonlinear regression (NL) procedure of version 6.1 of SHAZAM (White et al.). The fourth equation was deleted because of singularity of the variance matrix for all four equations, and parameters of that equation were obtained through the homogeneity and symmetry restrictions. The adding-up property is preserved by letting the sum of the four trend coefficients equal zero. By iterating over both the parameters and the error variance-covariance matrix, the estimates obtained are invariant to the equation chosen for deletion (Barten).

Table 1. Unconstrained Results, Almost Ideal Demand System

Parameter[a]	Estimate	Standard Error
α_{beef}	−0.502	0.157
$\gamma_{beef \cdot beef}$	0.091	0.019
$\gamma_{beef \cdot pork}$	−0.004	0.012
$\gamma_{beef \cdot poultry}$	−0.009	0.013
$\gamma_{beef \cdot fish}$	−0.078	
β_{beef}	0.216	0.039
τ_{beef}	−0.0029	0.0007
α_{pork}	0.732	0.144
$\gamma_{pork \cdot pork}$	0.040	0.0145
$\gamma_{pork \cdot poultry}$	0.0162	0.017
$\gamma_{pork \cdot fish}$	−0.052	
β_{pork}	−0.010	0.037
τ_{pork}	−0.0020	0.0008
$\alpha_{poultry}$	0.211	0.238
$\gamma_{poultry \cdot poultry}$	0.016	0.037
$\gamma_{poultry \cdot fish}$	−0.023	
$\beta_{poultry}$	−0.0006	0.063
$\tau_{poultry}$	0.0032	0.0011
$\gamma_{fish \cdot fish}$	0.152	
β_{fish}	0.116	
τ_{fish}	0.0016	

[a] Estimates of parameters without standard errors were obtained from the equality restrictions for symmetry, homogeneity, and adding-up.

Estimates of the parameters are given in table 1. The trend coefficients do appear to contribute significantly to the model, as the likelihood-ratio test statistic for the restriction that they are all zero is 35.19 (as compared with a critical value of $\chi^2_{3,\alpha=.05}$ of 11.345). They imply that, if the almost ideal model is the correct specification, there has been a negative trend affecting beef and pork and a positive one affecting chicken and fish.[7]

Elasticities of substitution are reported in table 2. Compensated price elasticities and expenditure elasticities are reported in table 3, part A, while uncompensated Marshallian elastici-

[5] Other applications of this system to agricultural data include Blanciforti, Green, and King; Chalfant; Hayes, Wahl, and Williams; and Moschini and Meilke. Hayes, Wahl, and Williams also consider the restriction that all meats are substitutes, in an examination of Japanese meats demand.

[6] Although LaFrance notes that treating expenditures as predetermined is inconsistent with the assumption that quantities are measured with errors, this is standard practice. The method we use does not depend on this assumption, however, and could accommodate a joint explanation of the first- and second-stage allocation decisions.

[7] Because the results from the nonparametric test for stable preferences were consistent with the existence of a stable demand system, the trends may be picking up misspecification of the almost ideal model. One way to decide whether one should instead disregard those results and conclude that significant trends imply structural change rather than misspecification is to analyze the implied behavior of elasticities in a variety of models. Our analysis of just one demand system cannot answer that question definitively, but the procedure followed below provides a means for such an analysis to incorporate prior beliefs.

Table 2. Elasticities of Substitution: Posterior Mean Values

Elasticity	Unconstrained	Concavity	Concavity and Substitutability
$\sigma_{\text{beef beef}}$	−1.13	−1.158	−1.207
		[0.003][a]	[0.004]
		(0.188)	(0.181)
$\sigma_{\text{beef pork}}$	0.953	0.934	0.902
		[0.003]	[0.004]
		(0.158)	(0.156)
$\sigma_{\text{beef poultry}}$	0.853	0.797	0.735
		[0.004]	[0.006]
		(0.225)	(0.221)
$\sigma_{\text{beef fish}}$	−0.049	0.072	0.252
		[0.005]	[0.007]
		(0.316)	(0.272)
$\sigma_{\text{pork pork}}$	−2.384	−2.405	−2.471
		[0.005]	[0.007]
		(0.282)	(0.280)
$\sigma_{\text{pork poultry}}$	1.268	1.186	1.087
		[0.008]	[0.011]
		(0.406)	(0.404)
$\sigma_{\text{pork fish}}$	0.013	0.147	0.375
		[0.009]	[0.011]
		(0.466)	(0.423)
$\sigma_{\text{poultry poultry}}$	−3.641	−3.876	−3.627
		[0.020]	[0.023]
		(1.092)	(0.994)
$\sigma_{\text{poultry fish}}$	0.549	0.965	0.95
		[0.019]	[0.020]
		(1.014)	(0.835)
$\sigma_{\text{fish fish}}$	−0.458	−1.216	−1.772
		[0.020]	[0.019]
		(1.181)	(0.795)

[a] The numerical standard errors of the estimated posterior means are reported in square brackets, and the corresponding standard deviations are in parentheses.

ties are given in table 4, part A. Recall that the latter make use of a constant level of meats expenditure, not total expenditure. These elasticities were calculated at the mean budget shares observed in the sample and were obtained without the inequality constraints.

The negative own-elasticities of substitution are as one would expect. The mostly positive elasticities of substitution indicate that the meats tend to be substitutes for one another at the midpoint of the sample. The unrestricted results that are contrary to prior belief are the negative elasticity of substitution, indicating complementarity, between fish and beef that is observed at the sample mean shares. Also contrary to expectations was the small positive elasticity between fish and pork and the fact that this elasticity also was often negative when the elasticities of substitution were evaluated at the individual observations.

Concavity holds at the mean and for the last eighteen observations in the sample; however, the own-elasticity of substitution for fish is positive for the first eleven data points, violating a necessary condition for concavity. For all but six observations, at least one elasticity of substitution between two goods was negative for either beef and fish or beef and pork, sometimes both at once. Finally, monotonicity holds with these estimates at every point in the sample.[8]

Using the Bayesian procedure described earlier, the probability that the monotonicity, concavity, and substitution restrictions hold for this demand system can be estimated. We followed the procedure outlined earlier to obtain a sample size of 10,000 replications (5,000 plus the antithetic replications) from the multivariate t-distribution with $\lambda = 4$ degrees of freedom, again

[8] A further indication that this model requires trend effects is that the results without the trend terms are completely inconsistent with the concavity or substitution restrictions. When the model was estimated with the trend coefficients set equal to zero and the estimated parameters were used to calculate elasticities, every observation violated both restrictions.

Table 3. Compensated Demand Elasticities

Quantity i	Price j Beef	Pork	Poultry	Fish
	Part A. Unconstrained Case			
Beef	−0.393	0.234	0.169	−0.010
Pork	0.331	−0.586	0.252	0.003
Poultry	0.297	0.312	−0.723	0.114
Fish	−0.017	0.003	0.109	−0.095
Expend. Elasticities	1.614	0.582	0.966	0.499
	Part B. Concavity Imposed			
Beef	−0.403	0.230	0.158	0.015
Pork	0.325	−0.591	0.235	0.031
Poultry	0.277	0.291	−0.769	0.200
Fish	0.025	0.036	0.191	−0.253
Expend. Elasticities	1.575	0.537	0.832	0.745
	Part C. Concavity and Substitutability Imposed			
Beef	−0.420	0.222	0.146	0.052
Pork	0.314	−0.607	0.216	0.078
Poultry	0.256	0.267	−0.720	0.197
Fish	0.088	0.092	0.189	−0.368
Expend. Elasticities	1.524	0.462	0.860	0.893

using SHAZAM. The in-sample results for $\hat{\theta}$ and $V(\hat{\theta})$ were used to specify the parameters of this distribution. Only four degrees of freedom were chosen to ensure that sampling would occur from a density with relatively "fat" tails, thereby assuring that the entire range of the actual posterior distribution is covered by the importance density. Monotonicity was checked by using each replication to predict budget shares for all twenty-nine observations; every replication was consistent with positive budget shares for each observation, so monotonicity was never violated. Concavity was checked for each replication by calculating substitution elasticities using the parameter values given by each replication and the mean observed budget shares. The eigenvalues were then calculated for each matrix of elasticities of substitution.

Consistency with the concavity restriction (a substitution matrix without positive eigenvalues) was found in over half of the replications. Had we been sampling from the exact posterior, the proportion consistent with concavity would have been the estimate of the probability that concavity holds, given diffuse priors. As noted earlier, however, that probability can be estimated using importance sampling to correct for not sampling from the exact posterior. Calculation of the ratio $\hat{p}_D$ in step (*e*) of section 4 yielded a value of 0.57. These results imply that the demand system is reasonably well behaved by this criterion. Table 5 summarizes all of the results in this section for the posterior probabilities that various restrictions are true.

Elasticities of substitution calculated using the concavity restriction are given in table 2, with numerical standard errors and standard deviations. The point estimates for the elasticities are obtained by estimating the means of the posterior distributions for each elasticity. Those distributions were found using the posterior distributions for the parameters, the mean budget shares, and the formulas for elasticities. The own

Table 4. Marshallian Demand Elasticities

Quantity i	Price j Beef	Pork	Poultry	Fish
	Part A. Unconstrained Case			
Beef	−0.955	−0.163	−0.151	−0.346
Pork	0.129	−0.729	0.136	−0.118
Poultry	−0.039	0.074	−0.914	−0.087
Fish	−0.191	−0.119	0.010	−0.199
	Part B. Concavity Imposed			
Beef	−0.951	−0.158	−0.154	−0.312
Pork	0.138	−0.723	0.129	−0.081
Poultry	−0.012	0.087	−0.934	0.028
Fish	−0.234	−0.147	0.044	−0.407
	Part C. Concavity and Substitutability Imposed			
Beef	−0.950	−0.153	−0.157	−0.264
Pork	0.153	−0.721	0.124	−0.018
Poultry	−0.044	0.056	−0.891	0.0187
Fish	−0.223	−0.127	0.011	−0.554

Table 5. Posterior Probabilities of Restrictions

Inequality Restriction	None	Concavity	Concavity and Substitutability
Concavity	0.57		
(n.s.e.)	0.010		
All substitutes	0.16	0.28	
(n.s.e.)	0.003		
$\sigma_{\text{beef pork}} > 0$	1	1	
$\sigma_{\text{beef poultry}} > 0$	0.999	1	
$\sigma_{\text{beef fish}} > 0$	0.427	0.59	
$\sigma_{\text{pork poultry}} > 0$	0.999	1	
$\sigma_{\text{pork fish}} > 0$	0.507	0.63	
$\sigma_{\text{poultry fish}} > 0$	0.713	0.89	
$\tau_{\text{beef}} < 0$	1	1	1
$\tau_{\text{pork}} < 0$	0.993	1	1
$\tau_{\text{poultry}} > 0$	0.983	.976	0.984
$\tau_{\text{fish}} > 0$	0.872	1	1
all τ	0.853	0.974	0.984

elasticities of substitution tend to be more negative than before (reflecting the concavity restriction), while imposing concavity alone was enough to make the average over all replications of each partial elasticity of substitution turn out to be positive. Expenditure and compensated price elasticities calculated using $\bar{\theta}$ are shown in table 3, part B, while table 4, part B contains corresponding uncompensated elasticities.

The system was less consistent with the restriction that all meats are substitutes. Imposing concavity and substitutability jointly reduces the parameter space D further; the probability that the joint restriction holds, given diffuse prior beliefs, falls to 0.16. (Alternatively, this can be viewed as a conditional probability of substitutability, given that concavity is imposed, of 0.16/0.57 = 0.28.) This relatively stronger rejection of prior beliefs casts some doubt on the estimated system being a valid representation of preferences. Nonetheless, posterior means were calculated for the remaining elasticities. These numbers are shown in table 2 for the substitution elasticities and in part C of tables 3 and 4 for the price elasticities. For comparison to the results in table 1, the constrained parameter estimates are reported in table 6.

It is important to note that violations of the substitution restriction are not due solely to the cases of complementarity relationships implied by $\hat{\theta}$—beef and fish or pork and fish. Any elasticity could be responsible. Unless the posterior density for any particular elasticity of substitution implies that it is positive with probability one, each can be responsible for any particular replication violating the constraint. We examined the posterior distributions for each elasticity and found that those two elasticities indeed were responsible for the lion's share of rejections. We never observed a replication that implied that beef and pork were complements at the mean budget shares, while the estimated probabilities that the substitution elasticity was positive for the beef-poultry and pork-poultry combinations were both greater than 0.99. In contrast, the probability that the elasticity of substitution was positive for the remaining cases of $\sigma_{beef \cdot fish}$, $\sigma_{pork \cdot fish}$, and $\sigma_{poultry \cdot fish}$ were 0.59, 0.63, and 0.89, respectively, suggesting that fish is most responsible for the violations.

One can also reexamine the trend results with an inequality-constrained model; although the trend coefficients do not affect elasticities directly, their point estimates are correlated with those of the γ_{ij} parameters and will be affected by imposing the inequality restrictions. The

Table 6. Inequality Constrained Coefficients

	Concavity	Concavity and Substitutability
α_{beef}	−0.4429	−0.3769
	[0.0043][a]	[0.0066]
	(0.1766)	(0.1689)
$\gamma_{beef\ beef}$	0.0867	0.0808
	[0.0005]	[0.0008]
	(0.0219)	(0.0195)
$\gamma_{beef\ pork}$	−0.0056	−0.0084
	[0.0003]	[0.0005]
	(0.0134)	(0.0123)
$\gamma_{beef\ poultry}$	−0.014	−0.0183
	[0.0004]	[0.0007]
	(0.0152)	(0.0153)
$\gamma_{beef\ fish}$	−0.0671	−0.0541
β_{beef}	0.2002	0.1823
	[0.0011]	[0.0016]
	(0.0433)	(0.0408)
τ_{beef}	−0.0032	−0.0036
	[0.0000]	[0.0000]
	(0.0008)	(0.0007)
α_{pork}	0.7856	0.8555
	[0.0044]	[0.0049]
	(0.1569)	(0.1252)
$\gamma_{pork\ pork}$	0.0401	0.0361
	[0.0004]	[0.0007]
	(0.0169)	(0.0155)
$\gamma_{pork\ poultry}$	0.0091	0.0042
	[0.0006]	[0.0008]
	(0.0197)	(0.0184)
$\gamma_{pork\ fish}$	−0.8348	−0.8958
β_{pork}	−0.1138	−0.1322
	[0.0011]	[0.0012]
	(0.0400)	(0.0312)
τ_{pork}	−0.0022	−0.0026
	[0.0000]	[0.0000]
	(0.0008)	(0.0006)
$\alpha_{poultry}$	0.3344	0.3125
	[0.0064]	[0.0099]
	(0.2432)	(0.2109)
$\gamma_{poultry\ poultry}$	0.0064	0.0162
	[0.0009]	[0.0014]
	(0.0391)	(0.0324)
$\gamma_{poultry\ fish}$	−0.3385	−0.3261
$\beta_{poultry}$	−0.0334	−0.0279
	[0.0016]	[0.0026]
	(0.0632)	(0.0547)
$\tau_{poultry}$	0.0025	0.0025
	[0.0000]	[0.0000]
	(0.0012)	(0.0010)
α_{fish}	0.3229	0.2089
$\gamma_{fish\ fish}$	1.2404	1.276
β_{fish}	−0.053	−0.0222
τ_{fish}	0.0029	0.0037

[a] The numerical standard errors for each element of $\bar{\theta}$ are reported in square brackets, and the corresponding standard deviations are in parentheses. Parameters without these measures were obtained from the equality restrictions, as in table 1.

negative point estimates for beef and pork that were reported in table 1 for the trend effects observed without the inequality restrictions are less than the resulting posterior mean values for those two coefficients. The trend coefficients in both the beef and pork equations rise in absolute value, as does the implied one in the omitted fish equation, when concavity and then substitutability are imposed. At the same time, the coefficient in the poultry equation falls slightly. All of the estimated means of the posterior distributions for the trend coefficients are more than twice the corresponding standard deviations, as shown in table 6, although this is barely true for poultry. The posterior probability of a negative value in the beef and pork equations was 1, which was the probability of a positive value in the fish equation; only for poultry does the posterior distribution have some probability on both sides of zero, although that value is small for negative values, as the estimated probability of a positive value in the poultry equation is over .95.

A sample size of 10,000 was sufficiently large to obtain accurate estimates for these expectations from the posterior distribution. This is reflected in the small numerical standard error estimates, obtained as described in step (f). These are well within tolerable ranges for applications.

Summary and Conclusions

An unfortunate byproduct of the use of demand systems that do not restrict substitution elasticities is that theoretical restrictions such as symmetry or homogeneity are often violated. More difficult to cope with are inequality restrictions. The familiar problem of curvature or monotonicity restrictions is the best example, but the signs of elasticities of substitution between goods are also good examples. In order to determine whether an estimated demand system is entirely consistent with prior beliefs, it is important to be able to impose or make inferences about inequality restrictions.

We showed that a Bayesian procedure handled this problem nicely. It produces constrained parameter estimates and also an estimate of the probability that the restrictions are true. For the demands for beef, pork, chicken, and fish in Canada, some support was found for the concavity of the consumer's expenditure function underlying an almost ideal demand system (p = 0.57), while the monotonicity restriction is definitely consistent with the model (p = 1). On the other hand, the sample information is less consistent with prior beliefs about substitution relationships; it reveals a low conditional probability (given concavity) that these four goods are all substitutes for each other (p = .28). Finally, whether or not these restrictions are imposed, the almost ideal demand system implies a negative trend affecting beef and pork consumption and a positive trend for chicken and fish.

Because the necessary integrals over the exact posterior density were quite complicated, Monte Carlo integration was used to estimate parameter values. The p.d.f. of the exact posterior was known but not recognizable, which made sampling from it difficult. This problem was overcome by importance sampling.

These findings concerning Canadian meat demand are conditional on the observed data and on the specification of the almost ideal demand system. All such inferences in demand systems are also conditional on separability and aggregation assumptions (Chalfant and Alston); but, if these assumptions are valid, the results must be interpreted as questioning either the prior belief that these goods are substitutes or the functional form for the demand system. If one is satisfied with the almost ideal model and unconcerned about the substitution restriction, the reasonably good results for concavity permit the use of the constrained elasticity estimates for policy analysis. The ability to produce a theoretically consistent set of elasticities is one of the main advantages of this procedure.

On the other hand, one might hesitate to use the estimated trends as evidence concerning taste changes if the lower probability for the restriction that all meats are substitutes is a cause for alarm. However, the results suggest that the almost ideal model cannot be estimated without the trend effects. Further research with other functional forms can help to address this question. It will be of interest to see how the probabilities we report change when another functional form is tried; Alston and Chalfant and the Monte Carlo studies to date certainly offer evidence that point estimates can change. In that light, the procedure and restrictions suggested in this study serve not only as a means to interpret the data but as a way of evaluating alternative functional forms for demand systems.

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