
NOTES D'ÉTUDES

ET DE RECHERCHE

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PRICE MODELS**

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Evaluating the Fit of Sticky Price Models¹

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Résumé :

Nous examinons les conséquences de l'introduction de coûts d'ajustement sur l'investissement, d'un taux variable d'utilisation du capital, de travail indivisible, et de consommations intermédiaires dans un modèle à prix visqueux soumis à une contrainte d'encaisses préalables. La combinaison de ces éléments permet au modèle d'échapper aux critiques qui sont traditionnellement adressées à cette classe de modèles. En utilisant le critère d'adéquation aux données formulé par Watson (1993), nous montrons que le modèle reproduit de façon satisfaisante la dynamique du produit, des heures et de l'investissement. Néanmoins, ce cadre se révèle incapable de répliquer le spectre de l'inflation. Cette conclusion malheureuse est robuste à de nombreuses spécifications alternatives.

Mots-clés : prix visqueux, analyse spectrale.

Abstract:

We examine the effects of introducing investment adjustment costs, variable capital utilization, indivisible labor, and material goods into a sticky price model subject to a cash-in-advance constraint. Combining these elements, the model overcomes the main criticisms traditionally addressed to this class of models. Under Watson (1993) goodness-of-fit criterion, the model does a very good job at replicating the dynamics of output, hours and investment. However, this framework dramatically fails at reproducing the spectrum of inflation. This unfortunate conclusion is robust to numerous alternative specifications.

Keywords: sticky prices, spectral analysis.

JEL Classification: E31, E32.

Résumé non technique :

Au cours de la décennie précédente, les modèles avec rigidité nominale sur les prix (prix visqueux) ont pris une place centrale en macroéconomie. Cependant, ces modèles souffrent d'importants défauts. En premier lieu, ils ne parviennent pas à engendrer des mouvements persistants du produit en réponse à un choc monétaire. Deuxièmement, ils engendrent un excès de volatilité du taux de croissance du produit aux fréquences hautes. Ces deux défauts combinés remettent singulièrement en question l'intérêt de ces modèles pour l'analyse macroéconomique.

En réaction à ces critiques, la littérature récente a tenté de renforcer les mécanismes de propagation internes de ces modèles. L'essentiel de cet effort a consisté à *(i)* réduire l'élasticité du coût marginal réel au produit et *(ii)* réduire la volatilité du produit découlant des chocs monétaires.

Dans ce papier, nous proposons d'évaluer quantitativement les conséquences de l'introduction de tous ou partie de ces éléments théoriques. Notre objectif est double : étudier dans quelle mesure les conclusions négatives auxquelles la littérature est parvenue sont robustes à des environnements plus complexes que le modèle à prix visqueux standard ; examiner la qualité des prédictions du modèle relatives à d'autres variables que le produit.

Les mécanismes pris en compte dans notre modèle devraient contribuer à améliorer ses performances en ce qui concerne le produit. Toutefois, il reste à voir si ces mécanismes lui permettent aussi d'obtenir de bons résultats pour les autres agrégats macroéconomiques usuellement étudiés dans la littérature.

La qualité de l'adéquation du modèle aux données est mesurée à l'aide d'une procédure formelle. Cette dernière consiste à rajouter une erreur au modèle de façon à réconcilier ses moments du second ordre avec ceux des données. Si l'erreur ajoutée n'est pas trop importante, alors on juge que le modèle est performant. Certains des paramètres clés du modèle sont estimés au moyen d'une méthode cohérente avec la procédure décrite ci-dessus. Les paramètres sont sélectionnés de façon à minimiser une distance pondérée entre les spectres théoriques et empiriques.

Nos résultats principaux sont alors les suivants. Premièrement, nous montrons que le modèle augmenté obtient des performances très satisfaisantes en ce qui concerne les taux de croissance du produit, de l'investissement et des heures travaillées. Cependant, il échoue complètement lorsqu'il s'agit de reproduire le comportement dynamique de l'inflation et de la part du travail dans la valeur ajoutée. Cette conclusion apparaît robuste à de nombreuses spécifications alternatives.

Non-technical summary:

Over the past decade, sticky price models of the business cycle have become one of the dominant workhorses of applied macroeconomics. However, these models suffer from major deficiencies. First they cannot generate persistent movements of output in response to money shocks, unless one is ready to

postulate unreasonably high degrees of nominal rigidity. Second standard sticky price models generate far too much volatility for output growth at very high frequencies. Taken together, these results cast serious doubts onto the ability of sticky price models to provide a convincing framework for macroeconomic analysis.

In response to these criticisms, recent researches have devoted important efforts to strengthen the internal propagation mechanisms of sticky price models. Much of these efforts consist in augmenting these models with elements that work towards *(i)* reducing the responsiveness of real marginal costs to output and *(ii)* diminishing the volatility of output originating from monetary shocks.

In this paper, we examine quantitatively the effects of introducing all or part of the above-mentioned elements into an otherwise standard sticky price model. Our objectives are twofold. First, we want to assess to what extent the negative conclusions reached in the literature are robust to an environment more complicated than the standard sticky price. Second, we want to examine the quality of our augmented sticky price model for other variables than solely output.

With regard to our first objective, the mechanisms taken into account in our model are expected to help improve its performances when it comes to output. However, and this leads us to our second objective, it remains to be seen whether this augmented model is able to generate realistic dynamics for other major macroeconomic variables traditionally studied in the business cycle literature.

The model's goodness-of-fit is assessed by resorting to a procedure which consists of augmenting the model with an approximation error designed to reconcile the second order moments of the model with those from the data. If the added error is small, then the model is judged to do a good job of accounting for these moments. Finally, some of the key parameters of our model, are estimated using a method consistent with the spirit of our retained goodness-of-fit criterion. The estimated parameters are selected so as to minimize a weighted distance between empirical and theoretical spectra.

Our main results are as follows. First, we formally show that the augmented sticky price model does a very good job of accounting for the dynamics of output growth, investment growth, and total hours growth. However, this model dramatically fails at providing a satisfying theory for inflation and markups dynamics. To check the robustness of this conclusion, we also consider versions of the model with fewer refinements than in the benchmark specification. We find that the identified problem still persists.

1 Introduction

Over the past decade, sticky price models of the business cycle have become one of the dominant workhorses of applied macroeconomics. However, much as their RBC counterparts, these models suffer from major deficiencies. First, as has been forcefully demonstrated by Chari et al. (2000), these models cannot generate persistent movements of output in response to money shocks, unless one is ready to postulate unreasonably high degrees of nominal rigidity. This in turn implies that sticky price models hit by monetary shocks cannot reproduce the peak in the spectral density of output growth at business cycle frequencies. Second, Ellison and Scott (2000) report that a standard sticky price model generates far too much volatility for output growth at very high frequencies. Taken together, these results cast serious doubts onto the ability of sticky price models to provide a convincing framework for macroeconomic analysis.

In response to these criticisms, recent researches have devoted important efforts to strengthen the internal propagation mechanisms of sticky price models. Much of these efforts consist in augmenting these models with elements that work towards (i) reducing the responsiveness of real marginal costs to output and (ii) diminishing the volatility of output originating from monetary shocks. Among these ingredients, prominent examples include the presence of materials goods, variable capacity utilization, and indivisible labor, which concern the first point, and investment adjustment costs, which concern the second point.

In this paper, we examine quantitatively the effects of introducing all or part of the above-mentioned elements into an otherwise standard sticky price model à la Calvo (1983). Our objectives are twofold. First, we want to assess to what extent the conclusions reached by Ellison and Scott (2000) are robust to an environment more complicated than the standard sticky price model which they considered. Second, we want to examine the quality of our augmented sticky price model for other variables than solely output.

In the spirit of Ellison and Scott (2000), the model's goodness-of-fit is assessed by resorting to Watson's (1993) test. This procedure consists of augmenting the model with an approximation error designed to reconcile the second order moments of the model with those from the data. If the added error is small, then the model is judged to do a good job of accounting for these moments. One key feature of this test is that it permits us to focus on particular frequencies along which dynamic general equilibrium models have been traditionally evaluated.

Finally, some of the key parameters of our model, are estimated using a method consistent with the spirit of Watson's (1993) test. This estimation strategy is a variant of the procedure proposed by Diebold et al. (1998). This method selects the estimated parameters so as to minimize a weighted distance between empirical and theoretical spectra. Given lack of a priori information, we estimate the parameter governing the degree of nominal rigidity, investment adjustment costs, and productivity shocks.

Interestingly, we find that our benchmark model requires a small amount of price stickiness.

Our main results are as follows. First, we formally show that the augmented sticky price model does a very good job of accounting for the dynamics of output growth, investment growth, and total hours growth. However, this model dramatically fails at providing a satisfying theory for inflation and markups dynamics. The model always generates far too much volatility for inflation, especially at business cycle frequencies, and not enough for markups, resulting in very high approximation errors. To check the robustness of this conclusion, we also consider versions of the model with fewer refinements than in the benchmark specification. We find that the identified problem still persists.

The remainder is as follows. Section 2 lays out our theoretical framework. Section 3 describes our results. First, the different theoretical components taken into account are discussed in detail. Second, the estimation procedure is outlined. Then, we resort to Watson’s (1993) test to assess the empirical fit of our model. The last section briefly concludes.

2 The Model

2.1 Households

We consider a discrete time economy¹ populated with a continuum of size one of identical, infinitely-lived agents. Each agent is endowed with one unit of time which can be devoted to labor or leisure. The agents own all the primary factors, namely physical capital, k_t , and labor, n_t . The representative agent’s goal in life is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{1,t}, c_{2,t}, n_t), \quad (1)$$

where $U(\cdot)$ is a well-behaved momentary utility function, \mathbb{E}_0 is the expectation operator, conditional on information available as of time $t = 0$, and $\beta \in (0, 1)$ is the subjective discount factor. Following Lucas and Stokey (1987) and Cooley and Hansen (1995), the detention of money is motivated by the requirement of accumulating cash balances to finance a subset of consumption expenditures. We resort to the familiar cash good-credit good construct, where $c_{1,t}$ represents consumption of cash goods, and $c_{2,t}$ stands for consumption of credit goods. Finally, n_t denotes labor supply.

The representative agent maximizes (1) subject to the sequence of constraints

$$c_{1,t} + c_{2,t} + m_t + i_t + a(u_t)k_t \leq w_t n_t + r_t u_t k_t + m_{t-1}/\pi_t + h_t + \tau_t, \quad (2)$$

$$c_{1,t} \leq m_{t-1}/\pi_t + \tau_t \quad (3)$$

¹A detailed technical appendix is available from the authors upon request.

$$k_{t+1} \leq (1 - \delta) k_t + \varphi(k_t, i_t, i_{t-1}). \quad (4)$$

Equation (2) is the budget constraint expressed in real terms, where i_t represents investment, r_t is the real rental rate of capital, u_t is the utilization rate of capital, w_t is the wage rate, h_t denotes profits redistributed by monopolistic firms, and $m_t \equiv M_t/P_t$ are the real cash balances accumulated for the next period, M_t and P_t being respectively the nominal cash balances and the aggregate price index. The term $\pi_t = P_t/P_{t-1}$ represents the (gross) inflation rate and $\tau_t \equiv T_t/P_t$ where T_t is a nominal transfer from the government. The function $a(\cdot)$, increasing and convex, reflects the fact that a higher utilization rate of physical capital calls for increased maintenance costs. We assume that $a(u) = 0$ and $a''(u)u/a'(u) = \nu_a > 0$, where u is the steady state utilization rate. Equation (3) is the cash in advance constraint, which requires that cash goods consumption be financed by previously accumulated cash balances plus current nominal transfers.

Finally equation (4) describes the law of motion for physical capital, which is assumed to depreciate linearly with constant rate $\delta \in (0, 1)$. The function $\varphi(\cdot)$ reflects the presence of adjustment costs in the accumulation of physical capital. As has been documented before², adjustment costs are necessary in sticky price models to prevent investment and output from being counterfactually volatile in response to monetary shocks. We defer until next section the precise description of the shape taken by $\varphi(\cdot)$.

2.2 Firms

The production side of the economy has three sectors. In the first one, competitive firms produce a homogeneous final good with the inputs of intermediate goods, according to the Dixit and Stiglitz (1977) CES technology

$$y_t = \left(\int_0^1 y_t(\varsigma)^{(\varepsilon-1)/\varepsilon} d\varsigma \right)^{\varepsilon/(\varepsilon-1)}, \quad (5)$$

where y_t is the quantity of final good produced in period t and $y_t(\varsigma)$ is the input of intermediate good ς . Intermediate goods are imperfectly substitutable, with substitution elasticity $\varepsilon > 1$. The zero profit condition for final good producers implies that the aggregate nominal price index obeys the relationship

$$P_t = \left(\int_0^1 P_t(\varsigma)^{1-\varepsilon} d\varsigma \right)^{1/(1-\varepsilon)}, \quad (6)$$

where $P_t(\varsigma)$ is the nominal price of intermediate good ς .

In the second sector, competitive firms produce materials goods by combining the same set of intermediate goods as above. They have access to the CES technology

$$q_t = \left(\int_0^1 q_t(\varsigma)^{(\varepsilon-1)/\varepsilon} d\varsigma \right)^{\varepsilon/(\varepsilon-1)}, \quad (7)$$

²See, for example, Chari et al. (2000).

where q_t is the produced quantity of material goods and $q_t(\varsigma)$ denotes the input of intermediate good ς . Notice that the technologies for producing final and materials goods share the same substitution elasticity between any two intermediate goods. Accordingly, the price of materials goods will be P_t .

Let $d_t(\varsigma)$ denote the total demand addressed to the producer of intermediate good ς . The above assumptions imply the following relationship

$$d_t(\varsigma) = \left(\frac{P_t(\varsigma)}{P_t} \right)^{-\varepsilon} (y_t + q_t). \quad (8)$$

In the third sector, monopolistic firms produce the intermediate goods. Each firm $\varsigma \in [0, 1]$ is the sole producer of intermediate good ς . Given a demand $d_t(\varsigma)$, it faces the following production possibilities

$$\min \left\{ \frac{k_t(\varsigma)^\phi (e^{z_t} n_t(\varsigma))^{1-\phi} - \kappa e^{z_t}}{1 - s_x}, \frac{x_t(\varsigma)}{s_x} \right\} \geq d_t(\varsigma), \quad (9)$$

where $k_t(\varsigma)$ and $n_t(\varsigma)$ are the inputs of capital and labor, respectively, $x_t(\varsigma)$ denotes the input of material goods, and κe^{z_t} is a fixed production cost. Here, the parameter s_x is the share of material goods in gross output. This specification is borrowed from Rotemberg and Woodford (1995). Finally, z_t is a productivity shock which evolves according to

$$z_t = \log g + z_{t-1} + \epsilon_t, \quad (10)$$

where $g - 1$ is the average growth rate and $\epsilon_t \sim iid(0, \sigma_\epsilon)$. Thus, productivity shocks follow a random walk with drift. Notice that to guarantee a well-behaved steady state, we make the fixed production cost grow at the same rate as technology.

With the above technology, monopolist ς will always choose materials inputs $x_t(\varsigma) = s_x d_t(\varsigma)$. The minimization of production costs then implies the relationships

$$(1 - s_x) w_t = (\psi_t - s_x) (1 - \phi) e^{z_t} \left(\frac{k_t(\varsigma)}{e^{z_t} n_t(\varsigma)} \right)^\phi, \quad (11)$$

$$(1 - s_x) r_t = (\psi_t - s_x) \phi \left(\frac{k_t(\varsigma)}{e^{z_t} n_t(\varsigma)} \right)^{\phi-1}, \quad (12)$$

where ψ_t is the real marginal cost. These conditions imply that in equilibrium, all the monopolists will face the same labor-capital ratio. Notice that when we ignore materials goods, i.e. set $s_x = 0$, the above equations collapse to the standard expressions derived by Yun (1996) or King and Wolman (1996).

Following Calvo (1983), we assume that in each period of time, a monopolistic firm can reoptimize its price with probability $1 - \theta$, irrespective of the elapsed time since it last revised its price. If a firm is not drawn to reoptimize, it simply rescales its price by the average (raw) inflation rate, i.e. sets $P_t(\varsigma) = \pi P_{t-1}(\varsigma)$, where π is the steady state value of π_t . As is well known, optimizing firms will all set

their price to a common value, which we denote P_t^* . Standard manipulations imply the relationship

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}_t \{ \lambda_{t+j} \psi_{t+j} (y_{t+j} + q_{t+j}) (\pi_{t,t+j}/\pi^j)^\varepsilon \}}{\sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}_t \{ \lambda_{t+j} (y_{t+j} + q_{t+j}) (\pi_{t,t+j}/\pi^j)^{\varepsilon-1} \}}, \quad (13)$$

where λ_t is the multiplier associated to the representative household's budget constraint, eq. (2), and $\pi_{t,t+j} = P_{t+j}/P_t$.

Since only a fraction $1 - \theta$ of firms can reoptimize and they all choose the same price P_t^* , it can be shown that the aggregate price index P_t evolves according to the well-known law³

$$P_t^{1-\varepsilon} = (1 - \theta) (P_t^*)^{1-\varepsilon} + \theta (\pi P_{t-1})^{1-\varepsilon}. \quad (14)$$

2.3 Equilibrium

The supply of cash balances evolves according to a standard growth rule, i.e. $M_t^s = e^{\mu_t} M_{t-1}^s$ with

$$\mu_t = \rho \mu_{t-1} + \xi_t, \quad (15)$$

where $0 < \rho < 1$, $\xi_t \sim iid((1 - \rho)\mu, \sigma_\xi)$, and μ is the steady state (average) value of μ_t . Arguably, this simple money growth rule is a poor representation of actual monetary policy. However, most former studies seeking to evaluate the fit of sticky price models have used money rules similar to eq. (15). Thus, to facilitate comparison with these studies, we stick to this simple specification.

Finally, it is assumed that nominal government transfers obey the relationship $T_t = M_t^s - M_{t-1}^s$. We can now define the equilibrium of our model economy.

Definition. *An equilibrium for the above described economy is a sequence of quantities $\{c_{1,t}, c_{2,t}, i_t, k_{t+1}, m_t, u_t, n_t, y_t, q_t, y_t(\varsigma), q_t(\varsigma), k_t(\varsigma), n_t(\varsigma), x_t(\varsigma); \varsigma \in [0, 1]\}_{t=0}^{\infty}$ and a sequence of prices $\{r_t, w_t, \pi_t, P_t, P_t^*, P_t(\varsigma); \varsigma \in [0, 1]\}_{t=0}^{\infty}$ such that*

1. *given k_0, m_{-1}, i_{-1} , a sequence of prices $\{r_t, w_t, \pi_t\}_{t=0}^{\infty}$, and a sequence of shocks $\{z_t, \xi_t\}_{t=0}^{\infty}$, the sequence $\{c_{1,t}, c_{2,t}, i_t, k_{t+1}, m_t, u_t, n_t\}_{t=0}^{\infty}$ is solution to the representative household's program;*
2. *given a sequence of prices $\{P_t(\varsigma); \varsigma \in [0, 1]\}_{t=0}^{\infty}$, and a sequence of shocks $\{z_t, \xi_t\}_{t=0}^{\infty}$, the sequence $\{y_t(\varsigma), q_t(\varsigma); \varsigma \in [0, 1]\}_{t=0}^{\infty}$ is solution to the representative final-goods and materials-goods producing firms' programs;*
3. *given a sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$, a sequence of quantities $\{y_t(\varsigma), q_t(\varsigma); \varsigma \in [0, 1]\}_{t=0}^{\infty}$, and a sequence of shocks $\{z_t, \xi_t\}_{t=0}^{\infty}$, the sequence $\{P_t^*, k_t(\varsigma), n_t(\varsigma), x_t(\varsigma); \varsigma \in [0, 1]\}_{t=0}^{\infty}$ is solution to the monopolists' program;*

³See King and Wolman (1996) for a complete derivation.

4. *markets clear, i.e. $c_{1,t} + c_{2,t} + i_t + a(u_t)k_t = y_t$, $\int_0^1 k_t(\varsigma)d\varsigma = u_t k_t$, $\int_0^1 n_t(\varsigma)d\varsigma = n_t$, $\int_0^1 x_t(\varsigma)d\varsigma = q_t$, and $M_t^s = M_t$;*

5. *the monetary authority obeys (15) and the sequence $\{P_t\}_{t=0}^\infty$ obeys (14).*

Given the presence of a stochastic trend in technical progress, the above conditions lead to a deterministic steady state in which $c_{1,t}$, $c_{2,t}$, i_t , k_{t+1} , m_t , and w_t all grow at the same rate while u_t and n_t are constant through time. To obtain a bounded steady state, trending variables dated t as well as k_{t+1} are divided through by e^{z_t} . We thus obtain a set of equilibrium conditions whose solution is stationary. The transformed system is loglinearized in the neighborhood of the deterministic steady state and solved with the undetermined coefficients method proposed by Uhlig (1999).

3 Results

3.1 The Data

In this section, we describe the data which we are interested in. A detailed description of data sources and construction is provided in appendix A. Two basic series which obviously ought to be included in an assessment of sticky price models are output and inflation. Additionally, given our emphasis on investment adjustment costs and labor indivisibility, it also seems natural to include total hours and investment in our study. Finally, we also include labor share in our empirical investigation. Following the seminal contribution of Galí and Gertler (1999), this series can be thought of as a proxy for real marginal costs. Unfortunately, because of the presence of materials goods and fixed production costs in our model, labor share and marginal cost do not exactly coincide. However, as explained by Rotemberg (1996), labor share is still a good proxy for the level of markups. Since sticky price models also provide a theory for markups, it seems useful to include the labor share in our study.

In all the estimates, we use U.S. data for the non-farm private business sector, over the time period 1965:I-1995:IV. Output is defined as value added. Total hours are defined as the product of average weekly hours and the employment rate. The nominal wage bill is computed as the product of nominal compensations with total hours. The labor share is the ratio of the nominal wage bill to nominal output. The price index is the implicit output price deflator. Additionally, we define investment as real consumption of durables plus real fixed private investment. Finally, we follow Ireland (2001) and define money as M2.

We collect the data in the vector $\mathbf{y}_t = (\Delta\hat{y}_t, \Delta\hat{l}_t, \Delta\hat{n}_t, \Delta\hat{\pi}_t, \Delta\hat{s}_t)'$, where s_t stands for the labor share, a letter with a hat refers to the demeaned natural logarithm of the associated variable, and Δ stands for the first difference filter. Since the focus of our study is in studying the ability of our model to replicate the spectral density matrix of \mathbf{y}_t , we must estimate the latter.

To do this, the approach taken in this paper consists of estimating a VAR process for \mathbf{y}_t so as to compute the implied spectrum using standard spectral analysis formulas. More precisely, the empirical spectral density matrix of \mathbf{y}_t is computed from the estimated parameters of a two-lag⁴ cointegrated VAR of $(\Delta\hat{y}_t, \hat{v}_t - \hat{y}_t, \hat{n}_t, \hat{\pi}_t, \hat{s}_t)'$. As in King et al. (1991), the latter implies that output and investment are cointegrated with cointegration vector $(1, -1)$, i.e. share the same stochastic trend. Moreover, this VAR takes total hours, inflation, and the labor share as stationary variables. These restrictions make our model and data sets comparable. However, it appears that total hours and the investment-output ratio exhibit small but significant trends. In order to avoid non stationarity problems in our estimation, we define these variables as deviations from deterministic trends.

3.2 Specifications

We now describe the models' specifications. As discussed before, the theoretical elements taken into account in our sticky price model work towards *(i)* reducing the responsiveness of real marginal costs to output and *(ii)* diminishing the volatility of output originating from monetary shocks.

First, notice that sticky price models share the common feature that firms set prices as a markup over marginal cost. In a standard sticky price model, marginal costs respond sharply to monetary policy shocks, translating into a large response of inflation for moderate degrees of nominal rigidity. This in turn implies that real output will not persistently deviate from its steady state path in this kind of models. This suggests that if sticky price models are to generate persistent movements of real output in response to monetary shocks, one must limit the ability of real marginal costs, and, hence, inflation, to respond sharply to these shocks. In our model, this role pertains to indivisible labor and materials goods⁵.

To begin with, let us specify households' preferences. As explained by Dotsey and King (2001), labor indivisibility helps reduce the elasticity of real marginal cost to output. This leads us to consider two alternative preferences families, according to whether labor is indivisible or not. In the first specification, we let

$$U(c_{1,t}, c_{2,t}, n_t) = \alpha \log(c_{1,t}) + (1 - \alpha) \log(c_{2,t}) - Bn_t, \quad (16)$$

where α represents the share of cash goods in utility and $B > 0$ is labor's marginal disutility. This is the standard form of labor indivisibility, as in Hansen (1985). Alternatively, we resort to a second

⁴The lag length selection was performed using sequential likelihood ratio tests. The results were confirmed by the usual information criteria.

⁵Variable capacity utilization plays a similar role. Since this mechanism has received great attention in recent research, and has been found to improve the performances of sticky price models, it is considered in all the subsequent specifications.

specification where

$$U(c_{1,t}, c_{2,t}, n_t) = \alpha \log(c_{1,t}) + (1 - \alpha) \log(c_{2,t}) + B \log(1 - n_t). \quad (17)$$

This particular utility function has been routinely used in the business cycle literature.

Additionally, we consider two alternative values for s_x . As with labor indivisibility, materials goods reduce the responsiveness of marginal costs to output. Notice that, as explained by Dotsey and King (2001), the effects of materials goods and labor indivisibility are self-reinforcing. Also, in the first specification, s_x is set to 50% as in Rotemberg and Woodford (1995)⁶. Alternatively, we set this parameter to zero, in which case materials goods are simply ignored.

When it comes to our second point, it has been well-documented that, absent adjustment costs to capital accumulation, standard sticky price models generate unreasonably high volatilities of investment and output in response to monetary shocks. Accordingly, we use two alternative specifications for the adjustment costs function. Following Christiano et al. (2001), our benchmark specification is assumed of the form

$$\varphi(k_t, i_t, i_{t-1}) = \left[1 - v \left(\frac{i_t}{i_{t-1}} \right) \right] i_t. \quad (18)$$

We suppose that $v(g) = v'(g) = 0$ and $g^2 v''(g) = \nu_v > 0$. As explained by Christiano et al. (2001), this form of adjustment costs penalizes fast changes in investment, thus contributing to generate a peak in the spectral density of investment growth at business cycle frequencies. Alternatively, we also use a more conventional specification, considered, among others, by King and Wolman (1996), i.e.

$$\varphi(k_t, i_t, i_{t-1}) = v \left(\frac{i_t}{k_t} \right) k_t. \quad (19)$$

In this case, we suppose that $v(i/k) = i/k$, $v'(i/k) = 1$, and $(i/k)v''(i/k) = -\nu_v < 0$, where i/k denotes the steady state investment to capital ratio. In contrast with the previous specification, this function penalizes large movements of investment with respect to the capital stock. This specification does not per se generate a peak in the spectrum of investment growth at business cycles frequencies. However, this mechanism should work towards reducing the volatility of investment in response to a monetary shock.

We thus consider a total of eight different specifications, depending on households' preferences, the presence of materials goods, and the nature of adjustment costs to physical capital accumulation. The different specifications and their components are summarized in table 1.

⁶Dotsey and King (2001) and Basu (1995) report even higher values. Naturally, the higher s_x , the lower the responsiveness of real marginal cost with respect to output.

3.3 Models Calibration

We now describe the models' calibration. The relevant parameter values are reported in table 2. In all the specifications considered, the parameters g , μ , β , ϕ , α , B , δ , ν_a , ρ , σ_ξ , and ε are calibrated on the basis of available a priori information.

We select g and μ so as to reproduce the average growth rates of output and money in our data sample. As is customary in the sticky price literature, we choose β and ϕ so that the average, annual, real interest rate is 4% and the capital share in value added is 30%. We follow Cooley and Hansen (1995) and set $\alpha = 0.84$. We select B so that, on average, households devote one third of their time endowment to the market. Naturally, B changes value according to whether momentary utility is defined by eq. (16) or eq. (17). The depreciation rate is set to 10% per annum. Finally, consistent with estimates reported in Basu and Fernald (1997), we set $\varepsilon = 10$, implying an average markup of 11%.

The money growth parameters ρ and ε_ξ are selected by fitting a simple $AR(1)$ to the demeaned growth rate of M2. The obtained results are

$$\mu_t = \underset{(0.0701)}{0.6390}\mu_{t-1} + \xi_t, \quad \sigma_\xi = 0.6491.$$

In a preliminary investigation, we tried to estimate the elasticity ν_a . We encountered similar problems as those reported by Christiano et al. (2001), in that our estimation algorithm tried to set ν_a to 0. This algorithm (to be detailed below) selects the parameters so as to minimize the discrepancy between models and data spectra. In our models, setting ν_a to 0 implies that the initial response of the rental rate of capital to a monetary shock is zero. This translates into a small initial response of the real marginal cost. As has been argued before, this small response is a prerequisite for reproducing the trend-reverting component of output. Unfortunately at this point, the model no longer admits a solution⁷. In order to avoid this problem in all specifications considered, we choose $\nu_a = 0.1$ which is small enough to help generate hump-shaped output dynamics.

3.4 Estimation

The remaining model's parameters, collected in the vector $\gamma = (\theta, \nu_v, \sigma_\varepsilon)'$, are selected by resorting to an estimation procedure in the spirit of Diebold et al. (1998) and Wen (1998). This method selects γ so as to minimize a weighted distance between empirical and theoretical second order moments of a relevant set of macroeconomic variables. Let \mathbf{x}_t denote the theoretical counterpart of the data which are contained in \mathbf{y}_t , and let $A_{\mathbf{y}}(e^{-i\omega})$ denote the spectral density matrix of \mathbf{y}_t and $A_{\mathbf{x}}(e^{-i\omega}; \gamma)$ denote the spectral density matrix of \mathbf{x}_t . Then, γ is selected so as to minimize $a(\gamma) = \text{tr}[Wv(\gamma)]$, where W is a

⁷More precisely, the transformed economy ceases to be stationary for sufficiently low values of ν_a .

prespecified weighting matrix and $v(\gamma)$ is defined as

$$v(\gamma) = \int_{-\pi}^{\pi} \varkappa(\omega) \odot D(e^{-i\omega}; \gamma) D(e^{-i\omega}; \gamma)' d\omega,$$

$$D(e^{-i\omega}; \gamma) = \text{diag}[A_{\mathbf{x}}(e^{-i\omega}; \gamma) - A_{\mathbf{y}}(e^{-i\omega})],$$

$$\varkappa(\omega) = A_{\mathbf{y}}(e^{-i\omega}) \oslash \int_{-\pi}^{\pi} A_{\mathbf{y}}(e^{-i\omega}) d\omega,$$

where \odot is an element by element multiplication operator, \oslash is an element by element division operator and $\text{diag}(\cdot)$ is an operator setting the off-diagonal elements of a square matrix to zero.

In the sequel, our weighting matrix W is a 5×5 diagonal matrix containing on its diagonal the inverse of the variances of the components of \mathbf{y}_t . This permits us to downsize the importance of those elements of \mathbf{y}_t that have a high variance relative to others. Finally, the role assigned to $\varkappa(\omega)$ is to weight the components of $v(\gamma)$ according to their relative importance in explaining $A_{\mathbf{y}}(e^{-i\omega})$. Thus, if a particular frequency ω is important in explaining the spectral density of \mathbf{y}_t , it will be granted relatively more attention than other frequencies. Since most of the variables contained in \mathbf{y}_t exhibit a peak at business cycles frequencies, this weighting scheme will emphasize these particular frequencies.

The spectral density matrix $A_{\mathbf{x}}(e^{-i\omega}; \gamma)$ is obtained from the state-space representation of the model's solution. The empirical spectral density matrix of \mathbf{y}_t is computed from the estimated parameters of the cointegrated VAR described above.

The estimated parameter values together with their standard errors⁸ are reported in table 3. Several interesting results emerge from our models' estimation. First, notice that the benchmark specification calls for a very moderate degree of nominal rigidity. Indeed, the point estimate of θ implies an average price fixity duration of less than two quarters. Second, comparing M1 and M2 shows that, as expected, the model requires a higher degree of nominal rigidity when materials goods are ignored. Additionally, since the degree of nominal rigidity is higher in model M2, productivity shocks are granted a smaller role in explaining the variance of our variables. In the same spirit, comparing M1 and M3 or M1 and M5 shows that labor indivisibility or our benchmark specification for adjustment costs help reduce the required degree of nominal rigidity.

Additionally, the estimated values for ν_v are close of slightly lower than that reported by Christiano et al. (2001) in comparable specifications of their model. Notice also that σ_ϵ increases when labor is

⁸Let $\hat{\beta}$ denote the estimated VAR parameters, whose true value is β^0 , and let $b(\gamma, \beta) = \partial a / \partial \gamma$. Similarly, let $\hat{\gamma}$ denote the estimate of γ , and let γ^0 denote γ 's true value. Assume that $\sqrt{T}(\hat{\beta} - \beta^0) \stackrel{d}{\sim} N(0, \Sigma_\beta)$. Now, define

$$A = -[\partial b(\gamma^0, \beta^0) / \partial \gamma']^{-1} [\partial b(\gamma^0, \beta^0) / \partial \beta'].$$

It follows from these definitions that $\sqrt{T}(\hat{\gamma} - \gamma^0) \stackrel{d}{\sim} N(0, A \Sigma_\beta A')$. In practice, all the derivatives are evaluated at the point estimates of β and γ . The matrix Σ_β is estimated as indicated by Hamilton (1994).

divisible. In these models, labor is less responsive to the different shocks, translating into a smaller variance of output. This reduction is compensated by an increase in σ_ϵ .

Finally, notice that the estimated values of ν_v are very stable across models M5, M6, M7, and M8. In these models, this parameter is readily interpretable as the elasticity of investment-capital ratio with respect to Tobin's q . Additionally, the point estimate is very close to the value reported by Jermann (1998). In the same set of models, the value for σ_ϵ is always found very small. From a general point of view, the degree of nominal rigidity in these models is always important. Thus the contribution of monetary shocks to the variance of the variables under study increases relative to the preceding specifications, and this leaves a smaller role for productivity shocks.

3.5 Assessing the Goodness-of-Fit

We now propose to quantify the goodness-of-fit of our four specifications. To do this, we simply resort to Watson's (1993) test. This test consists in decomposing the performances of a model into the frequency domain. The procedure amounts to augmenting the model with an approximation error designed to reconcile the second order moments of the model with those from the data. If the added error is small, then the model is judged to do a good job of accounting for these moments. Watson (1993) proposes a lower bound on the ratio of the mean square approximation error to the variance of actual data. We will refer to this lower bound as the relative mean square approximation error (RMSAE)⁹. Using this test enables us to tell which version of the model best accounts for the dynamic properties of the data. Here, we will study the spectral properties of the process $(\hat{y}_t, \hat{i}_t, \hat{n}_t, \hat{\pi}_t, \hat{s}_t)'$, either taken in first difference or HP filtered.

We first propose to study the RMSAEs over the frequency range $[0, \pi]$ so as to get a feel for the overall behavior of our models. However, as has been often acknowledged, DGE models are not necessarily meant to account for all the dynamic movements in the data. Naturally, in this paper, our primary focus is on the business cycle. Thus, in a second step, we restrict our attention to the frequency band $[\pi/16, \pi/3]$. This interval insulates the frequencies typically attached to business cycle, i.e. cyclical movements the reproduction period of which runs from 6 to 32 quarters. Additionally, we complete this study by considering HP filtered variables. Resorting to this particular filter is standard practice in the business cycle literature.

The estimated RMSAEs together with their standard errors¹⁰ are reported in table 4. The benchmark model and data spectra are reported in figure 1.

The benchmark model does an excellent job on output, investment and hours. M1 perfectly reproduces

⁹See appendix B for a brief description.

¹⁰The standard errors are computed using similar principles as those outlined in footnote 8.

the peak of the spectral density of these three variables in the range of business cycles frequencies without overestimating these densities for higher frequencies. This sharply contrasts with the findings of Ellison et Scott (2001).

To help explain the dynamic properties of the benchmark model, we take a look at the impulse response functions of our variables to the two theoretical shocks (figures 2 and 3). We see that the model produces hump-shaped responses of both output, investment and hours in response to a monetary shock. In particular, the instantaneous response of output is not excessive. This point explains the relatively low spectrum of output at high frequencies. Then, output growth is still positive for two periods after impact, hence the variance of output growth is essentially concentrated at business cycles frequencies. The line of reasoning is quite similar for investment and hours. Notice that the role played by the technological shocks is somewhat secondary for business cycles movements of all three variables. We see that the instantaneous response of output to a productivity shocks is very small compared to the response to a monetary shock¹¹. The variance of output growth generated by productivity shocks is concentrated at very low frequencies, thus lowering the RMSAE for frequencies under $\pi/16$.

Hence, the major criticisms addressed to a sticky price model are no longer valid: a sticky price model augmented with some adequate propagation mechanisms is able to almost perfectly reproduce the behavior of output, hours and investment, for whatever frequency range considered.

In contrast, the benchmark model does a very poor job for both inflation and labor share. To begin with, notice that the model largely overestimates the variance of inflation growth and is unable to reproduce the inertial behavior of prices growth. This explains why the RMSAE is well beyond one. Notice that this deficiency is essentially due to monetary shocks which generate much larger movements of inflation than productivity shocks. Notice also that the weakness of the model with regard to inflation is particularly flagrant at business cycle frequencies. This explains why the RMSAE for inflation is so large over the range $[\pi/16, \pi/3]$ with the first-difference filter or with the HP filter over $[0, \pi]$. Additionally, the benchmark model also largely underestimates the volatility of labor share. Recall that in this model, labor share and real marginal cost do not coincide. Thus, the failure of our model at reproducing the dynamic behavior of s_t does not explain why the model performs so poorly with regard to inflation.

To assess the relative importance of each of our ingredients, let us now compare the benchmark model with the other specifications. This comparison permits us to draw two main conclusions. First, the benchmark model is the best one concerning output, investment and hours. Comparing M1 to M2, we see that the presence of material goods helps minimize the RMSAE on output and hours. Comparing M1 and M3, we see that the indivisible labor hypothesis helps the model reproduce the behavior of hours

¹¹Notice additionally that the instantaneous response of hours is negative, suggesting the presence of a recessionary effect of technological shocks, as in Gali (1999). However, this initial response is very small compared with the empirical response reported by Gali (1999).

growth. In the case of divisible labor, the volatility of hours is slightly underestimated. Comparing M1 to M5, we see that the model with the usual specification of adjustment costs on physical capital accumulation severely fails at reproducing the behavior of output, hours and investment. The usual specification, which only penalizes the movement of investment relative to the available stock of capital, does not dampen enough the volatility of investment growth at high frequencies. This translates into excessive theoretical volatilities of output and hours at high frequencies. This point suggests that our benchmark specification for adjustment costs is the most important of our three additional mechanisms. If we suppress indivisible labor (M3) or materials goods (M2) from the benchmark model, the fit for output is still satisfying, allowing the model to escape the two critiques formulated in the introduction. On the contrary, a model with the usual form of adjustment costs (M5 to M8) is unable to escape the critique of Ellison and Scott (2001): too large a part of output growth fluctuations is concentrated at very high frequencies.

Second, whatever the specification considered the fit of the model for inflation and labor share is extremely poor. All the versions largely overestimate the variance of inflation and underestimate the variance of the labor share. This result is valid on the whole range of frequencies and even more so at business cycles frequencies (first differenced variables on $[\pi/16, \pi/3]$ or HP filtered variables on $[0, \pi]$). Interestingly, the model exhibiting the poorest performances with regard to inflation is our benchmark model. This suggests that the mechanisms considered in this paper induce a trade-off between obtaining a good fit for real variables and obtaining a good fit for inflation. However, this conclusion needs to be mitigated since all the sticky price models which are considered here have difficulties in modelling inflation dynamics.

4 Conclusion

In this paper, we have proposed to examine quantitatively the effects of introducing materials goods, variable capacity utilization, indivisible labor, and investment adjustment costs into an otherwise standard sticky price model à la Calvo (1983), subject to a cash-in-advance constraint. The first three mechanisms work towards reducing the responsiveness of real marginal costs to output, thus generating persistent dynamics of output. The last mechanism contributes to diminishing the volatility of output and investment originating from monetary shocks. Additionally, the particular specification of adjustment costs considered in the benchmark model helps concentrate the volatility of these two variables at business cycle frequencies.

Combined together, these mechanisms allow the model to reproduce the spectral density of output growth, investment growth, and hours growth with a relatively small approximation error. Interestingly, using an estimation strategy designed to minimize the discrepancy between empirical and theoretical

spectra, the complete model is found to require only a moderate degree of nominal rigidity in order to reproduce the dynamics of output.

Unfortunately, these elements cannot help the model replicate the spectral density of inflation and labor share, especially at business cycle frequencies. This deficiency is robust to various combinations of our theoretical ingredients. As a general matter, all the models considered generate too much volatility for inflation, and not enough for labor share. This negative result is all the more troubling as sticky price should in principle provide a convincing theory of inflation dynamics.

It is worth stressing that in our model, the monetary authority obeys a simple stochastic growth rule. It might be interesting for further research to investigate the robustness of our conclusions in economic environments where the monetary authority is allowed to respond to inflation and output gaps.

References

- Basu, S., 1995. Intermediate goods and business cycles: Implications for productivity and welfare. *American Economic Review*, 85, 512–531.
- Basu, S., Fernald, J., 1997. Returns to scale in U.S. production: Estimates and implications. *Journal of Political Economy*, 105, 249–283.
- Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12, 383–398.
- Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2000. Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem? *Econometrica*, 68, 1151–1179.
- Christiano, L.J., Eichenbaum, M., Evans, C.L., 2001. Nominal rigidities and the dynamic effects of a shock to monetary policy. Working paper 8403, NBER.
- Cooley, T.F., Hansen, G.D., 1995. Money and the business cycle. In: Cooley, T. F. (Ed.), *Frontiers of Business Cycle Research*, chapter 7, pages 175–216. Princeton University Press.
- Diebold, F.X., Ohanian, L.E., Berkowitz, J., 1998. Dynamic equilibrium economies: A framework for comparing models and data. *Review of Economic Studies*, 65, 433–451.
- Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. *American Economic Review*, 67, 297–308.
- Dotsey, M., King, R.G., 2001. Pricing, production and persistence. Working paper 8407, NBER.
- Ellison, M., Scott, A., 2000. Sticky prices and volatile output. *Journal of Monetary Economics*, 46, 621–632.
- Gali, J., 1999. Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 89, 249–271.
- Gali, J., Gertler, M., 1999. Inflation dynamics: A structural econometrics analysis. *Journal of Monetary Economics*, 44, 195–222.
- Hamilton, J.D., 1994. *Time Series Analysis*. Princeton University Press, Princeton.
- Hansen, G.D., 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16, 309–327.

- Ireland, P.N., 2001. Sticky-price models of the business cycle: Specification and stability. *Journal of Monetary Economics*, 47, 3–18.
- Jermann, U.J., 1998. Asset pricing in production economies. *Journal of Monetary Economics*, 41, 257–275.
- King, R.G., Plosser, C.I., Stock, J.H., Watson, M.W., 1991. Stochastic trends and economic fluctuations. *American Economic Review*, 81, 819–840.
- King, R.G., Wolman, A.L., 1996. Inflation targeting in a St. Louis model of the 21th century. *Federal Reserve Bank of St. Louis Review*, 78, 83–107.
- Lucas, R.E., Stokey, N.L., 1987. Money and interest in a Cash-in-Advance economy. *Econometrica*, 55, 491–513.
- Rotemberg, J.J., Woodford, M., 1995. Dynamic general equilibrium models with imperfectly competitive product markets. In: Cooley, T. F. (Ed.), *Frontiers of Business Cycle Research*, chapter 9, pages 243–293. Princeton University Press, Princeton.
- Uhlig, H., 1999. A toolkit for analyzing nonlinear dynamic stochastic models easily. In: Marimon, R., Scott, A. (Eds.), *Computational Methods for the Study of Dynamic Economies*, pages 30–61. Oxford University Press, Oxford.
- Watson, M.W., 1993. Measures of fit for calibrated models. *Journal of Political Economy*, 101, 1011–1041.
- Wen, Y., 1998. Can a real business cycle pass the Watson test?. *Journal of Monetary Economics*, 42, 185–203.

Appendix

A Data

So as to define our macroeconomic variables, we start from the following time series over the time period 1964:IV-1995:IV:

[1] : *gross domestic product, business, nonfarm;*

[2] : *real gross domestic product, business, nonfarm;*

[3] : *real consumption of durable goods;*

[4] : *real fixed private investment*

[5] : *average weekly hours;*

[6] : *total non farm employment;*

[7] : *money stock, M2;*

[8] : *civilian population over 16;*

[9] : *nominal compensation, business, non farm.*

The quarterly series [1] to [4] are taken from the BEA database (series [1], table 1.1, series [2] – [4], table 1.2); series [2] – [4] are in chained-type 1996 \$. Series [5] – [6] and [8] – [9] are from the BLS. They are available at monthly frequency, and are transformed to quarterly frequency by simply taking the average value over a quarter. Finally, the quarterly series [7] is from the FRED database. All series except [8] are seasonally adjusted. We then define the aggregate series:

$$i_t = ([3]_t + [4]_t)/[8]_t,$$

$$y_t = [2]_t/[8]_t,$$

$$\pi_t = \Delta \log([1]_t/[2]_t),$$

$$n_t = [5]_t * [6]_t/[8]_t,$$

$$M_t = [7]_t/[8]_t,$$

$$s_t = [9]_t * n_t/[1]_t.$$

B Measure of Fit

This appendix briefly describes Watson's (1993) test. Assuming that the empirical data \mathbf{y}_t and simulated data \mathbf{x}_t are jointly stationary, we can define the error induced by the model, \mathbf{u}_t , as the difference between the two data sets, i.e. $\mathbf{u}_t = \mathbf{y}_t - \mathbf{x}_t$. Then, the method consists in minimizing the variance of the error \mathbf{u}_t so that the spectral density matrix of \mathbf{x}_t corresponds to that of \mathbf{y}_t . Given the definition of \mathbf{u}_t , we can

define the spectral density matrix of \mathbf{u}_t at frequency ω by the formula

$$A_{\mathbf{u}}(z) = A_{\mathbf{y}}(z) + A_{\mathbf{x}}(z) - A_{\mathbf{x}\mathbf{y}}(z) - A_{\mathbf{x}\mathbf{y}}(z)', \quad z = e^{-i\omega},$$

where a prime denotes the transpose-conjugate operation and $A_{\mathbf{x}\mathbf{y}}(z)$ is the cross-spectrum of the model and data. As mentioned in the main text, the spectral density of the data is built from the cointegrated VAR previously estimated. Additionally, we resort to the state space form of the approximate solution to the model to compute the theoretical spectral density of \mathbf{x}_t . In contrast, the cross spectral density $A_{\mathbf{x}\mathbf{y}}(z)$ cannot be estimated. It is chosen so as to minimize a weighted trace of the variance of \mathbf{u}_t subject to the requirement that the spectral density matrix of $(\mathbf{x}'_t, \mathbf{y}'_t)'$ be positive semidefinite at all frequencies. Notice that contrary to other tests, the approximation error \mathbf{u}_t is neither interpreted as an extraction error nor as an expectation error. It is correlated with both the empirical data and the simulated data.

For each frequency, we can determine a lower bound of the variance of the approximation error divided by the variance of the data. Let $r(\omega)$ denote this bound and $r_j(\omega)$ denote the j th component of r . In the same fashion, let $[A_{\mathbf{u}}(z)]_{jj}$ and $[A_{\mathbf{y}}(z)]_{jj}$ denote the (j, j) elements of matrices $A_{\mathbf{u}}(z)$ and $A_{\mathbf{y}}(z)$, respectively. We can then define

$$r_j(\omega) = \frac{[A_{\mathbf{u}}(z)]_{jj}}{[A_{\mathbf{y}}(z)]_{jj}}, \quad z = e^{-i\omega}.$$

Watson (1993) proposes to integrate separately both the numerator and denominator of the above expression, defining so the relative mean square approximation error (RMSAE) which the model induces compared with the data. The smaller it is, the better the model reproduces the spectral behavior of the data.

Table 1. Model specifications

Model codes	Indivisible labor	Materials goods	Benchmark adjustment costs
M1	yes	yes	yes
M2	yes	no	yes
M3	no	yes	yes
M4	no	no	yes
M5	yes	yes	no
M6	yes	no	no
M7	no	yes	no
M8	no	no	no

Table 2. Benchmark Calibration

Parameter	Value	Interpretation
β	0.9948	Subjective discount factor
α	0.8400	Share of cash goods in utility
g	1.0046	Technology growth rate
μ	1.0136	Money growth rate
B	{2.1565; 3.0808}	Preference parameter
δ	0.0250	Depreciation rate
ε	10.000	Inverse demand elasticity
ϕ	0.3000	Share of capital in value added
ν_a	0.1000	Elasticity of $a(\cdot)$
s_x	{0.5000; 0.0000}	Share of materials inputs
ρ	0.6390	Autocorrelation of money growth
σ_ξ	0.6491	S.E. of monetary shock

Notes: The first value of s_x corresponds to the models including materials goods and the second value corresponds to models that abstract from this element. Additionally, the first value of B corresponds to the models with divisible labor and the second value corresponds to models with indivisible labor.

Table 3. Estimated Parameters

	Prob. of no adjust.	Elasticity of $v(\cdot)$	S.E. of technology
Models	θ	ν_v	σ_ϵ
M1	0.4519 (0.0132)	0.7119 (0.0486)	0.3964 (0.0670)
M2	0.5824 (0.0115)	0.6940 (0.0460)	0.3318 (0.0744)
M3	0.6453 (0.0138)	1.1253 (0.0876)	0.6910 (0.0150)
M4	0.7542 (0.0116)	1.0874 (0.0907)	0.4652 (0.0440)
M5	0.7452 (0.1353)	0.2529 (0.0373)	$1.0954 \times 1e - 7$ ($3.5576 \times 1e - 6$)
M6	0.8105 (0.1044)	0.2595 (0.0384)	$1.2185 \times 1e - 5$ ($1.3074 \times 1e - 5$)
M7	0.7734 (0.0116)	0.2621 (0.0037)	$9.4497 \times 1e - 7$ ($2.5149 \times 1e - 7$)
M8	0.8356 (0.0936)	0.2734 (0.0433)	$4.2507 \times 1e - 5$ ($4.9299 \times 1e - 5$)

Notes: The standard errors of structural shocks are in percentage. The values in parentheses are the standard errors of the estimated parameters. Models codes as in table 1

Table 4. Results of Watson's (1993) Test

Frequencies, filter	M1					M2				
	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}
[0, π], Δ	0.2259 (0.0549)	0.0739 (0.0279)	0.5314 (0.1286)	2.9592 (0.4253)	0.9444 (0.0778)	0.2302 (0.0606)	0.0745 (0.0263)	0.6435 (0.1816)	2.9544 (0.4188)	1.2252 (0.0930)
$[\pi/16, \pi/3]$, Δ	0.0999 (0.0333)	0.0129 (0.0065)	0.3880 (0.1048)	4.9295 (0.7794)	1.1969 (0.0879)	0.0993 (0.0395)	0.0158 (0.0057)	0.4582 (0.1644)	4.8875 (0.7796)	1.4324 (0.1080)
[0, π], HP	0.0993 (0.0282)	0.0254 (0.0259)	0.4153 (0.1131)	3.8356 (0.5546)	1.1223 (0.0630)	0.0934 (0.0277)	0.0260 (0.0203)	0.4713 (0.1768)	3.8016 (0.5548)	1.3383 (0.0710)
Frequencies, filter	M3					M4				
	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}
[0, π], Δ	0.2576 (0.0748)	0.1017 (0.0259)	0.5774 (0.1472)	2.4514 (0.3151)	1.2604 (0.1454)	0.2650 (0.0846)	0.1097 (0.0251)	0.6805 (0.1966)	2.3509 (0.2914)	1.9454 (0.1908)
$[\pi/16, \pi/3]$, Δ	0.1743 (0.0840)	0.0273 (0.0083)	0.4273 (0.1265)	4.0562 (0.6435)	1.8847 (0.2340)	0.1784 (0.1028)	0.0342 (0.0135)	0.5011 (0.1880)	3.8461 (0.6092)	2.5797 (0.3605)
[0, π], HP	0.1462 (0.0600)	0.0306 (0.0089)	0.4231 (0.1173)	3.3342 (0.4788)	1.7563 (0.1858)	0.1329 (0.0685)	0.0367 (0.0180)	0.4741 (0.1762)	3.1911 (0.4537)	2.4049 (0.3406)
Frequencies, filter	M5					M6				
	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}
[0, π], Δ	0.5854 (0.1794)	0.0846 (0.0389)	0.9159 (0.2671)	1.3955 (0.1336)	1.5107 (0.0869)	0.6082 (0.1838)	0.0882 (0.0414)	1.2863 (0.3848)	1.4431 (0.1435)	1.8456 (0.1367)
$[\pi/16, \pi/3]$, Δ	0.2537 (0.1598)	0.0530 (0.0496)	0.3646 (0.1499)	1.6688 (0.2624)	1.4811 (0.1366)	0.2601 (0.1606)	0.0645 (0.0556)	0.5015 (0.2374)	1.7321 (0.2856)	1.8409 (0.2334)
[0, π], HP	0.1526 (0.1004)	0.0732 (0.0734)	0.3429 (0.1208)	1.6316 (0.1988)	1.4434 (0.1358)	0.1530 (0.0954)	0.0919 (0.0814)	0.4199 (0.1953)	1.6779 (0.2161)	1.7815 (0.2401)
Frequencies, filter	M7					M8				
	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}	\hat{y}	\hat{i}	\hat{n}	$\hat{\pi}$	\hat{s}
[0, π], Δ	0.6506 (0.1941)	0.0845 (0.0427)	0.7875 (0.2228)	1.4529 (0.1446)	2.2107 (0.1870)	0.6839 (0.2000)	0.0921 (0.0470)	1.0835 (0.3193)	1.5166 (0.1569)	2.8554 (0.2747)
$[\pi/16, \pi/3]$, Δ	0.2848 (0.1739)	0.0688 (0.0591)	0.3065 (0.1104)	1.7477 (0.2897)	2.2642 (0.3379)	0.2943 (0.1746)	0.0895 (0.0665)	0.3939 (0.1752)	1.8287 (0.3194)	2.9867 (0.5177)
[0, π], HP	0.1653 (0.1043)	0.0997 (0.0859)	0.3014 (0.0810)	1.6912 (0.2189)	2.1787 (0.3563)	0.1681 (0.0983)	0.1308 (0.0942)	0.3377 (0.1280)	1.7484 (0.2408)	2.8230 (0.5532)

Notes: The values in parentheses are the standard errors computed using sampling errors on the estimated VAR coefficients. Models codes as in table 1. The sign Δ refers to the first-difference filter, and HP refers to the Hodrick-Prescott filter with smoothing parameter set to 1600.

Notes for Figures 1 to 3

1. **Figure 1:** benchmark model and data spectra. All the spectra have been multiplied 10^5 .
2. **Figure 2:** impulse response functions of relevant macroeconomic variables to a monetary policy shock of $\sigma_\xi\%$ in the benchmark economy.
3. **Figure 3:** impulse response functions of relevant macroeconomic variables to a technology policy shock of $\sigma_\epsilon\%$ in the benchmark economy.

Figure 1: Model and Data Spectra in the Benchmark Economy

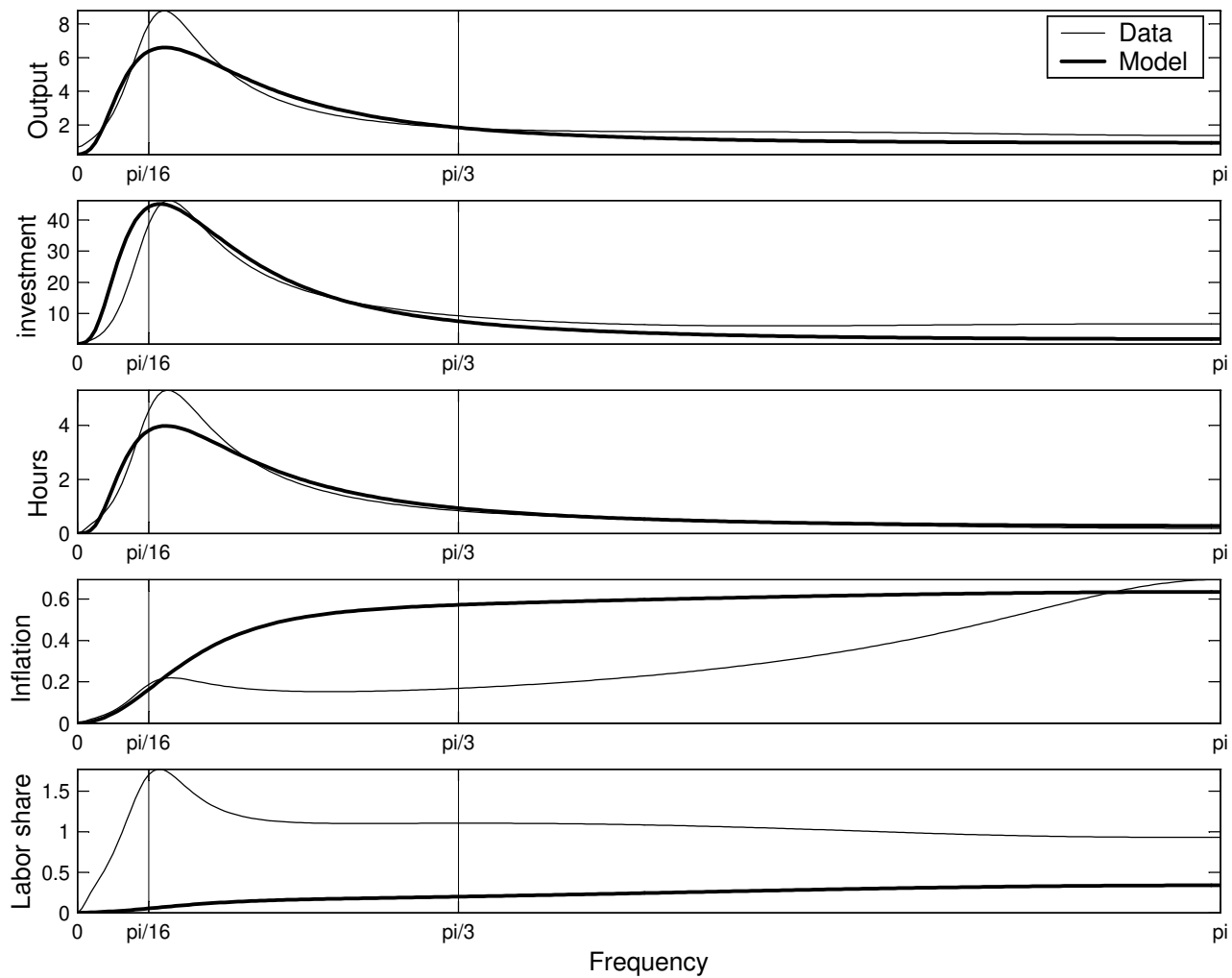


Figure 2: Impulse Response to a Money Supply Shock

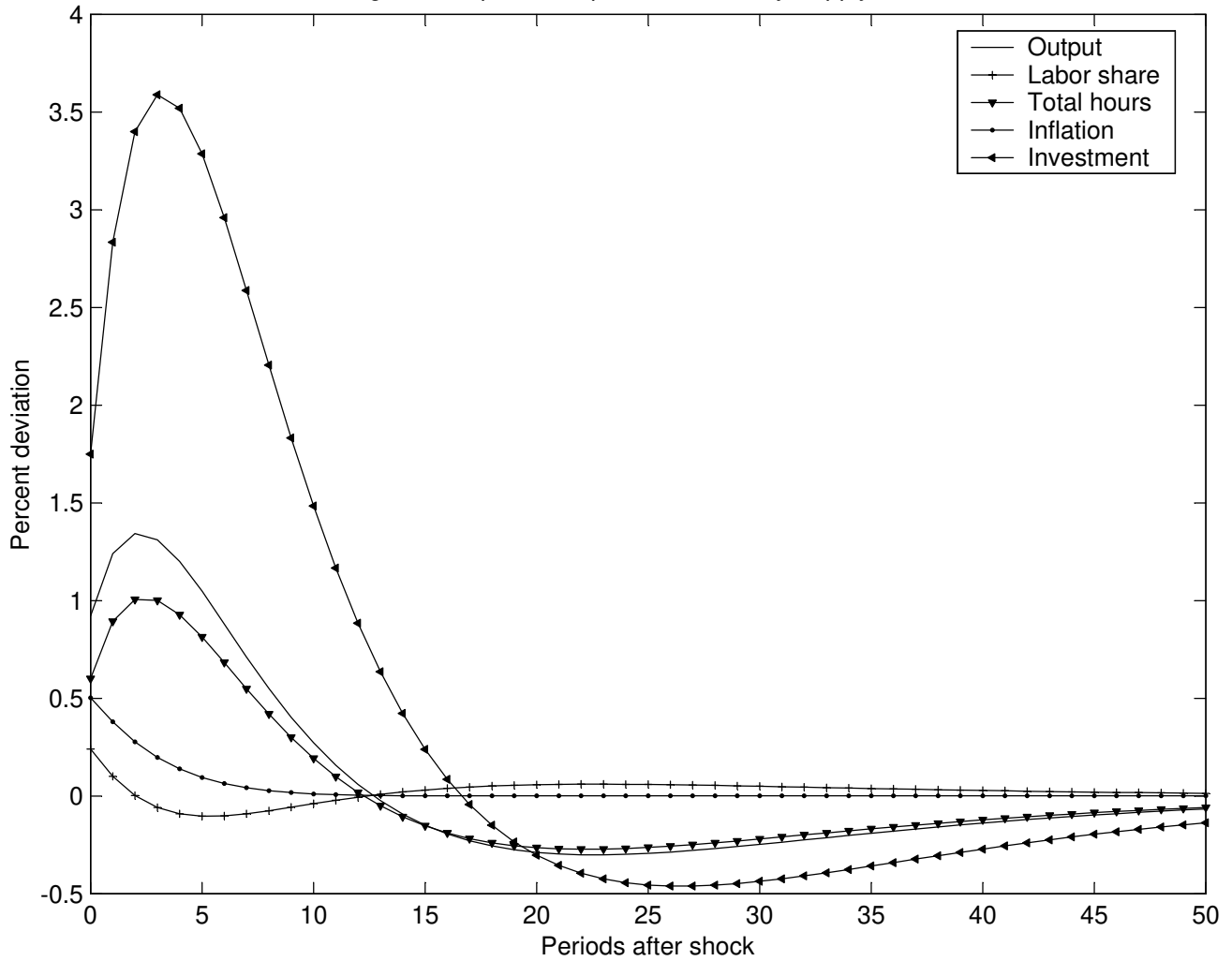
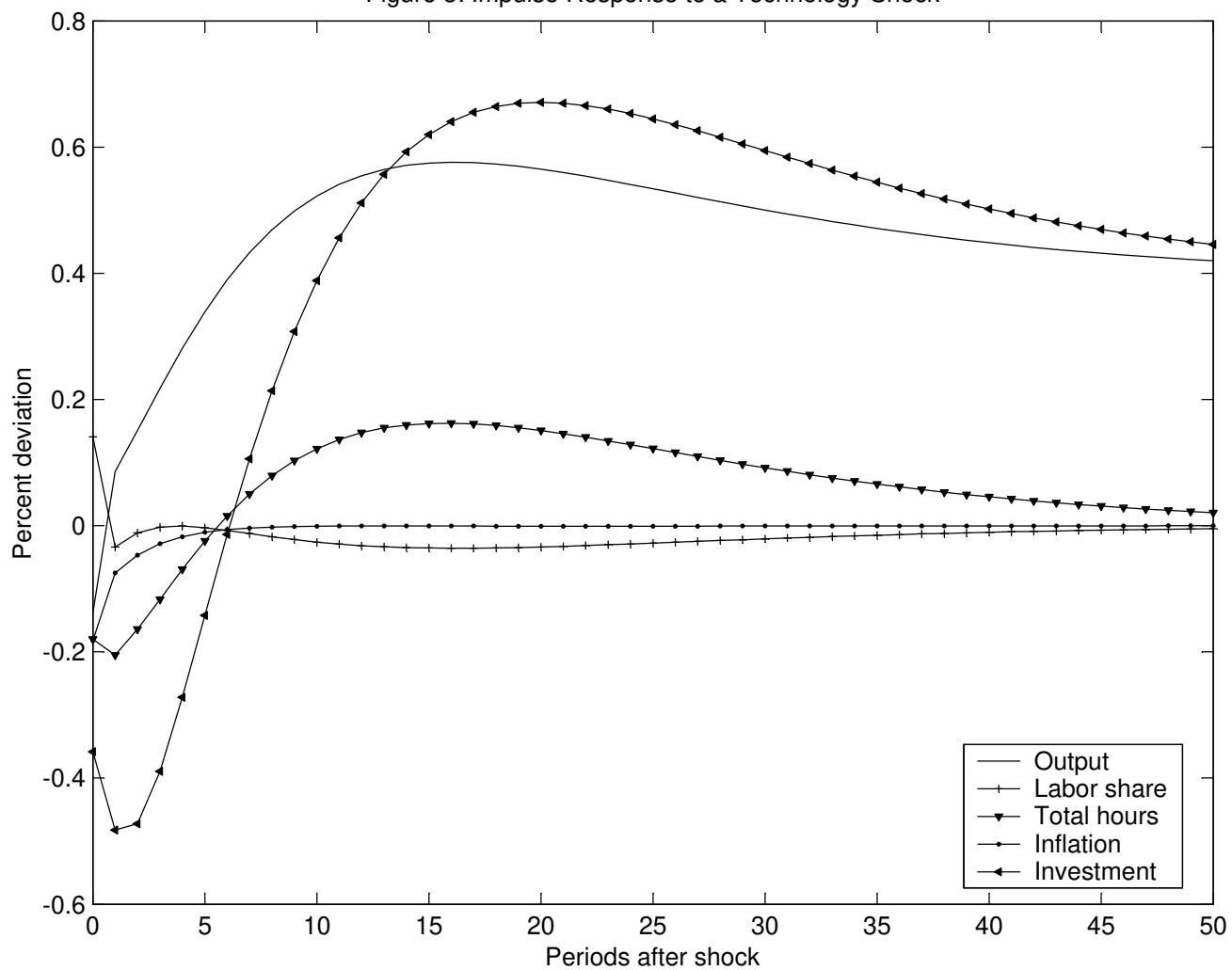


Figure 3: Impulse Response to a Technology Shock



Notes d'Études et de Recherche

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