Evaluation and Tuning of Robust PID Controllers

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Abstract

A general controller evaluation method is introduced, based on four performance and robustness criteria including both low-, mid- and high-frequency properties. According to this method, optimal PI and PID controllers are designed. In industrial applications derivative action is often omitted, due to noise sensitivity. In this paper is shown that when a low-pass filter is included in the design of a PID controller, the control activity as well as the noise sensitivity can be significantly reduced compared to common design rules. These benefits are reached without deterioration of stability margins or low frequency performance. Two sets of simple tuning rules for stable non-oscillating plants are proposed. One set is suitable for automatic tuning. The other one is based on a step response and leaves the operator with just one parameter to tune. This rule makes it in fact easier to tune a PID controller than a PI controller close to the optimum.

Furthermore, it is pointed out that a well tuned PID controller with a second order filter often offers the same performance to the same price in form of control activity as a modern $\mathcal{H}_\infty$ controller. For a delayed plant a Smith predictor can be introduced. However, when a PI controller is used, it is shown to be more profitable to provide the controller with derivative action than with a Smith predictor. On the other hand, together with a PID controller the Smith predictor may improve performance to some extent for plants with moderate delays.

1 Introduction

Within the control community a huge number of results have been presented, where new or modified control design strategies have been compared with existing methods. Too often such comparisons are not objective, since only some aspects of performance and robustness issues are considered. Typically step responses and stability margins are compared, neglecting the cost in terms of e.g. high frequency robustness and control activity.

One design methodoften applied within the area of robust control is the LQG/LTR approach [53]. The pass-band robustness (stability margins) is then improved, but at the cost of deterioration of high frequency loop gain and roll-off. Another example is the $\mathcal{H}_\infty$ loopshaping strategy [35], which typically is directed to good high frequency properties, but sometimes misses the impaired mid frequency robustness it might bring about. The coupling between the low frequency (LF), mid frequency (MF) and high frequency (HF) ranges is obviously important. Improvement of performance in terms of e.g. reduced integrated square error from process disturbances implies either reduced pass-band robustness (stability margins), or reduced HF robustness due to increased lead action in the controller. Thus, performance optimization cannot be considered without looking at robustness in both the MF and the HF domain, which is also pointed out in e.g. [48] and [49].

Based on these observations, a method for general and objective evaluation of controllers for all kinds of plants has during the last years been introduced, see e.g. [31, 23, 22, 27, 26]. It is based on four criteria, related to vital performance and robustness characters, including both LF, MF and HF goals. This method can be used to compare controllers of different structures, but it can also serve as a guideline for appropriate control design procedures. That such a method for critical analysis of available tuning techniques is needed is witnessed in e.g. [30]. Compared to [26] a more complete version of the method is presented here. An additional HF criterion is introduced to handle roll-off. Moreover, the MF criterion is extended to handle high gain controllers such as the Smith predictor.

The PID controller is by far the most common controller in use [61, 34, 19]. It has for decades been practically important and has even been called The ”Process Industries Default” controller. In recent days it has also been object for increased interest from the research community [9, 30]. Unfortunately the derivative part has in practice often been shut off because of lack of a simple and reliable tuning method considering measurement noise sensitivity [21].

This paper presents some results reached by applying the presented evaluation method to PID controllers. One of the key results is that by including a filter in the design, demanded performance and stability margin can be achieved by much lower control activity than with the filter added afterwards. That means that derivative action can be introduced without harmful sensitivity to sensor noise. In [21] it is argued that the filter must be an inherent part of a PID controller. Similar thoughts are also expressed in [31, 32, 39, 33]. The investigations in this paper show that significant improvements compared to most standard methods e.g. [65, 4, 20, 40] can be obtained. These results are valid for all kinds of plants, but this paper is focused on the large group of stable non-oscillating ones.

A reformulation of the classical PID controller is introduced, motivated by the fact that the optimal controller mostly has a pair of complex zeros, whose best location is easy to find, see [22]. Rules for this part of the design are presented in two versions. Both can be used manually and then allow the user to adjust two of the tuning parameters and still get results close to the optimum. This means that the user has some freedom to manage the important trade-off between performance, robustness and control activity. For automatic tuning the rules for the zeros are supplemented by rules for the integral and high frequency gains. These rules, though simplified compared to [26], have been shown to result in almost optimal control. All demanded plant knowledge can be found by a relay experiment [2, 3, 54, 62] and a step response. An extremely simple method for manual tuning is based only on a simple step response.

Furthermore, it is shown that the PID controller can always offer significant performance improvements, compared to the PI controller, to moderately higher control cost. This is true for all kinds of plants, independent of whether their dynamics are due to lag or time delay.

The PID controller may also be augmented by a higher order low-pass filter. Significant improvements of noise sensitivity and HF robustness are then obtained with only very marginal deterioration of low- or mid-frequency properties. Actually a PID controller with a second order filter is often well competitive with a PID-weighted $\mathcal{H}_\infty$-controller and superior to the more common PI-weighted one.

Finally, it is asserted by the evaluation method that it is more profitable to provide a PI controller with derivative action than with a Smith predictor, also in cases when the plant has a significant time delay. On the other hand it is demonstrated that a Smith predictor may improve performance in a system with a PID controller when the plant has a delay of moderate size.

The paper is organized as follows. After this introduction, a description of the proposed evaluation method and its criteria is given in Section 2. New formulations of the PI and PID controllers with their parameters are presented in Section 3. In Section 4 the trade-off between LF and HF characteristics for PI- and PID-controlled systems is elucidated. Improvements of some system properties, for different kinds of plants, due to optimizing of the controller parameter are also presented. Simple tuning rules for PI as well as for PID controllers are given in Section 5. Strictly proper PID controllers are discussed in Section 6 together with PI- and PID-weighted $\mathcal{H}_\infty$-controllers. The benefits of Smith predictors are investigated in Section 7.

2 Evaluation criteria

As was pointed out in the introduction, improvement of a controller design in one respect will very often bring deterioration in another one. Obviously, different system qualities depend on each other. Both LF, MF and HF properties thus have to be investigated, when performance and robustness issues are compared for different regulators. A method for comparison of two controllers must, if it claims to be fair, guarantee that all aspects that are not immediately compared are equally restricted during the comparison. The evaluation method proposed here will fulfill this demand. Four suitable criteria are defined. They are mainly related to the frequency domain, but do also have some time domain interpretations.

Consider the SISO system in Figure 1, where a plant $G(s)$ is controlled by a controller $K(s)$. It has three inputs, the reference signal $r(t)$, the process disturbance $v(t)$ and the measurement noise $w(t)$. Relevant outputs are the controlled output $y(t)$, the control signal $u(t)$ and the control error $e(t) = r(t) - y(t)$. Also introduce the *loop transfer function*

$$L(s) = G(s)K(s)$$

and the following four sensitivity functions with corresponding closed loop transfer functions, which have related output and input signals as indices.

Sensitivity function	$S(s) = \dfrac{1}{1 + L(s)} = G_{er}(s)$
Complementary sensitivity function	$T(s) = \dfrac{L(s)}{1 + L(s)} = G_{yr}(s) = G_{yw}(s)$
Disturbance sensitivity function	$S_v(s) = \dfrac{G(s)}{1 + L(s)} = G_{yv}(s)$
Control sensitivity function	$S_u(s) = \dfrac{K(s)}{1 + L(s)} = G_{ur}(s) = G_{uw}(s)$

Generally seen, a controller can be strictly proper or just proper. When integral action

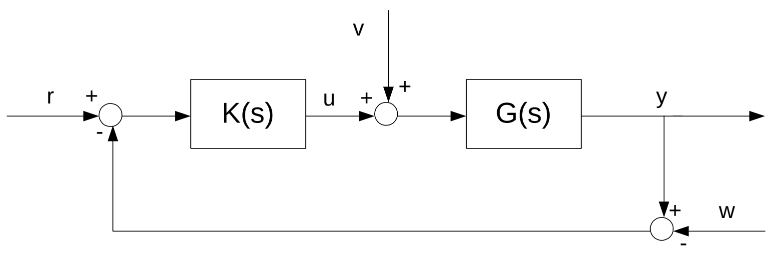

Figure 1: Closed loop SISO system with plant $G(s)$ and controller $K(s)$.

is included, it has the following asymptotic properties

$$K(s) \rightarrow \begin{cases} \frac{k_i}{s} & s \rightarrow 0 \\ \\ \frac{k_\infty}{s^m} & s \rightarrow \infty \end{cases} \tag{1}$$

where k_i is the *integral gain*, k_∞ is the *high frequency gain* and m is the *rolloff rate* of the controller.

Performance criterion The first of the proposed evaluation criteria, related to the low frequency LF region, can be defined as

$$J_v = ||\frac{1}{s}G_{yv}||_\infty = ||\frac{1}{s}S_v(s)||_\infty \tag{2}$$

This is a measure of the systems ability to handle low frequency load disturbances, a frequency domain alternative to the more common criteria based on some function of the error signal [5, 17]. As soon as the controller includes integral action it is a finite quantity, which has the advantage of being almost independent of the plant model. In fact, at low frequencies where $L(s) \gg 1$, $S_v(s) \approx K^{-1}(s) \approx s/k_i$ according to (1). For servo problems, a more relevant criterion is obtained by replacing G_{yv} with G_{er} in (2).

Stability margin Two classical measures are still common to characterize the mid frequency MF robustness, the *phase margin* φ_m and the *gain margin* G_m [59, 18, 11, 15, 41]. However, in recent years a restriction of the maximum sensitivity function

$$||S||_\infty = \max_\omega |S(j\omega)| \leq M_S \tag{3}$$

has been more and more accepted as an exclusive robustness measure, [5, 28, 55, 45]. The reason is that $||S||_\infty$ is equal to the inverse of the minimal distance from the loop transfer function to the critical point $(-1, 0)$ in the Nyquist plot. In many situations it is also a fully sufficient MF robustness measure, but there are exceptions.

With demands on further damping of the step response or increased phase margin but preserved system response, it could be worthwhile to add a restriction on the maximum complementary sensitivity function

$$||T||_\infty \leq M_T \tag{4}$$

especially for plants with integral action, see [49, 44, 51].

For high gain controllers, e.g. when a Smith predictor structure is involved, the loop transfer function $L(j\omega)$ has to be further restricted above the frequency ω_{180_L}, where the

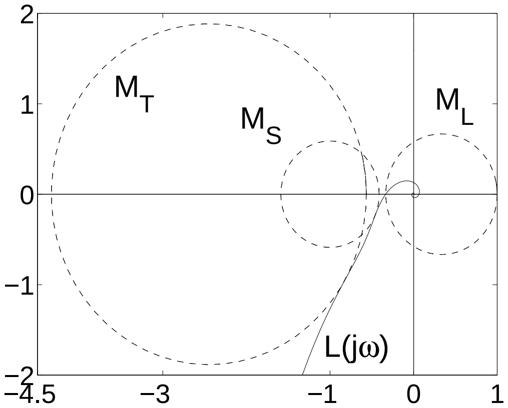

Figure 2: The M_S-circle ($M_S = 1.7$), the M_T-circle ($M_T = 1.3$) and the M_L-circle ($m_{G_m} = 3$) which together define GM_{SL}.

loop has a phase lag of $180°$. Otherwise the system may be very sensitive to reduced time delays, see [25] and Section 7. A restriction

$$\max_{\omega} \left| L(j\omega) - \frac{1}{m_{G_m}} \right| \leq \frac{2}{m_{G_m}} \qquad \omega \geq \omega_{180_L} \tag{5}$$

means that the loop at frequencies above ω_{180_L} has to remain inside a circle with radius $2/m_{G_m}$ and centre in $1/m_{G_m}$. This implies that the gain margin $G_m \geq m_{G_m}$.

These three restrictions on $L(j\omega)$ are illustrated in Figure 2 by three circles in the complex plane, the M_S-circle representing the limit on $|S(j\omega)|$, the M_T-circle representing the limit on $|T(j\omega)|$ and the M_L-circle representing the limit on $|L(j\omega)|$ at frequencies above ω_{180_L}. The values of the constants in this figure are equal to the default values in this paper. They are $M_S = 1.7$, $M_T = 1.3$ and $m_{G_m} = 3.0$ In this figure is also shown a typical Nyquist plot for a well-behaved system. It has two tangential points at the frequencies where the constraints on S and T are just fulfilled, and enters the M_L- circle at the point $(-1/m_{G_m}, 0)$. Compare the Nyquist plot constraint corresponding to a line proposed in [60].

To get one measure which inludes all three restrictions the *Generalized Maximum Sensitivity*

$$GM_{SL} = \max \left(\|S\|_\infty,\ \alpha\|T\|_\infty,\ \gamma \max_{\omega} \left(W_L(j\omega)\ |L(j\omega) - 1/m_{G_m}| \right) \right) \tag{6}$$

is introduced. The weight function $W_L(j\omega) = 0$ for $\omega < \omega_{180_L}$ and $W_L(j\omega) = 1$ for $\omega \geq \omega_{180_L}$, $\alpha = M_S/M_T$ and $\gamma = M_S m_{G_m}/2$. When there is equality in at least one of the restrictions ((3))–((5)) this means that $GM_{SL} = M_S$. Hence the GM_{SL} criterion converts the restrictions (4), (5) to corresponding M_S levels.

Undoubtedly, GM_{SL} is a somewhat complex criterion, but there are good reasons for including each of its elements as was motivated above. However, the third criterion is not necessary for plain PI and PID controllers, since these controllers do not imply a high gain property which motivates the restriction (5). In such cases, including all evaluations in this paper except those in Section 7, GM_{SL} is simplified to

$$GM_S = \max\left(\|S\|_\infty,\ \alpha\|T\|_\infty\right) \tag{7}$$

Control activity When a reasonable stability margin is established, design of a control system is typically a question of trade-off between performance and control activity. It is therefore suitable to introduce a cost criterion related to the mid to high frequency MHF region, around or slightly above the closed loop bandwidth, where the maximum of the control sensitivity is mostly to be found, cf. Figure 4.

$$J_u = \|G_{ur}\|_\infty = \|G_{uw}\|_\infty = \|S_u(s)\|_\infty \tag{8}$$

High frequency robustness and noise sensitivity In the HF region two demands are especially relevant, robustness against model uncertainties and reduction of sensitivity to sensor noise. For plants with significant uncertainty due to e.g. varying time delay or unmodeled high frequency resonances, the complementary sensitivity function $T(s)$ must be kept small according to the Small Gain Theorem [13]. Furthermore, the measurement noise is transferred to the output by $G_{yw} = T(s)$, which gives another reason to keep $T(s)$ small.

Now, $T(s) = G(s)K(s)S(s)$ and consequently $TG^{-1} = S_u = G_{ur}$, why keeping $S_u(s)$ low is keeping $T(s)$ low as well. Then a relevant criterion for the HF domain can be formulated as

$$J_{HF} = \|s^m G_{ur}\|_\infty = \|s^m G_{uw}\|_\infty = \|s^m S_u(s)\|_\infty \tag{9}$$

For high frequencies, where $S(s) \approx 1$, $S_u \approx K \approx k_\infty s^{-m}$ according to (1). Hence this criterion, just as J_v, is almost independent of the nominal plant model. When $m = 0$, which is valid for PI and PID controllers without extra filters, $J_{HF} = J_u$.

Evaluation procedure In all controller design, independent of method, the user has to modify a set of tuning parameters ρ. An objective method to evaluate a control system in some respect is then to keep three of the four introduced criteria constant and equal, or at least bounded upwards , and then modify ρ to the minimum of the fourth criterion. Then evaluation of LF performance is accomplished by solving the constrained optimization problem

$$\min_\rho J_v(\rho) \quad GM_S(\rho) \le C_1 \quad J_u(\rho) \le C_2 \quad J_{HF}(\rho) \le C_3 \tag{10}$$

where the constants C_i may be given different values. The default value of C_1 in this paper is 1.7, while the values of C_2 and C_3 may vary. The last restriction is relevant only for strictly proper controllers.

By this optimization procedure completely different controllers may be compared under equal conditions. In fact it is straightforward to include even sampled-data controllers with different sampling periods in such a comparison [**?**]. An evaluation method like this one is wanted in [56]. A similar idea, but with other criteria and with more vague constraints, is presented in [42] and another one in [34].

The expression *optimal controller* is from now on used for a controller which is optimized according to (10) with all available controller parameters included in the tuning vector ρ. For instance a PI controller has two free parameters. When both are used in the optimization the optimal PI controller is achieved. In this paper Matlab Optimization Toolbox is used for the computation.

3 The PID controller

As was noted above the proposed evaluation method can be used to compare all kinds of controllers, but the constrained optimization procedure (10) may also be used as a synthesis tool for a given class of controllers. A systematic investigation has been made to find the most effective parameters for PID and related controllers and to find simple rules for computing optimal values in different situations. During the work it has become more and more obvious that the optimal values are not always those that could be expected according to common recommendations. It has also turned out that essentially the same parameters are suitable for several different kinds of low order controllers, see [27].

Formulation and parameters

There are many ways to formulate the transfer function of a PID controller and to choose its design parameters. The controller discussed in this paper is a one-degree-of-freedom one, see Figure 1, and hence the intention is mainly to design it for good rejection of load disturbances. This is motivated by the fact that most PID controllers work as regulators [50, 7]. When good servo properties are demanded, the controller can always be augmented by a filter in the feedforward path.

The traditional PID controller with the three parameters *proportional gain* K, *integral time constant* T_i and *derivative time constant* T_d has the drawback that it is not proper. To bound the high frequency gain, it is mostly augmented by a low-pass filter on the derivative part and may then be formulated as

$$K_{PID}(s) = K(1 + \frac{1}{sT_i} + \frac{sT_d}{1 + sT_f}) \tag{11}$$

However, in [24, 22] it has been shown that a PID controller, optimized with the filter included and all parameters free, typically implies complex zeros in the controller. These results have made it natural to reformulate the PID controller with a first order low-pass filter as

$$K_{PID}(s) = k_i \frac{1 + 2\zeta\tau s + (\tau s)^2}{s(1 + s\tau/\beta)} \tag{12}$$

where the four tuning parameters are *damping* ζ and *natural frequency* $1/\tau$ of the controller zeros, *integral gain* k_i and *high frequency gain* $k_\infty = K_{PID}(\infty) = k_i\tau\beta$. The variable $\beta = k_\infty/(\tau k_i)$ is introduced here just for convenience. A more general formulation of the PID controller than the traditional one is also recommended in [21].

Correspondingly, the PI controller may be formulated as

$$K_{PI}(s) = k_i \frac{1 + \tau s}{s} \tag{13}$$

Translation from the parameters in (12) to the traditional parameters in (11) is straightforward. Note also that with $\beta = 1$ (low high frequency gain) and $\zeta = 1$ (double zero) the PID controller becomes a PI controller.

Plant knowledge

To obtain useful tuning rules for PI and PID controllers, the demands on plant knowledge must presumably be very moderate.

The parameter κ was introduced by Åström and coworkers in [16] as a measure of the difficulty to control a process. For stable plants it is defined as

$$\kappa = \frac{|G(j\omega_{180_G})|}{|G(0)|} \tag{14}$$

where ω_{180_G} is the frequency at which the plant has a phase lag of $180°$. This normally gives κ numbers in the interval $[0, 1]$. The higher the value is, the more complex and hard to control the plant is. Note that κ is the inverse of the gain margin that a P controller would give with $K(s) = 1/|G(0)|$.

However, there are plants for which neither κ nor ω_{180_G} are defined, since the plant phase never reaches $-180°$. To meet such a situation κ may be modified to

$$\kappa_{150} = \frac{|G(j\omega_{150_G})|}{|G(0)|} \tag{15}$$

where ω_{150_G} is the frequency at which the plant has a phase lag of $150°$. Together with ω_{150_G} and $|G(0)|$, this number has been used in this paper to characterize plant dynamics and to formulate a set of simple tuning rules. These characteristics can be found by a relay experiment including hysteresis [2] and a step response. Together they also fulfill the demand on three items of information about the plant that are necessary for tuning of a PID controller according to [57, 9].

The models used for the investigations in this paper are the same as in [26]. They are essentially those recommended in groups 1–5 in [6], suggested as standard benchmark models for testing of PID controllers. Among them are all kinds of stable non-oscillating plants, such as minimum and non-minimum phase plants, plants of high and low orders, plants with multiple and spread poles etc.

4 Some results for optimal PI and PID controllers

In many design methods for PID controllers presented in the literature, the ratios between the time constants in (11) have been more or less fixed and not really utilized in the design procedure. Introduce

$$a = \frac{T_i}{T_d} \qquad b = \frac{T_d}{T_f} \tag{16}$$

Ever since Ziegler and Nichols presented their tuning rules in [65] the standard value $a = 4$ has often been used without further motivation, see for example [2, 38, 58, 14]. This value corresponds to a double zero in (11) when $T_f = 0$. Regarding b there are no well-founded recommendations at all to be found, since the filter has mostly not been looked upon as a part of the design. It has just been added afterwards and there are hints in e.g. [4, 50, 47, 46] about a value of b in the vicinity of 10, motivated by a demand not to let the filter influence the closed loop properties too much, especially not concerning the mid-frequency robustness (stability margins).

Filter design

When a and b are fixed to 4 and 10 respectively, only two parameters are left for the optimization. Compared to this case Figure 3 shows that great improvements in the properties of a PID controlled system can be achieved for a second order plant with time delay, when the four parameters are all optimized. The left part shows the trade-off between process disturbance compensation J_v and control activity J_u, while the right part shows the correspondence between normalized bandwidth ω_b/ω_{180_G} and control activity J_u. In all cases $GM_S = 1.7$. The same figure shows that fixing a to 4 brings rather marginal deterioration

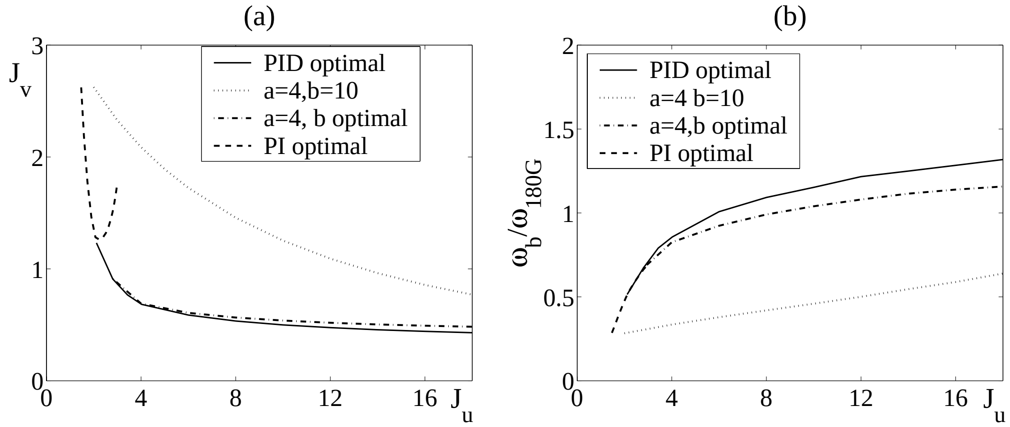

Figure 3: (a) J_v and (b) ω_b/ω_{180_G} as functions of J_u for the plant $G(s) = e^{-0.3s}/(1+s)^2$ controlled by different PI(D) controllers. In all cases $GM_S = 1.7$, while J_v is minimized for each J_u value, corresponding to different C_2 in (10).

compared to the optimal case. This is confirmed by results from optimization of the serial PID controller (a PI-block in series with a PD-block). For this controller a double zero is shown to give the best trade-off between J_v and J_u, see [22]. A double zero corresponds to $a = 4$ when b is infinite (no filter) and just above when b is finite. For stable non-oscillating plants, the optimal value of a in a PID-controller (11) is about 2.5 [22].

Optimal versus fixed low-pass filter The dramatic reduction of J_v, as well as the corresponding enlargement of the bandwidth without J_u-shift, comes from including b in the tuning vector ρ, that is from including the filter in the design. The optimal values of b has shown mostly to be rather 3 to 5 than 10, somewhat increasing with J_u and κ. It is obvious from Figure 3 that the low frequency properties J_v of a control system can be significantly improved without loss of high frequency robustness or stability margin GM_S and without increased control cost J_u, just by adjustment of the filter constant. Note also that derivative action, i.e. PID control instead of PI control, can decrease J_v significantly at moderate J_u values.

The trade-off between J_v and J_u and the normalized bandwidth For an optimal PI controller there is mostly a minimum in the J_v/J_u-graph. This has been shown by investigations, see [8], [22] and Figure 3 (a), but can also, at least for simple cases, be shown theoretically.

For the optimal PID case there is normally no such minimum. However, it has been noted that the graph tends to be more horizontal when J_u grows. Then it can be argued that it is no use to increase J_u above a certain level, because the reward in decreased J_v is too small. This tendency towards non-decreasing J_v is more obvious and the "economic level” $J_{u_{ec}}$ is lower the more complex the plant is (higher κ value), see [26]. This ”economic limit” is obviously not very sharp, but for the plant in Figure 3 it has been estimated to approximately 10.

It should be emphasized that an optimal PID controller, working at its ”economic level”, can always offer better system properties than an optimal PI controller, and still $J_{u_{ec}}$ is very reasonable. *Moreover, it is seen from Figure 3 (a) that when a PID controller with $a = 4$ and $b = 10$ offers the same performance J_v as the optimal PI controller, the demanded control activity J_u is almost 5 times higher. This is one reason why derivative action is not used in most industrial applications. Note that b is even fixed to 10 in many commercial PID controllers.*

The comparison between the controllers can alternatively be done for ω_b. From Figure 3 it is remarkable how well the bandwidth follows the inverse of the J_v/J_u-graph. The exception is the PI case, where no tendency to maximum can be seen in the bandwidth. It is also worth noting that for those values of J_u where the PI controller works at its best, the PID-graphs typically come close to the PI-graphs. This is what could be expected from Section 3, since then $\beta \to 1$ and $\beta = 1$, $\zeta = 1$ corresponds to a PI controller.

The relation between J_u and k_∞ According to (8), the control activity criterion J_u is equal to the maximum of the control sensitivity function $S_u(\omega)$. For high frequencies $|S(\omega)| \approx 1$ and hence $|S_u(\omega)| \approx |K(\omega)| \approx k_\infty/\omega^m$, cf. (9).

For the PID controller (11) or (12) with $m = 0$, maximum of $S_u(\omega)$ typically occurs when $\omega \to \infty$ due to the derivative action, and then $J_u = k_\infty$.

In the PI controller case for which also $m = 0$, there is no derivative action and then the maximum of $S_u(\omega)$ occurs for lower frequencies, most oftenly just above the closed loop bandwidth. However, also for the PI controller there is a close relation between J_u and k_∞, so that higher values of J_u corresponds to higher values of k_∞, at least for plants

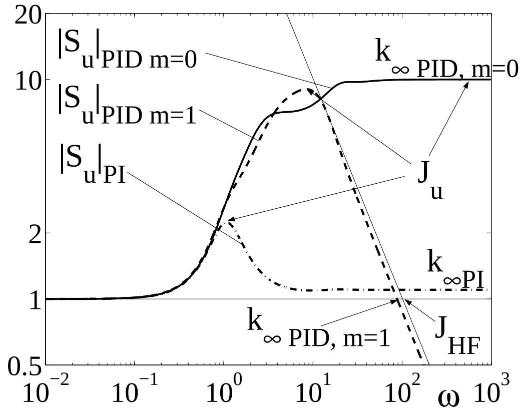

Figure 4: Plant $G = e^{-0.3s}/(1+s)^2$ optimally controlled by a PI controller (dashdotted), a PID controller with $m = 0$ (solid) and a PID controller with $m = 1$ (dashed). The figure illustrates the relations between k_∞, J_u and J_{HF}.

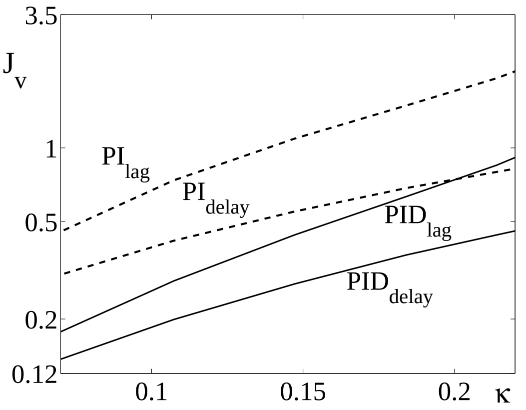

Figure 5: J_v/κ-graphs for G_{lag} + PID, G_{lag} + PI, G_{delay} + PID and G_{delay} + PI.

with not too high κ values, see [22].

When the PID controller is augmented by an extra low-pass filter so that $m > 0$, it has a roll-off and $S_u(\omega)$ has a maximum at finite frequencies also for this controller. Then k_∞ can be found approximately at the intersection with the frequency axis. Figure 4 illustrates the relations between J_u, J_{HF} and k_∞.

Similarities between lag and time delayed plants

It is sometimes argued that the derivative part of the PID controller is more profitable for a lag plant than for a plant with time delay [7, 50]. This statement is only valid to some extent as may be concluded from Figure 5 and 6. Two plants are compared

$$G_{lag}(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)} \quad \alpha = 0.3, 0.4...1.0 \tag{17}$$

and

$$G_{delay}(s) = \frac{e^{-sL_d}}{(1+s)(1+0.2s)} \tag{18}$$

where L_d is varied to give the same κ values for the two types of plants.

Figure 5 shows that, independent of κ, J_v will, for both types of plants, decrease to approximately half of its value for the optimal PI-controller, when derivative action is introduced. In the lag case $J_{v_{PID}}/J_{v_{PI}}$ varies from 0.39 for small κ values to 0.46 when $\kappa = 0.25$. In the delay case the same variation goes from 0.45 to 0.58. In all cases moderate control activities have been used. For the PID controllers $J_u = 10$, which is close to $J_{u_{ec}}$ for these plants, and for PI the optimal J_u value (2–4) is chosen.

In Figure 6 the J_v/J_u relations and the process disturbance step responses are shown for two pairs of plants with equal κ values controlled with and without derivative action. The J_v/J_u relations show that the two kinds of plants behave equal for reasonable control

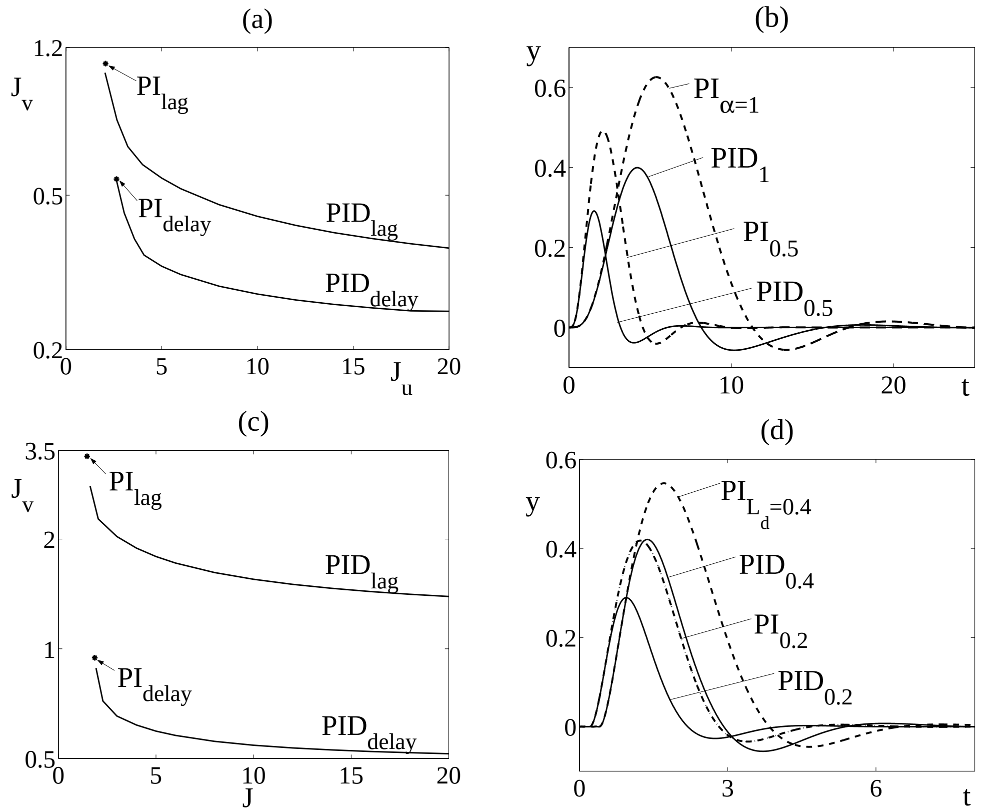

Figure 6: Left: J_v/J_u relations for G_{lag} and G_{delay} with (a) $\kappa = 0.147$ $(\alpha = 0.5, L_d = 0.211)$ and (c) $\kappa = 0.249$ $(\alpha = 1.0, L_d = 0.405)$. Right: Process disturbance step responses for corresponding plants (b) G_{lag} + PID ($J_u = 10$) and G_{lag} + PI, (d) G_{delay} + PID ($J_u = 10$) and G_{delay} + PI.

activities. The step responses show in all cases significantly smaller integrated errors for the PID controllers ($J_u = 10$) than for the PI controllers (optimal J_u).

Since the κ values of the two compared plants are equal in pairs and have the same LF gain (=1), they are supposed to be equally difficult to control. It is obvious that the plants with delay in these examples show better trade-off between J_v and J_u and more favorable step responses (for the same J_u) than the corresponding lag plants, despite the same demand on GM_S. However, the profit offered by the derivative action is almost the same for the two kinds of plant dynamics.

The reason why derivative action is argued to be more profitable for plants with lag than for time delayed plants is possibly that very large control activity J_u reduces J_v significantly for plants with lag. This is unfortunately not the case for time delayed plants, due to their non-minimum phase behaviour. However, a reasonably large control activity means that the differences between PI and PID control are comparable for plants with the same κ value, independently of whether the plant dynamics are characterized by lag or

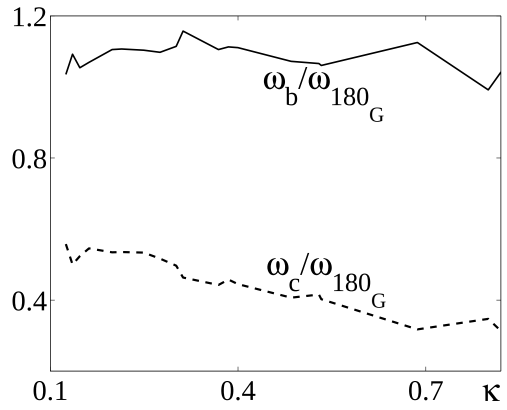

Figure 7: Normalized closed loop bandwidth ω_b and gain crossover frequency ω_c as functions of κ for optimal PID controllers with $GM_S = 1.7$ and $J_u = 8.0$.

time delay.

Some optimal system properties

There are some system properties that can be immediately observed when PID control is applied and the filter is designed, so that the closed loop system is optimized.

Bandwidth and crossover frequency It may be argued that the closed loop bandwidths ω_b presented so far are often rather small. The same assertion is valid for the open loop crossover frequencies ω_c. This is a consequence of the optimization of $J_v \approx 1/k_i$, the relatively strong demands on MF robustness (stability margin) and the moderate control activity. According to Figure 7 the crossover frequency decreases slightly with increasing κ, $\omega_c \approx (0.6 - 0.35\kappa)\omega_{180_G}$, while the resulting closed loop bandwidth is independent of this value, $\omega_b \approx 1.1\omega_{180_G}$ (ω_{180_G} = plant phase crossover frequency). This means that $w_b \approx 2 - 3$ times ω_c, which is considerably larger than the rule $\omega_c \leq \omega_b \leq 2\omega_c$, found in the literature [13].

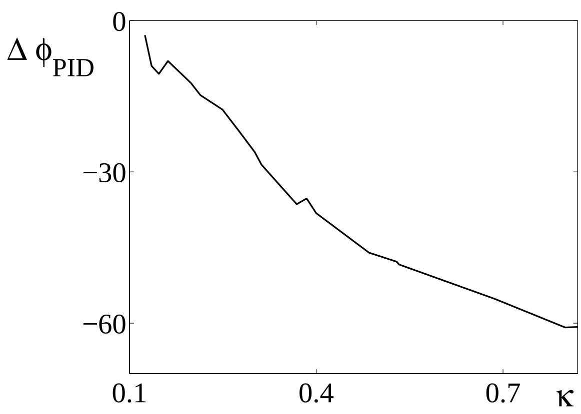

Figure 8: Controller phase shift ($\Delta\varphi_{PID}$) at ω_c as a function of κ for optimal PID controllers with $GM_S = 1.7$ and $J_u = 8.0$.

Controller phase shift Sometimes it has been asserted that when a phase margin of $45°$ (approximately corresponding to $M_T = 1.3$) is demanded, a well-tuned PID controller conserves the phase shift of a minimum phase plant at a frequency where it is around $-135°$ [1, 7]. This means that the controller at the open loop gain crossover frequency ω_c will have no significant phase lag. According to [1] it is even likely to have a positive phase at this frequency.

Figure 8 shows the phase shift $\Delta\varphi$ found for optimal PID controllers at this frequency for different values of κ. It implies that $\Delta\varphi$ at ω_c, introduced by an optimal PID controller, increases negatively with κ, roughly as $\Delta\varphi = 5 - 100\kappa$. This result is valid for all kinds of plants investigated in this paper including minimum phase plants with higher κ values. The optimization means a minimization of J_v, which approximately corresponds to a maximization of the integral gain k_i. It is well known that large integral action brings good disturbance compensation but also large negative phase shifts.

The PID controller zeros The PID controller zeros are in (12) characterized by two parameters, ζ the damping ratio and τ the inverse of the natural frequency.

For the optimal ζ no significant dependence has been found neither on κ nor on J_u for the stable non-oscillating plants investigated in this and other papers. This implies that $a = T_i/T_d \approx (2\zeta)^2$ is also constant. As can be seen from Figure 9(a), $\zeta = 0.75$, corresponding to $a \approx 2.25$ (significantly less than 4), is always a good approximation. Fixing ζ to this value and optimizing the remaining parameters will always give results that are almost impossible to separate from the optimal ones.

Also the optimal natural frequency $1/\tau$ is almost independent of J_u, but for this parameter, normalized by ω_{150_G}, there is a linear dependence on κ_{150_G} that can not be neglected, see Figure 9 (b).

Another way to normalize τ is to use a character from the step response of the plant. The ratio of τ to the *equivalent time constant* T_{63}, the time it takes for the response to reach 63% of its final value, shows a very marginal dependence on κ. Figure 9 (c) shows $\tau/T_{63} \approx 0.35 \approx 1/3$.

These results offer a good starting point for a presentation of simple tuning rules for the PID controller, which is given in the next section.

5 Tuning rules for PI and PID controllers

Based on experience collected from comprehensive investigations on optimization of PI and PID controllers for many kinds of *stable non-oscillating plants*, some recommendations on tuning of these controllers can now be formulated. The plant knowledge needed

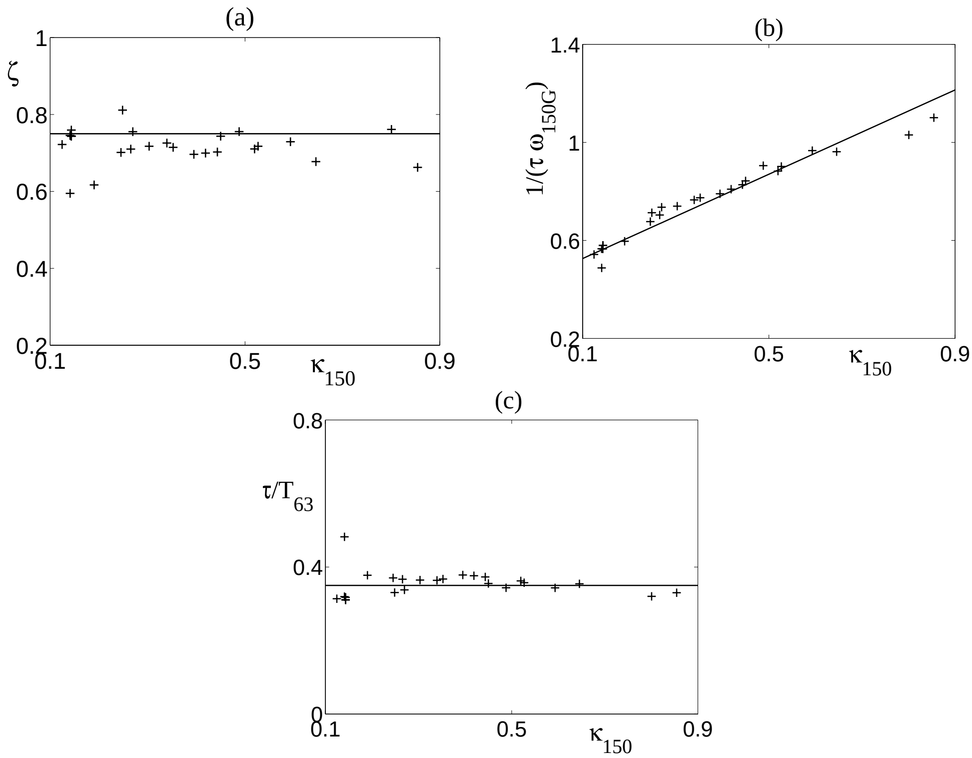

Figure 9: Normalized zero parameters of the PID controller as functions of κ_{150}, optimal outcomes ($+$) and linear approximations.

is expressed by κ_{150}, the corresponding frequency ω_{150_G} and sometimes the equivalent time constant T_{63}. According to e.g. [2, 62], this knowledge can be found by an experiment with a relay including hysteresis, giving ω_{150_G} and $|G(j\omega_{150})|$, together with a step response, giving $|G(0)|$ and T_{63}.

A set of rules based on κ may be found in [26] for stable non-oscillating plants with $\kappa \geq 0.1$, completed by another set for the same kind of plants with $\kappa < 0.1$ or no κ defined. Here just one set of rules is given, which will work for all stable non-oscillating plants independent of κ. The only exception is first order plants with time delays. For these plants a special recommendation is given in [26]. In the same paper there is also a set of tuning rules for plants with integral action.

PI controllers

For the PI-controller (13) there are only two parameters to be tuned. The aim in the design of this controller is to reach the minimum in the J_v/J_u graph, see Figure 3(a). The controller zero may then be positioned by one of the following rules:

$$\frac{1}{\tau} = \omega_{150_G}(0.06 + 1.6\kappa_{150} - 0.6\kappa_{150}^2) \tag{19}$$

or

$$\tau = T_{63}(0.70 - 0.45\kappa_{150}) \tag{20}$$

The first rule will for most stable non-oscillating plants give a difference from the optimal value of less than 5%, the second one slightly more.

The remaining parameter, the integral gain k_i, can either be used for tuning the system to the demanded stability margin e.g. $GM_S = 1.7$ or to the desired damping of a step response. It may also be computed from the formula

$$k_i = \frac{\omega_{150_G}}{|G(0)|} * (0.2 + \frac{0.075}{\kappa_{150} + 0.05}) \tag{21}$$

This formula gives a difference less than 5% from the optimal values of k_i and J_v for most plants, a little more for some plants with very small κ values. GM_S falls in the interval $[1.60, 1.85]$.

PID controllers

The zeros of the PID controller (12) may be computed with good accuracy by the simple formulas

$$\zeta = 0.75 \tag{22}$$

$$\frac{1}{\tau} = \omega_{150_G}(0.44 + 0.86\kappa_{150}) \tag{23}$$

or when $\kappa \geq 0.1$

$$\tau = 0.35T_{63} \approx T_{63}/3 \tag{24}$$

The remaining parameters may still be freely used by the operator to take care of special demands on proper control activity ($J_u \approx k_\infty$) and desired rejection of process disturbances ($k_i \approx 1/J_v$).

However, the following formula for k_∞ will give a J_u value close to the economic level $J_{u_{ec}}$

$$k_\infty = \frac{1}{|G(0)|} * \max(3 + \frac{2}{\kappa_{150}}, 25) \tag{25}$$

Finally k_i can be adjusted to demanded stability or computed by the formula

$$k_i = \frac{\omega_{150_G}}{|G(0)|} * (\frac{0.45}{\kappa_{150} + 0.07} - 0.1) \tag{26}$$

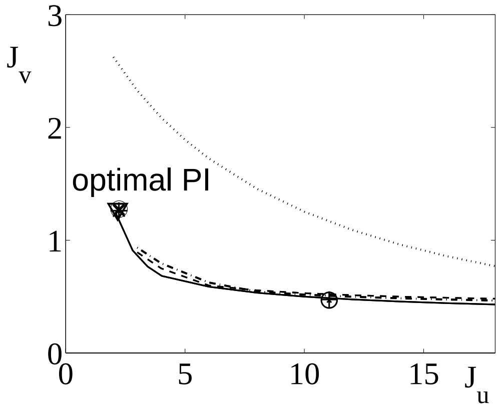

Figure 10: The J_v/J_u relations for $G = e^{-0.3s}/(1+s)^2$ controlled in different ways, that is with optimal PID controller (solid), PID with $a = 4$, $b = 10$ (dotted), PID with locked zeros (23) (dashed) or (24) (dashdotted). So far k_i is optimal. Furthermore PID is tuned by the complete set (22), (23)/(24), (25) and (26)(+/o). The PI controllers are optimal (*) and tuned by (19)/(20) and (21) (x/v) (both coincide with *).

The parameter β in (12) is introduced just for convenience. It is short for $k_\infty/(\tau k_i)$. Note that $\beta \approx b\sqrt{a}$, see [22].

When automatic tuning is preferred, with use of (22), (23) or (24), (25) and (26), the resulting J_v value will mostly differ from the optimal one (at the same J_u value) with less than 5%, slightly more for plants with very small κ values ($\kappa < 0.1$) or no κ numbers defined. The stability margin GM_S will fall in the interval $[1.65, 1.85]$, in most cases close to 1.7.

Figure 10 shows some results from tuning by the presented rules. It is obvious that the rules for the zeros in the PID case can be used over a large interval of J_u values. It is interesting that the two rules for τ give almost identical results in both the PI and the PID case. Observe also that tuning all accessible parameters by the proposed rules offers results that come very close to the optimal ones.

Extremely simple tuning rule *From a step response giving T_{63} the zeros of a PID controller can be fixed by (22) and (24). To find a suitable high frequency gain, try $\beta =$ 5-8. The only remaining parameter k_i can then be tuned to demanded stability (damping).*

The economic level $J_{u_{ec}}$ typically corresponds to $\beta =$ 5-8. If more high frequency gain can be accepted β can be slightly increased. That leaves the user with the freedom to handle the trade-off between control activity (k_∞) and performance (k_i).

Using this rule, it can not be argued any more that a PID controller is more difficult to design than a PI controller. In fact PID tuning based on a step response (T_{63}) is simpler, since there is no minimum to reach as in the PI case. For each choice of control activity selected by β an optimal k_i can be found. In the PI case, there are two parameters to adjust simultaneously to get to the optimum, cf. Figure 3(a).

The results of this simple tuning procedure for plants with $\kappa \geq 0.1$, including e.g. plants with highly non-minimum phase behaviour, has shown to be remarkably close to the optimal ones.

6 Strictly proper PID controllers and PID weighted $\mathcal{H}_\infty$ controllers

Sometimes a PID controller with a first order low-pass filter (a second order compensator) can not fulfill the given property demands. Typically there is a need for more roll-off than can be offered by the plant to compensate for significant model uncertainties or measurement noise [10]. In such cases the PID controller can be augmented by a low-pass filter of higher order. Another way to meet demands on roll-off is to introduce one or more weighting functions in the plant model and then optimize the controller with an $\mathcal{H}_\infty$ strategy.

PID controller with a second order filter

When the ordinary first order filter in the PID controller (12) is exchanged by a second order filter, the controller can be formulated as (the index *ro* means roll-off)

$$K_{PID_{ro}}(s) = k_i \frac{1 + 2\zeta\tau s + (\tau s)^2}{s(1 + 2\zeta_f \frac{\tau}{\beta} s + (\frac{\tau}{\beta} s)^2)} \tag{27}$$

This formulation opens up for complex poles, with the damping ratio ζ_f and the undamped frequency β/τ, as well as for complex zeros.

In Figure 11 results for a representative plant model $G(s) = (1 - 0.5s)/(1 + s)^3$ are given. Except for the study of the J_v/J_u relation in Figure 11(a), J_u has been kept constant at $J_u = 7.0$, which is close to $J_{u_{ec}}$ for this plant. A PID controller with a first order filter has been optimized under these conditions and is shown for comparison. The damping ratio ζ_f of the second order filter has been varied in a wide range. The more it is decreased, the more effective the roll-off is, corresponding to lower control activity and better attenuation of measurement noise. For example it can be gathered from Figure 11(c) that the gain from the measurement noise to the control signal at $\omega = 100$ rad/s is reduced by a factor about 100, and the high frequency criterion J_{HF} goes from from 787 to 19, when ζ_f decreases from 2.0 to 0.1, with $J_{HF} = 65$ for $\zeta_f = 0.5$.

However, it should be noted that there are less favorable effects coming up when the reduction of ζ_f is driven too far. In the complementary sensitivity function T a resonance is growing. According to the Small Gain Theorem [13, 64], such peaks should be avoided

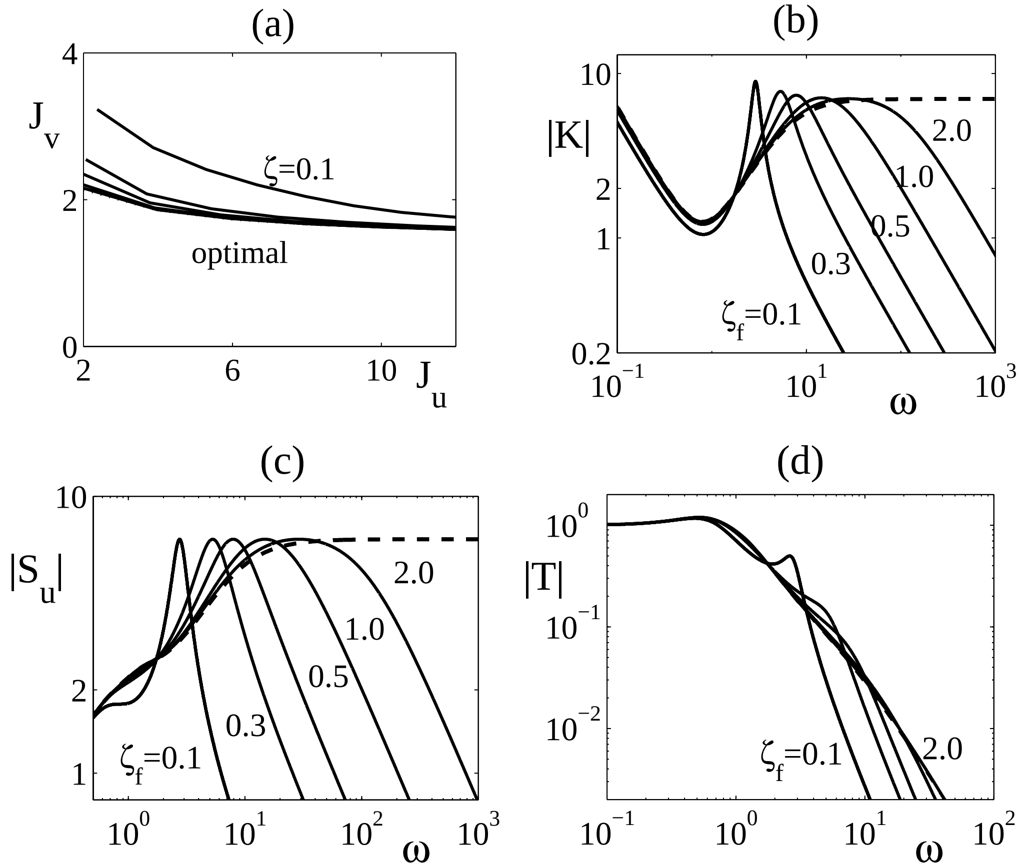

Figure 11: (a) J_v/J_u relations, (b) controller gains, (c) control sensitivity functions and (d) complementary sensitivity functions for $G(s) = (1 - 0.5s)/(1 + s)^3$ controlled by optimal PID controllers augmented by a first (dashed) or a second order filter with different damping ratios ζ_f. In (b)–(d) $J_u = 7.0$ and $GM_S = 1.7$.

for robustness reasons. Also the relation between J_v and J_u is growing poorer for $\zeta_f = 0.1$ and 0.3.

From Figure 11 the conclusion can be drawn that a PID controller may well be augmented by a second order low-pass filter with complex poles. Without significant deterioration of low frequency performance and with retained stabitity margin ($GM_S = 1.7$), the damping ratio can be reduced to $\zeta_f = 0.5$ or even to $\zeta_f = 0.3$. The tuning rules for PID_{ro} may then be the same as those given for PID in Section 5 with the addition

$$\zeta_f = 0.5 \tag{28}$$

This means that also for this strictly proper controller, at least three of the five tuning parameters can be computed by very simple rules, while the two basic parameters k_i and k_∞ (or β) are left for the user.

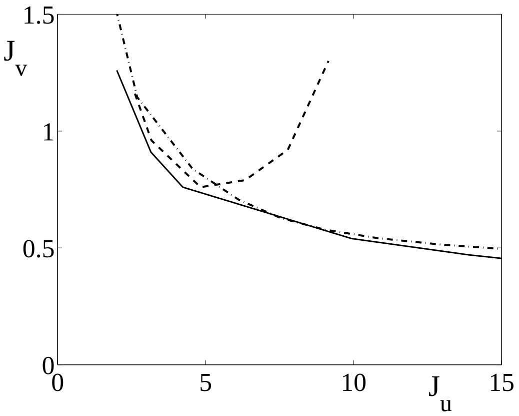

Figure 12: J_v as a function of J_u for $G = e^{-0.3s}/(1+s)^2$ and an optimal PID_{ro} controller with $\zeta_f = 0.5$ (dashdotted) or alternatively a $\mathcal{H}_\infty$ controllers with a PI weight $\mathcal{H}_{\infty PI}$ (13) (dashed) or a PID weight $\mathcal{H}_{\infty PID}$ (12) (solid).

PID weighted $\mathcal{H}_\infty$ controller

To find a suitable $\mathcal{H}_\infty$ controller, the loop shaping procedure described by MacFarlane and Glover in [35], is applied. The main idea is to augment the plant with a weight function W, and modify this function until a desired open loop shape is obtained for the augmented plant $\overline{G} = WG$. The controller $\overline{K}$ thus found is then combined with the corresponding weight function to give the final controller $K_{\mathcal{H}_\infty} = W\overline{K}$. It is quite common to use a PI filter (13) as the weight function. However, when somewhat higher control activity can be accepted, a PID filter (12) is an interesting alternative.

Figure 12 shows the J_v/J_u relations for $G = e^{-0.3s}/(1+s)^2$ and three controllers, all of them optimized with constrained J_u and $GM_S = 1.7$. The PI weighted $\mathcal{H}_\infty$ controller $\mathcal{H}_{\infty PI}$, tuned by k_i and $k_\infty = k_i\tau$, shows the same characteristic minimum as the PI controller, cf. Figure 3(a). Here the optimal J_u value is around 5. The result for the PID weighted $\mathcal{H}_\infty$ controller $\mathcal{H}_{\infty PID}$, is surprisingly similar to that of the optimal strictly proper PID controller PID_{ro}, except for very low values of J_u. Here the $\mathcal{H}_\infty$ controllers are optimized with all weighting filter parameters free, while PID_{ro} is optimized with fixed $\zeta = 0.8$ and $\zeta_f = 0.5$. However, the two sets of controllers have the same GM_S and J_{HF} values. In fact, the value of J_{HF} obtained for PID_{ro} has been introduced as a constraint in the optimization of the weighting filter in the $\mathcal{H}_\infty$ design.

Note that the $\mathcal{H}_\infty$ loop shaping procedure is only a tool to obtain the aim, i.e. to minimize J_v and to obtain a fair comparison between different controllers. Included in the description by MacFarlane and Glover is a scaling factor α. The theoretically optimal robustness corresponds to $\alpha = 1$, but for practical reasons α slightly larger than one, for instance 1.05, is often chosen. When the $\mathcal{H}_\infty$ optimizations were carried through, it was found that GM_S did not hit the limit 1.7, when the restriction on J_{HF} was added (to achieve a fair comparison to PID_{ro}). Although, by allowing somewhat larger α, this could be taken care of. In fact, small variations in α resulted in dramatically different

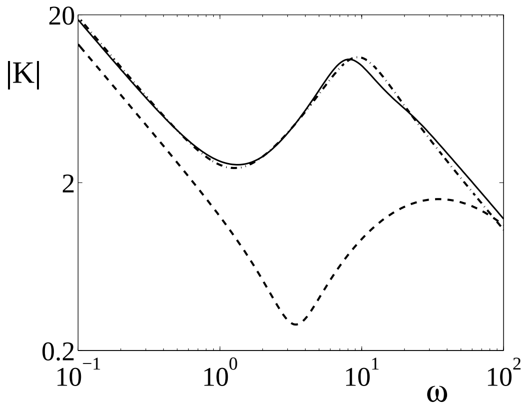

Figure 13: Gains of PID_{ro}, $J_u = 10$ (dashdotted), $\mathcal{H}_{\infty PID}$, $J_u = 10$ (solid) and $\mathcal{H}_{\infty PI}$, $J_u = 5$ (dashed), for $G = e^{-0.3s}/(1+s)^2$.

optimal solutions with respect to low frequency performance. This implies that α is an important and sensitive tuning parameter for $\mathcal{H}_\infty$ controller design.

The controller gains of $\mathcal{H}_{\infty PID}$, $\mathcal{H}_{\infty PI}$ and PID_{ro} are compared in Figure 13. For all of them $GM_S = 1.7$, for $\mathcal{H}_{\infty PID}$ and PID_{ro} $J_u = 10$ and $J_{HF} = 122$, whereas for $\mathcal{H}_{\infty PI}$, the optimal value $J_u = 5$ has been chosen. As can be seen, the graphs for PID_{ro} and $\mathcal{H}_{\infty PID}$ almost coincide while $\mathcal{H}_{\infty PI}$ has a quite different behaviour.

Finally, Figure 14 shows some simulations. The reference and process disturbance step responses and the control signals after a reference step are shown for five cases, with the same plant $G = e^{-0.3s}/(1+s)^2$ and different controllers. Just as could be expected, both the disturbance and reference step responses are very similar for the two PID controllers, PID_{ro} and PID_{opt}. However, it is interesting to see how well also the responses from $\mathcal{H}_{\infty PID}$ agree with those two. On the contrary, the $\mathcal{H}_{\infty PI}$ has a somewhat slower reference response with less overshoot, and a little greater maximum error regarding disturbance rejection. The PI controller is the loser in both respects. The control activity, on the other hand, is greatest for PID_{opt}, equal for PID_{ro} and $\mathcal{H}_{\infty PID}$ and somewhat smaller for $\mathcal{H}_{\infty PI}$. For PI it is very small but also very sluggish. Taken together, the PID_{ro} and $\mathcal{H}_{\infty PID}$ controllers are superior, but the competition between those two must be judged as undecided. Note, however, the higher dimension of the $\mathcal{H}_{\infty PID}$ controller. With no restrictions on J_{HF}, $\mathcal{H}_{\infty PID}$ can do marginally better with respect to J_v ($\approx 2\%$), but with much larger J_{HF}.

7 PID controllers with Smith predictors

When a plant has a significant time delay it might be a bit tricky to control, and a number of methods have been tried. Very often a traditional or modified Smith predictor has been recommended, see [52, 43, 37, 63, 36]. Here another contribution will be given.

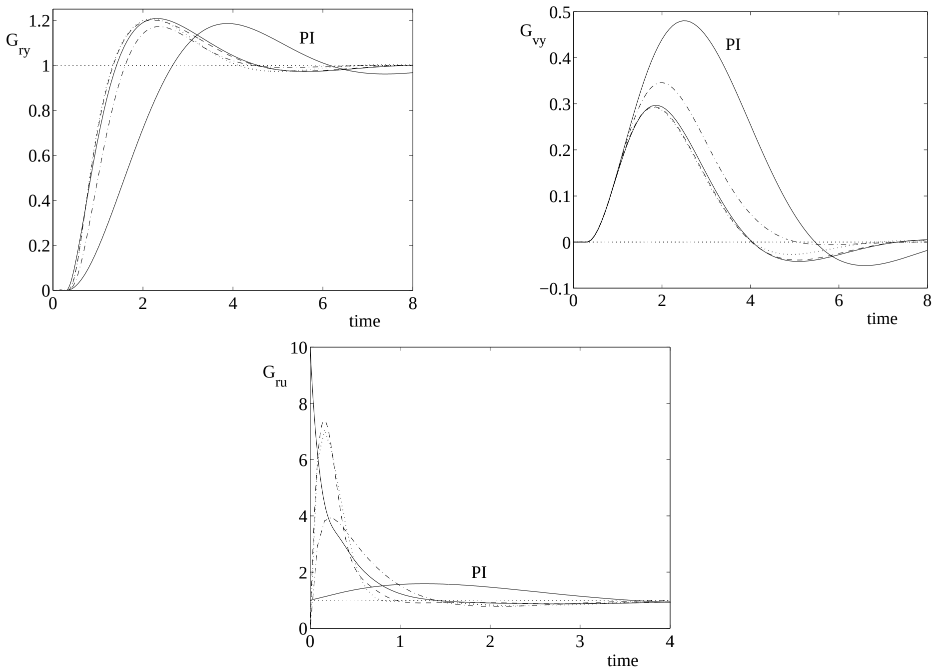

Figure 14: Reference step responses (left), process disturbance step responses (middle) and control signals after a reference step (right) from five systems with the following controllers: $\mathcal{H}_{\infty PID}$ (solid), $\mathcal{H}_{\infty PI}$ (dashdotted), PID_{ro} (dashed), PID_{opt}(dotted) and PI(solid). $J_u = 10$ except for $\mathcal{H}_{\infty PI}$ with $J_u = 5$ and PI with $J_u = 2$.

Limitation of the loop gain

The third demand involved in the definition (6) of GM_{SL} is a limitation of the loop gain for frequencies from ω_{180_L} (where the loop has a phase lag of $180°$) and upwards. For most controllers this criterion can well be weakened to a demand on minimum gain margin G_m or even be excluded (7), but when a Smith predictor is combined with a PI or especially a PID controller there is a tendency towards unhealthy high loop gain in this frequency range. Figure 15 and 16 show Nyquist plots for the loop gains when a PI and a PID controller is or is not augmented by a Smith predictor and the loop gain is limited in different ways. For the PI case in Figure 15 the high frequency limitation on $|L|$ is insignificant, since the gain never reaches the M_L-circle; the bound on G_m is strong enough. However, the tendency to growing gain in the range above ω_{180_L} for the controller with predictor, compared to the one without, is clear. Figure 16 shows the PID case. Note that the plot for bounded $|L|$ follows the M_L-circle, while the one corresponding to the weaker bound on G_m goes far beyond.

Define the *Maximum Delay margin* M_{D_m} as the time delay corresponding to the phase margin φ_m ($M_{D_m} = \varphi_m/\omega_c$, where ω_c is the open loop crossover frequency). Then a

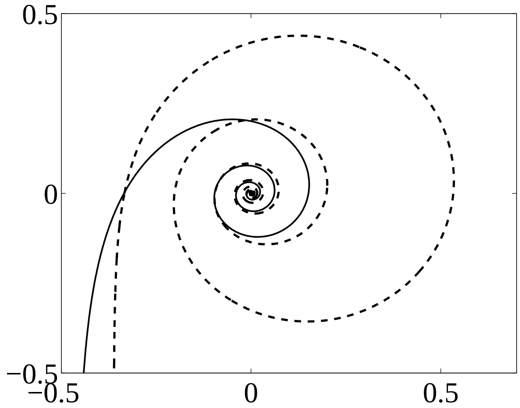

Figure 15: Nyquist plots for $G = e^{-10s}/(1+s)^3$ and an optimal PI controller with (dashed) and without (solid) Smith predictor.

minimum Delay margin m_{D_m} can be defined in the same way corresponding to a second crossover frequency above ω_{180_L}. Such an m_{D_m} often occurs when a Smith predictor is included in a PID controller and the loop is bounded only by G_m and not by the M_L-circle. For $G = e^{-sL_d}/(1+s)^3$ the delay margins become

L_d [sec]	M_{D_m}	m_{D_m}
4	5.45	-1.49
10	9.97	-2.01

Obviously this design is very sensitive to negative uncertainties in the time delay [51, 29]. However, applying the proposed M_L bound implies that the loop is restricted to stay inside the unit circle for $\omega > \omega_{180_L}$, which means that a reduced time delay can never bring the system to instability.

PI and PID controllers with Smith predictors

Figure 17 illustrates how J_v varies with J_u for some optimal controllers with and without Smith predictors (SP) included. It shows that the benefit of providing a PI controller

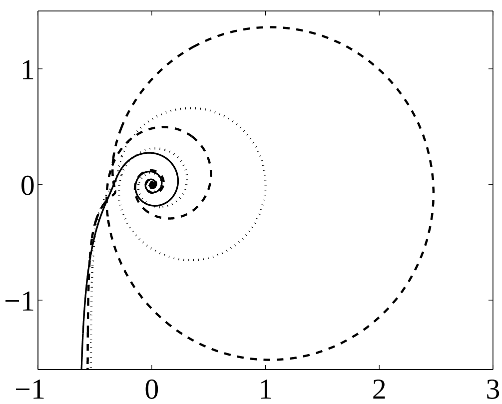

Figure 16: Nyquist plots for $G = e^{-4s}/(1+s)^3$ with an optimal PID controller without (solid) and with Smith predictor with limited G_m (dashed) or limited $|L|$ (dotted), $J_u = 6$.

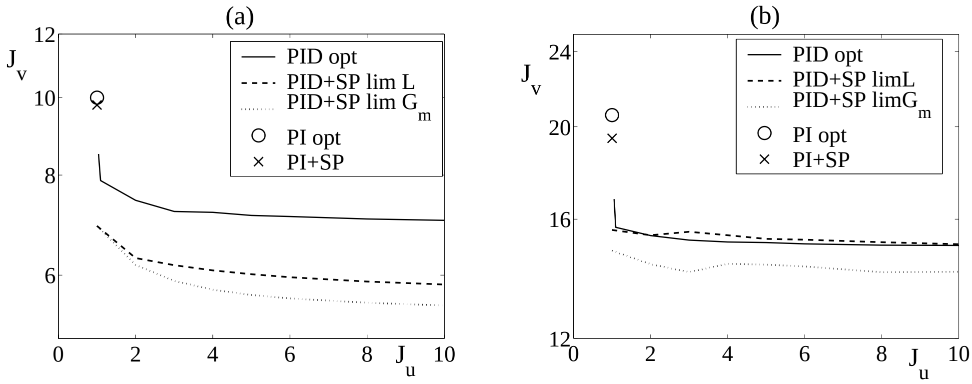

Figure 17: J_v as a function of J_u for $G = e^{-sL_d}/(1+s)^3$ and optimal PID without and with Smith predictor (SP) and bounded $|L|$ or just limited G_m and, furthermore, for PI without and with Smith predictor. (a) $L_d = 4.0$ and (b) $L_d = 10.0$.

with a Smith predictor is marginal, both when the delay is of medium size ($L_d = 4$) and when it is large ($L_d = 10$). For a PID controller, on the other hand, the introduction of a predictor implies some improvements for a plant with medium delay as in Figure 17(a). However, when the delay is large as in Figure 17(b), these improvements are eaten up by the bound on $|L|$ introduced by GM_{SL}. With the demand on bounded loop gain above ω_{180_L} weakened to a demand on the gain margin, the improvement can be driven a little further but to the prize of poor high frequency robustness, see Figure 16. Further details including step responses are given in [22, 25].

To conclude it can be stated that a PID controller with or without a Smith predictor can offer significantly better properties than a PI controller with Smith predictor for all kinds of plants, also for plants with significant delays. There is more to be won by including derivative action in a PI controller than by introducing a Smith predictor structure. The improvements in J_v for the examples in Figure 17 are given in the following table.

L_d [sec]	$PI \to PI + Smith$	$PI \to PID$	$PID \to PID + Smith$
1	1.4%	36%	15%
4	2.3%	30%	16%
10	6.4%	27%	0%

8 Conclusions

In this paper a general method for evaluation of controllers has been presented. The evaluation strategy involves four criteria expressing significant performance and robustness system properties.

Based on this method simple tuning rules have been introduced for stable plants with real poles. Corresponding rules for plants with integral action can be found in [26].

It has also been shown that the advantages in terms of improved performance offered by a PID controller, compared to the simpler PI controller, are just as good for a plant with time delay as for a plant with lag when the same dynamic complexity (in terms of κ number) is considered. However, it is essential for the properties of the resulting system that the low-pass filter in the PID controller is designed as an inherent part of the controller and not added afterwards. It is also crucial that all accessible parameters are used in the optimization of the controller, possibly except for the damping of the controller zeros, which without loss of optimality may be fixed to 0.75 for stable non-oscillating plants.

When a low-pass filter of the second order with complex poles is included in the controller, this can offer increased robustness against model uncertainties and better rejection of measurement noise, compared to a controller with a first order filter. These advantages are reached without deterioration of the low frequency properties or the stability margin. Actually it has been shown that such a strictly proper PID controller can offer system properties that are well comparable with those of an $\mathcal{H}_\infty$ loop shaping controller.

For plants with significant but not too great time delay a Smith compensator can be an attractive alternative to the plain PID controller, presupposed that the complete controller is optimized. However, when the primary controller is of PI type more profit can be won by providing it with a derivative part than by augmenting it with a Smith predictor.

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