Open access • Journal Article • DOI:10.1080/10255842.2012.688818

Evaluation of a transient, simultaneous, arbitrary Lagrange-Euler based multi-physics method for simulating the mitral heart valve. — Source link

Daniel M. Espino, Duncan E.T. Shepherd, David W.L. Hukins

Institutions: University of Birmingham

Published on: 03 Jan 2014 - Computer Methods in Biomechanics and Biomedical Engineering (Taylor & Francis)

Topics: Mitral valve

Evaluation of a transient, simultaneous, arbitrary Lagrange-Euler based multi-physics method for simulating the mitral heart valve

Espino, Daniel M; Shepherd, Duncan E T; Hukins, David W L

DOI:
10.1080/10255842.2012.688818

License:
Other (please specify with Rights Statement)

Document Version
Peer reviewed version

Citation for published version (Harvard):
Espino, DM, Shepherd, DET & Hukins, DWL 2014, 'Evaluation of a transient, simultaneous, arbitrary Lagrange-Euler based multi-physics method for simulating the mitral heart valve', *Computer Methods in Biomechanics and Biomedical Engineering*, vol. 17, no. 4, pp. 450-458. https://doi.org/10.1080/10255842.2012.688818

Evaluation of a transient, simultaneous, Arbitrary Lagrange-Euler based multi-physics method for simulating the mitral heart valve

Daniel M Espino*, Duncan ET Shepherd, David WL Hukins

School of Mechanical Engineering, University of Birmingham, Birmingham, UK B15 2TT

*Corresponding author:

Daniel Espino

School of Mechanical Engineering,

University of Birmingham,

Birmingham,

UK

B15 2TT

e-mail: daniel.m.espino@gmail.com

tel. +44 (0) 121 414 4258

fax. +44 (0) 121 414 3958

Evaluation of a transient, simultaneous, Arbitrary Lagrange-Euler based multi-physics method for simulating the mitral heart valve

A transient multi-physics model of the mitral heart valve has been developed which allows simultaneous calculation of fluid flow and structural deformation. A recently developed contact method has been applied to enable simulation of systole (the stage when blood pressure is elevated within the heart to pump blood to the body). The geometry was simplified to represent the mitral valve within the heart walls in two dimensions. Only the mitral valve undergoes deformation. A moving ALE mesh is used to allow true fluid-structure interaction. The fluid-structure interaction model requires blood flow to induce valve closure by inducing strains in the region of 10 to 20%. Model predictions were found to be consistent with existing literature and will undergo further development.

Keywords: Fluid Structure Interaction, Hertzian contact, Large strain, Mitral valve, Multi-physics modelling.

1. Introduction

The aim of this study was to evaluate the application of a recently developed method for two-dimensional hydrodynamic flow driven contact, to a simple model of the mitral heart valve within a left ventricle. As the physical mechanism leading to contact (i.e. valve closure) is fluid flow (Bellhouse, 1972; Caro *et al*., 1978) a simultaneous and transient fluid-structure interaction (FSI) simulation was performed. A finite element (FE) method was used for simulations, including solution of computational fluid dynamics (CFD).

Recently, several FSI mitral valve models have been developed which include simulation of valve closure (Cheng & Zhang, 2010; Lau *et al*., 2010; Einstein *et al*., 2005a & 2005b; Kunzelman *et al*., 2007; Vigmostad *et al*., 2010; Yin *et al*., 2010). However, these FSI simulations use penalty methods that require alterations to certain properties, such as bulk modulus, being set to 1% of its true value (Lau *et al*., 2010). A commercially available multi-physics package (Comsol Multi-physics v3.3, Comsol Ltd, London, UK) overcomes such limitations, enabling true simultaneous coupling of distinct physical states, including FSI. However, implementation of contact modelling has limitations for transient studies. An applicable transient contact method was, therefore, developed (Espino *et al*., 2012a) and applied to simultaneous FSI simulations (Espino *et al*., 2012b).

Simultaneous FSI solutions are well suited to heart valve modelling as instabilities may occur using iterative approaches (see Peskin, 1972 & 1977). In heart valves, flow of blood provides the loading conditions for valve opening and closure (Caro *et al*., 1978), and flow patterns affect how the valve opens or closes (Bellhouse, 1972). The motion of the valve structure then alters the flow patterns; early valve closure during the inflow stage in the heart is dependent on flow patterns set up behind the valve leaflets (Caro *et al*., 1978). Thus, mechanical replacements (Stijnen *et al*., 2004), aortic (De Hart *et al*., 2000 & 2003a) and mitral valves (Van Loon *et al*., 2006) have been modelled under simultaneous FSI conditions.

Simultaneous FSI simulations calculate the reaction force that the fluid exerts on the structure on shared boundaries (Dowell & Hall, 2001; Wall *et al*., 2006; Van de Vosse *et al*., 2003). Lagrange multipliers are used to determine these forces. A weak formulation is necessary to determine Lagrange multipliers. Two-way coupling is achieved by constraining the fluid velocity to be equivalent to the structural time-dependent deformation (Dowell & Hall, 2001; Wall *et al*., 2006; Van de Vosse *et al*., 2003). An Arbitrary-Lagrange-Euler (ALE) mesh is used to enable both FE and CFD analysis (Donea *et al*., 1982; Formaggia & Nobile, 1999). We have previously described a new large strain contact method for FSI modelling (Espino *et al*., 2012b).

In this paper, a simplified two-dimensional FSI model of the mitral valve has been solved using this new method. The model has been used to evaluate the potential of a true multi-physics model to overcome limitations encountered when using other FSI strategies.

2. Methods

2.1 Geometry

Contact was simulated for a two-dimensional simplified model of the mitral valve within the left ventricle of a heart (figure 1). The two-dimensional model simplified the left ventricle as half an ellipse, which is truncated. A small section of 'aorta' (outflow tract from the left ventricle) was added to the original model, in order to mimic the outflow tract from the heart. The two contacting leaflets are in the position of the mitral valve. In contrast to the published diastolic model (Al-Atabi *et al*., 2010), the mitral valve leaflets are set to the same geometry and their tips are set as touching.

2.2 Material properties

The two contacting structures were considered to be isotropic, homogenous and to have a linear stress-strain relationship. Fluid properties matched those of blood assuming blood to be an incompressible and Newtonian fluid (a valid assumption under large scale flow, as occurs through the left ventricle out towards the aorta) (Caro *et al*., 1978). All properties used (table 1) were obtained from literature (Clark, 1973; Kunzelman & Cochran, 1992; Barber *et al*., 2001; De Hart *et al*., 2000; 2003a; Kunzelman *et al*., 1993; Votta *et al*., 2002; Maisano *et al*., 1999; Redaelli *et al*., 2001; Bellhouse, 1972; Caro *et al*., 1978; Kaur, 2007; Millard *et al*., 2011).

2.3 Boundary conditions

For *fluid boundaries* (figure 1), pressure was applied from the boundary at the apex of the heart, i.e. the truncated segment of the ellipse, using equation 1:

$$P = P_p\left(\frac{t}{T}\right) + \ 100 \qquad 1$$

where P, P_p and t, refer to the pressure, peak pressure (16 kPa) and time, respectively. A total time (T) of 0.3 s was set for this simulation, in order to mimic systolic timing in the left ventricle of the heart.

A time-dependent outflow velocity was applied at the boundary of a simplified aorta (equation 2). All other boundaries of the left ventricle were set as no-slip (i.e. 0 m/s for the non-moving ventricular walls), with the exception of the boundaries of the valve leaflets in contact with the fluid domain. Fluid at these shared boundaries was set to have a velocity equivalent to the velocity of the moving structure; i.e. the valve leaflet, according to equation 3.

$$v = v_p\left(\frac{t}{T}\right) \qquad 2$$

$$u = \frac{\partial x}{\partial t}, \qquad v = \frac{\partial y}{\partial t} \qquad 3$$

where u, v, and v_p refer to *x*- and *y-axis* and peak *y-axis* velocities (the latter was set at 1.5 m/s), respectively. Note, the *x*- and *y-axis* define two orthogonal axes of a Cartesian coordinate system, where the former is parallel to inflow and outflow boundaries of the model and the latter is perpendicular to these as shown in figure 1b.

For *structural boundaries*, mitral valve leaflet tips and annular edges were restricted from moving (figure 1). Forces were induced by fluid dynamics and contact. The force on the leaflet boundaries was induced by fluid flow and led to valve closure (see *Solving Fluid-Structure Interaction* section below). A boundary contact force (F_c) enabled contact modelling (equation 4). Friction was not included in the model.

$$F_c = \left(\frac{l}{L}\left(\sum B_{n+1} - \sum B_n\right)\right) + \sum B_n \qquad 4$$

where the summation of the local contact force at the n^{th} contact vertex, B_n, was calculated for each contact vertex as described elsewhere (Espino *et al*., 2012a). L defines the total length between the contact vertices and l the distance along this length of a given point. The force between adjacent contact vertices was defined by linear interpolation.

2.4 Solving Fluid-Structure Interaction

Simultaneous solution of fluid and structure, and their interaction, requires constraints that enforce the coupling. The velocity constraint defined by equation 3 coupled fluid flow to structural changes. Equal and opposite reaction forces from the fluid on the structure ensured a two-way coupling. These forces are equivalent to Lagrange multipliers determined using a (non-ideal) weak formulation of fluid dynamics. Fluid dynamics were solved using the continuity and incompressible Navier-Stokes equations, assuming Newtonian flow, using a full stress tensor. Corner smoothing and anisotropic streamline diffusion were included to improve simulation of flow around corners and flow-field predictions without requiring increase mesh resolution. Further detail on these techniques is provided elsewhere (Espino *et al*., 2012b).

A moving ALE mesh was applied to boundaries shared by the valve leaflets (structure) and blood (fluid). This enabled the mesh to follow structural changes over leaflets and displace freely over the meshed blood. No re-meshing was used but Winslow smoothing was applied to improve the resultant mesh (Winslow, 1966). All other boundaries had a fixed mesh (figure 1b). A total of 11 variables were solved for each node (table 2). These variables included: solid domain deformation; fluid domain pressure, velocities (and the corresponding Lagrange multipliers) as well as ALE-mesh domain displacement (and Lagrange multipliers).

2.5 Analysis

The finite element analysis package Comsol Multi-physics (v3.3, Comsol Ltd, London, UK) was used to solve the FSI model under time dependent conditions. The structural mechanics package was used to analyse the leaflets. This enabled the use of a large deformation setting allowing determination of Green strains and Cauchy stresses, as reported previously (Espino *et al*., 2012a). Further details on such stresses and strains are available elsewhere (Appleton, 1996).

A direct UMFPACK solver (i.e. one that uses an unsymmetric multifrontal method) was used for the time-dependent simulation. Transition from one time step to the next occurred once the estimated model error was below a set tolerance, as discussed previously (Espino *et al*., 2012a). Time-stepping was defined using a backward differentiation formula (Heath, 1997), which uses an interpolating polynomial function to determine the subsequent time-step to solve (table 2).

3. Results

Peak stress concentrations on the leaflets were found at the restrained tips. Other regions of high stress included the restrained base of the leaflets. Regions of greater curvature also had higher stresses than flatter regions (figures 2 & 3). Peak von Mises leaflet stresses at 0.3 s (i.e. peak applied pressure) were in the region of 0.5 MPa (table 3). Peak Cauchy leaflet stresses at 0.3 s occurred along the *x-axis* and were in the region of 0.6 MPa with a corresponding peak Green strains of the order of 0.2 (table 3). Peak strains were predicted towards the leaflet free edge and lowest strain towards the leaflet annulus. Contact stresses developed at leaflet contact boundaries were approximately an order of magnitude lower than the stresses generated within the leaflets (figure 3 & table 3). Contact between the two leaflets increased during systolic pressure rise.

Recirculation behind the posterior leaflet developed with increasing pressure with time (figure 2) during outflow through the aorta. Peak velocity in the ventricle was 2.4 m/s at 16.1 kPa pressure. The corresponding Reynolds numbers were low (table 4). Peak vorticity was $1 \times 10^{3}\ s^{-1}$ which is defined as the curl of the velocity field (Granger, 1985).

4. Discussion

A transient simultaneous FSI mitral valve model has been developed using an ALE mesh and includes contact modelling. Recently FE (Prot *et al*., 2010; Wenk *et al*., 2010) and FSI (Cheng & Zhang, 2010; Lau *et al*., 2010; Einstein *et al*., 2005a & 2005b; Kunzelman *et al*., 2007; Vigmostad *et al*., 2010; Yin *et al*., 2010) mitral valve models have been developed. The FSI models have included constraints that impose limitations on the physical model. However, FSI heart valve models have been developed that do not require such limitations, including the development of methods for contact modelling for a simplified mitral valve model (Van Loon *et al*., 2006). Heart valve FSI models have been also developed for models of aortic (De Hart *et al*., 2003a) and mechanical valves (Stijnen *et al*., 2004). Such FSI models simultaneously calculate the fluid and solid interaction (Van de Vosse *et al*., 2003) and are, therefore, often referred to as 'true' multi-physics models (Cross *et al*., 2007).

FSI simulations enable certain limitations of FE analyses to be overcome. For example, FE analysis of heart valves requires the tissue density to be set to ten-times physiological values to replicate the "effective" mass-density (Kunzelman *et al*., 1993). In our model similar, but physiological, leaflet and blood density values were used. One advantage is that this leads to negligible buoyancy effects (De Hart *et al*., 2003b).

The two-dimensional, simultaneous, FSI mitral valve model, described here, has several limitations including linear pressure and velocity increases and simplified geometry. Material properties of leaflets were implemented as linear and homogenous and the fluid was assumed Newtonian with constant viscosity. However, this study aimed to determine the feasibility of developing a suitable ALE-based, simultaneous, mitral valve transient FSI simulation with true multi-physics coupling. While the geometry was simplified, we included a heart chamber as in our diastolic model (Al-Atabi *et al*., 2010). It has since been shown that the left ventricle should be included in FSI models of the mitral valve for prediction of physiological fluid dynamics (Lau *et al*., 2010). Many mitral valve FSI models have ignored this, instead simulating a two-dimensional beam in a conduit (van Loon *et al*., 2006) or three dimensional models of either the mitral valve (Einstein *et al*., 2005a & 2005b; Kunzelman *et al*., 2007) or a chorded synthetic polyurethane replacement in a conduit (Watton *et al*., 2007; Yin *et al*., 2010). The importance of the left ventricle to flow patterns in the heart is discussed elsewhere (Bellhouse, 1972). Having determined the value of this method for mitral valve modelling, future development will aim to include a non-Newtonian blood description, non-linear material properties, and equations defining pressure and velocity. The geometry will be developed into three-dimensions. This would enable modelling of non symmetric effects across the valve, as occur during ischemic mitral regurgitation (Wenk *et al*., 2010. Such changes will increase the computing requirements for modelling; however, it is feasible to develop a model of this kind. Critically, such models may enable limitations encountered by other FSI strategies to be overcome, e.g. penalty methods that lead to properties, such as bulk modulus, being set to 1% of its true value (Kunzelman *et al*., 2007; Lau *et al*., 2010).

The current two-dimensional model is limited in that it cannot be used to simulate chordae tendineae directly. An example of this limitation, numerically, is the small gap (75 μm) predicted between the leaflets towards their free edge beneath the main contact zone of the leaflets. However, the effect of chordae has been approximated, as the tip of each leaflet free edge was restrained. Thus, the restraining function of marginal chordae (see Al-Atabi *et al*., 2012; Espino *et al*., 2005; Obadia *et al*., 1997) is included. However, basal chordae insert away from the leaflet free edge and, although not critical to coaptation, they support higher loads than marginal chordae (Lomholt *et al*., 2002). Stress distributions may be altered by their modelling, with three-dimensional models more likely to be more sensitive to their inclusion. This is because a two dimensional model represents the central portions of the anterior and posterior leaflets where no chordae insert. However, three-dimensional FE (Votta *et al*. 2002) and FSI (Lau *et al*., 2010) models have not always included basal chordae. This might be because simulating the chordal restraining effect can lead to predictions similar to those of a chorded FE model (Dal Pan *et al*., 2005).

Our anterior leaflet model did not include the full radial 'sigmoidal' shape of the anterior leaflet (Rodriguez *et al*., 2005). However, our model predicted compressive stresses towards the annulus that Rodriguez *et al*. (2005) anticipated due to the anterior leaflet's shape. The posterior leaflet is concave towards the left ventricle, but 'flatter' than the anterior leaflet at the end of systole (Tibayan *et al*., 2004). This represents a limitation of our model as several basal chordae insert into the posterior leaflet and may maintain this flatter shape. Therefore, they may restrict some of the posterior leaflet deformation predicted by our model. However, a certain amount of annular shifting of both valve leaflets occurs under loading (Green *et al*., 1999). Thus, we would expect that adding such a restraining effect would reduce, but not remove, the posterior leaflet annular shift. As the shape of the mitral valve is believed to reduce stress (Salgo *et al*., 2002), adding anatomical complexity to our model may reduce our predicted peak stresses. However, our peak stresses were in the range of 0.5 MPa, comparable to peak stresses predicted by most other models ranging from 0.25 to 0.6 MPa (Dal Pan *et al*., 2005; Einstein *et al*., 2005b; Kunzelman *et al*., 1993; Lau *et al*., 2010; Votta *et al*., 2002). Our peak radial stresses (i.e. oriented from the annulus to free edge) were around 0.36 MPa, which also fall within such a range.

Despite the limitations of our model, the predictions are consistent with experimental results. For example, our model predicted peak radial strains of 12.6%, at a pressure equivalent to 120 mmHg (16 kPa). This compares to strain measurements of up to 14% *ex vivo* at 120 mmHg (Chen *et al*., 2004) and 22% *in vivo* (Sacks *et al*., 2006) at the higher pressure of 150 mmHg (20 kPa). Our model predicted peak strains towards the leaflet free edge and lowest strains towards the annulus, consistent with an increase in radial strain from the annulus towards the leaflet free edge (Chen *et al*., 2004). Moreover, flow patterns predicted by our model showed flow directed out of the aorta, with some flow towards/around the valve leaflets. This is consistent with systolic flow patterns (Reul *et al*., 1981).

The rheological properties used to simulate blood in our model were physiological for large scale flow. Our model also includes flow out of the aorta, which provides a flow condition that the mitral valve is exposed to during systole. As we develop this model further we plan to develop our model verification. We previously defined an experimental model to provide some valve opening validation (Al-Atabi *et al*., 2010). We plan to further validate our future models using results from our previous studies on mitral valves and their failure (Espino *et al*., 2005 & 2007) along with development of the experimental validation method (Al-Atabi *et al*., 2010).

ACKNOWLEDGEMENTS

The authors would like to thank the British Heart Foundation for a Junior Fellowship (FS/05/32) awarded to DME during the early stages of this investigation. The research leading to these results has received funding from the [European Community's] Seventh Framework Programme [FP7/2007-2013] under a Marie Curie Intra-European Fellowship for Career Development, grant agreement n° [252278], awarded to DME.

References

Al-Atabi M, Espino DM, Hukins DWL. 2010. Computer and experimental modelling of blood flow through the mitral valve of the heart. J Biomech Sci Eng. 5(1):78-84.

Al-Atabi M, Espino DM, Hukins DWL, Buchan KG. 2012. Biomechanical assessment of surgical repair of the mitral valve. Proc Inst Mech Eng H. **In Press**. DOI: 10.1177/0954411912437116

Appleton A. 1996. Thermodynamic and mechanical properties of matter. London: Appleton.

Barber JE, Kasper FR, Ratliff NB, Cosgrove DM, Griffin BP, Vesely I. 2001. Mechanical properties of myxomatous mitral valves. J Thorac Cardiovasc Surg. 122(5):955-962.

Bellhouse BJ. 1972. The fluid mechanics of heart valves. In: Cardiovascular fluid mechanics. Volume 1. Bergel DH (ed). London: Academic Press.

Caro CG, Pedley TJ, Schroter RC, Seed WA. 1978. The mechanics of the circulation. Oxford: Oxford University Press.

Chen L, McCulloch AD, May-Newman K. 2004. Nonhomogeneous deformation in the anterior leaflet of the mitral valve. Ann Biomed Eng. 32(12):1599-1606.

Cheng YG, Zhang H. 2010. Immersed boundary method and lattice Boltzmann method coupled FSI simulation of mitral leaflet flow. Comput Fluids. 39(5):871-881.

Clark RE. 1973. Stress-strain characteristics of fresh and frozen human aortic and mitral leaflets and chordae tendineae. J Thorac Cardiovasc Surg. 66(2):202-208.

Cross M, Croft TN, McBride D, Slone AK, Williams AJ. 2007. Multiphysics modelling and simulation: progress and challenges. Canada: NAFEMS World Congress.

Dal Pan F, Donzella G, Fucci C, Schreiber M. 2005. Structural effects of an innovative surgical technique to repair heart valve defects. J Biomech. 38(12):2460–2471.

De Hart J, Peters GW, Schruers PJ, Baaijens FP. 2000. A two-dimensional fluid-structure interaction model of the aortic valve. J Biomech. 33(9):1079-1088.

De Hart J, Peters GW, Schreurs PJ, Baaijens FP. 2003a. A three-dimensional computational analysis of fluid–structure interaction in the aortic valve. J Biomech. 36(1):103-112.

De Hart J, Baaijens FP, Peters GW, Schreurs PJ. 2003b. A computational fluid-structure interaction analysis of a fiber-reinforced stentless aortic valve. J Biomech. 36(5):699-712.

Donea J, Giuliani S, Halleux JP. 1982. An arbitrary Lagrangian–Eulerian finite element method for transient dynamic fluid–structure interactions. Comput Methods Appl Mech Engrg. 33(1-3):689 –723.

Dowell EH, Hall KC. 2001. Modelling of Fluid-structure interaction. Annu Rev Fluid Mech. 33(1):445-490.

Einstein DR, Kunzelman KS, Reinhall PG, Nicosia MA, Cochran RP. 2005a. The relationship of normal and abnormal microstructural proliferation to the mitral valve closure sound. J Biomech Eng 127(1):134-147.

Einstein DR, Kunzelman KS, Reinhall PG, Nicosia MA, Cochran RP. 2005b. Non-linear fluid-coupled computational model of the mitral valve. J Heart Valve Dis. 14(3):376-385.

Espino DM, Shepherd DET, Hukins DWL, Buchan K. 2005. The role of chordae tendineae in mitral valve competence. J Heart Valve Dis. 14(5):603-609.

Espino DM, Shepherd DET, Buchan KG. 2007. Effect of mitral valve geometry on valve competence. Heart Vessels. 22(2):109-115.

Espino DM, Shepherd DET, Hukins DWL. 2012a. Development of a transient large strain contact method for biological heart valve simulations. Comput Methods Biomech Biomed Engin. **In Press.** DOI: 10.1080/10255842.2011.623676

Espino DM, Shepherd DET, Hukins DWL. 2012b. Transient large strain contact modelling: a comparison of contact technique for simultaneous fluid-structure interaction. Comput Fluids. **Submitted.**

Formaggia L, Nobile F. 1999. A stability analysis for the arbitrary Lagrangian Eulerian formulation with finite elements. East–West J Numer Math. 7(2):105–132.

Granger RA. 1985. Fluid mechanics. New York: Holt, Rinehart and Winston.

Green GR, Dagum P, Glasson JR, Nistal JF, Daughters GT, Ingels NB, Miller C. 1999. Restricted posterior leaflet motion after mitral ring annuloplasty. Ann Thorac Surg. 68(6):2100-2106.

Heath MT. 1997. Scientific computing - an introductory survey. New York: McGraw-Hill.

Kaur H. 2007. Determination of material properties of natural heart valves. (MEng thesis). Birmingham: University of Birmingham.

Kunzelman KS, Cochran RP. 1992. Stress/strain characteristics of porcine mitral valve tissue: parallel versus perpendicular collagen orientation. J Cardiac Surg. 7(1):71-78.

Kunzelman KS, Cochran RP, Chuong C, Ring WS, Verrier ED, Eberhart RC. 1993. Finite element analysis of the mitral valve. J Heart Valve Dis. 2(3):326-340.

Kunzelman KS, Einstein DR, Cochran RP. 2007. Fluid-structure interaction models of the mitral valve: function in normal and pathological states. Phil Trans R Soc B. 362(1484):1393-1406.

Lau KD, Diaz V, Scambler P, Burriesci G. 2010. Mitral valve dynamics in structural and fluid-structure interaction models. Med Eng Phys. 32(9):1057-1064.

Lomholt M, Nielsen SL, Hansen SB, Andersen NT, Hasenkam JM. 2002. Differential tension between secondary and primary mitral chordae in an acute in-vivo porcine model. J Heart Valve Dis. 11(3):337-345.

Maisano F, Redaelli A, Pennati G, Fumero R, Torracca L, Alfieri O. 1999. The hemodynamic effects of double-orifice valve repair for mitral regurgitation: a 3-D computational model. Eur J Cardio-thorac Surg. 15(4):419-425.

Millard L, Espino DM, Shepherd DET, Hukins DWL, Buchan KG. 2011. Mechanical properties of chordae tendineae: variation with chordal type and age. J Mech Med Biol. 11(1):221-230.

Obadia JF, Cendrine C, Chassignolle JF, Janier M. 1997. Mitral subvalvular apparatus - different functions of primary and secondary chordae. Circulation. 96(9):3124-3128.

Peskin CS. 1972. Flow patterns around heart valves: a numerical method. J Comput Phys. 10(2):252–270.

Peskin CS. 1977. Numerical analysis of blood flow in the heart. J Comput Phys. 25(3):220–252.

Prot V, Skallerud B, Sommer G, Holzapfel GA. 2010. On modeling and analysis of healthy and pathological human mitral valves: two case studies. J Mech Behav Biomed Mater. 3(2):167-177.

Redaelli A, Guadagni G, Fumero R, Maisano F, Alfieri O. 2001. A computational study of the hemodynamics after "edge to edge" mitral valve repair. J Biomech Eng. 123(6):565-570.

Reul H, Talukder N, Mueller EW. 1981. Fluid mechanics of the natural mitral valve. J Biomech. 14(5):361-372.

Rodriguez F, Langer F, Harrington KB, Tibayan FA, Zasio MK, Liang D, Daughter GT, Ingels NB, Miller DC. 2005. Effect of cutting second-order chordate on in-vivo anterior mitral leaflet compound curvature. J Heart Valve Dis. 14(5):592-602.

Sacks MS, Enomoto Y, Graybill JR, Merryman WD, Zeeshan A, Yoganathan AJ, Levy RJ, Gorman RC, Gorman III JH. 2006. In-vivo dynamic deformation of the mitral valve anterior leaflet. Ann Thorac Surg. 82(4):1369–1378.

Salgo IS, Gorman III JH, Gorman RC, Jackson BM, Bowen FW, Plappert T, St John Sutton MG, Edmunds LH. 2002. Effect of annular shape on leaflet curvature in reducing mitral leaflet stress. Circulation. 106(6)711-717.

Stijnen JMA, De Hart J, Bovendeerd PHM, Van de Vosse FN. 2004. Evaluation of a fictitious domain method for predicting dynamic response of mechanical heart valves. J Fluids Struct. 19(6):835-850.

Tibayan FA, Rodriguez F, Langer F, Zasio MK, Bailey L, Liang D, Daughters GT, Karlsson M, Ingels Jr. NB, Miller DC. 2004. Increases in mitral leaflet radii of curvature with chronic ischemic mitral regurgitation. J Heart Valve Dis. 13(5):772-778.

Van de Vosse FN, De Hart J, Van Oijen CHGA, Bessems D, Gunther TWM, Segal A, Wolters BJBM, Stijnen JMA, Baaijens FPT. 2003. Finite-element-based computational methods for cardiovascular fluid-structure interaction. J Eng Math. 47(3-4):335–368.

van Loon R, Anderson PD, van de Vosse FN. 2006. A fluid-structure interaction method with solid-rigid contact for heart valve dynamics. J Comput Phys. 217(2):806-823.

Vigmostad SC, Udaykumar HS, Lu J, Chandran KB. 2010. Fluid-structure interaction methods in biological flows with special emphasis on heart valve dynamics. Int J Numer Meth Biomed Engng. 26(3-4):435-470.

Votta E, Maisano F, Soncini M, Redaelli A, Montevecchi FM, Alfieri O. 2002. 3-D computational analysis of the stress distribution on the leaflets after edge-to-edge repair of mitral regurgitation. J Heart Valve Dis. 11(6):810-822.

Wall W, Gerstenberger A, Gamnitzer P, Forster C, Ramm E. 2006. Large deformation Fluid-structure interaction – advances in ALE methods and new fixed grid approaches. In: Fluid-structure interaction. Bungartz HJ, Shafer M (Eds.). Berlin: Springer.

Watton PN, Luo XY, Wang X, Bernacca GM, Molloy P, Wheatley DJ. 2007. Dynamic modelling of prosthetic chorded mitral valves using the immersed boundary method. J Biomech. 40(3):613-626.

Wenk JF, Zhang Z, Cheng G, Malhotra D, Acevedo-Bolton G, Burger M, Suzuki T, Saloner DA, Wallace AW, Guccione JM, Ratcliffe MB. 2010. The first finite element model of the left ventricle with mitral valve: insights into ischemic mitral regurgitation. Ann Thorac Surg. 89(5):1546–1553.

Winslow AM. 1966. Numerical solution of the quasilinear poisson equation in a nonuniform triangle mesh. J Comput Phys. 1(2):149-172.

Yin M, Luo XY, Wang TJ, Watton PN. 2010. Effects of flow vortex on a chorded mitral valve in the left ventricle. Int J Numer Meth Biomed Engng. 26(3-4):381-404.

TABLES

Table 1. Material properties.

AL Young's modulus (MPa)	PL Young's modulus (MPa)	Poisson's ratio	Density (kg/m^3)	Viscosity (Pa s)
5	2	0.33	1060	5×10^{-3}

AL defines anterior leaflet, and PL posterior leaflet.

Table 2. Transient settings for solving simulations.

Total degrees of freedom solved	Number of Elements	Lagrange element type	BDF Max.	AL contact vertices	PL contact vertices
9876	1150	P2P1	4	21	21

BDF: backward differentiation formula; see Heath (1997).

AL defines anterior leaflet, and PL posterior leaflet.

Table 3. Maximum and minimum values for stress, strain and contact pressure under a given loading pressure, per time step.

time (s)		Pressure (kPa)	von Mises stress (kPa)	Cauchy stress (kPa)				Green strain			Contact pressure (Pa)
				σ_x	σ_y	σ_z	σ_{xy}	ε_x	ε_y	ε_{xy}	
0.1	max	5.43	140	160	170	64.8	56.0	0.06	0.065	0.034	8.22×10^{3}
	min	5.43	9.70	-28.0	-24.7	1.90	-64.0	-0.03	-0.03	-0.016	-4.09×10^{-22}
0.2	max	10.8	314	368	274	120	98.1	0.13	0.099	0.057	1.33×10^{4}
	min	10.8	6.43	-45.8	-50.7	-12.8	-114	-0.049	-0.059	-0.028	-1.58×10^{-10}
0.3	max	16.1	521	583	359	197	134	0.194	0.126	0.0759	1.74×10^{4}
	min	16.1	7.90	-68.8	-73.7	-35.1	-157	-0.063	-0.08	-0.041	-1.1×10^{-15}

Table 4. Maximum and minimum values for flow parameters per given time step, including the *x-axis* and *y-axis* Lagrange multipliers (λ_5 and λ_6, respectively).

time (s)		Pressure (kPa)	*x*-velocity (m/s)	*y*-velocity (m/s)	velocity field (m/s)	Vorticity (1/s)	cell Reynold's number	λ_5 (10^3)	λ_6 (10^3)
0.1	max	5.43	0.16	0.812	0.8121	359	0.989	6.00	0.45
	min	5.43	-0.239	-0.0576	1.30×10^{-19}	-633	1.39×10^{-19}	-6.00	-6.00
0.2	max	10.8	0.329	1.61	1.61	710	1.96	11.8	0.90
	min	10.8	-0.495	-0.115	1.95×10^{-18}	-1.38×10^{3}	2.07×10^{-18}	-11.8	-12.0
0.3	max	16.1	0.504	2.39	2.39	1.07×10^{3}	2.93	17.6	1.40
	min	16.1	-0.764	-0.173	5.30×10^{-18}	-2.28×10^{3}	5.60×10^{-18}	-17.7	-17.6

FIGURES

Figure 1. Geometry of the mitral valve and applied boundary conditions. (a) Meshed left ventricle model (including the mitral valve and aorta). (b) Boundary conditions applied to the ventricle & aorta: the inflow boundary was defined using a pressure condition, the outflow boundary was defined by a velocity condition, a no-slip condition was applied to remaining ventricle boundaries (note, the dotted boundary applies a neutral condition; i.e. it does not affect the simulation). (c) Mitral valve boundary conditions: 1. FSI condition, 2. constrained, 3. contact. Note, AL and PL denote the anterior and posterior leaflet of the mitral valve, respectively.

Figure 2. Flow patterns and valve leaflet stress at (a) 0.1, (b) 0.2 and (c) 0.3 s. The colour coded bar defines the valve leaflet von Mises stress (Pa).

Figure 3. Deformation at 0.3 s of (a) mitral valve leaflets including von Mises stress distribution and (b) leaflet contact boundaries including contact pressure. The colour coded bar denotes von Mises stress (Pa) in (a) and pressure (Pa) in (b).