# Evaluation of automotive forward collision warning and collision avoidance algorithms 

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#### Abstract

Collision warning/collision avoidance (CW/CA) systems target a major crash type and their development is a major thrust of the Intelligent Vehicle Initiative. They are a natural extension of adaptive cruise control systems already available on many car models. Many CW/CA algorithms have recently been proposed but the existing literature mainly focuses on algorithm development. Evaluations of these algorithms have been usually based on subjective ratings. The main contribution of this paper is the utilization of a naturalistic driving data set for the evaluation of CW/CA algorithms. We first collect manual driving data from the ICCFOT project, then process the data by Kalman smoothing, and finally identify 'threatening' and 'safe' data sets according to vehicle brake inputs and vehicle range behavior. Five CW/CA algorithms published in the literature are evaluated against the identified data sets. The performance of these algorithms is determined through a performance metric commonly used in signal detection and information retrieval under unbalanced data population.


Keywords: Active safety; Human model; Collision avoidance; Intelligent vehicles

## 1. Introduction

Frontal collision warning/collision avoidance (CW/CA) systems target a major crash type-rear-end crashes with a moving or parked vehicle - which account for $35 \%$ of all accidents [1]. These active safety systems are a natural extension of adaptive cruise control (ACC) systems due to their similarity in hardware requirement. Therefore, CW/CA systems are expected to take off quickly, in a fashion similar to the success of vehicle stability control systems. Due to the difference in product segmentation (safety vs. comfort) however, many car manufacturers are much more cautious in CW/CA product design and launching. In addition to legal/liability concerns, this conservativeness is mainly due to the heavier reliance of CW/CA systems on human follow-up actions. In the case of ACC, the product is designed to keep the vehicle operating in regions where human intervention is normally unnecessary (safe time headway, small relative speed). Therefore, the characteristics of the human driver are relatively unimportant. In the case of CW/CA however, the human driver is always in control and could encroach on safe driving boundaries. Furthermore, the driver is responsible for reacting to the warning signal in a proper fashion solely or together with the automatic braking system. Therefore,

[^0]designing a CW/CA system that can accommodate a wide range of human characteristics is non-trivial.

Since the early 1990s, many CW/CA algorithms and systems have been proposed, mostly by industrial researchers. Doi et al. [2] studied the effectiveness of a rear-end collision avoidance system capable of working on both straight and curved sections of highway. They identified four key elements: forward looking sensor (laser radar), path estimation, collision prediction and automatic brake control. Fujita et al. [3] proposed a radar-based automatic braking system to prevent the vehicle from a rear-end collision or to reduce the impact speed without adverse effects on normal driving. Araki et al. [4] developed a rear end collision avoidance system by integrating CCD cameras, a laser radar and a fuzzy learning algorithm from the driver's brake timing. Barber et al. [5] presented two collision warning algorithms based on time-to-collision, range, range rate and relative acceleration. Seiler et al. [6] derived a collision warning algorithm using parameters estimated from a tire-road friction estimation scheme.

The existing literature reveals two major technical challenges for CW/CA systems, namely the development of a reliable and all-weather target detection system and the trade-off between false/nuisance alarms (false-positive) and missed detections (false-negative). The second challenge is related to the first, and is more complicated because it depends on human perceptions, and has to encompass widely varying driving situations and human characteristics. Thus, for any fixed (non-adaptive) CW/CA system, disagreement between the human drivers and system response always exists. How to minimize the rate of false/nuisance alarms without significantly raising the rate of missed detection is a question that has been feverishly pursued. However, since the evaluation of CW/CA systems has been largely done in a subjective manner, few impartial comparisons of multiple algorithms have been reported.

Obtaining high-quality, naturalistic driving data suitable for the evaluation of $\mathrm{CW} / \mathrm{CA}$ systems is a non-trivial task. At least three conditions have to be satisfied:
(i) The vehicles have to be instrumented with high quality sensors to measure crucial signals (e.g. forward speed, range to a lead vehicle, range rate).
(ii) The test vehicles have to operate and feel like regular vehicles.
(iii) The data collection process has to span a long period of time so that natural driving behavior can be recorded.

Recently, the University of Michigan Transportation Research Institute conducted an 'Intelligent cruise control field operational test (ICC FOT)' and collected driving data for more than 110 thousand miles from 107 drivers [7]. Roughly $61 \%$ of the data was collected when these vehicles were driven manually, while the remainder was collected during conventional or adaptive cruise control. Available real-time measurements include range, range rate, host vehicle speed, brake on/off and throttle opening. In general, other factors such as vehicles in adjacent lanes and the curvature of the road might influence the driver's perception and decision in vehicle speed regulation. However, we ignore these factors in this paper because they are not available in the ICCFOT database. When more complete data, such as that from the on-going 'Automotive Collision Avoidance Systems Field Operational Test' [8] becomes available, a more comprehensive evaluation will become possible.

Once naturalistic driving data is collected, criteria can be set to identify a 'threatening' data set, under which a warning signal/avoidance action should be triggered; as well as 'safe' situations, under which a driver would either continue to drive, coast down, or apply a mild brake, and thus warning/avoidance actions would not be warranted. In the data selection, a 'threatening' situation can be identified based on pre-determined threshold indices (e.g. time to impact, braking level). Similarly, a non-overlapping 'safe' data set can be identified. We can then use these two data sets for impartial evaluations of the performance of $\mathrm{CW} / \mathrm{CA}$ algorithms.

This proposed evaluation process is different from the prevailing approach - designing a matrix of test maneuvers in a well controlled environment, and reporting drivers' opinion as the main evaluation results. The proposed approach complements the existing approach by providing assessment under more naturalistic driving conditions.

Before we elaborate on the process, a possible statistics issue needs to be mentioned. As pointed out by Parasuraman et al. [9], a detection system that has a high true-negative (no warning $\mid$ safe) and a low false-positive (warning|safe) rate does not necessarily result in a good design. For example, when the positive (threatening) situation is happening far less often than negative (safe) situation, one might need to sacrifice true-positive rate (warning|threatening) in order to further reduce the number of false-positive cases. This happens to be the case for $\mathrm{CW} / \mathrm{CA}$ systems - threatening driving situations are rare. A performance index to deal with such 'unbalanced' data has to be addressed (e.g. [10]).

The remainder of this paper is organized as follows. In section 2, we will explain the data extraction process and present statistical snapshots of the data. The signal detection algorithm will be illustrated in section 3. The basic ideas of four collision warning/avoidance logics, followed by their evaluation results, are explained in section 4. Finally, conclusions are given in section 5 .

## 2. Data extraction and processing

The ICCFOT database [7] contains driving data from 107 drivers for a total of more than 110 thousand miles on highways as well as on urban streets. About 68 thousand miles were driven under manual control, which represents a rich collection of driving data. Each driver drove the vehicle for $2-5$ weeks, which we assume to be long enough to reveal their naturalistic driving behavior. Measured variables include range, host vehicle velocity, braking on/off and throttle opening at a sampling rate of 10 Hz . Variables of interest for CW/CA analysis are shown in figure 1 . Two important derived variables are: time headway which is defined as the range over host vehicle speed, and time-to-impact (TTI), defined as the range over absolute value of range rate. TTI is defined only when the range rate is negative.

Among the 107 drivers, data from 15 drivers with a long manual-driving dataset were selected. Since we are interested in data only when a lead vehicle exists, the data stream must be divided into segments. A vehicle interaction segment starts when a lead vehicle appears in front of the host vehicle and ends when it vanishes. When a new lead vehicle appears at a shorter range (cut-in) or longer range (lane change of either host or lead vehicle), we will start a new segment. The ICCFOT database has the 'new target' field indicating the change of lead vehicle to help facilitate the data segmentation.


Figure 1. Variables of interest.

Once the segments are defined, the Kalman smoothing technique described below is used to obtain the range rate and host vehicle acceleration signals. First, the car-following situation described in figure 1 is put into a kinematic equation [11]

$$
\begin{align*}
\dot{\mathbf{x}}(t) & =\mathbf{A} \cdot \mathbf{x}(t)+\mathbf{G} \cdot \mathbf{w}(t) \\
\mathbf{y}(t) & =\mathbf{C} \cdot \mathbf{x}(t)+\mathbf{N} \cdot \mathbf{v}(t) \tag{1}
\end{align*}
$$

where $\mathbf{x}(t)=\left[\begin{array}{lllll}V_{F}(t) & a_{F}(t) & R(t) & V_{L}(t) & a_{L}(t)\end{array}\right]^{T}, \mathbf{w}(t)$ represents the white and random jerk signals for both the lead and host vehicles, and $\mathbf{v}(t)$ is the white measurement noise of the output measurement $\mathbf{y}(t)$. The matrices of equation (1) are shown in the following:

$$
\mathbf{A}=\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{array}\right], \mathbf{G}=\left[\begin{array}{ll}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1
\end{array}\right], \mathbf{C}=\left[\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{array}\right], \mathbf{N}=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]
$$

The Kalman filtering for this dynamic system is implemented in the discrete-time as follows:

$$
\begin{align*}
\hat{\mathbf{x}}(k+1) & =\mathbf{A}_{d f} \cdot \hat{\mathbf{x}}(k)+\mathbf{L} \cdot[\mathbf{y}(k)-\mathbf{C} \cdot \hat{\mathbf{x}}(k)] \\
& =\left(\mathbf{A}_{d f}-\mathbf{L} \cdot \mathbf{C}\right) \cdot \hat{\mathbf{x}}(k)+\mathbf{L} \cdot \mathbf{y}(k)=\mathbf{A}_{k d f} \cdot \hat{\mathbf{x}}(k)+\mathbf{L} \cdot \mathbf{y}(k) \tag{2}
\end{align*}
$$

where the subscript $d f$ indicates the matrices are in discrete-time and are for the forward filtering. Since the forward filtered data contains undesirable time delays, a backward Kalman filter is constructed to cancel its effect, which is the Kalman filter for the duo system:

$$
\left[\begin{array}{c}
\mathbf{x}(k+1)=\mathbf{A}_{d f} \cdot \mathbf{x}(k)+\mathbf{G}_{d f} \cdot \mathbf{w}(k) \\
\mathbf{y}(k)=\mathbf{C} \cdot \mathbf{x}(k)+\mathbf{N} \cdot \mathbf{v}(k)
\end{array}\right] \Longleftrightarrow\left[\begin{array}{c}
\mathbf{z}(k+1)=\left[\mathbf{A}_{d f}^{T} \mathbf{A}_{d f}\right]^{-1} \mathbf{A}_{d f}^{T} \cdot \mathbf{z}(k) \\
-\left[\mathbf{A}_{d f}^{T} \mathbf{A}_{d f}\right]^{-1} \mathbf{A}_{d f}^{T} \cdot \mathbf{G}_{d f} \cdot \mathbf{w}(k) \\
\mathbf{q}(k)=\mathbf{C} \cdot \mathbf{z}(k)+\mathbf{N} \cdot \mathbf{v}(k)
\end{array}\right]
$$

The input to the backward filter is the same as that of forward filter except it is time-reversed. Initial values for forward (backward) are calculated from the initial (final) values of the measurement $\mathbf{y}$. The average of the two filtered results is then used as the smoothed, undelayed data. Since the Kalman smoother output has larger error at the beginning and end of each data segment compared with forward filter output, data in the first and last 2.5 seconds of each segment are discarded. Besides, if the Kalman smoothed velocity or range signal is smaller than 0 at any time, the whole segment is discarded. On the surface this decision seems to be throwing away important and interesting data. However, since we know the ICCFOT database does not contain any collision cases, this criterion is added to guard against bad filtering. The final number of data points accepted is about 7.6 million. This is equivalent to roughly 211 hours of driving, which we assume to be long enough to represent naturalistic driving. Figure 2 shows the relative frequency distribution (the ratio of the absolute frequency to the total number of data points in a frequency distribution) of the host vehicle acceleration and the conditional relative frequency distribution of the host vehicle acceleration when the brake is applied.

From figure 2a, it can be seen that the distribution is quite Gaussian-like although the data points are from 15 drivers and are from widely-varying driving conditions. When we focus on the data points when the throttle is off and the brake is applied, the distribution is skewed toward the negative side (figure 2 b ). The data points in figure 2 b are much more important and are examined more closely.


Figure 2. Relative frequency distribution of acceleration distribution of accepted data.

Figure 3 is the log-plot of conditional cumulative frequency distribution for the braking cases. Only those data points with zero throttle and brake applied are included in this figure. It can be seen that among all the data collected, the deceleration level shown here is as severe as 0.65 g . However, most of the time, mild deceleration is used. The cumulative frequency distribution is somewhat wavy but can be roughly approximated by a straight line. It can be seen that decelerating at a rate steeper than 0.23 g happens in less than $1 \%$ approximately of all the braking cases.


Figure 3. Cumulative relative frequency distribution of decelerating cases.


Figure 4. Cumulative relative frequency distribution of accelerating cases.

Just to show the other side of the story, we also present the statistics of the opposite case, when the brake is off and throttle is applied. Figure 4 shows that accelerating at a rate harder than 0.20 g only happened in $1 \%$ of all acceleration cases.
Figure 5 shows the time domain plot of one example segment with severe braking. The host vehicle starts at high speed, catches up to a slower vehicle and has to brake heavily at the end. The circles mark the occurrence of 0.23 g or harder braking (i.e. 'severe braking'). It is commonly recognized as a rule of thumb that when the time-to-impact (TTI) becomes less than 9 to 10 seconds, the drivers start to pay close attention to the lead vehicle [12]. This observation seems to be true for most of this dataset. Usually, the TTI plots drop to below the 9 to 10 second level, shortly before a severe braking is activated.


Figure 5. An example $1 \%$-hard braking case in the time domain.

The populations of data points with 'severe braking' are given in table 1. Among all accepted data points from 15 drivers, 0.23 g-or-harder braking and negative range rate cases account for only $0.129 \%$ of the total population. We hypothesize that a warning signal/avoidance action should be triggered, because a human driver applies an elevated-level of braking. The data set with negative range rate and 0.23 g -or-harder braking is thus called the 'threatening' cases in the remainder of this paper. We understand that there may be two more factors that should be considered: (1) The heavy braking might be trigged by reasons other than range keeping (the so-called 'alter-control'), and (2) the warning/avoidance probably should be triggered several hundred milliseconds before a human driver took action. However, the 'alter-control' cases cannot be clearly identified using available signals and thus are difficult to remove. Seemingly clearly non-threatening data points (outliers with large range) were manually removed. We also decided not to take the human delays into consideration to make these 'threatening' cases more conservative.

According to detection theories (to be discussed in the next section), once a 'threatening' dataset is defined, a non-overlapping ('safe') dataset should also be defined. The easiest way is to define these two sets to be completely complementary. However, such a definition makes the data separation extremely crisp and hard. Therefore, we decide to define the 'safe' situation to include the cases when
(i) The driver is applying throttle, coasting (zero throttle and zero braking), or is applying brake at 0.052 g (i.e. $50 \%$-braking) or lower.
(ii) The range rate is less than 0 .

The idea is that a warning signal/avoidance action should not be triggered by the points in the 'safe' dataset, because the situation is so benign and common that a warning/control would be perceived by a large population of drivers to be nuisance or false. The definition of the two sets is shown in figure 6. Since the 'safe' and 'threatening' sets are not overlapping,

Table 1. Number of data points related to braking.

| Condition | \# points | $\%$ |
| :--- | ---: | ---: |
| Overall | $7,648,326$ | 100.000 |
| Braking | 865,850 | 11.321 |
| 0.23 g Braking | 11,833 | 0.155 |
| 0.23 g Braking \& RR $<0$ | 9,879 | 0.129 |
| 'Threatening' | 9,189 | 0.120 |



Figure 6. Separation of 'threatening' and 'safe' data sets.
a well designed CW/CA algorithm might be able to separate these two sets with both low false positive rate and low false negative rate. The total number of data points included in the 'safe' set is about 3.4 million.

## 3. Signal detection theory

CW/CA is essentially a signal detection and decision-making problem. Signal detection theory [13] assumes that the whole parameter domain is denoted by $\Theta$ (see figure 7), which is divided into two disjoint subsets, $\Theta_{0}$ (safe) and $\Theta_{1}$ (threatening), from which two hypotheses $H_{0}$ and $H_{1}$ about an unknown parameter $\theta$ emerge:

$$
\begin{align*}
& H_{0}: \theta \in \Theta_{0} \\
& H_{1}: \theta \in \Theta_{1} \tag{3}
\end{align*}
$$

A detection algorithm tries to decide whether $H_{0}$ or $H_{1}$ is true by using $q$, a function of measurement $y$. Because of various uncertainties, the measurement may not be always consistent and two probability distribution functions $f\left(q \mid H_{0}\right)$ and $f\left(q \mid H_{1}\right)$ exist (see figure 8 ). A detection algorithm might use a threshold value $\eta$; for example, if $q$ is higher than $\eta$, it decides $H_{1}$, else it decides $H_{0}$.

Due to the overlap between the two probability density functions, two types of errors exist, as shown in figure 9: false alarm and miss. False alarm means the detection algorithm decides 'threatening' but the situation is actually safe. Miss indicates the case when the detection


Figure 7. Probability distribution functions associated to each hypothesis.


Figure 8. Application of criterion $\eta$.


Figure 9. False alarm rate and miss rate.
algorithm fails to detect a threatening situation when it occurs. The major challenge in the $\mathrm{CW} / \mathrm{CA}$ logic development is to find a function $q(y)$ that can achieve satisfactory separation of the two conditional distribution functions.
The performance of a detection algorithm can be evaluated based on the measurement and detection threshold it used. Imagined decision results are shown in the confusion matrix shown in table 2. Here, 'Negative' and 'Positive' mean 'safe' and 'threatening' for our CW/CA problem, respectively. Based on the confusion matrix, the rates in table 3 can be calculated. If the positive and negative data sets have similar populations, accuracy will be a good performance index, which reveals the overall rate of correct decisions. However, if the two data sets have very different populations, a detector that favors the detection accuracy in the dominant data set will always have high overall accuracy even if it works extremely poorly with the other data set. One such example is that someone could design a (fake) CW/CA algorithm that never issues a warning signal. Since normal driving data is predominately safe, such an algorithm might still enjoy an extremely high accuracy. Therefore, we need to define a performance index that works better under unbalanced situations. The geometrical mean of TP and $P$ is one of the preferred indices in such cases [10] and will be used in this paper.

Table 2. Confusion matrix.

|  | Actual data |  |  |
| :--- | :---: | :---: | :---: |
| Number of predictions | Negative <br> (safe) | Positive <br> (threatening) |  |
| Prediction | Negative (safe) | a | c |
|  | Positive (threatening) | b | d |

Table 3. Rates calculated from confusion matrix.

| Rate | Definition |
| :--- | :---: |
| Accuracy | $(\mathrm{a}+\mathrm{d}) /(\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d})$ |
| P (Precision) | $\mathrm{d} /(\mathrm{b}+\mathrm{d})$ |
| TP (True Positive rate) | $\mathrm{d} /(\mathrm{c}+\mathrm{d})$ |
| FN (False Negative rate) | $\mathrm{c} /(\mathrm{c}+\mathrm{d})$ |
| TN (True Negative rate) | $\mathrm{a} /(\mathrm{a}+\mathrm{b})$ |
| FP (False Positive rate) | $\mathrm{b} /(\mathrm{a}+\mathrm{b})$ |
| Geometric mean | $\sqrt{T P \cdot P}$ |

## 4. Evaluation of CW/CA algorithms

There have been many papers published on collision warning/avoidance algorithms over the last decade. However, few provide enough information for a complete reconstruction and evaluation. In this paper, we will compare four algorithms proposed from researchers at Mazda, Honda, JHU and Jaguar. These four CW/CA algorithms all calculate a threshold distance based on vehicle motion and human characteristic variables (e.g. range rate, host vehicle velocity, host and lead vehicle accelerations, human delays). When the measured range is smaller than this threshold distance, a warning or avoidance signal is triggered. The terms frequently used in these CW/CA logics are summarized in table 4.

First, the Mazda avoidance logic [2] is explained. Figure 10 shows an interpretation on Mazda's logic. Note that this is an imagined worst case. Here, $x(y)$ axis indicates time $\tau$ (vehicle velocities). The measurements ( $V_{F}, V_{L}$ ) are used as initial conditions at $\tau=0$. The scenario assumes that the lead (host) vehicle maintains the current velocity $V_{L}\left(V_{F}\right)$ during the time $\tau_{2}\left(\tau_{1}+\tau_{2}\right)$ and then engages emergency braking whose slope is $-\alpha_{2}\left(-\alpha_{1}\right)$. The colored area between the two velocity profiles (of each lead vehicle and host vehicle) is the required safety range minus the minimum range. This scenario continues until both vehicles come to

Table 4. Frequently used terms in CW/CA logics.

| Meaning | Form |
| :--- | :---: |
| Host vehicle stopping distance $\left(R_{H}\right)$ | $R_{H}=\frac{1}{2} \frac{V_{F}^{2}}{\left(-\alpha_{F}\right)}, \alpha_{F}<0, V_{F} \geq 0$ |
| Lead vehicle stopping distance $\left(R_{L}\right)$ | $R_{L}=\frac{1}{2} \cdot \frac{V_{L}^{2}}{\left(-\alpha_{L}\right)}, \alpha_{L}<0, V_{L} \geq 0$ |
| Constant relative acceleration range margin $\left(R_{C R A}\right)$ | $R_{C R A}=\frac{1}{2} \cdot \frac{\dot{R}^{2}}{(-\ddot{R})}, \ddot{R}<0, \dot{R}<0$ |
| Reaction time margin $\left(R_{t d}\right)$ | $R_{t d}=\tau_{V_{F}}, V_{F}, \tau_{V_{F}}>0$ |
| Range rate margin $\left(R_{R R}\right)$ | $R_{R R}=\tau_{\dot{R}} \cdot(-\dot{R}), \tau_{\dot{R}}>0, \dot{R}<0$ |
| Minimum range $\left(R_{\text {min }}\right)$ | $R_{\text {min }}=\operatorname{constant}>0$ |



Figure 10. An interpretation to Mazda's logic.
a full stop.

$$
\begin{equation*}
R_{\text {braking }}=f\left(\dot{R}, V_{F}\right)=\frac{1}{2}\left(\frac{V_{F}^{2}}{\alpha_{1}}-\frac{V_{L}^{2}}{\alpha_{2}}\right)+V_{F} \tau_{1}-\dot{R} \tau_{2}+R_{\min } \tag{4}
\end{equation*}
$$

Their parameters are: $\alpha_{1}=6\left(m / \sec ^{2}\right), \alpha_{2}=8\left(m / \sec ^{2}\right), \tau_{1}=0.1 \mathrm{sec}, \tau_{2}=0.6 \mathrm{sec}$ and $R_{\text {min }}=5 \mathrm{~m}$.

The Honda logic consists of a warning algorithm and an avoidance algorithm [3]. The avoidance logic has two parts and the switching between them depends on whether the estimated lead vehicle time to stop is shorter than the reaction time of the host vehicle driver. In mathematical form, they are:

$$
\left\{\begin{array}{l}
R_{\text {warning }}=f(\dot{R})=-2.2 \cdot \dot{R}+6.2  \tag{5}\\
R_{\text {braking }}=f\left(\dot{R}, V_{F}\right)=\left\{\begin{array}{cl}
-\tau_{2} \cdot \dot{R}+\tau_{1} \tau_{2} \alpha_{1}-0.5 \alpha_{1} \tau_{1}^{2}, & V_{F} \geq 11.67(\mathrm{~m} / \mathrm{s}) \\
\tau_{2} \cdot V_{F}-0.5 \alpha_{1}\left(\tau_{2}-\tau_{1}\right)^{2}-\frac{V_{L}^{2}}{2 \alpha_{2}}, & V_{F}<11.67(\mathrm{~m} / \mathrm{s})
\end{array}\right.
\end{array}\right.
$$

The Honda's warning algorithm is a straight line in the range rate-range plane, indicating a time-to-impact consideration. Their braking logic has two parts selected by estimated shortest time-to-lead-vehicle-stop. If the lead vehicle is not expected to stop within $\tau_{2}$, the first part is selected; otherwise, the second part is used. Both of the scenarios assume that the lead vehicle is engaging the emergency braking and the host vehicle engages emergency braking after reaction time $\tau_{1}$ and estimate safety range until $\tau_{2}$. Their suggested parameters are: $\alpha_{1}=7.8\left(m / \mathrm{sec}^{2}\right), \alpha_{2}=7.8\left(m / \mathrm{sec}^{2}\right), \tau_{1}=0.5 \mathrm{sec}$ and $\tau_{2}=1.5 \mathrm{sec}$.

The Jaguar logic also contains a warning algorithm and a braking algorithm [5].

$$
\begin{aligned}
& R_{\text {warning }}=f(R, \dot{R}, \ddot{R})=\left\{-4 \cdot\left[\frac{-4 \cdot \dot{R}}{} \quad \text { if } V_{L}=0\right.\right. \\
& R_{\text {braking }}=\frac{\dot{R}}{2} a \dot{R}^{2}
\end{aligned}
$$




Figure 11. Interpretations to Honda's logic.

The warning logic consists of two parts. For fixed objects, the warning criterion is simply 4 -second time-to-impact. For moving vehicles, the warning criterion calculates the time to collision assuming the instantaneous relative acceleration is maintained into the future, which is depicted in figure 12. The condition to guarantee the existence of real solutions is

$$
\begin{equation*}
\dot{R}^{2}+2 R \cdot \ddot{R} \geq 0 \Longleftrightarrow R \leq \frac{1}{2} \cdot \frac{\dot{R}^{2}}{(-\ddot{R})} \tag{7}
\end{equation*}
$$

For the braking logic, the suggested value for parameter ' $a$ ' is 0.2 .
In the collision warning logic developed by NHTSA and the Applied Physics Laboratory of the Johns Hopkins University - (JHU APL) [14], the variable 'Time to lead vehicle stop' is an important criterion used in their logic. The algorithm is presented below.

$$
D_{\text {thresh }}=2[m]+V_{H} \cdot 0.1[s]
$$

$$
D_{\text {miss }}=\left\{\begin{array}{cc}
R+\Delta R_{1}+\Delta R_{2}+\Delta R_{3} & T_{L S} \geq T_{R} \\
R+\Delta R_{1}+\Delta R_{4} & T_{L S}<T_{R}
\end{array}\right.
$$

$$
\Delta R_{1}=\dot{R} T_{R}+\frac{1}{2}\left(a_{L}-a_{F}\right) T_{R}^{2}
$$

$$
\Delta R_{2}=\left[\dot{R}+\left(a_{L}-a_{F}\right) T_{R}\right]\left(T_{L S}-T_{R}\right)+\frac{1}{2}\left(a_{L}-a_{F \max }\right)\left(T_{L S}-T_{R}\right)^{2}
$$

$$
\Delta R_{3}=\left[\dot{R}+\left(a_{L}-a_{F}\right) T_{R}+\left(a_{L}-a_{F \max }\right)\left(T_{L S}-T_{R}\right)\right]\left(T_{H S}-T_{L S}\right)
$$

$$
\begin{equation*}
+\frac{1}{2}\left(0-a_{F \max }\right)\left(T_{H S}-T_{L S}\right)^{2} \tag{8}
\end{equation*}
$$

$$
\Delta R_{4}=\left[\dot{R}+\left(a_{L}-a_{F}\right) \cdot T_{R}\right]\left(T_{M}-T_{R}\right)+\frac{1}{2}\left(a_{L}-a_{F \max }\right)\left(T_{M}-T_{R}\right)^{2}
$$

$$
T_{L S}\left(V_{F}, \dot{R}, a_{L}\right)=\frac{V_{F}+\dot{R}}{-a_{L}} \quad a_{L}<0
$$

$$
T_{H S}\left(V_{F}, a_{F}\right)=\left\{\begin{array}{cl}
T_{R}+\frac{V_{F}+a_{F} T_{R}}{-a_{F \max }} & V_{F}+a_{F} T_{R} \geq 0 \\
\frac{V_{F}}{-a_{F}} & V_{F}+a_{F} T_{R}<0
\end{array}\right.
$$

$$
T_{M}=\frac{\dot{R}+\left(a_{L}-a_{F}\right) T_{R}}{a_{F \max }-a_{L}}+T_{R}
$$





Figure 12. Concept of Jaguar's collision warning logic for moving targets.


Figure 13. JHU-APL algorithm for $T_{L S} \geq T_{R}$.

The suggested parameter values are: $a_{F \max }=-0.5 \mathrm{~g}$ and $T_{R}=1.5 \mathrm{sec}$. The basic idea of this algorithm is explained in the following because the original publication did not provide much detail. When the time to lead vehicle stop $\left(T_{L S}\right)$ is longer than the human reaction time $\left(T_{R}\right)$, this logic divides the time-to-stop into three segments: 0 to $T_{R}\left(\Delta R_{1}\right), T_{R}$ to $T_{L S}\left(\Delta R_{2}\right)$ and $T_{L S}$ to $T_{H S}$ (time to host vehicle stop) $\left(\Delta R_{3}\right)$ as shown in figure 13. The logic calculates the range consumed within each segment from the final relative velocity of the previous segment and the duration of the segment. When $T_{L S}$ is shorter than $T_{R}$, the logic only considers two segments: 0 to $T_{R}\left(\Delta R_{1}\right)$ and $T_{R}$ to $T_{M}\left(\Delta R_{4}\right)$ (see figure 14). Within the first segment ( 0 to $T_{R}$ ), the range calculation is the same for both cases. In figure 14, the area below the x -axis should have been subtracted from the overall range calculation because the vehicle speed should not become negative. We assume that this compensation is not included in the original JHU-APL algorithm because its calculation is somewhat complicated. The proposed threshold range calculation is simpler and offers extra safety margin by neglecting this area. The JHU-APL algorithm also includes an additional rule to improve implementation robustness: the warning signal will be triggered only if $D_{\text {miss }}$ is smaller than $D_{\text {thresh }}$ for two out of the last three detections.


Figure 14. JHU-APL algorithm for $T_{L S}<T_{R}$.


Figure 15. Headway margin.

Given any data point from the identified data sets, each of the four CW/CA algorithms described above will calculate its threshold distance and decide whether the situation is safe or threatening. The difference between the calculated threshold range and the measured range (as shown in figure 15) indicates how much range margin is left for a safe situation, or how severe the situation is for a threatening case. We normalize the range margin using the host vehicle speed to define the time headway margin THM.

$$
\begin{equation*}
T H M=\frac{R_{\text {data }}-R_{\text {safe }}}{V_{\text {data }}} . \tag{9}
\end{equation*}
$$

When $T H M$ is larger (less) than 0 , the $\mathrm{CW} / \mathrm{CA}$ logic decides that the situation is safe (threatening). Distributions of time headway margin for the two data sets are shown in figure 16. The dashed line shows the distribution of the 'threatening' data set and the solid line is for the 'safe' data set. As a benchmark, the simple TTI $=10$ second criterion was included in this figure. All the plots in figure 16 show significant overlap between the two distributions (i.e., all the algorithms fail to separate the 'threatening' and 'safe' data sets). TTI, even though extremely simple, separate the data sets better than most of these algorithms.

Table 5 shows the calculated confusion matrix. Several things are noticeable:
(1) The two data sets are quite unbalanced - by a factor of about 380 .
(2) Most of these algorithms correctly classify the majority of 'safe' test data to be safe; but none was able to correctly classify $55 \%$ of the 'threatening' data.

As a comparison, the simple TTI algorithm has the best rate for correctly classifying threatening data - perhaps there is lesson to be learned here. It seems quite clear that many of these algorithms were designed to reduce false-alarms. However, they all suffer an unacceptably high rate of missed detections.

Most of the logics achieve high accuracy (column 3) as in table 6. However, most of them have extremely poor precision performance. The JHU-APL logic achieved both high accuracy and high precision and seems to perform best overall. However, its true positive rate is quite low - the logic failed to detect the majority ( $87 \%$ ) of the 'threatening' data. The extremely low true-positive rate indicates that the JHU-APL algorithm (together with the Honda avoidance algorithm) was designed to deliver low rate of false/nuisance alarms. The side effect is that many human drivers may find these algorithms fail to provide warning/avoidance action in


Figure 16. Relative frequency distributions of the time headway margins of $\mathrm{CW} / \mathrm{CA}$ logics.
a timely fashion. For the threatening scenarios we collected, collisions may still be avoidable. However, the awakened driver or the automatic braking system might have to apply braking more severe than 0.23 g , which could cause problems for surrounding vehicles.

As explained in section 3, for this unbalanced detection case, geometric mean of TP (True Positive) and P (Precision) is chosen as the main performance index.

Higher precision means less false alarms and higher true-positive indicates better capability of detecting threatening situations. Based on this metric, the JHU-APL algorithm performs the best. There is, nevertheless, still a great deal of room for improvement.

It should be noted that the evaluation results presented in this section depend heavily on the collected data sets. If new data becomes available or new criterions to separate threatening

Table 5. Confusion matrices of evaluated CW/CA logics.

|  |  | Data |  |  |  |
| :--- | :--- | ---: | ---: | ---: | ---: |
| Prediction |  | Safe |  | Threatening |  |
| Safe | TTI $=10$ | 3215739 | $92.27 \%$ | 1396 | $15.19 \%$ |
|  | Mazda | 2400552 | $68.88 \%$ | 4310 | $46.90 \%$ |
|  | Honda(w) | 3379455 | $96.97 \%$ | 4557 | $49.59 \%$ |
|  | Honda(a) | 3475378 | $99.72 \%$ | 8718 | $94.87 \%$ |
|  | Jaguar | 3425114 | $98.28 \%$ | 6506 | $70.80 \%$ |
|  | JHU-APL | 3482357 | $99.92 \%$ | 8010 | $87.17 \%$ |
| Threatening | TTI =10 | 269480 |  |  |  |
|  | Mazda | 1084667 | $31.73 \%$ | 7793 | $84.81 \%$ |
|  | Honda(w) | 105764 | $3.03 \%$ | 4879 | $53.10 \%$ |
|  | Honda(a) | 9841 | $0.28 \%$ | 471 | $50.41 \%$ |
|  | Jaguar | 60105 | $1.72 \%$ | 2683 | $29.13 \%$ |
|  | JHU-APL | 2862 | $0.08 \%$ | 1179 | $12.83 \%$ |

Table 6. Performance indices of evaluated CW/CA logics.

|  | TP | Precision | Accuracy | g-mean (TP,P) |
| :--- | ---: | ---: | ---: | ---: |
| TTI $=10$ | $83.22 \%$ | $3.33 \%$ | $91.31 \%$ | $16.65 \%$ |
| Mazda | $41.08 \%$ | $0.85 \%$ | $82.63 \%$ | $5.90 \%$ |
| Honda(w) | $39.23 \%$ | $4.10 \%$ | $96.50 \%$ | $12.69 \%$ |
| Honda(a) | $3.29 \%$ | $12.33 \%$ | $99.57 \%$ | $6.37 \%$ |
| Jaguar | $18.72 \%$ | $3.29 \%$ | $97.74 \%$ | $7.85 \%$ |
| JHU-APL | $8.98 \%$ | $21.89 \%$ | $99.56 \%$ | $14.02 \%$ |

and/or safe data sets applied, the result is likely to be different. In addition, our data only reflects driving data based on US drivers. Collecting similarly rich data sets that represent an international market is also very important.

Finally, we wish to point out that the four algorithms evaluated in this paper might have been designed based on very different philosophies - ranging from avoiding all collisions, collision mitigation, to mimicking the behavior of an average driver. No matter what the design philosophy is, these CW/CA algorithms, once realized in a product, will be subject to consumers' 'acceptance test'. The naturalistic driving data used in this paper represent an objective and non-intrusive acceptance test. This is because the 'safe' and 'threatening' data sets were obtained such that any 'false alarm' and 'miss' indicates deviation from human drivers' naturalistic behavior. Consumers likely will not know, or understand the design philosophy of a CW/CA algorithm, but they will for sure judge its performance-deviation from their naturalistic behavior is a good indication of whether they will agree with the CW/CA's warning and control actions.

## 5. Conclusion

Five CW/CA logics are evaluated using naturalistic driving data in this paper. The data were queried from the ICCFOT databases, based on the driving record of 15 human drivers. Based on the observed driving behavior, two data sets were identified: representing safe and threatening driving situations. Since the data sets were unbalanced, the geometric mean of true-positive rate and precision was used as the main performance index in this study. The evaluation results show that the JHU-APL logic achieved the best performance but its performance is still less than satisfactory, especially because of its low true-positive rate. All the other three logics suffer from extremely low precision and low true-positive rates. In comparison, the extremely simple TTI index achieves much higher true-positive rate than all evaluated algorithms. Continued refinement of these CW/CA logics seems to be necessary.

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