NOTES

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Evaluation of tortuosity in acoustic porous materials saturated by air

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Tortuosity is an important parameter for the prediction of the acoustical properties of porous sound absorbing materials. The evaluation of tortuosity by resistivity measurements is now used in several laboratories, although this method presents several drawbacks. In particular, the complete saturation by a conducting fluid of a porous foam having a high flow resistivity is difficult to obtain without partially damaging the structure of the cells. A simple technique based on ultrasonic wavespeed measurements in a material saturated by air is described. This method has been used previously only for water or superfluid helium saturated materials.

Tortuosity is an intrinsic property of porous frames. Different representations of this quantity can be found in the works of Biot,[1] Brown,[2] and Johnson.[3–5] A synthesis is given in a book by Allard.[6] The tortuosity α_∞ (we use the symbol introduced by Johnson) is related to the dispersion of the microscopic molecular velocity $v(M)$ of an inviscid fluid which flows through the frame[5–6]

$$\alpha_\infty=\overline{v^2}(M)/\overline{v}(M)^2. \tag{1}$$

The added mass ρ_α defined by Biot[1] is related to α_∞ by the following equation:[6]

$$\rho_a=(\alpha_\infty-1)\rho_0\phi, \tag{2}$$

where ρ_0 is the density of the fluid and ϕ the porosity of the frame. The tortuosity may be evaluated from the following expression:

$$\alpha_\infty=\phi\, r_s/r_f, \tag{3}$$

where r_s and r_f, respectively, are the resistivities of the material saturated by a conducting fluid and of the fluid. The link between this evaluation and Eq. (1) is based on the analogy between the velocity field in an inviscid fluid due to a pressure gradient, and the velocity field of the ions in a conducting fluid due to an electrical potential.[2]

Tortuosity is also related to the velocity of the acoustic waves in an inviscid fluid which saturates a rigid frame[3]

$$\alpha_\infty=C_i^2/C_{is}^2, \tag{4}$$

where C_i is the wave speed in the fluid and C_{is} the wave speed in the same fluid when it saturates the frame. For the case of a viscous fluid, air for instance, viscosity and thermal exchanges with the frame must be taken into account. At high frequencies the complex wave number is related to tortuosity by[7]

$$k=\frac{\omega}{C_0}\sqrt{\alpha_\infty}\left[1+\frac{\delta(1-j)}{2}\left(\frac{1}{\Lambda}+\frac{\gamma-1}{B\Lambda}\right)\right], \tag{5}$$

where C_0 is the celerity in the free air, ω the radian frequency, γ the specific heat ratio, B the square root of the Prandtl number, and Λ and Λ' are the characteristic viscous and thermal lengths.[5,8]

The viscous skin depth δ is given by

$$\delta=\sqrt{2\eta/\omega\rho_0}, \tag{6}$$

where η is the viscosity of air.

The loss angle φ of the complex wave number can be written as

$$\varphi=\frac{\delta}{2}\left(\frac{1}{\Lambda}+\frac{\gamma-1}{B\Lambda'}\right), \tag{7}$$

and the velocity inside the porous material C, and the tortuosity α_∞ are respectively given by

$$C=\frac{C_0}{\sqrt{\alpha_\infty}}\left[1-\frac{\delta}{2}\left(\frac{1}{\Lambda}+\frac{\gamma-1}{B\Lambda'}\right)\right], \tag{8}$$

$$\alpha_\infty=\left(\frac{C_0}{C}\right)^2(1-2\varphi). \tag{9}$$

Tortuosity can be evaluated from Eq. (9). The ratio C_0/C can be evaluated from the increase of the time of flight of an ultrasonic pulse between two transducers when a layer of material is inserted, and φ from the damping of the pulse. The high-frequency range is the frequency domain, where δ is much smaller than the characteristic lengths Λ and Λ', and φ, given by Eq. (7), is small in this range of frequencies. Then, in the evaluation of α_∞ by Eq. (9), φ only provides a small correction, and a crude estimate of such quantity is sufficient. For most of the porous

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TABLE I. The two measurements of tortuosity and the loss angle of the wave number, for three partially reticulated plastic foams.

Material	Thickness (mm)	α_∞ (resistivity measurement)	α_∞ (velocity measurement)	φ wave number loss angle
Plastic foam (1)	4.0 and 8.0	1.5 ± 0.1	1.52 ± 0.05	0.01
Plastic foam (2)	4.3 and 5.5	1.7 ± 0.1	1.44 ± 0.05	0.08
Plastic foam (3)	3.8 and 5.2	1.5 ± 0.1	1.42 ± 0.05	0.04

sound absorbing materials saturated by air, the frequency can be safely chosen between 50 and 100 kHz. Recent measurements reported by Nagy[9] indicate that for higher frequencies the loss angle predicted by Eq. (7) is underestimated, due to other dissipation mechanisms, but the difference is generally weak or not noticeable up to 100 kHz.

The ultrasonic measurements were performed by using narrowband 75 kHz piezoelectric transducers manufactured by Vermon SA (France). The transducers were fed in the standard pulse echo technique by the 5058 PR emitter/receiver from Panametrics providing 900 V peak-to-peak voltage pulses. Such high-voltage peaks were needed because the porous plastic foams used in this work were highly attenuating for ultrasound waves. Due to the very high attenuation that occurred, only thin samples (between 5 and 10 mm) have been tested. The digitizing rate is equal to 100 kHz which is large enough to not induce noticeable error in the measurement of the time of flight. Due to the similarity of the signals obtained with and without sample, the time-of-flight measurements were achieved by using a cross-overlapping procedure directly performed on the oscilloscope. From these wave speed measurements made on three different plastic foams (enclosed results in Table I), the tortuosity parameter α_∞ was calculated with the help of Eq. (9). It was also measured by using a resistivity method. Also present in the table are the values of the damping coefficient φ. This parameter was measured by dealing with two samples with different thicknesses made of the same sample. This two-step procedure was needed to take into account the reflected waves at the interface of the porous plate. An acceptable precision on these measurements was not trivial to achieve because of difficulties in measuring the thickness of the samples. The agreement between both measurements is good for materials (1) and (3); the measured values are compatible in spite of the small error intervals. For material (2) the resistivity measurements provide a value of α_∞ which is too large, due to a noncomplete saturation of the porous frame by the conducting fluid. The new method is simple to use, and is nondestructive. It may be noticed that similar work was presented more than ten years ago (3), for porous materials saturated by water or superfluid helium. It is believed that it is the first time that such results are reported for air saturated materials.

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