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# Event Discrimination Using Phase Correlation in $\Phi$ -OTDR System Based on Coherent Detection

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**Abstract:** We report a novel method of event discrimination using phase correlation in  $\Phi$ -OTDR system based on coherent detection. The phase change demodulated from the Rayleigh backscattering light is the result of the event acting on the optical fiber. Then, the correlation coefficient between the phase change of every point along the fiber and that of the fiber end is obtained. A vibration event acting on the fiber induces the change of the correlation coefficient. So, the number of vibration events is determined by the number of changes of the correlation coefficient. In addition, it proves that our proposed method is able to discriminate two close events even when they are in the range of half-pulsewidth.

Index Terms: Event discrimination, phase correlation,  $\Phi$ -OTDR, spatial resolution limit.

# 1. Introduction

Phase optical time domain reflectometer( $\Phi$ -OTDR) is based on the interference effect of the Rayleigh backscattering light in the optical fiber. When events such as vibration, tension or sound field act on the fiber, the refractive index and the fiber length will be changed. Then the OTDR trace of interference is altered accordingly. So it is very suitable for dynamic measurement due to the advantages such as high sensitivity, fast response speed, long sensing distance, etc [1]–[5].

In  $\Phi$ -OTDR system, the spatial resolution is one of the important parameters. It determines whether a  $\Phi$ -OTDR system can finely discriminate two events which are close to each other. Normally, the spatial resolution of  $\Phi$ -OTDR is determined by the width of the probe pulse light and the relationship can be expressed as cw/2n, where c is the velocity of light in vacuum, w is the pulse width and n is the refractive index of the fiber [6]–[7]. Over the past decades, research efforts have been made to develop a series of techniques all towards improving the spatial resolution of  $\Phi$ -OTDR. Z. Qin *et al.* have proposed a wavelet denoising method to improve the performance of  $\Phi$ -OTDR, which allows to shorten the pulse width of  $\Phi$ -OTDR to 5 ns [8]. T. Zhu *et al.* have proposed a two-dimensional edge detection method to improve the signal-to-noise ratio and spatial



Fig. 1. Scheme of  $\Phi$ -OTDR system based on coherent detection.

resolution. And the resulting spatial resolution breaks the traditional limit [9]. These works provide a detailed description of event discrimination in the form of amplitude.

Since the perturbation on the fiber has linearly relationship with the optical path difference of the Rayleigh backscattering light, we can achieve quantitative measurement by demodulating the phase change of the Rayleigh backscattering light [10]–[13]. So more and more researchers have paid attention to the phase demodulation in  $\Phi$ -OTDR system. F. Pang *et al.* have researched on the distribution of the phase change along the optical fiber in the  $\Phi$ -OTDR system based on coherent detection and use the phase property to achieve fading discrimination. But the spatial resolution of the method is limited by the pulse duration [14]. In this article, we use the  $\Phi$ -OTDR system based on coherent detection to achieve the phase demodulation of vibration event. Then we calculate the correlation coefficient between the phase change of arbitrary point on the optical fiber and that of the fiber end. The number of events is distinguished from the change of the correlation coefficient. The simulation results show that this method can not only realize event discrimination of dynamic events acting on the optical fiber, but also achieve the spatial resolution which breaks the traditional limit which is no less than the half pulse width.

# 2. Principle

Fig. 1 is a typical schematic diagram of a  $\Phi$ -OTDR system based on coherent detection. The laser setup(LD) has very narrow linewidth (normally less than 10 kHz). The light from LD is divided into two parts with a 90:10 optical coupler (OC). The 90% light is modulated into the probe optical pulse by an acoustic-optic modulator(AOM). Then the probe pulse light is amplified by an erbium-doped optical fiber amplifier (EDFA) and injected into the optical fiber. The 10% light is used as local reference light. It mixes with the Rayleigh backscattering light generated by the probe pulse light in optical fiber via a 50:50 OC and received by a balanced photodetector (BPD). Then the beat signal output from the BPD is transferred to digital signal by an oscilloscope (OSC) for further processing. By synchronizing the AOM and OSC, each probe pulse can generate an OTDR trace. If there is no perturbation acting on the optical fiber, all the traces keep the same. But if the optical fiber is disturbed or the fiber-winding PZT is being driven by the signal source, the OTDR trace at the corresponding position will change simultaneously.

#### 2.1 Rayleigh Backscattering Light and Phase Demodulation

For a coherent detection system, we can assume that the amplitude and phase of the reference light are constant. And the signal light at the surface of BPD in a  $\Phi$ -OTDR system is an integrated field of the returned light from a number of Rayleigh scattering centers within one pulse width in the optical fiber. When AC coupling mode is used, the detection result can be expressed as [15]:

$$E_B(z,t) = E_0 e^{-2\alpha z} \int_z^{z+\Delta z} \bar{r} f(z) \exp\left(j\frac{4\pi}{\lambda} \int_0^x \Delta n(y,t) \, dy\right) dx \tag{1}$$

where  $E_0$  is the amplitude, *t* is a variable which is related to the repeat time of pulse, *z* is the fiber length,  $\alpha$  is the fiber attenuation factor,  $\Delta z$  is the pulse width in length,  $\bar{r}$  is the scattering factor which contains Rayleigh scattering coefficient, reverse capture coefficient and the detection response rate, *f* is waveform function,  $\Delta n$  is the change of fiber refractive index.



Fig. 2. The (a) Phase change and (b) Correlation coefficient of phase change along fiber for single point event.

What should be point out is that, in many cases, both the refractive index and the fiber length would be modulated under a perturbation. However, since the signal of  $\Phi$ -OTDR is correlated with the phase of Rayleigh scattering light, from the perspective of the phase change, both the changes of refractive index and fiber length can be represented by an equivalent change of refractive index  $\Delta N$ .

So (1) is rewritten as:

$$E_B(z, t) = E_0 e^{-2\alpha z} \int_z^{z+\Delta z} \bar{r} f(z) \exp\left(j\frac{4\pi}{\lambda} \int_0^x \Delta N(y, t) \, dy\right) dx \tag{2}$$

Then the phase of received signal is directly demodulated from  $E_B$ :

$$\Phi(z, t) = actan(E_B(z, t)) \pm k\pi, k = 0, 1, 2, ...$$
(3)

where *k* is determined by the phase unwrapping algorithm.

Because the amplitude of the perturbation is related to the change of phase among different pulses, in order to measure the perturbation quantitatively, we derive the phase change by the following relationship:

$$\varphi(z,t) = \Phi(z,t) - \Phi(z,0) \tag{4}$$

where  $\varphi(z, 0)$  is the initial phase value induced by the perturbation.

#### 2.2 Phase Correlation

When the probe pulse light passes through the perturbation point, all the optical signals after the perturbation point are modulated [12]. Thus the phases of optical signal at all positions after the perturbation point are similar to each other before the probe pulse light meets another perturbation. Assuming (*e*) is phase change at the fiber end, then the correlation coefficient between the phase change (*z*) of arbitrary position and (*e*) can be expressed as:

$$\rho(z) = \frac{\sum_{i=1}^{M} \left[ \left( \varphi(z, i) - \frac{\sum_{i=1}^{M} \varphi(z, i)}{M} \right) \left( \varphi(e, i) - \frac{\sum_{i=1}^{M} \varphi(e, i)}{M} \right) \right]}{\sqrt{\sum_{i=1}^{M} \left( \varphi(z, i) - \frac{\sum_{i=1}^{M} \varphi(z, i)}{M} \right)^2 \sum_{i=1}^{M} \left( \varphi(e, i) - \frac{\sum_{i=1}^{M} \varphi(e, i)}{M} \right)^2}, i = \frac{t}{T}$$
(5)

where *M* is the pulse number, *T* is the pulse period. For a fiber without perturbation, the phase change  $\varphi(z, t)$  is zero for all positions in theory. But when the fiber is under perturbation, the phase change  $\varphi(z, t)$  for positions after the perturbation point will vary with the perturbation. As shown in Fig. 2(a), we assume a perturbation is applied between points A and B on the fiber. When the optical pulse meets the perturbation event in AB region, the phase change  $\varphi(z, t)$  begins to vary. At first,



Fig. 3. The correlation coefficient of phase change along fiber for double point events.

the variation is small, since only a little part of the probe pulse light overlaps with the perturbation region. Then the amplitude of the phase change increases gradually with the increment of the overlap length between the optical pulse and the perturbation region. When the whole probe pulse passes through the perturbation region, the amplitude of phase change would reach maximum and keep constant along the fiber.

When we focus on the phase correlation, according to (5), the correlation coefficient between the phase change of every point on the fiber and that of the fiber end is calculated, and the result is shown as Fig. 2(b). Before the AB region, the value of phase change is 0, because there is no relativity between the phase change of this region and the fiber end. After the optical pulse encounters AB region, the correlation coefficient is increasing quickly until it reaches to the maximum. From the comparison between Fig. 2(a) and (b), it can be seen that the slope of the change of correlation coefficient is much sharper than that of the phase change. This is because the correlation coefficient in (5) is normalized and it is only correlated with the tendency of the phase variation. Obviously, when the correlation coefficient jumps from 0 to 1, there must be one perturbation event on the fiber.

If there is a signal fading or other noise at the end of the fiber, the calculated phase change of the fiber end can't be used as  $\varphi(e)$  in (5). Fortunately, we can tell from Fig. 2(a) whether there is false value of phase change at the end of the fiber. When the false value is at the fiber end, we should use a normal value of phase change near the end as  $\varphi(e)$  in (5).

The diagrammatic sketch of the correlation coefficient when there are two perturbation events is shown as Fig. 3. Two perturbations locate at AB region and CD region respectively. The phase change of an arbitrary point is affected by all perturbation events before this point. Thus, the correlation coefficient has two jumps near the point A and point C respectively. Conversely, we know that there are two events acting on the fiber for two jump values or the intermediate value.

It should be pointed out that when the length of AD is shorter than half of the pulse width, it is nearly impossible to discriminate the two perturbations with the traditional method for a  $\Phi$ -OTDR system. However, since the correlation coefficient reaches a constant value very soon after only a little part of the optical pulse enters the perturbation region. It is obvious in AB region of Fig. 2. For two events, if the correlation coefficient obtained with (5) still has two obvious steps more than zero when the length of AD is shorter than the half pulse width, the spatial resolution limited by the pulse width would be broken.

# 3. Simulation

Based on the above analysis, the method of using phase correlation for event discrimination is verified by simulation with MATLAB software.



Fig. 4. (a) The phase change along fiber for single vibration. (b) The correlation coefficient derived from Fig. (a).

Due to the limit of the computing capability, we set the pulse width as 1ns. The constant  $E_0$  of (2) is assigned to be 1. The attenuation coefficient  $\alpha$  is 0.2 dB/km. Variable f(z) is a rectangular pulse and is assigned to be 1 in the range of pulse width. Wavelength  $\lambda$  is 1550 nm. The initial value  $\Delta N(z, 0)$  is a continuous uniform distribution in the range of [-0.001,0.001] on the whole fiber. The pulse number *M* is 80.

Perturbation would lead to a variation of  $\Delta N$ . The variation of  $\Delta N$  with *t* is directly superimposed on initial N(z, 0). The simulation is made for two cases: single–point-vibration and double–point-vibration. For convenience, we assume two sinusoidal signals representing two vibrations on the fiber, which are:

$$\Delta N_1(z, t) = \Delta N(z, 0) + 0.00005 * \sin(2 * 10^3 t)$$
(6)

$$\Delta N_2(z, t) = \Delta N(z, 0) + 0.001 * \sin(6 * 10^3 t)$$
(7)

#### 3.1 Single-Point-Vibration

We set the vibration source at [0.4264 m 0.4279 m]. The variation of N modulated by vibration is given in (6). Fig. 4(a) shows the phase change with vibration time t along the fiber length. When the optical pulse has not encountered the vibration event, all the phase change values are 0. At 0.2264 m, due to the influence of vibration, the phase change begins to vary. And the amplitude of the phase change has a fluctuating ascending trend until the back edge of the pulse leaves the vibration region. But, when we observe the trend carefully, we find that the amplitude of phase change in Fig. 4(a) does not vary linearly with the fiber length. That is because the overlap length between the optical pulse and the perturbation region increases gradually and the distribution of fiber refractive index is inhomogeneous. If fading occurs at the vibration region, this ascending trend may be chaotic. However, the phase change after the vibration point varies periodically with vibration and its amplitude keeps constant. Fig. 4(b) is the correlation coefficient calculated according to (5) for the data of Fig. 4(a). Because (5) is a normalized correlation calculation, the calculated value ranges from -1 to 1. The value reflects the degree of similarity between  $\varphi(z)$  and  $\varphi(e)$ . The closer it is to 1, the higher is the degree of similarity between the two phase changes. As can be seen from Fig. 4(b), the correlation coefficient fluctuates at [0 m 0.226 m], where no vibration exists. The reason for the fluctuation is that very subtle difference phase change between different optical pulses is introduced by the computer calculation and the normalized calculation magnifies the nuance between the true value of the phase change and zero. So we record the correlation coefficient of this region as its average value 0. From 0. 229 m to 0. 23 m, the correlation coefficient increases from 0 to 1 quickly. Then we get a steep valley between 0.258 m and 0.262 m. But the correlation coefficient after 0.262 m is restored to 1 quickly. The reason for the valley is an interference fading between 0.258 m and 0.262 m. So the valley can be ignored and the correlation coefficient between 0.23 m and 0.6 m can be considered as always 1. Therefore, on the whole correlation coefficient diagram, we consider that there is only one step from 0 to 1, which means that there is one vibration event on the whole fiber.





Fig. 5. (a) The phase change along fiber for double vibrations. (b) The correlation coefficient derived from Fig. (a).



Fig. 6. (a) The phase change along fiber for double vibrations whose distance is within half pulse width. (b) The correlation coefficient derived from Fig. (a).

## 3.2 Double-Point-Vibration

Two vibration sources are set at [0.2093 m 0.2109 m] and [0.5233 m 0.5248 m] respectively. The variation of  $\Delta N$  induced by the first vibration is in (6) and that induced by the second vibration is in (7). Fig. 5(a) is the phase change induced by the two vibration sources. From Fig. 5(a), we can see that there are two obvious stable regions after the first vibration point, each of which corresponds to a vibration event. Fig. 5(b) is obtained from Fig. 5(a). Similar to Fig. 4(b), the correlation coefficient at [0 m 0.009 m] is fluctuant around 0, since there is no vibration event. And there are two stable steps along the fiber whose values are 0.46 and 1 respectively. From these two steps, we can determine that there are two different vibration events on the optical fiber.

#### 3.3 High Discrimination Resolution

From Figs. 2(b), 3, 4(b) and 5(b), we can see that the value of the correlation coefficient has a quick increment after the front edge of the optical pulse enters the vibration region. Then the value remains nearly unchanged until another vibration is encountered. As long as the step induced by the stable value can be identified, two close events on the fiber can be distinguished. Therefore, this method of vibration discrimination using phase correlation also provides us with a new method to improve the event discrimination resolution. In this simulation, we set the two vibration sources at [0.4448 m 0.4463 m] and [0.5233 m 0.5248 m] respectively. The distance between the two vibration sources is less than 0.08 m. The resulting phase change and correlation coefficient are shown in Fig. 6(a) and (b).

In Fig. 6(a), the amplitude of the phase change fluctuates continuously from 0.2448 m to 0.5248 m. Thus, we cannot discriminate the two vibration events directly. However, by calculating the correlation coefficient for Fig. 6(a) with (5), we obtain Fig. 6(b). Again, in the interval [0 m 0.23 m], the correlation coefficient can be taken as 0. In the interval [0.252 m 0.325 m] and the interval [0.524 m; 0.6 m], the correlation coefficients are stable and their values are 0.48 and 1 respectively. Therefore, we judge that there are two events in Fig. 6(b), causing the correlation coefficient to climb from 0 to 0.48 and then from 0. 48 to 1. Because these two vibration events are separated



Fig. 7. The correlation coefficient along fiber for two vibrations whose distances are (a) 0.08 m, (b) 0.07 m, (c) 0.06 m and (d) 0.05 m respectively.

from 0.08 m, which is less than half pulse width 0.1 m, we think that the discrimination resolution of  $\Phi$ -OTDR breaks the traditional spatial resolution limit.

Finally, Fig. 7 compares the correlation coefficients of two vibration points separated by 0.08 m, 0.07 m, 0.06 m and 0.05 m respectively. As can be seen from Fig. 7, two vibration events with 0.07 m interval can also be clearly distinguished. For the interval of 0.06 m, it is difficult to determine the median value of the stable correlation coefficient. When the interval decreases to 0.05 m, the intermediate value cannot be discriminated completely. So the closer the two events are, the more difficult it is to distinguish them.

# 4. Discussion and Conclusion

In our simulation, the pulse width of  $\Phi$ -OTDR is only 1ns which is much less for most of practical application system. It is because the minimum unit of simulation is equal to wavelength of the laser [15] and the amount of calculation is too large when the pulse width is wider. For example, when the pulse width of our simulation is 1 ns, there is 129000 units in one pulse width and we need at least three days to finish one calculation. So the setting of pulse width is limited by the calculation capability. But the method of event discrimination using phase correlation is suitable for arbitrary pulse width.

For fast vibration signal, we think it is necessary to consider the phase change's difference of Rayleigh backscattering light at different positions behind the vibration as backscattering light passes through the vibration region. The farther away from the vibration region, the greater the difference. The faster the signal is, the more obvious is the difference. Because the velocity of light is fast, normally the difference is not significant. However, if the vibration is very fast and the distance between the vibration region and fiber end is very long, the phase change of the scattering signal from the fiber end will be different from that of the scattering signal right behind the vibration position. In this case, the correlation coefficient obtained with (5) decreases slowly with the distance between the calculation position and the fiber end. However, event discrimination is confirmed by the jump of the correlation coefficient. So the fast signal also can be identificated.

For weak signal, as long as the phase change can be correctly demodulated, our method is valid. It is because the normalized correlation operation amplifies the information carried by the weak signal.

For noise caused by laser, PD, etc., it is complicated. Usually, laser leads to noise in the form of phase and PD leads to the noise in the form of amplitude. But, in fact, whether the noise in the form of phase or amplitude, what we detected is in the form of amplitude and what we finally solved is in the form of phase. The detected result contains the noise originally in the form of both amplitude and phase. In the practical application, when the noise is increased, the number of sampling points along the fiber where the phase can be demodulated correctly is reduced. As the number of correct demodulation for phase change decreases, the distribution of correlation coefficient becomes more and more irregular. When jump can't be distinguished, event discrimination can't be done.

In this paper, we have proposed the method of event discrimination using phase correlation in the  $\Phi$ -OTDR system. We give the simulation model for  $\Phi$ -OTDR and the method of analyzing the correlation coefficient. Through the simulation by MATLAB, we realize the event discrimination of single-point-vibration event and double-point-vibration event. The single-point-vibration event has two steady correlation coefficients: 0 and 1. And the double-point-vibration event has three steady correlation coefficients: 0, an intermediate value and 1. The number of vibration events is the number of correlation coefficients minus one. Furthermore, our proposed method can distinguish double vibration events whose distance is less than half pulse width.

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