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Event-triggered communication for passivity and synchronization of multi-weighted coupled neural networks with and without parameter uncertainties

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Abstract: A multi-weighted coupled neural networks (MWCNNs) model with event-triggered communication is studied in this paper. On the one hand, the passivity of the presented network model is studied by utilizing Lyapunov stability theory and some inequality techniques, and a synchronization criterion based on the obtained output-strict passivity condition of MWCNNs with event-triggered communication is derived. On the other hand, some robust passivity and robust synchronization criteria based on output-strict passivity of the proposed network with uncertain parameters are presented. At last, two numerical examples are provided to testify the effectiveness of the output-strict passivity and robust synchronization results.

1 Introduction

In recent decades, plenty of scholars have attracted widespread concern about dynamical behaviors of complex networks, e.g., passivity, synchronization, and stability, due to their extensive applications in various domains such as secure communication, brain science, pattern recognition and so on [1]. Recently, a lot of research achievements have been given on synchronization and passivity in complex networks [2–11]. Wang *et al*. [11] proposed a network model that is complex dynamical network with time-varying delay, and the passivity-based synchronization problem for this network was studied. With in-depth research of complex networks, more and more fresh elements are being added. For example, event-triggered control is a very effective strategy in complex networks, which is a discrete-time control method. It can conquer the shortcomings of the continuous control method and reduce the unnecessary transmission of the communication media during the process of information exchanging. In recent years, some interesting results on event-triggered passivity and synchronization of complex networks have been gained [12–27]. The authors in [16] designed two kinds of distributed event-triggering laws and some criteria that insure the event-based exponential synchronization were established.

Coupled neural networks (CNNs), as a special complex networks, have received the concern of scholars from different domains, and passivity and synchronization of CNNs have become hot issues [28–42]. The finite-time passivity problem for adaptive CNNs with undirected and directed topologies was studied in [28]. Zhang *et al*. [30] considered the memristor-based coupling recurrent neural networks with time-varying delays and impulses, some sufficient conditions for synchronization were obtained by means of a generalized Halanay differential inequality. In [40], the authors put forward a network model of partially CNNs with event-triggered communication and gave some conditions for solving the synchronization and passivity problems of this kind of network.

In practice, many systems, such as transportation networks, social networks, and so on, in the real world should be modeled by multi-weighted complex networks (MWCNs). In recent years, the passivity and synchronization for MWCNs have been considered [43–47]. In [46], Wang *et al*. established some conditions about passivity and passivity-based synchronization for MWCNs by employing some inequality techniques and Lyapunov functional approach. Moreover, multi-weighted CNNs (MWCNNs) have become a very hot topic due to their broad application foreground. Some standards were established to ensure passivity and synchronization for MWCNNs [48–50]. Tang *et al*. [49] investigated the passivity and exponential synchronization for two classes of MWCNNs with and without time delays. It is regrettable that the passivity and synchronization for MWCNNs with event-triggered communication have rarely been considered.

Due to the effect of environmental noise and model error, it is usually impossible to obtain the accurate value of parameters in CNNs in many cases. Therefore, it is of great significance and interesting to research the dynamical behaviors of CNNs with parameter uncertainties [51–54]. The authors [51] concerned the problems of generalized synchronization and pinning control for delayed CNNs with and without parameter uncertainties. Similarly, the environmental noise and model error also exist in the MWCNNs. Thus, it is worthwhile taking uncertain parameters into account when researching MWCNNs. Unfortunately, the event-triggered synchronization and passivity of MWCNNs with parameter uncertainties have not yet been investigated.

Inspired by the above discussion, this paper aims to study the event-triggered passivity and passivity-based synchronization problems of MWCNNs with and without parameter uncertainties. The main contributions of our work can be outlined as follows.

(1) By employing some new inequality techniques and designing appropriate event-triggered condition and Lyapunov functional, some sufficient conditions for passivity of MWCNNs with event-triggered communication are established.

(2) Considering the precise parameter values of network cannot be obtained in some situations, we also purse the study on the robust passivity of MWCNNs with parameter uncertainties, and present several event-triggered robust passivity criteria for the considered network.

(3) Based on the obtained output-strict passivity results, two sufficient conditions for ensuring the event-triggered synchronization and robust synchronization of MWCNNs without and with parameter uncertainties are derived respectively.

2 Preliminaries

2.1 Definitions and Notations

Let $\digamma = (\alpha, \delta, \widetilde{G})$ be a weighted directed digraph, where $\delta \subseteq \alpha \times \alpha$ is directed set of edge and $\alpha = \{1, 2, \cdots, N\}$ is a set of nodes. $(i, j) \in \delta$ represents an edge of graph $\digamma$. $\mathcal{N}_i = \{j \in \alpha | (i, j) \in \delta\}$ is the neighbor set of node i. The connection adjacency matrix $\widetilde{G} = (\widetilde{G}_{ij})_{N\times N}$ of the graph $\digamma$ satisfies $\widetilde{G}_{ii} = 0$ and $\widetilde{G}_{ij} > 0 \Longleftrightarrow j \in \mathcal{N}_i$. $\|\cdot\|_2$ is the Euclidean norm.

Definition 2.1. (See [3]) A system with output $y(t) \in \mathbb{R}^p$ and input $u(t) \in \mathbb{R}^p$ is called to be dissipative if there is a storage function $\Pi : [0, +\infty) \to [0, +\infty)$ satisfying

$$\int_{t_a}^{t_b} \Upsilon(u(t), y(t))dt \geqslant \Pi(t_b) - \Pi(t_a)$$

for any $t_a, t_b \in [0, +\infty)$ and $t_a \leqslant t_b$, where $\Upsilon(u, y)$ is supply rate.

Definition 2.2. (See [3]) The system is called to be passive if it is dissipative and

$$\Upsilon(u(t), y(t)) = y^T(t)u(t).$$

Definition 2.3. (See [3]) If a system is dissipative and

$$\Upsilon(u(t), y(t)) = y^T(t)u(t) - \kappa_1 y^T(t)y(t) - \kappa_2 u^T(t)u(t),$$

where $\kappa_1 > 0, \kappa_2 > 0$ and $\kappa_1 + \kappa_2 > 0$, then the system is called to be strictly passive. In addition, the system is called to be input-strictly passive if $\kappa_2 > 0$ and output-strictly passive if $\kappa_1 > 0$.

3 Event-triggered passivity analysis of MWCNNs

3.1 Network model

The MWCNNs to be studied in this section is described by

$$\begin{aligned}\dot{w}_i(t) =& - Aw_i(t) + \sum_{s=1}^{m}\sum_{j=1}^{N} d_s G_{ij}^s \Gamma^s w_j(t) + L + Hf(w_i(t)) \\ &+ Bx(w_i(t-\tau(t))) + v_i(t) + u_i(t), \end{aligned} \tag{1}$$

where $i = 1, 2, \cdots, N$; $m \in \mathbb{N}^+$; the state vector of the ith node is $w_i(t) = (w_{i1}(t)), w_{i2}(t)), \cdots, w_{in}(t)) \in \mathbb{R}^n$; $u_i(t) \in \mathbb{R}^n$ signifies the input vector; $A = \mathrm{diag}(a_1, a_2, \cdots, a_n) > 0$; $L = (L_1, L_2, \cdots, L_n)^T$ means the external input; $B \in \mathbb{R}^{n\times n}, H \in \mathbb{R}^{n\times n}$ are known matrices; the inner coupling matrix of the sth coupling form is $\Gamma^s \in \mathbb{R}^{n\times n} > 0$; $0 < d_s \in \mathbb{R}$, $0 < \widetilde{d}_s \in \mathbb{R}$ represent the overall coupling strengths of the sth coupling form for no event-triggered configuration and event-triggered one; $f(w_i(t)) = (f_1(w_{i1}(t)), f_2(w_{i2}(t)), \cdots, f_n(w_{in}(t)))^T$, $x(w_i(t - \tau(t))) = (x_1(w_{i1}(t - \tau(t))), x_2(w_{i2}(t - \tau(t))), \cdots, x_n(w_{in}(t - \tau(t))))^T$ stand for the activation functions; $\tau(t)$ signifies the time varying delay which meets $0 \leqslant \tau(t) \leqslant \tau$ and $\dot{\tau}(t) \leqslant \varepsilon < 1$; $\widetilde{G}^s = (\widetilde{G}_{ij}^s)_{N\times N}$ with $\widetilde{G}_{ii}^s = 0, \widetilde{G}_{ij}^s > 0$ is the weight matrix of the controller in the sth coupling form; the element in the matrix $G^s = (G_{ij}^s)_{N\times N}$ in the sth coupling form has the following definition: if there exists a connection between node i and node j, then $G_{ij}^s > 0$ and $G_{ij}^s = G_{ji}^s$, otherwise $G_{ij}^s = 0 (i \neq j)$, and $G_{ii}^s = -\sum_{j=1, j\neq i}^{N} G_{ij}^s$, $i = 1, 2, \cdots, N$; $v_i(t) \in \mathbb{R}^n$ represents the control input, which is given as follows:

$$v_i(t) = \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{d}_s \widetilde{G}_{ij}^s \big(w_i(t) - w_j(t)\big).$$

The increasing event-triggered time sequence is denoted by $\{t_k^i\}_{k=1}^{\infty}$, where $t_{k+1}^i - t_k^i > 0, i = 1, 2, \cdots, N$. According to the event-triggered method, the control input $v_i(t)$ can be rewritten as:

$$v_i(t) = \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{d}_s \widetilde{G}_{ij}^s \big(w_i(t_k^i) - w_j(t_k^j)\big), \tag{2}$$

where $t \in [t_k^i, t_{k+1}^i)$, $w_i(t_k^i)$ represents the state of node i at t_k^i, t_k^i is the event-triggered time instant of node i.

Remark 1. As is well known, we can get in touch with others through various contact ways such as facebook, wechat, email, QQ, message and so on. Each contact method has different weights which stands for different coupling form. Therefore, the social network can be reflected more accurately by the model of MWCNs. Actually, many networks can be modeled by MWCNs, such as communication networks, complex biology networks, transportation networks. As a special type of MWCNs, MWCNNs received more and more attention in recent years. More exactly, the nodes in the MWCNNs are coupled by multiple coupling forms compared with the traditional CNNs with single weight. Therefore, it is more reasonable to describe some real-world networks by utilizing the MWCNNs. However, there are only a few results reported on the passivity and synchronization of MWCNNs [48, 49], and it is worth devoting more efforts on this topic. As far as we know, the event-triggered passivity and synchronization for MWCNNs have not been considered before, which motivates our research work in this paper. To the best of our knowledge, this is the first paper toward studying the passivity and synchronization of MWCNNs based on event-triggered mechanism.

The event-triggered measure error can be defined as $\vartheta_i(t) = w_i(t_k^i) - w_i(t)$ and

$$t_{k+1}^i = \inf\Bigg\{ t : t > t_k^i, \|\vartheta_i(t)\|_2 > \sigma \Bigg\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s \big(w_i(t_k^i) - w_j(t_k^j)\big) \Bigg\|_2 \Bigg\} \tag{3}$$

for any $t \in [t_k^i, t_{k+1}^i)$ and the constant $\sigma > 0$.

The activation functions $f_\iota(\cdot)$ and $x_\iota(\cdot)$ are assumed to meet global Lipschitz condition in this paper. In other words, there exist $0 < \varpi_\iota \in \mathbb{R}, 0 < \chi_\iota \in \mathbb{R}$ such that

$$|f_\iota(b) - f_\iota(a)| \leqslant \varpi_\iota |b - a|,$$
$$|x_\iota(b) - x_\iota(a)| \leqslant \chi_\iota |b - a|$$

for any $a, b \in \mathbb{R}$ and $\iota = 1, 2, \cdots, n$. Denote $\hat{\varpi} = \mathrm{diag}\Big(\varpi_1^2, \varpi_2^2, \cdots, \varpi_n^2\Big), \hat{\chi} = \mathrm{diag}\Big(\chi_1^2, \chi_2^2, \cdots, \chi_n^2\Big)$.

3.2 Passivity analysis

Assume that an equilibrium solution vector of network (1) is $r^* = (r_1^*, r_2^*, \cdots, r_n^*)^T$. Then, we can get

$$0 = - Ar^* + L + Bx(r^*) + Hf(r^*).$$

Letting $z_i(t) = w_i(t) - r^*$, we have

$$\begin{aligned}\dot{z}_i(t) =& - Az_i(t) + H\hat{f}(z_i(t)) + \sum_{s=1}^{m}\sum_{j=1}^{N} d_s G_{ij}^s \Gamma^s z_j(t) + u_i(t) \\ &+ \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{d}_s \widetilde{G}_{ij}^s \big(z_i(t_k^i) - z_j(t_k^j)\big) \\ &+ B\hat{x}(z_i(t - \tau(t))), \end{aligned} \tag{4}$$

in which $t \in [t_k^i, t_{k+1}^i)$, $\hat{x}(z_i(t - \tau(t))) = x(w_i(t - \tau(t))) - x(r^*) = (x_1(w_{i1}(t - \tau(t))) - x_1(r_1^*), x_2(w_{i2}(t - \tau(t))) - x_2(r_2^*), \cdots,$

$x_n(w_{in}(t-\tau(t))) - x_n(r_n^*))^T$; $\hat{f}(z_i(t)) = f(w_i(t)) - f(r^*) = (f_1(w_{i1}(t)) - f_1(r_1^*), f_2(w_{i2}(t)) - f_2(r_2^*), \cdots, f_n(w_{in}(t)) - f_n(r_n^*))^T$.

Define the following output vector $y_i(t) \in \mathbb{R}^n$ for the network (4):

$$y_i(t) = Q_1 z_i(t) + Q_2 u_i(t),$$

in which $Q_1 \in \mathbb{R}^{n\times n}, Q_2 \in \mathbb{R}^{n\times n}$ are known matrices.

As $\vartheta_i(t) = w_i(t_k^i) - w_i(t)$, the error system (4) can be rewritten as:

$$\begin{aligned}\dot{z}_i(t) =& -Az_i(t) + H\hat{f}(z_i(t)) + u_i(t) + \sum_{s=1}^{m}\sum_{j=1}^{N} d_s G_{ij}^s \Gamma^s z_j(t) \\ &+ \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{d}_s \widetilde{G}_{ij}^s \big(z_i(t) + \vartheta_i(t) - z_j(t) - \vartheta_j(t)\big) \\ &+ B\hat{x}(z_i(t-\tau(t))). \qquad (5)\end{aligned}$$

According to the distributed event-triggered condition (3), we can obtain

$$\begin{aligned}\|\vartheta_i(t)\|_2 \leqslant& \ \sigma \left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s \left(z_i\left(t_k^i\right) - z_j\left(t_k^j\right)\right)\right\|_2 \\ \leqslant& \ \sigma \left[\left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s(\vartheta_i(t) - \vartheta_j(t))\right\|_2 \right.\\ &\left. + \left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s(z_i(t) - z_j(t))\right\|_2\right] \\ \leqslant& \ \sigma \left[\left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s\vartheta_i(t)\right\|_2 + \left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s\vartheta_j(t)\right\|_2\right. \\ &\left. + \left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s z_i(t)\right\|_2 + \left\| \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i} \widetilde{G}_{ij}^s z_j(t)\right\|_2\right] \\ \leqslant& \ \sigma \left[\sum_{s=1}^{m}\widetilde{g}_s\|\vartheta(t)\|_2 + \sum_{s=1}^{m}\widetilde{g}_s\|\vartheta(t)\|_2 + \sum_{s=1}^{m}\widetilde{g}_s\|z(t)\|_2 \right.\\ &\left. + \sum_{s=1}^{m}\widetilde{g}_s\|z(t)\|_2\right] \\ =& \ 2\sigma\sum_{s=1}^{m}\widetilde{g}_s\big(\|\vartheta(t)\|_2 + \|z(t)\|_2\big), \qquad (6)\end{aligned}$$

where $\widetilde{g}_s = \max\{\sum_{j\in\mathcal{N}_i}\widetilde{G}_{ij}^s\}$, $\vartheta(t) = \left(\vartheta_1^T(t), \vartheta_2^T(t), \cdots, \vartheta_N^T(t)\right)^T$ and $z(t) = \left(z_1^T(t), z_2^T(t), \cdots, z_N^T(t)\right)^T$. Then,

$$\|\vartheta(t)\|_2 \leqslant 2\sigma N\sum_{s=1}^{m}\widetilde{g}_s\big(\|\vartheta(t)\|_2 + \|z(t)\|_2\big).$$

Therefore,

$$\|\vartheta(t)\|_2 \leqslant \frac{2\sigma N\sum_{s=1}^{m}\widetilde{g}_s}{1 - 2\sigma N\sum_{s=1}^{m}\widetilde{g}_s}\|z(t)\|_2, \qquad (7)$$

where $0 < \sigma < \frac{1}{2N\sum_{s=1}^{m}\widetilde{g}_s}$.

Remark 2. In this paper, the Zeno behavior can be excluded. In other words, the inter-event intervals $t_{k+1}^i - t_k^i$ have a positive lower bounded for all $i = 1, 2, \cdots, N$, where $k > 0$. Actually, for $t \in [t_k^i, t_{k+1}^i)$ and the constant $\sigma > 0$, we have

$$\begin{aligned}\|\dot{\vartheta}_i(t)\|_2 \leqslant& \left\| -Az_i(t) + \sum_{s=1}^{m}\sum_{j=1}^{N} d_s G_{ij}^s\Gamma^s z_j(t) + H\hat{f}(z_i(t)) \right.\\ &\left. + B\hat{x}(z_i(t-\tau(t)))\right\|_2 + \hat{U}_i(t_k^i, t_k^j) \\ \leqslant& \lambda_{\max}(A)\left\|w_i(t_k^i) - \vartheta_i(t) - r^*\right\|_2 + |\lambda_{\max}(H)|\varpi\left\|w_i(t_k^i)\right.\\ &\left. - \vartheta_i(t) - r^*\right\|_2 + |\lambda_{\max}(B)|\chi\left\|z_i(t-\tau(t))\right\|_2 \\ &+ \sum_{s=1}^{m}\sum_{j=1}^{N} d_s|G_{ij}^s|\lambda_{\max}(\Gamma^s)\left\|z_j(t)\right\|_2 + \hat{U}_i(t_k^i, t_k^j) \\ \leqslant& \left(\lambda_{\max}(A) + |\lambda_{\max}(H)|\varpi\right)\left\|\vartheta_i(t)\right\|_2 \\ &+ \left(\lambda_{\max}(A) + |\lambda_{\max}(H)|\varpi\right)\left\|w_i(t_k^i) - r^*\right\|_2 \\ &+ |\lambda_{\max}(B)|\chi\left\|z_i(t-\tau(t))\right\|_2 + \hat{U}_i(t_k^i, t_k^j) \\ &+ \sum_{s=1}^{m}\sum_{j=1}^{N} d_s|G_{ij}^s|\lambda_{\max}(\Gamma^s)\left\|z_j(t)\right\|_2 \\ \leqslant& \ \alpha\|\vartheta_i(t)\|_2 + \beta,\end{aligned}$$

where $\varpi = \max_{\iota=1,2,\cdots,n}\{\varpi_\iota\}$, $\chi = \max_{\iota=1,2,\cdots,n}\{\chi_\iota\}$, $\alpha = \lambda_{\max}(A) + |\lambda_{\max}(H)|\varpi$, $\beta(t) = \left(\lambda_{\max}(A) + |\lambda_{\max}(H)|\varpi\right)\left\|w_i(t_k^i) - r^*\right\|_2 + |\lambda_{\max}(B)|\chi\left\|z_i(t-\tau(t))\right\|_2 + \sum_{s=1}^{m}\sum_{j=1}^{N} d_s|G_{ij}^s|\lambda_{\max}(\Gamma^s)\left\|z_j(t)\right\|_2 + \hat{U}_i(t_k^i, t_k^j)$, $\hat{U}_i(t_k^i, t_k^j) = \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{d}_s\widetilde{G}_{ij}^s\left\|w_i(t_k^i) - w_j(t_k^j)\right\|_2$ and $\beta = \sup_{t_k^i \leqslant t < t_{k+1}^i}\beta(t)$.

By solving the inequality $\|\dot{\vartheta}_i(t)\|_2 \leqslant \alpha\|\vartheta_i(t)\|_2 + \beta$. Thus, we can get

$$\|\vartheta_i(t)\|_2 \leqslant \frac{\beta}{\alpha}\left(e^{\alpha(t-t_k^i)} - 1\right).$$

According to the event-triggered condition (3), when the event of node i is triggered, we have

$$\|\vartheta_i(t_{k+1}^i)\|_2 \geqslant \sigma\left\|\sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{G}_{ij}^s\big(w_i(t_k^i) - w_j(t_k^j)\big)\right\|_2.$$

So, one obtains

$$\sigma\left\|\sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{G}_{ij}^s\big(w_i(t_k^i) - w_j(t_k^j)\big)\right\|_2 \leqslant \frac{\beta}{\alpha}\left(e^{\alpha(t_{k+1}^i - t_k^i)} - 1\right).$$

After simple calculation, one can get

$$\begin{aligned}t_{k+1}^i - t_k^i \geqslant& \frac{1}{\alpha}\ln\left(1 + \frac{\alpha\sigma\left\|\sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{G}_{ij}^s\big(w_i(t_k^i) - w_j(t_k^j)\big)\right\|_2}{\beta}\right) \\ >& \ 0,\end{aligned}$$

which implies the network does not exhibit Zeno behavior.

Theorem 3.1. The error system (5) with the event-triggered condition (3) realizes output-strict passivity if there are a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^{m}\widetilde{g}_s}$, a real number $\kappa_1 > 0$ and a matrix $0 < \Omega \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix} \Psi_1 & \Xi_1 \\ \Xi_1^T & \Psi_2 \end{pmatrix} \leqslant 0, \tag{8}$$

where $\Psi_1 = I_N \otimes \Big(\Omega HH^T\Omega + \Omega BB^T\Omega + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + \hat{\varpi} + \frac{\hat{\chi}}{1-\varepsilon} - 2\Omega A + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\Big(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\Big)I_n + \kappa_1 Q_1^TQ_1\Big) + \sum_{s=1}^{m}d_s[G^s\otimes(\Omega\Gamma^s+\Gamma^s\Omega)]$, $\Xi_1 = I_N\otimes\Big(\kappa_1Q_1^TQ_2 + \Omega - Q_1^T\Big)$, $\Psi_2 = I_N\otimes\Big(\kappa_1Q_2^TQ_2 - Q_2^T - Q_2\Big)$.

Proof: The Lyapunov functional $V(t)$ for network (5) is chosen as:

$$V(t) = \sum_{i=1}^{N} z_i^T(t)\Omega z_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\chi_j^2}{1-\varepsilon}\int_{t-\tau(t)}^{t} z_{ij}^2(s)ds, \tag{9}$$

Take the upper right derivative of $V(t)$ with regard to *t*, we have

$$\begin{aligned} D^+V(t) = &-\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{(1-\dot{\tau}(t))\chi_j^2}{1-\varepsilon}z_{ij}^2(t-\tau(t)) + \sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\chi_j^2}{1-\varepsilon}z_{ij}^2(t) \\ &+ 2\sum_{i=1}^{N} z_i^T(t)\Omega\Bigg[-Az_i(t) + \sum_{s=1}^{m}\sum_{j=1}^{N} d_sG_{ij}^s\Gamma^s z_j(t) \\ &+ u_i(t) + H\hat{f}(z_i(t)) + B\hat{x}(z_i(t-\tau(t))) \\ &+ \sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{d}_s\widetilde{G}_{ij}^s(z_i(t)+\vartheta_i(t)-z_j(t) \\ &-\vartheta_j(t))\Bigg]. \end{aligned} \tag{10}$$

By utilizing some related inequalities, we can get

$$\begin{aligned} &2\sum_{i=1}^{N} z_i^T(t)\Omega H\hat{f}(z_i(t)) \\ &\leqslant z^T(t)\Big[I_N\otimes\Big(\Omega HH^T\Omega + \hat{\varpi}\Big)\Big]z(t), \end{aligned} \tag{11}$$

$$\begin{aligned} &2\sum_{i=1}^{N} z_i^T(t)\Omega B\hat{x}(z_i(t-\tau(t))) \\ &\leqslant \sum_{i=1}^{N} z_i^T(t-\tau(t))\hat{\chi}z_i(t-\tau(t)) + \sum_{i=1}^{N} z_i^T(t)\Omega BB^T\Omega z_i(t) \\ &= z^T(t-\tau(t))(I_N\otimes\hat{\chi})z(t-\tau(t)) \\ &\quad + z^T(t)\Big[I_N\otimes\Big(\Omega BB^T\Omega\Big)\Big]z(t), \end{aligned} \tag{12}$$

$$\begin{aligned} &2\sum_{i=1}^{N} z_i^T(t)\Omega\sum_{s=1}^{m}\sum_{j=1}^{N} d_sG_{ij}^s\Gamma^s z_j(t) \\ &= z^T(t)\sum_{s=1}^{m} d_s[G^s\otimes(\Omega\Gamma^s+\Gamma^s\Omega)]z(t). \end{aligned} \tag{13}$$

Based on (6), we can get

$$\begin{aligned} &2\sum_{i=1}^{N} z_i^T(t)\Omega\sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{d}_s\widetilde{G}_{ij}^s\big(z_i(t)-z_j(t)\big) \\ &\leqslant \sum_{s=1}^{m}\widetilde{d}_s\sum_{i=1}^{N}\Bigg[z_i^T(t)\Omega^2z_i(t) + \Bigg\|\sum_{j\in\mathcal{N}_i}\widetilde{G}_{ij}^s\Big(z_i(t)-z_j(t)\Big)\Bigg\|_2^2\Bigg] \\ &\leqslant \sum_{s=1}^{m}\widetilde{d}_s z^T(t)\Big(I_N\otimes\Omega^2\Big)z(t) + 4\sum_{i=1}^{N}\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\|z(t)\|_2^2 \\ &= \sum_{s=1}^{m}\widetilde{d}_s z^T(t)\Big(I_N\otimes\Omega^2\Big)z(t) + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2 z^T(t)z(t). \end{aligned} \tag{14}$$

According to (7), we have

$$\begin{aligned} &2\sum_{i=1}^{N} z_i^T(t)\Omega\sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{d}_s\widetilde{G}_{ij}^s\big(\vartheta_i(t)-\vartheta_j(t)\big) \\ &\leqslant 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}z^T(t)z(t) \\ &\quad + \sum_{s=1}^{m}\widetilde{d}_s z^T(t)\Big(I_N\otimes\Omega^2\Big)z(t). \end{aligned} \tag{15}$$

It follows from (9)-(15) that

$$\begin{aligned} D^+V(t) \leqslant\ & z^T(t)\Bigg\{I_N\otimes\Bigg(\Omega HH^T\Omega + \Omega BB^T\Omega + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 \\ &+ \hat{\varpi} - 2\Omega A + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\Big(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\Big)I_n\Bigg) \\ &+ \sum_{s=1}^{m} d_s[G^s\otimes(\Omega\Gamma^s+\Gamma^s\Omega)]\Bigg\}z(t) + 2z^T(t)(I_N\otimes\Omega)u(t) \\ &+ z^T(t-\tau(t))(I_N\otimes\hat{\chi})z(t-\tau(t)) - \frac{1-\dot{\tau}(t)}{1-\varepsilon} \\ &\times z^T(t-\tau(t))(I_N\otimes\hat{\chi})z(t-\tau(t)) + \frac{1}{1-\varepsilon} \\ &\times z^T(t)(I_N\otimes\hat{\chi})z(t) \\ \leqslant\ & z^T(t)\Bigg\{I_N\otimes\Bigg(-2\Omega A + \Omega HH^T\Omega + \Omega BB^T\Omega + \hat{\varpi} \\ &+ 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\Big(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\Big)I_n \\ &+ \frac{\hat{\chi}}{1-\varepsilon}\Bigg) + \sum_{s=1}^{m} d_s\left[G^s\otimes(\Omega\Gamma^s+\Gamma^s\Omega)\right]\Bigg\}z(t) \\ &+ 2z^T(t)(I_N\otimes\Omega)u(t). \end{aligned} \tag{16}$$

Furthermore,

$$\begin{aligned}
&D^+V(t) + \kappa_1 y^T(t)y(t) - 2y^T(t)u(t) \\
&\leqslant z^T(t)\Bigg\{I_N \otimes \Bigg(-2\Omega A + \Omega H H^T\Omega + \Omega B B^T \Omega + \hat{\varpi} \\
&+ 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n \\
&+ \frac{\hat{\chi}}{1-\varepsilon}\Bigg) + \sum_{s=1}^{m} d_s[G^s\otimes(\Omega\Gamma^s + \Gamma^s\Omega)]\Bigg\}z(t) \\
&+ 2z^T(t)\left[I_N \otimes \left(\Omega - Q_1^T\right)\right]u(t) - 2u^T(t)\Big(I_N \\
&\otimes Q_2^T\Big)u(t) + \kappa_1\Big[z^T(t)\left(I_N\otimes\left(Q_1^TQ_1\right)\right)z(t) \\
&+ z^T(t)\left(I_N\otimes\left(Q_1^TQ_2\right)\right)u(t) + u^T(t)\Big(I_N \\
&\otimes\left(Q_2^TQ_1\right)\Big)z(t) + u^T(t)\left(I_N\otimes\left(Q_2^TQ_2\right)\right)u(t)\Big] \\
&= \zeta^T(t)\begin{pmatrix}\Psi_1 & \Xi_1\\ \Xi_1^T & \Psi_2\end{pmatrix}\zeta(t),
\end{aligned} \tag{17}$$

where $\Psi_1 = I_N \otimes \Big(\Omega H H^T \Omega + \Omega B B^T\Omega + 2\sum_{s=1}^m \widetilde{d}_s\Omega^2 + \hat{\varpi} - 2\Omega A + \frac{\hat{\chi}}{1-\varepsilon} + 4N\sum_{s=1}^m \widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n + \kappa_1 Q_1^TQ_1\Big) + \sum_{s=1}^m d_s[G^s\otimes(\Omega\Gamma^s + \Gamma^s\Omega)]$, $\Xi_1 = I_N\otimes\Big(\kappa_1 Q_1^TQ_2 + \Omega - Q_1^T\Big)$, $\zeta(t) = \left(z^T(t), u^T(t)\right)^T$, $\Psi_2 = I_N\otimes\Big(\kappa_1 Q_2^TQ_2 - Q_2^T - Q_2\Big)$. Therefore,

$$D^+V(t) \leqslant 2y^T(t)u(t) - \kappa_1 y^T(t)y(t).$$

One obtains

$$V(t_b) - V(t_a) \leqslant 2\int_{t_a}^{t_b} y^T(z)u(z)dz - \int_{t_a}^{t_b}\kappa_1 y^T(z)y(z)dz$$

for any $t_a, t_b \in [0, +\infty)$ and $t_b \geqslant t_a$. Namely,

$$\widetilde{V}(t_b) - \widetilde{V}(t_a) \leqslant \int_{t_a}^{t_b}\left(y^T(z)u(z) - \frac{\kappa_1}{2}y^T(z)y(z)\right)dz,$$

where $\widetilde{V}(t) = \frac{V(t)}{2}$. □

Theorem 3.2. The error system (5) realizes input-strict passivity under the condition (3), if there are a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^m \widetilde{g}_s}$, a real number $\kappa_2 > 0$ and a matrix $0 < \Omega \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix}\Psi_3 & \Xi_2\\ \Xi_2^T & \Psi_4\end{pmatrix} \leqslant 0, \tag{18}$$

where $\Psi_3 = I_N \otimes \Big(-2\Omega A + \Omega H H^T\Omega + \Omega B B^T\Omega + \hat{\varpi} + \frac{\hat{\chi}}{1-\varepsilon} + 2\sum_{s=1}^m \widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^m \widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n\Big) + \sum_{s=1}^m d_s [G^s\otimes(\Omega\Gamma^s + \Gamma^s\Omega)]$, $\Xi_2 = I_N\otimes\left(\Omega - Q_1^T\right)$, $\Psi_4 = I_N\otimes\Big(\kappa_2 I_n - Q_2 - Q_2^T\Big)$.

Proof: Selecting the same Lyapunov functional as (9), one can get

$$\begin{aligned}
&D^+V(t) + \kappa_2 u^T(t)u(t) - 2y^T(t)u(t) \\
&\leqslant z^T(t)\Bigg\{I_N\otimes\Bigg(-2\Omega A + \Omega H H^T\Omega + \Omega B B^T\Omega + \hat{\varpi} \\
&+ 2\sum_{s=1}^m \widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^m\widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n \\
&+ \frac{\hat{\chi}}{1-\varepsilon}\Bigg) + \sum_{s=1}^m d_s\left[G^s\otimes\left(\Omega\Gamma^s + \Gamma^s\Omega\right)\right]\Bigg\}z(t) \\
&+ 2z^T(t)\left[I_N\otimes\left(\Omega - Q_1^T\right)\right]u(t) + \kappa_2 u^T(t)u(t) \\
&- 2u^T(t)\left(I_N\otimes Q_2^T\right)u(t) \\
&= \zeta^T(t)\begin{pmatrix}\Psi_3 & \Xi_2\\ \Xi_2^T & \Psi_4\end{pmatrix}\zeta(t),
\end{aligned} \tag{19}$$

where $\Psi_3 = I_N \otimes \Big(-2\Omega A + \Omega H H^T\Omega + \Omega B B^T\Omega + \hat{\varpi} + \frac{\hat{\chi}}{1-\varepsilon} + 2\sum_{s=1}^m \widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^m \widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n\Big) + \sum_{s=1}^m d_s [G^s\otimes(\Omega\Gamma^s + \Gamma^s\Omega)]$, $\Xi_2 = I_N\otimes\left(\Omega - Q_1^T\right)$, $\Psi_4 = I_N\otimes\Big(\kappa_2 I_n - Q_2 - Q_2^T\Big)$, $\zeta(t) = \left(z^T(t), u^T(t)\right)^T$.

By referring to the proof of Theorem 3.1, we have

$$\widetilde{V}(t_b) - \widetilde{V}(t_a) \leqslant \int_{t_a}^{t_b}\left(y^T(z)u(z) - \frac{\kappa_2}{2}u^T(z)u(z)\right)dz,$$

where $\widetilde{V}(t) = \frac{V(t)}{2}$. □

As a special case of input-strict passivity, it is very easy to deduce the following passivity criterion. To avoid repetition, we omit the proof for this result.

Theorem 3.3. The system (5) with the condition (3) realizes passivity if there are a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^m \widetilde{g}_s}$ and a matrix $0 < \Omega \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix}\Psi_3 & \Xi_2\\ \Xi_2^T & \Psi_5\end{pmatrix} \leqslant 0, \tag{20}$$

where $\Psi_3 = I_N \otimes \Big(-2\Omega A + \Omega H H^T\Omega + \Omega B B^T\Omega + \hat{\varpi} + \frac{\hat{\chi}}{1-\varepsilon} + 2\sum_{s=1}^m \widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^m \widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n\Big) + \sum_{s=1}^m d_s [G^s\otimes(\Omega\Gamma^s + \Gamma^s\Omega)]$, $\Xi_2 = I_N\otimes\left(\Omega - Q_1^T\right)$, $\Psi_5 = -I_N\otimes\Big(Q_2 + Q_2^T\Big)$.

3.3 *Synchronization analysis*

Definition 3.1. The network (1) is synchronized under the event-triggered condition (3), if

$$\lim_{t\to+\infty}\|w_i(t) - r^*\|_2 = 0,$$

under the condition $u_i(t) = (0, 0, \cdots, 0) \in \mathbb{R}^n$, for all $i = 1, 2, \cdots, N$.

Theorem 3.4. Assume that $\Re : [0, +\infty) \to [0, +\infty)$ is continuously differentiable and meets

$$\nu_1(\|z(t)\|_2) \leqslant \Re(t) \leqslant \nu_2(\|z(t)\|_\tau), \tag{21}$$

where $\|z(t)\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0}\|z(t+\theta)\|_2$, the continuous functions $\nu_1, \nu_2 : [0, +\infty) \to [0, +\infty)$ are strictly monotonically increasing, $\nu_1(e)$ and $\nu_2(e)$ are positive for $e > 0$ with $\nu_1(0) = \nu_2(0) = 0$, then the network (1) is synchronized under the event-triggered condition (3) if it is output-strictly passive with regard to $\Re(t)$ and matrix $Q_1 \in \mathbb{R}^{n\times n}$ is nonsingular.

Proof: If system (5) is output-strictly passive with regard to storage function $\Re(t)$, then there is a positive constant κ_1 satisfying

$$\Re(t+\varsigma) - \Re(t) \leqslant \int_t^{t+\varsigma} y^T(z)u(z)dz - \int_t^{t+\varsigma} \kappa_1 y^T(z)y(z)dz,$$

where $t \in [0, +\infty)$ and $0 < \varsigma \in \mathbb{R}$. Hence,

$$\frac{\Re(t+\varsigma) - \Re(t)}{\varsigma} \leqslant \frac{\int_t^{t+\varsigma} y^T(z)u(z)dz}{\varsigma} - \frac{\int_t^{t+\varsigma} \kappa_1 y^T(z)y(z)dz}{\varsigma}.$$

Let $\varsigma \to 0$, we can get

$$D^+\Re(t) \leqslant y^T(t)u(t) - \kappa_1 y^T(t)y(t).$$

Since $u(t) = 0$, we have

$$\begin{aligned} D^+\Re(t) &\leqslant -\kappa_1 z^T(t)(I_N \otimes (Q_1^T Q_1))z(t) \\ &\leqslant -\kappa_1 \lambda_{\min}\Big((I_N \otimes (Q_1^T Q_1))\Big)\|z(t)\|_2^2, \end{aligned} \tag{22}$$

where $D^+\Re(t)$ is the upper right derivative of $\Re(t)$ along the solution of error system (5). From (21) and (22), we can get that the system (5) is asymptotically stable. Therefore, the network (1) is synchronized under the event-triggered condition (3) in the sense of Definition 3.1. □

Based on Theorems 3.1 and 3.4, we can obtain the conclusion as follows.

Corollary 3.1. If there exist a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^m \widetilde{g}_s}$, a real number $\kappa_1 > 0$, two matrices $0 < \Omega \in \mathbb{R}^{n\times n}$ and $Q_1 \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix} \Psi_1 & \Xi_1 \\ \Xi_1^T & \Psi_2 \end{pmatrix} \leqslant 0, \tag{23}$$

where Q_1 a nonsingular matrix, $\Psi_1 = I_N \otimes (-2\Omega A + \Omega H H^T \Omega + \Omega B B^T \Omega + \hat{\varpi} + \frac{\hat{\chi}}{1-\varepsilon} + 2\sum_{s=1}^m \widetilde{d}_s \Omega^2 + 4N\sum_{s=1}^m \widetilde{d}_s(\widetilde{g}_s)^2(1 + \frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2})I_n + \kappa_1 Q_1^T Q_1) + \sum_{s=1}^m d_s[G^s \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]$, $\Xi_1 = I_N \otimes (\kappa_1 Q_1^T Q_2 + \Omega - Q_1^T)$, $\Psi_2 = I_N \otimes (\kappa_1 Q_2^T Q_2 - Q_2^T - Q_2)$. Then the network (1) is synchronized under the event-triggered condition (3).

Remark 3. Generally speaking, most of CNNs cannot achieve passivity and synchronization by themselves in many situations. Thus, it is necessary to adopt some control methods in these cases. Recently, event-triggered control method has been turned out a successful strategy for achieving passivity and synchronization of networks, which has many advantages, such as less data transfer, better resource utilization, smaller state error and so on. Therefore, some scholars have investigated event-triggered passivity and synchronization of CNNs [40–42]. Due to the above mentioned advantages of event-triggered control strategy, it is also meaningful to investigate passivity and synchronization of MWCNNs based on event-triggered mechanism. However, there is no results reported on this topic until now. In this section, several event-triggered passivity and passivity-based synchronization criteria are firstly established for MWCNNs by employing some new inequality techniques and Lyapunov functional method.

Remark 4. Due to the introduction of multi-weighted coupling term in the network model, the problem for achieving the event-triggered passivity and synchronization of the considered network becomes more complex, which cannot be dealt with by using the traditional event-triggered control techniques for CNNs with single weights. For overcoming this difficulty, a novel event-triggered condition (3) is designed by utilizing the own characteristics of our network model, which is an improvement of the existing event-triggered condition in literatures [13, 16]. Moreover, some new inequality techniques need to be employed in (6) and the proof of our theoretical results because of the multi-weighted coupling term and the new designed event-triggered condition. In Theorems 3.1, 3.2, 3.3 and Corollary 3.1, several criteria are proposed for achieving the passivity and synchronization of MWCNNs under the event-triggered condition (3), which are dependent on the multi-weighted coupling term.

4 Event-triggered robust passivity analysis of MWCNNs

4.1 Network model

In this section, we study the network model (1), in which the parameters A, B, H are changed within a certain range of accuracy as follows:

$$\begin{cases} A_I := \{A = \mathrm{diag}(a_i) : A^- \leqslant A \leqslant A^+, 0 < a_i^- \\ \qquad \leqslant a_i \leqslant a_i^+, i = 1, 2, \cdots, n, \forall A \in A_I\}; \\ B_I := \{B = (b_{ij})_{n\times n} : B^- \leqslant B \leqslant B^+, b_{ij}^- \leqslant b_{ij} \\ \qquad \leqslant b_{ij}^+, i = 1, 2, \cdots, n, j = 1, 2, \cdots, n, \\ \qquad \forall B \in B_I\}; \\ H_I := \{H = (h_{ij})_{n\times n} : H^- \leqslant H \leqslant H^+, h_{ij}^- \leqslant h_{ij} \\ \qquad \leqslant h_{ij}^+, i = 1, 2, \cdots, n, j = 1, 2, \cdots, n, \\ \qquad \forall H \in H_I\}. \end{cases} \tag{24}$$

Additionally, for convenience, we denote

$$\begin{aligned} \tilde{b}_{ij} &= \max\{|b_{ij}^-|, |b_{ij}^+|\}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, n; \\ \tilde{h}_{ij} &= \max\{|h_{ij}^-|, |h_{ij}^+|\}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, n; \\ \xi_B &= \sum_{i=1}^n \sum_{j=1}^n \tilde{b}_{ij}^2;\ \xi_H = \sum_{i=1}^n \sum_{j=1}^n \tilde{h}_{ij}^2. \end{aligned}$$

4.2 Robust passivity analysis

Theorem 4.1. The system (5) with the ranges of parameters defined by (24) realizes robust output-strict passivity under the condition (3), if there exist a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^m \widetilde{g}_s}$, a real number $\kappa_1 > 0$ and a matrix $0 < \Omega = \mathrm{diag}(\omega_1, \omega_2, \cdots, \omega_n) \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix} \hat{\Psi}_1 & \hat{\Xi}_1 \\ \hat{\Xi}_1^T & \hat{\Psi}_2 \end{pmatrix} \leqslant 0, \tag{25}$$

where $\hat{\Psi}_1 = I_N \otimes \Big(-2\Omega A^- + (\xi_H + \xi_B)\Omega^2 + \frac{\hat{\chi}}{1-\varepsilon} + \kappa_1 Q_1^T Q_1 + \hat{\varpi} + 2\sum_{s=1}^m \widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^m \widetilde{d}_s(\widetilde{g}_s)^2\Big(1 + \frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\Big)I_n\Big) + \sum_{s=1}^m d_s[G^s \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]$, $\hat{\Xi}_1 = I_N \otimes \Big(\kappa_1 Q_1^T Q_2 + \Omega - Q_1^T\Big)$, $\hat{\Psi}_2 = I_N \otimes \Big(\kappa_1 Q_2^T Q_2 - Q_2^T - Q_2\Big)$.

Proof: Choose the same Lyapunov functional $V(t)$ as (9). Similarly, one has

$$D^+V(t)=\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\chi_j^2}{1-\varepsilon}z_{ij^2(t)}-\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{(1-\dot{\tau}(t))\chi_j^2}{1-\varepsilon}z_{ij}^2(t-\tau(t)) + 2\sum_{i=1}^{N}z_i^T(t)\Omega\bigg[-Az_i(t)+H\hat{f}(z_i(t))+u_i(t) + B\hat{x}(z_i(t-\tau(t)))+\sum_{s=1}^{m}\sum_{j=1}^{N}d_sG_{ij}^s\Gamma^s z_j(t) +\sum_{s=1}^{m}\sum_{j\in\mathcal{N}_i}\widetilde{d}_s\widetilde{G}_{ij}^s\big(z_i(t)+\vartheta_i(t)-z_j(t)-\vartheta_j(t)\big)\bigg]. \quad (26)$$

Obviously, we have

$$\begin{aligned}&-2\sum_{i=1}^{N}z_i^T(t)\Omega Az_i(t)\\ &=-2\sum_{i=1}^{N}\sum_{p=1}^{n}\omega_p a_p z_{ip}^2(t)\\ &\leqslant -2\sum_{i=1}^{N}\sum_{p=1}^{n}\omega_p a_p^- z_{ip}^2(t)\\ &=z^T(t)\Big[I_N\otimes\Big(-2\Omega A^-\Big)\Big]z(t), \quad (27)\end{aligned}$$

$$\begin{aligned}&2\sum_{i=1}^{N}z_i^T(t)\Omega H\hat{f}(z_i(t))\\ &\leqslant\sum_{i=1}^{N}z_i^T(t)\Omega HH^T\Omega z_i(t)+\sum_{i=1}^{N}z_i^T(t)\hat{\varpi}z_i(t)\\ &\leqslant z^T(t)\Big[I_N\otimes\Big(\xi_H\Omega^2+\hat{\varpi}\Big)\Big]z(t), \quad (28)\end{aligned}$$

$$\begin{aligned}&2\sum_{i=1}^{N}z_i^T(t)\Omega B\hat{x}(z_i(t-\tau(t)))\\ &\leqslant\sum_{i=1}^{N}z_i^T(t-\tau(t))\hat{\chi}z_i(t-\tau(t))+\sum_{i=1}^{N}z_i^T(t)\Omega BB^T\Omega z_i(t)\\ &=z^T(t-\tau(t))(I_N\otimes\hat{\chi})z(t-\tau(t))\\ &\quad+z^T(t)\Big[I_N\otimes\Big(\xi_B\Omega^2\Big)\Big]z(t). \quad (29)\end{aligned}$$

By (13)-(15), (27)-(29), we know that

$$\begin{aligned}D^+V(t)\leqslant{}& z^T(t)\bigg\{I_N\otimes\bigg(-2\Omega A^-+(\xi_H+\xi_B)\Omega^2+\hat{\varpi}\\ &+2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2+4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n\bigg)\\ &+\sum_{s=1}^{m}d_s\left[G^s\otimes\left(\Omega\Gamma^s+\Gamma^s\Omega\right)\right]\bigg\}z(t)+2z^T(t)(I_N\\ &\otimes\Omega)u(t)+z^T(t-\tau(t))(I_N\otimes\hat{\chi})z(t-\tau(t))\\ &+\frac{1}{1-\varepsilon}z^T(t)(I_N\otimes\hat{\chi})z(t)-\frac{1-\dot{\tau}(t)}{1-\varepsilon}\\ &\times z^T(t-\tau(t))(I_N\otimes\hat{\chi})z(t-\tau(t))\\ \leqslant{}& z^T(t)\bigg\{I_N\otimes\bigg(-2\Omega A^-+(\xi_H+\xi_B)\Omega^2+\hat{\varpi}\\ &+2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2+\frac{\hat{\chi}}{1-\varepsilon}+4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\bigg(\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\\ &+1\bigg)I_n\bigg)+\sum_{s=1}^{m}d_s[G^s\otimes(\Omega\Gamma^s+\Gamma^s\Omega)]\bigg\}z(t)\\ &+2z^T(t)(I_N\otimes\Omega)u(t). \quad (30)\end{aligned}$$

Furthermore,

$$\begin{aligned}&D^+V(t)-2y^T(t)u(t)+\kappa_1y^T(t)y(t)\\ &\leqslant z^T(t)\bigg\{I_N\otimes\bigg(-2\Omega A^-+(\xi_H+\xi_B)\Omega^2+\hat{\varpi}+2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2\\ &+\frac{\hat{\chi}}{1-\varepsilon}+4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n\bigg)+\sum_{s=1}^{m}d_s[G^s\\ &\otimes(\Omega\Gamma^s+\Gamma^s\Omega)]\bigg\}z(t)+2z^T(t)\left[I_N\otimes\left(\Omega-Q_1^T\right)\right]u(t)\\ &-2u^T(t)\left(I_N\otimes Q_2^T\right)u(t)+\kappa_1\Big[z^T(t)\left(I_N\otimes\left(Q_1^TQ_1\right)\right)z(t)\\ &+z^T(t)\left(I_N\otimes\left(Q_1^TQ_2\right)\right)u(t)+u^T(t)\left(I_N\otimes\left(Q_2^TQ_1\right)\right)z(t)\\ &+u^T(t)\left(I_N\otimes\left(Q_2^TQ_2\right)\right)u(t)\Big]\\ &=\zeta^T(t)\begin{pmatrix}\hat{\Psi}_1 & \hat{\Xi}_1\\ \hat{\Xi}_1^T & \hat{\Psi}_2\end{pmatrix}\zeta(t), \quad (31)\end{aligned}$$

where $\hat{\Psi}_1=I_N\otimes\Big(-2\Omega A^-+(\xi_H+\xi_B)\Omega^2+\hat{\varpi}+2\sum_{s=1}^{m}\widetilde{d}_s+\frac{\hat{\chi}}{1-\varepsilon}\Omega^2+4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1+\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right)I_n+\kappa_1Q_1^TQ_1\Big)+\sum_{s=1}^{m}d_s[G^s\otimes(\Omega\Gamma^s+\Gamma^s\Omega)]$, $\hat{\Xi}_1=I_N\otimes\left(\kappa_1Q_1^TQ_2+\Omega-Q_1^T\right)$, $\zeta(t)=\left(z^T(t),u^T(t)\right)^T$, $\hat{\Psi}_2=I_N\otimes\left(\kappa_1Q_2^TQ_2-Q_2^T-Q_2\right)$.

By referring to the proof of Theorem 3.1, we can get

$$\widetilde{V}(t_b)-\widetilde{V}(t_a)\leqslant\int_{t_a}^{t_b}\left(y^T(z)u(z)-\frac{\kappa_1}{2}y^T(z)y(z)\right)dz, \quad (32)$$

where $\widetilde{V}(t)=\frac{V(t)}{2}$. □

Theorem 4.2. The system (5) with the parameter uncertainties given by (24) under the event-triggered condition (3) realizes robust input-strict passivity, if there exist a constant $0<\sigma<$

$\widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^{m}\widetilde{g}_s}$, a real number $\kappa_2 > 0$ and a matrix $0 < \Omega = \text{diag}(\omega_1, \omega_2, \cdots, \omega_n) \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix} \hat{\Psi}_3 & \hat{\Xi}_2 \\ \hat{\Xi}_2^T & \hat{\Psi}_4 \end{pmatrix} \leqslant 0, \tag{33}$$

where $\hat{\Psi}_3 = I_N \otimes \Big(-2\Omega A^- + (\xi_H + \xi_B)\Omega^2 + \hat{\varpi} + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + \frac{\hat{\chi}}{1-\varepsilon} + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1 + \frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right) I_n\Big) + \sum_{s=1}^{m} d_s[G^s \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]$, $\hat{\Xi}_2 = I_N \otimes \left(\Omega - Q_1^T\right)$, $\hat{\Psi}_4 = I_N \otimes (\kappa_2 I_n - Q_2^T - Q_2)$.

Proof: Choose the same Lyapunov functional as (9), we get

$$\begin{aligned}
&D^+V(t) - 2y^T(t)u(t) + \kappa_2 u^T(t)u(t) \\
&\leqslant z^T(t)\Big\{ I_N \otimes \Big(-2\Omega A^- + (\xi_H + \xi_B)\Omega^2 + \hat{\varpi} + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 \\
&\quad + \frac{\hat{\chi}}{1-\varepsilon} + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1 + \frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right) I_n \Big) + \sum_{s=1}^{m} d_s[G^s \\
&\quad \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]\Big\} z(t) + 2z^T(t)\left[I_N \otimes \left(\Omega - Q_1^T\right)\right] u(t) \\
&\quad - 2u^T(t)\left(I_N \otimes Q_2^T\right) u(t) + \kappa_2 u^T(t)u(t) \\
&= \zeta^T(t) \begin{pmatrix} \hat{\Psi}_3 & \hat{\Xi}_2 \\ \hat{\Xi}_2^T & \hat{\Psi}_4 \end{pmatrix} \zeta(t),
\end{aligned} \tag{34}$$

where $\hat{\Psi}_3 = I_N \otimes \Big(-2\Omega A^- + (\xi_H + \xi_B)\Omega^2 + \hat{\varpi} + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + \frac{\hat{\chi}}{1-\varepsilon} + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1 + \frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right) I_n\Big) + \sum_{s=1}^{m} d_s[G^s \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]$, $\hat{\Xi}_2 = I_N \otimes \left(\Omega - Q_1^T\right)$, $\hat{\Psi}_4 = I_N \otimes \left(\kappa_2 I_n - Q_2^T - Q_2\right)$, $\zeta(t) = \left(z^T(t), u^T(t)\right)^T$. Then the following proof run as in Theorem 3.1, we can get

$$\widetilde{V}(t_b) - \widetilde{V}(t_a) \leqslant \int_{t_a}^{t_b} \left(y^T(z)u(z) - \frac{\kappa_2}{2}u^T(z)u(z) \right) dz,$$

where $\widetilde{V}(t) = \frac{V(t)}{2}$. □

Similarly, we have the following conclusion.

Theorem 4.3. The system (5) with the ranges of parameters given by (24) realizes robust passivity under the condition (3), if there exist a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^{m}\widetilde{g}_s}$ and a matrix $0 < \Omega = \text{diag}(\omega_1, \omega_2, \cdots, \omega_n) \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix} \hat{\Psi}_3 & \hat{\Xi}_2 \\ \hat{\Xi}_2^T & \hat{\Psi}_5 \end{pmatrix} \leqslant 0, \tag{35}$$

where $\hat{\Psi}_3 = I_N \otimes \Big(-2\Omega A^- + (\xi_H + \xi_B)\Omega^2 + \hat{\varpi} + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + \frac{\hat{\chi}}{1-\varepsilon} + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(1 + \frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2}\right) I_n\Big) + \sum_{s=1}^{m} d_s[G^s \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]$, $\hat{\Psi}_5 = -I_N \otimes \left(Q_2^T + Q_2\right)$, $\hat{\Xi}_2 = I_N \otimes \left(\Omega - Q_1^T\right)$.

4.3 Robust synchronization analysis

Definition 4.1. If for all $A \in A_I, B \in B_I, H \in H_I$,

$$\lim_{t\to+\infty} \|w_i(t) - r^*\|_2 = 0, \ i = 1, 2, \cdots, N$$

holds, then the network (1) with the parameter uncertainties given by (24) realizes robust synchronization under the conditions (3) and $u_i(t) = (0, 0, \cdots, 0) \in \mathbb{R}^n, i = 1, 2, \cdots, N$.

Theorem 4.4. Assume that $\hat{\Re} : [0, +\infty) \to [0, +\infty)$ is continuously differentiable and meets

$$\nu_1(\|z(t)\|_2) \leqslant \hat{\Re}(t) \leqslant \nu_2(\|z(t)\|_\tau), \tag{36}$$

where $\|z(t)\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0}\|z(t+\theta)\|_2$, the continuous functions $\nu_1, \nu_2 : [0, +\infty) \to [0, +\infty)$ are strictly monotonically increasing and $\nu_1(e)$, $\nu_2(e)$ are positive for $e > 0$ with $\nu_1(0) = \nu_2(0) = 0$. Then, the MWCNNs (1) with the uncertain parameters defined by (24) under the event-triggered condition (3) is robustly synchronized if it is output-strictly passive with regard to $\hat{\Re}(t)$ and matrix $Q_1 \in \mathbb{R}^{n\times n}$ is nonsingular.

Proof: If system (5) with the uncertain parameters defined by (24) is robustly output-strictly passive with regard to storage function $\hat{\Re}(t)$, then

$$\hat{\Re}(t+\varsigma) - \hat{\Re}(t) \leqslant \int_t^{t+\varsigma} y^T(z)u(z)dz - \int_t^{t+\varsigma} \kappa_1 y^T(z)y(z)dz,$$

where $t \in [0, +\infty)$ and $0 < \varsigma \in \mathbb{R}$. Thus,

$$\frac{\hat{\Re}(t+\varsigma) - \hat{\Re}(t)}{\varsigma} \leqslant \frac{\int_t^{t+\varsigma} y^T(z)u(z)dz}{\varsigma} - \frac{\int_t^{t+\varsigma} \kappa_1 y^T(z)y(z)dz}{\varsigma}.$$

Let $\varsigma \to 0$, one has

$$D^+\hat{\Re}(t) \leqslant y^T(t)u(t) - \kappa_1 y^T(t)y(t).$$

Since $u_i(t) = (0, 0, \cdots, 0) \in \mathbb{R}^n$, we can get

$$\begin{aligned}
D^+\hat{\Re}(t) &\leqslant -\kappa_1 z^T(t)\left(I_N \otimes \left(Q_1^T Q_1\right)\right) z(t) \\
&\leqslant -\kappa_1 \lambda_{\min}\left(I_N \otimes \left(Q_1^T Q_1\right)\right) \|z(t)\|_2^2.
\end{aligned} \tag{37}$$

It follows from (36) and (37) that the system (5) with the uncertain parameters given by (24) is robustly asymptotically stable. Then, the network (1) with the uncertain parameters defined by (24) is robustly synchronized under the event-triggered condition (3) in the sense of Definition 4.1. □

Based on Theorems 4.1 and 4.4, we can obtain the following corollary.

Corollary 4.1. If there exist a constant $0 < \sigma < \widetilde{\sigma} = \frac{1}{2N\sum_{s=1}^{m}\widetilde{g}_s}$, a real number $\kappa_1 > 0$, two matrices $0 < \Omega = \text{diag}(\omega_1, \omega_2, \cdots, \omega_n) \in \mathbb{R}^{n\times n}$ and $Q_1 \in \mathbb{R}^{n\times n}$ such that

$$\begin{pmatrix} \hat{\Psi}_1 & \hat{\Xi}_1 \\ \hat{\Xi}_1^T & \hat{\Psi}_2 \end{pmatrix} \leqslant 0, \tag{38}$$

where Q_1 is a nonsingular matrix, $\hat{\Psi}_1 = I_N \otimes \Big(-2\Omega A^- + \frac{\hat{\chi}}{1-\varepsilon} + (\xi_H + \xi_B)\Omega^2 + 2\sum_{s=1}^{m}\widetilde{d}_s\Omega^2 + 4N\sum_{s=1}^{m}\widetilde{d}_s(\widetilde{g}_s)^2\left(\frac{\sigma^2}{(\widetilde{\sigma}-\sigma)^2} + 1\right) I_n + \hat{\varpi} + \kappa_1 Q_1^T Q_1\Big) + \sum_{s=1}^{m} d_s[G^s \otimes (\Omega\Gamma^s + \Gamma^s\Omega)]$, $\hat{\Xi}_1 = I_N$

$\otimes\left(\kappa_1 Q_1^T Q_2+\Omega-Q_1^T\right), \hat{\Psi}_2=I_N\otimes\left(\kappa_1 Q_2^T Q_2-Q_2^T-Q_2\right)$. Then the network (1) with the parametric uncertainties defined by (24) under the event-triggered condition (3) is robustly synchronized.

Remark 5. In this paper, the dimensions of the matrix conditions in Theorems 3.1, 3.2, 3.3 and Corollary 3.1 are $2nN$. After using the Schur complement, the dimension of the linear matrix inequation (LMI) for each matrix condition will reach $5nN$. For the matrix conditions in Theorems 4.1, 4.2, 4.3 and Corollary 4.1, the dimensions of corresponding LMIs is $3nN$. Therefore, these LMI conditions are dependent on the number of nodes N and dimension of each node n. Generally speaking, if the network size n and N are very large, these LMI conditions may be difficult to solve by using the existing MATLAB LMI Toolbox. As far as we know, this paper is only the first step towards investigating the event-triggered passivity and synchronization of MWCNNs with and without parameter uncertainties. In future work, we will try our best to establish some less conservative conditions by designing more effective controllers or using some other novel methods.

Remark 6. As is well known, synchronization of complex network is a very interesting topic. Actually, there are different communication constraints for synchronization. Event-triggered sampling, as an important constraint, is investigated in this paper. More precisely, several event-triggered synchronization criteria are established for MWCNNs. Recently, some other kinds of constraints, such as partial information transmission and impulse, has attracted more and more attention [4, 5, 21]. For example, the authors in [5] analyzed the partial-information-based synchronization of complex networks; Li studied impulsive synchronization of stochastic neural networks via controlling partial states in [21]. In the future, it would be very interesting to incorporate these novel constraints with the MWCNNs model considered in our paper. Moreover, the work in [4, 5, 21] inspires us to do some research on robust partial-information-based synchronization and robust impulsive synchronization of MWCNNs, which would be also a research topic of our future work.

5 Numerical examples

Example 5.1. Given the following MWCNNs with event-triggered communication:

$$\begin{aligned}\dot{w}_i(t)=&-Aw_i(t)+\sum_{s=1}^{3}\sum_{j\in\mathcal{N}_i}\tilde{d}_s\tilde{G}_{ij}^s\left(w_i(t)-w_j(t)\right)+L+u_i(t)\\&+Hf(w_i(t))+\sum_{s=1}^{3}\sum_{j=1}^{6}d_sG_{ij}^s\Gamma^s w_j(t)\\&+Bx(w_i(t-\tau(t))),\end{aligned}\qquad(39)$$

where $i=1,2,\cdots,6; A=\mathrm{diag}(7,9,12.5); d_1=0.5, d_2=0.8, d_3=0.9; \tilde{d}_1=0.05, \tilde{d}_2=0.03, \tilde{d}_3=0.01; \tau(t)=0.05-0.04e^{-t}$, $\varepsilon=0.04; f_\iota(a)=\frac{|a+1|-|a-1|}{4}, x_\iota(a)=\frac{|a+1|-|a-1|}{8}, \iota=1,2,3$; $L=(0,0,0)^T; \Gamma^1=\mathrm{diag}(0.01,0.05,0.07), \Gamma^2=\mathrm{diag}(0.6,0.9,0.4)$, $\Gamma^3=\mathrm{diag}(0.3,0.8,0.2); u_i(t)=(0.2i\sin 2t, 0.4i\sin 2t, 0.6i\sin 2t)^T$. The matrices $G^s, \tilde{G}^s, B, H, Q_1, Q_2$ are chosen as follows:

$$G^1=\begin{pmatrix}-0.22&0.05&0.07&0.04&0&0.06\\0.05&-0.2&0.08&0.06&0.01&0\\0.07&0.08&-0.24&0.04&0.02&0.03\\0.04&0.06&0.04&-0.16&0&0.02\\0&0.01&0.02&0&-0.03&0\\0.06&0&0.03&0.02&0&-0.11\end{pmatrix},$$

$$G^2=\begin{pmatrix}-0.14&0.05&0.04&0.03&0&0.02\\0.05&-0.19&0.06&0.01&0.07&0\\0.04&0.06&-0.23&0.05&0.06&0.02\\0.03&0.01&0.05&-0.13&0&0.04\\0&0.07&0.06&0&-0.13&0\\0.02&0&0.02&0.04&0&-0.08\end{pmatrix},$$

$$G^3=\begin{pmatrix}-0.15&0.06&0.04&0.03&0&0.02\\0.06&-0.21&0.05&0.01&0.09&0\\0.04&0.05&-0.15&0.03&0.02&0.01\\0.03&0.01&0.03&-0.09&0&0.02\\0&0.09&0.02&0&-0.11&0\\0.02&0&0.01&0.02&0&-0.05\end{pmatrix},$$

$$\tilde{G}^1=\begin{pmatrix}0&0.06&0.09&0.12&0.03&0.12\\0.03&0&0.15&0.03&0.15&0.03\\0.09&0.03&0&0.09&0.06&0.15\\0.03&0.03&0.15&0&0.12&0.09\\0.09&0.18&0.03&0.03&0&0.09\\0.06&0.03&0.15&0.06&0.12&0\end{pmatrix},$$

$$\tilde{G}^2=\begin{pmatrix}0&0.06&0.18&0.03&0.06&0.09\\0.15&0&0.03&0.09&0.06&0.09\\0.15&0.06&0&0.03&0.12&0.06\\0.03&0.06&0.09&0&0.15&0.09\\0.15&0.06&0.12&0.06&0&0.03\\0.15&0.06&0.09&0.03&0.09&0\end{pmatrix},$$

$$\tilde{G}^3=\begin{pmatrix}0&0.06&0.03&0.09&0.12&0.12\\0.09&0&0.15&0.06&0.03&0.09\\0.06&0.12&0&0.03&0.06&0.15\\0.09&0.15&0.03&0&0.06&0.09\\0.03&0.06&0.09&0.12&0&0.12\\0.18&0.06&0.12&0.03&0.03&0\end{pmatrix},$$

$$B=\begin{pmatrix}0.05&0.07&0.09\\0.08&0.06&0.05\\0.09&0.06&0.03\end{pmatrix}, H=\begin{pmatrix}0.6&0.3&0.8\\0.9&0.7&0.5\\0.5&0.4&0.3\end{pmatrix},$$

$$Q_1=\begin{pmatrix}0.8&0.4&0.7\\0.5&0.3&0.6\\0.4&0.2&0.8\end{pmatrix}, Q_2=\begin{pmatrix}0.5&0.3&0.6\\0.2&0.4&0.7\\0.6&0.3&0.9\end{pmatrix}.$$

Evidently, $\hat{\varpi}=\mathrm{diag}(0.25,0.25,0.25), \hat{\chi}=\mathrm{diag}(0.0625,0.0625,0.0625)$ and $r^*=(0,0,0)^T$. Take $\sigma=0.054$ in the triggering event condition (3). By making use of MATLAB, the following matrix Ω and a constant κ_1 satisfying (8) can be computed:

$$\kappa_1=0.0081,$$

$$\Omega=\begin{pmatrix}1.5575&-0.7689&0.4317\\-0.7689&1.9896&0.5151\\0.4317&0.5151&1.8167\end{pmatrix}.$$

In light of Theorem 3.1, the system (39) is event-triggered output-strictly passive in the sense of Definition 2.3. The change processes of state, input, output and event-triggered time instants are shown in Fig. 1 and Fig. 2 respectively. Fig. 3 shows the related dynamical change processes of the storage function $\widetilde{V}(t)=\frac{V(t)}{2}$ and the integral of $\int_{t_a}^{t_b}\left(y^T(z)u(z)-\frac{\kappa_1}{2}y^T(z)y(z)\right)dz$, which is an intuitively reflection of output-strict passivity in the sense of Definition 2.3.

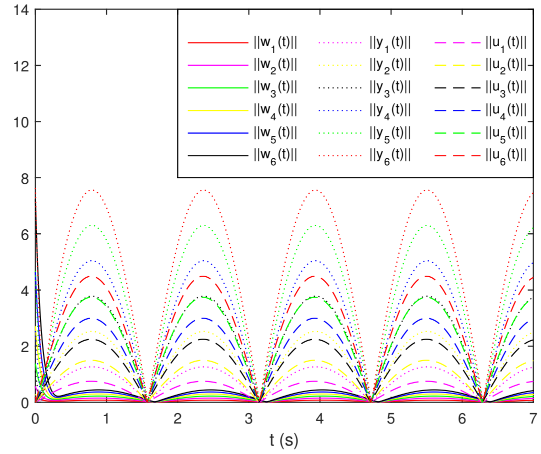

Fig. 1: The evolution of the state vectors $\|w_i(t)\|$, output vector $\|y_i(t)\|$ and input vector $\|u_i(t)\|$, $(i = 1, 2, \cdots, 6)$.

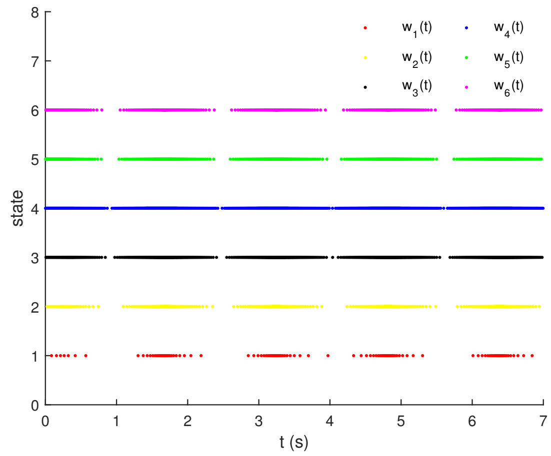

Fig. 2: The evolution of the event-triggered time instants of the state vectors $w_i(t)$, $(i = 1, 2, \cdots, 6)$.

Example 5.2. Given the following MWCNNs with event-triggered communication:

$$\begin{aligned}\dot{w}_i(t) =& - Aw_i(t) + \sum_{s=1}^{3} \sum_{j\in\mathcal{N}_i} \widetilde{d}_s \widetilde{G}^s_{ij} (w_i(t) - w_j(t)) + L \\ &+ Hf(w_i(t)) + \sum_{s=1}^{3} \sum_{j=1}^{6} d_s G^s_{ij} \Gamma^s w_j(t) \\ &+ Bx(w_i(t - \tau(t))), \end{aligned} \quad (40)$$

where $i=1,2,\cdots,6$; $d_1 = 0.3, d_2 = 0.6, d_3 = 0.7$; $\widetilde{d}_1 = 0.01$, $\widetilde{d}_2 = 0.02$, $\widetilde{d}_3 = 0.03$; $\tau(t) = 0.08 - 0.06e^{-t}$, $\varepsilon = 0.06$; $L = (0,0,0)^T$; $f_\iota(a) = \frac{|a+1|-|a-1|}{4}$, $x_\iota(a) = \frac{|a+1|-|a-1|}{8}$, $\iota=1,2,3$; $\Gamma^1 = \text{diag}(0.01, 0.05, 0.06)$, $\Gamma^2 = \text{diag}(0.04, 0.07, 0.08)$, $\Gamma^3 = \text{diag}(0.01, 0.09, 0.03)$, and the parameters A, B, H are given as the following ranges:

$$\begin{cases} A_I := \{A = \text{diag}(a_i) : A^- \leqslant A \leqslant A^+, 0 < 5i \leqslant a_i \leqslant 6i, i \\ \qquad = 1,2,3, \forall A \in A^I\}; \\ B_I := \{B = (b_{ij})_{n\times n} : B^- \leqslant B \leqslant B^+, \frac{1}{i+j+3} \leqslant b_{ij} \\ \qquad \leqslant \frac{1}{i+j+5}, i = 1,2,3, j = 1,2,3, \forall B \in B_I\}; \\ H_I := \{H = (h_{ij})_{n\times n} : H^- \leqslant H \leqslant H^+, \frac{1}{i+j+4} \leqslant h_{ij} \\ \qquad \leqslant \frac{1}{i+j+6}, i = 1,2,3, j = 1,2,3, \forall H \in H_I\}. \end{cases}$$

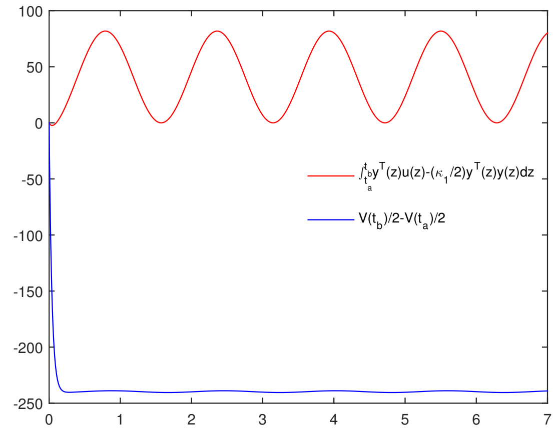

Fig. 3: The evolution of the integral of supply rate $\int_{t_a}^{t_b} \left(y^T(z)u(z) - \frac{\kappa_1}{2} y^T(z)y(z) \right) dz$, and storage function $\frac{V(t_b)}{2} - \frac{V(t_a)}{2}$.

In addition, the matrices $G^s, \widetilde{G}^s, Q_1, Q_2$ are chosen as follows:

$$G^1 = \begin{pmatrix} -0.19 & 0.04 & 0.06 & 0.03 & 0.05 & 0.01 \\ 0.04 & -0.2 & 0.07 & 0.05 & 0.04 & 0 \\ 0.06 & 0.07 & -0.23 & 0.05 & 0.01 & 0.04 \\ 0.03 & 0.05 & 0.05 & -0.14 & 0 & 0.01 \\ 0.05 & 0.04 & 0.01 & 0 & -0.1 & 0 \\ 0.01 & 0 & 0.04 & 0.01 & 0 & -0.06 \end{pmatrix},$$

$$G^2 = \begin{pmatrix} -0.2 & 0.06 & 0.04 & 0.05 & 0.03 & 0.02 \\ 0.06 & -0.17 & 0.05 & 0.02 & 0.04 & 0 \\ 0.04 & 0.05 & -0.2 & 0.04 & 0.05 & 0.02 \\ 0.05 & 0.02 & 0.04 & -0.14 & 0 & 0.03 \\ 0.03 & 0.04 & 0.05 & 0 & -0.12 & 0 \\ 0.02 & 0 & 0.02 & 0.03 & 0 & -0.07 \end{pmatrix},$$

$$G^3 = \begin{pmatrix} -0.13 & 0.05 & 0.02 & 0.04 & 0.01 & 0.01 \\ 0.05 & -0.2 & 0.06 & 0.02 & 0.07 & 0 \\ 0.02 & 0.06 & -0.13 & 0.02 & 0.02 & 0.01 \\ 0.04 & 0.02 & 0.02 & -0.11 & 0 & 0.03 \\ 0.01 & 0.07 & 0.02 & 0 & -0.1 & 0 \\ 0.01 & 0 & 0.01 & 0.03 & 0 & -0.05 \end{pmatrix},$$

$$\widetilde{G}^1 = \begin{pmatrix} 0 & 0.05 & 0.08 & 0.11 & 0.02 & 0.11 \\ 0.02 & 0 & 0.14 & 0.02 & 0.14 & 0.02 \\ 0.08 & 0.02 & 0 & 0.08 & 0.05 & 0.14 \\ 0.02 & 0.02 & 0.14 & 0 & 0.11 & 0.08 \\ 0.08 & 0.17 & 0.02 & 0.03 & 0 & 0.08 \\ 0.05 & 0.02 & 0.15 & 0.06 & 0.12 & 0 \end{pmatrix},$$

$$\widetilde{G}^2 = \begin{pmatrix} 0 & 0.05 & 0.17 & 0.02 & 0.05 & 0.07 \\ 0.15 & 0 & 0.03 & 0.09 & 0.06 & 0.09 \\ 0.14 & 0.07 & 0 & 0.02 & 0.11 & 0.05 \\ 0.03 & 0.06 & 0.09 & 0 & 0.15 & 0.09 \\ 0.14 & 0.06 & 0.10 & 0.06 & 0 & 0.03 \\ 0.13 & 0.04 & 0.06 & 0.04 & 0.07 & 0 \end{pmatrix},$$

$$\widetilde{G}^3 = \begin{pmatrix} 0 & 0.06 & 0.03 & 0.09 & 0.12 & 0.12 \\ 0.09 & 0 & 0.15 & 0.06 & 0.03 & 0.09 \\ 0.06 & 0.12 & 0 & 0.03 & 0.06 & 0.15 \\ 0.09 & 0.15 & 0.03 & 0 & 0.06 & 0.09 \\ 0.03 & 0.06 & 0.09 & 0.12 & 0 & 0.12 \\ 0.18 & 0.06 & 0.12 & 0.03 & 0.03 & 0 \end{pmatrix},$$

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$$Q_1 = \begin{pmatrix} 0.7 & 0.3 & 0.6 \\ 0.4 & 0.2 & 0.5 \\ 0.3 & 0.1 & 0.7 \end{pmatrix}, Q_2 = \begin{pmatrix} 0.4 & 0.2 & 0.3 \\ 0.1 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.8 \end{pmatrix}.$$

Evidently, $\hat{\varpi} = \text{diag}(0.25, 0.25, 0.25)$, $\hat{\chi} = \text{diag}(0.0625, 0.0625, 0.0625)$ and $r^* = (0, 0, 0)^T$. Take $\sigma = 0.059$ in the triggering event condition (3). By making use of MATLAB, the following matrix Ω and a constant κ_1 satisfying (38) can be computed:

$$\kappa_1 = 0.0833,$$

$$\Omega = \begin{pmatrix} 2.5325 & 0 & 0 \\ 0 & 1.4540 & 0 \\ 0 & 0 & 1.0882 \end{pmatrix}.$$

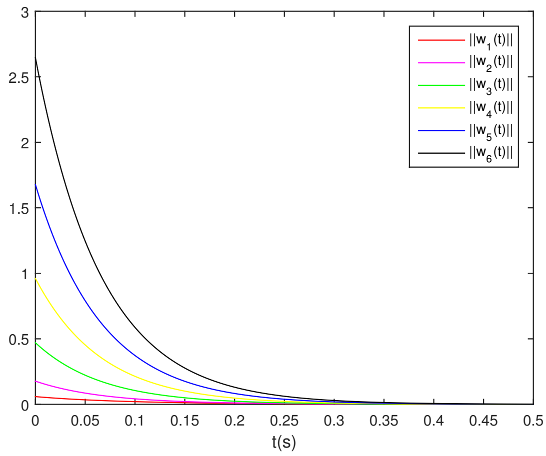

Fig. 4: The evolution of the state vectors $\|w_i(t)\|$, $(i = 1, 2, \cdots, 6)$.

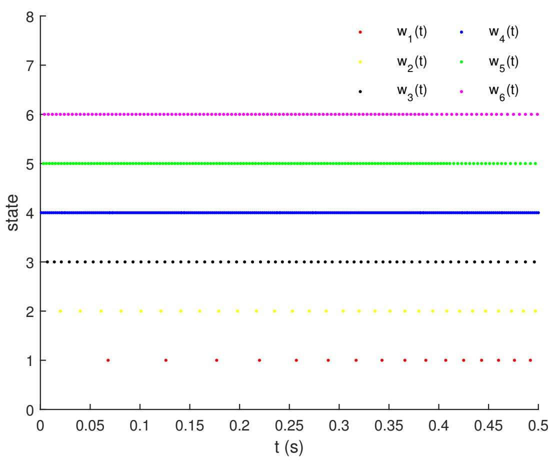

Fig. 5: The evolution of the event-triggered time instants of the state vectors $w_i(t)$, $(i = 1, 2, \cdots, 6)$.

According to Corollary 4.1, the system (40) is event-triggered robustly synchronized in the sense of Definition 4.1. The change processes of the state vectors and event-triggered time instants are shown in Fig. 4 and Fig. 5 respectively.

6 Conclusion

In this paper, the event-triggered passivity and synchronization of MWCNNs with and without parameter uncertainties have been studied. Under the help of the Lyapunov functional method, we have provided several sufficient conditions to ensure the passivity of the proposed network model and have acquired a synchronization criterion based on the obtained output-strict passivity condition for the presented network model. Besides, the robust passivity and robust synchronization have also been investigated for the MWCNNs with event-triggered communication and uncertain parameters. Finally, two illustrative examples have been shown to testify the validity of the passivity and synchronization conditions.

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7 References

1 V. Bevelevich, Classical Network Synthesis, Van Nostrand, New York, 1968
2 Wang J. L., Wu H. N., Huang T., Ren S. Y., Wu J. and Zhang X. X.: 'Analysis and control of output synchronization in directed and undirected complex dynamical networks', *IEEE Transactions on Neural Networks and Learning Systems*, 2018, **29**, pp. 3326–3338
3 Wang J. L., Wu H. N., Huang T., Ren S. Y., Wu J.: 'Passivity and output synchronization of complex dynamical networks with fixed and adaptive coupling strength', *IEEE Transactions on Neural Networks and Learning Systems*, 2018, **29**, (2018) pp. 364–376
4 Huang C., Ho D. W. C., Lu J. Q., Kurths J.: 'Pinning synchronization in T-S fuzzy complex networks with partial and discrete-time couplings', *IEEE Transactions on Fuzzy Systems*, 2015, **23**, pp. 1274–1285
5 Huang C., Ho D. W. C., Lu J. Q.: 'Partial-information-based synchronization analysis for complex dynamical networks', *Journal of the Franklin Institute*, 2015, **352**, pp. 3458–3475
6 Fan H., Zhao Y., Feng J.: 'Cluster synchronisation in non-linearly coupled impulsive networks with non-identical nodes and time-varying delays', *IET Control Theory & Applications*, 2016, **10**, pp. 762–771
7 Zhou H., Li W.: 'Synchronisation of stochastic-coupled intermittent control systems with delays and LÃl'vy noise on networks without strong connectedness', *IET Control Theory & Applications*, 2019, **13**, pp. 36–49
8 Forbes J. R., Damaren C. J.: 'Hybrid passivity and finite gain stability theorem: stability and control of systems possessing passivity violations', *IET Control Theory & Applications*, 2010, **4**, pp. 1795–1806
9 Zhang W., Li C., Yang S., Yang X.: 'Exponential synchronisation of complex networks with delays and perturbations via impulsive and adaptive control', *IET Control Theory & Applications*, 2019, **13**, pp. 395–402
10 Yang S., Li C., Huang T.: 'State-dependent impulsive synchronisation of complex dynamical networks with non-linear coupling', *IET Control Theory & Applications*, 2018, **12**, pp. 1189–1200
11 Wang J. L., Wu H. N., Huang T.: 'Passivity-based synchronization of a class of complex dynamical networks with time-varying delay', *Automatica*, 2015, **56**, pp. 105–112
12 Dong Y., Chen J. W., Xian J. G.: 'Event-triggered control for finite-time lag synchronisation of time-delayed complex networks', *IET Control Theory & Applications*, 2018, **12**, pp. 1916–1923
13 Wang A., Dong T., Liao X.: 'Event-triggered synchronization strategy for complex dynamical networks with the markovian switching topologies', *Neural Networks*, 2016, **74**, pp. 52–57
14 Hu A., Cao J., Hu M., Guo L.: 'Cluster synchronization of complex networks via event-triggered strategy under stochastic sampling', *Physica A: Statistical Mechanics and its Applications*, 2015, **434**, pp. 99–110
15 Li H., Liao X., Chen G., Hill D. J., Dong Z., Huang T.: 'Event-triggered asynchronous intermittent communication strategy for synchronization in complex dynamical networks', *Neural Networks*, 2015, **66**, pp. 1–10
16 Zhou B., Liao X., Huang T.: 'Event-based exponential synchronization of complex networks', *Cognitive Neurodynamics*, 2016, **10**, pp. 423–436

17 Dai H., Chen W., Jia J., Liu J., Zhang Z.: 'Exponential synchronization of complex dynamical networks with time-varying inner coupling via event-triggered communication', *Neurocomputing*, 2017, **245**, pp. 124–132
18 Fei Z., Gao H., Zheng W. X.: 'New synchronization stability of complex networks with an interval time-varying coupling delay', *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2009, **56**, pp. 499–503
19 Fei Z., Guan C., Gao H.: 'Exponential synchronization of networked chaotic delayed neural network by a hybrid event trigger scheme', *IEEE Transactions on Neural Networks and Learning Systems*, 2018, **29**, pp. 2558–2567
20 Guan C., Fei Z., Hamid R. K., Shi P.: 'Finite-time H_∞ synchronization for switched neural networks via quantized feedback control', *IEEE Transactions on Systems, Man and Cybernetics: Systems*, 2019, Article in Press, doi: 10.1109/TSMC.2019.2917497
21 Li Y. Y., 'Impulsive synchronization of stochastic neural networks via controlling partial states', *Neural Processing Letters*, 2016, **46**, pp. 59–69
22 Wang X., Fei Z., Gao H., Yu J.: 'Integral-based event-triggered fault detection filter design for unmanned surface vehicles', *IEEE Transactions on Industrial Informatics*, 2019, Article in Press, doi: 10.1109/TII.2019.2900230
23 Fei Z., Wang X., Liu M., Yu J.: 'Reliable control for vehicle active suspension systems under event-triggered scheme with frequency range limitation', *IEEE Transactions on Systems, Man and Cybernetics: Systems*, 2019, Article in Press, doi: 10.1109/TSMC.2019.2899942
24 Zhu F., Xia M., Antsaklis P. J.: 'On passivity analysis and passivation of event-triggered feedback systems using passivity indices', *IEEE Transactions on Automatic Control*, 2017, **62**, pp. 1397–1402
25 Li B., Wang Z., Ma L.: 'An event-triggered pinning control approach to synchronization of discrete-time stochastic complex dynamical networks', *IEEE Transactions on Neural Networks and Learning Systems*, 2018, **29**, pp. 5812-5822
26 Nowzari C., Garcia E., Corts J.: 'Event-triggered communication and control of networked systems for multi-agent consensus', *Automatica*, 2019, **105**, pp. 1–27
27 Issa A. S., Chakravarty A., Kar I.: 'Improved event-triggered adaptive control of non-linear uncertain networked systems', *IET Control Theory & Applications*, 2019, **13**, pp. 2146–2152
28 Wang J. L., Zhang X. X., Wu H. N., Huang T., Wang Q.: 'Finite-time passivity of adaptive coupled neural networks with undirected and directed topologies', *IEEE Transactions on Cybernetics*, 2018, Article in Press, doi: 10.1109/TCYB.2018.2882252
29 Lin S. R., Huang Y. L., Wang J. L., Ren S. Y., Liu D. F.: 'Passivity of coupled neural networks with different dimensional nodes', *2018 IEEE 8th Annual International Conference on CYBER Technology in Automation, Control, and Intelligent Systems (CYBER)*, Tianjin, China, 2018, pp. 775–780
30 Zhang W., Li C., Huang T., He X.: 'Synchronization of memristor-based coupling recurrent neural networks with time-varying delays and impulses', *IEEE Transactions on Neural Networks and Learning Systems*, 2015, **26**, pp. 3308–3313
31 Zhang H., Gong D., Chen B., Liu Z.: 'Synchronization for coupled neural networks with interval delay: a novel augmented Lyapunov-Krasovskii functional method', *IEEE Transactions on Neural Networks and Learning Systems*, 2013, **24**, pp. 58–70
32 Cao J. D., Chen G. R., Li P.: 'Global synchronization in an array of delayed neural networks with hybrid coupling', *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 2008, **38**, pp. 488–498
33 Li N., Cao J. D.: 'Passivity and robust synchronisation of switched interval coupled neural networks with time delay', *International Journal of Systems Science*, 2016, **47**, pp. 2827–2836
34 Huang Y. L., Ren S. Y., Wu J. G., Xu B. B.: 'Passivity and synchronization of switched coupled reaction-diffusion neural networks with non-delayed and delayed couplings', *International Journal of Computer Mathematics*, 2018, Article in Press, doi:10.1080/00207160.2018.1463437
35 Ren S. Y., Wu J. G., Wang S. X., Huang Y. L.: 'Passivity and pinning control of coupled neural networks with and without time-varying delay', *Transactions of the Institute of Measurement and Control*, 2017, **40**, pp. 2708–2717
36 Tan M. C.: 'Stabilization of coupled time-delay neural networks with nodes of different dimensions', *Neural Processing Letters*, 2016, **43**, pp. 255–268
37 Huang Y. L., Xu B. B., Ren S. Y.: 'Analysis and pinning control for passivity of coupled reaction-diffusion neural networks with nonlinear coupling', *Neurocomputing*, 2018, **272**, pp. 334–342
38 Huang Y. L., Qiu S. H., Ren S. Y.: 'Finite-time synchronization and passivity of coupled memristive neural networks', *International Journal of Control*, 2019, Article in Press, doi: 10.1080/00207179.2019.1566640
39 Huang Y. L., Wang S. X., Ren S. Y.: 'Pinning exponential synchronization and passivity of coupled delayed reaction-diffusion neural networks with and without parametric uncertainties', *International Journal of Control*, 2019, **92**, pp. 1167–1182
40 Huang C., Wang W., Cao J., Lu J.: 'Synchronization-based passivity of partially coupled neural networks with event-triggered communication', *Neurocomputing*, 2018, **319**, pp. 134–143
41 Lv S., He W., Qian F., Cao J.: 'Leaderless synchronization of coupled neural networks with the event-triggered mechanism', *Neural Networks*, 2018, **105**, pp. 316–327
42 Li L., Ho D. W. C., Cao J., Lu J.: 'Pinning cluster synchronization in an array of coupled neural networks under event-based mechanism', *Neural Networks*, 2016, **76**, pp. 1–12
43 Qiu S. H., Huang Y. L., Ren S. Y.: 'Finite-time synchronization of multi-weighted complex dynamical networks with and without coupling delay', *Neurocomputing*, 2018, **275**, pp. 1250–1260
44 Wang Q., Wang J. L., Huang Y. L., Ren S. Y.: 'Generalized lag synchronization of multiple weighted complex networks with and without time delay', *Journal of the Franklin Institute*, 2018, **355**, pp. 6597–6616
45 Wang J. L., Qin Z., Wu H. N., Huang T., Wei P. C.: 'Analysis and pinning control for output synchronization and H_∞ output synchronization of multiweighted complex networks', *IEEE Transactions on Cybernetics*, 2019, **49**, pp. 1314–1326
46 Wang J. L., Xu M., Wu H. N., Huang T.: 'Passivity analysis and pinning control of multi-weighted complex dynamical networks', *IEEE Transactions on Network Science and Engineering*, 2019, **6**, pp. 60–73
47 Wang J. L., Wei P. C., Wu H. N., Huang T., Xu M.: 'Pinning synchronization of complex dynamical networks with multiweights', *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2017, Article in Press, doi: 10.1109/TSMC.2017.2754466
48 Wang J. L., Xu M., Wu H. N., Huang T.: 'Finite-time passivity of coupled neural networks with multiple weights', *IEEE Transactions on Network Science and Engineering*, 2018, **5**, pp. 184–197
49 Tang H. A., Wang J. L., Wang L., Hu X., Zhou Y. and Duan S.: 'Impulsive control for passivity and exponential synchronization of coupled neural networks with multiple weights', *Journal of the Franklin Institute*, 2019, Article in Press, doi: 10.1016/j.jfranklin.2019.05.010
50 Lin S., Huang Y., Ren S.: 'Event-triggered passivity and synchronization of delayed multiple-weighted coupled reaction-diffusion neural networks with non-identical nodes', *Neural Networks*, 2020, **121**, pp. 259–275.
51 Huang Y., Chen W., Ren S., Zheng Z.: 'Analysis and pinning control for generalized synchronization of delayed coupled neural networks with different dimensional nodes', *Journal of the Franklin Institute*, 2018, **355**, pp. 5968–5997
52 Huang Y., Qiu S., Ren S., Zheng Z.: 'Fixed-time synchronization of coupled Cohen-Grossberg neural networks with and without parameter uncertainties', *Neurocomputing*, 2018, **315**, pp. 157–168.
53 Zheng C., Cao J.: 'Robust synchronization of coupled neural networks with mixed delays and uncertain parameters by intermittent pinning control', *Neurocomputing*, 2014, **141**, pp. 153–159
54 Li N., Cao J.: 'Periodically intermittent control on robust exponential synchronization for switched interval coupled networks', *Neurocomputing*, 2014, **131**, pp. 52–58