Event-Triggered Pinning Control of Switching Networks

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Abstract—This paper investigates event-triggered pinning control for the synchronization of complex networks of nonlinear dynamical systems. We consider networks described by time-varying weighted graphs and featuring generic linear interaction protocols. Sufficient conditions for the absence of Zeno behavior are derived and exponential convergence of a global normed error function is proven. Static networks are considered as a special case, wherein the existence of a lower bound for interevent times is also proven. Numerical examples demonstrate the effectiveness of the proposed control strategy.

Index Terms—Network analysis and control, networked control systems, switched systems.

I. INTRODUCTION

NETWORKS of dynamical systems are a suitable model for many distributed phenomena in biology, social sciences, physics, economics, and engineering [1], [2] and have attracted much research interest in the last few decades [1]–[5].

Pinning control is a strategy to steer the collective behavior of a multiagent system in a desired manner. In pinning control problems, the goal is for a set of interconnected dynamical systems to synchronize onto a given reference trajectory. The reference trajectory is supposed to be a solution of the uncoupled agents' dynamics, known *a priori*, and corresponding to some control objective. A small fraction of the agents is selected in order to receive direct feedback control. Such agents are called *pins* or *pinned agents*. The remaining agents are influenced only through their connections with other agents.

Research on pinning control has been carried out from physical and engineering perspectives. The focus is usually on the design of adaptive pinning controllers [6]–[8], or on finding criteria for the optimal selection of the agents to control [9]–[11], or on finding sufficient conditions for synchronization [12]–[14]. Such conditions usually relate to the agents' dynamics, the network topology, and the pinning scheme.

In many scenarios of multiagent coordination, the assumption that the network topology is constant over time is unrealistic. Topology variations are due to imperfect communication between agents, or simply the existence of a proximity range beyond which communication is not possible. A large number of papers investigate synchronization [15]–[18] or pinning control [19]–[21] under time-varying interaction topologies. Note that communication failures can usually be regarded as switching events. Therefore, a pinning control algorithm, which is intended to be robust against such failures, can be designed by considering the controlled network as a switched system.

Pinning control algorithms have been traditionally designed under the hypothesis of continuous-time communication. In many realistic network systems, however, such hypothesis is not verified. Also, synchronized sampled communication is hard to obtain. Event-triggered control was introduced to limit the amount of communication instances for feedback systems [22]. Recently, event-triggered control has been extended to multiagent systems [23]–[29].

In a realistic multiagent control problem, several challenges are present at the same time: nonlinear dynamics, exogenous reference signals, limited communication capacity, and time-varying interaction topology. In [30], the authors addressed the problem of event-triggered pinning synchronization considering linear diffusive coupling and unweighted network topologies. In this paper, a more general setup is considered, namely, *weighted switching topologies with generic linear interactions* are investigated. A model-based and event-triggered pinning control law is designed, which drives the agent states to an *a priori* specified common reference trajectory. We derive a set of sufficient conditions under which Zeno behavior [31] is avoided and the agents achieve exponential convergence to the reference trajectory. Static networks are studied as a special case, for which we also prove that there exists a lower bound for the interevent times in the sequences of updates of the control signals. Differently from most existing works on event-triggered multiagent control, we envision an implementation of the control algorithm which does not require agents to exchange state measurements at each update time. Agents exchange state measurements only when they establish their connection. When an agent updates its control signal to a new value, it is required to broadcast its value to its neighbors in the network. In this

way, it is possible for neighboring agents to predict each others' states consistently.

The rest of this paper is organized as follows. In Section II, we introduce some notation, formalisms, and properties that are used in later sections. In Section III, we define the mathematical model adopted to describe the network to be controlled: we state the control objective and we give the expressions of the proposed event-triggered control law. In Section IV, we prove that the closed-loop system is well-posed and achieves the control objective. In Section V, we provide some numerical examples to illustrate the effectiveness of the proposed control strategy, and compare it to a time-triggered control strategy. Section VI presents some considerations on the robustness of the proposed algorithm. Section VII concludes this paper with a summary of our results and some possible future developments.

II. PRELIMINARIES

A. Notation and Mathematical Background

For $x \in \mathbb{R}^n$, we denote $x_{[N]} := [x^\top, \ldots, x^\top]^\top \in \mathbb{R}^{nN}$. For a symmetric square matrix A, $A > 0$ denotes that A is positive definite and $A \geq 0$ denotes that it is positive semidefinite.

Definition 2.1: A function $f : (t, x) \in \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is *globally Lipschitz* with *Lipschitz constant* L_f if for all $t \in \mathbb{R}$ and all $x, y \in \mathbb{R}^n$, it holds that $\|f(t, x) - f(t, y)\| \leq L_f \|x - y\|$.

The Kronecker product is denoted as $\otimes$. We recall some useful lemmas, which can easily be derived from [32].

Lemma 2.1: Let $A \in \mathbb{R}^{n \times n}$ have eigenvalues $\lambda_1, \ldots, \lambda_n$ and eigenvectors $v_1, \ldots, v_n$, and $B \in \mathbb{R}^{m \times m}$ have eigenvalues $\mu_1, \ldots, \mu_m$ and eigenvectors $u_1, \ldots, u_m$. Then, $A \otimes B$ has eigenvalues $\lambda_i \mu_j$ and eigenvectors $v_i \otimes u_j$, $i = 1, \ldots, n$, $j = 1, \ldots, m$.

Lemma 2.2: Consider $0 \leq A_1, A_2 \in \mathbb{R}^{N \times N}$, $0 < B_1, B_2 \in \mathbb{R}^{n \times n}$, and $0 < c_1, c_2 \in \mathbb{R}$. Then, $c_1(A_1 \otimes B_1) + c_2(A_2 \otimes B_2) > 0$ if and only if $A_1 + A_2 > 0$.

Proof: Preliminarily, note that from Lemma 2.1, we have that $A \geq 0, B > 0$ implies $A \otimes B \geq 0$ and $A \otimes B > 0 \iff A > 0$, while from the equality $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$, we have $(x \otimes y)^\top[c_1(A_1 \otimes B_1) + c_2(A_2 \otimes B_2)](x \otimes y) = c_1(x^\top A_1 x)(y^\top B_1 y) + c_2(x^\top A_2 x)(y^\top B_2 y)$ for any $x \in \mathbb{R}^N, y \in \mathbb{R}^n$. Now suppose $x^\top(A_1 + A_2)x = 0$. Since $A_1, A_2 \geq 0$, this implies $x^\top A_1 x = x^\top A_2 x = 0$ which, in turn, implies $(x \otimes y)^\top[c_1(A_1 \otimes B_1) + c_2(A_2 \otimes B_2)](x \otimes y) = c_1(x^\top A_1 x)(y^\top B_1 y) + c_2(x^\top A_2 x)(y^\top B_2 y) = 0$. Vice-versa, suppose that $A_1 + A_2 > 0$. Then, at least one of $A_1, A_2 \geq 0$ must be positive definite and, consequently, at least one of $c_1(A_1 \otimes B_1), c_2(A_2 \otimes B_2) \geq 0$ must be positive definite, which implies that their sum is positive definite. ∎

B. Graph Theory

We define a *graph* as a pair $\mathcal{G} = (\mathcal{V}, W)$ consisting of a set of *nodes* $\mathcal{V} = \{1, \ldots, N\}$ and a time-varying matrix $W(t) = \{w_{ij}(t) \geq 0\} \in \mathbb{R}^{N \times N}$. A graph is *undirected* if the *weight* $w_{ij}(t) = w_{ji}(t)$ for all $i, j \in \mathcal{V}$ and at all times t; it is *simple* if $w_{ii}(t) \equiv 0$ for all $i \in \mathcal{V}$.

In a simple undirected graph, two nodes i and j are said to be *neighbors* or *adjacent* at time t if $w_{ij}(t) > 0$. The value $d_i(t) =$

$\sum_{j=1}^{N} w_{ij}(t)$ is the *degree* of node i at time t. A *path* between nodes i and j is a sequence of nodes, starting in i and ending in j or vice-versa, such that every two consecutive nodes are adjacent. A graph is *connected* if there exists a path between any two of its nodes. If a graph is not connected, then its nodes can be partitioned into subsets such that all resulting subgraphs are connected. Each such subgraph is called a *component* of the original graph.

The Laplacian matrix $L(t) = \{l_{ij}(t)\} \in \mathbb{R}^{N \times N}$ is defined as

$$l_{ij}(t) = \begin{cases} d_i(t), & \text{if } i = j, \\ -w_{ij}(t), & \text{if } i \neq j. \end{cases}$$

The Laplacian of any simple undirected graph is symmetric with zero row sum and, therefore, the vector $1_{[N]}$ is always an eigenvector with a zero eigenvalue. Also, it can be shown that the Laplacian of such graphs is positive semidefinite and that it has as many zero eigenvalues as components of the graph. In particular, when the graph is connected, the Laplacian has exactly one zero eigenvalue [5].

C. Pinning Control

For the pinning control problem, we extend the graph formalism as follows.

Definition 2.2: Consider a simple and undirected graph $\mathcal{G} = (\mathcal{V}, W)$ and a time-varying matrix $P(t) = \{p_{ij}(t) \geq 0\}$ with $p_{ij}(t) \equiv 0$ for all $i \neq j$. The triple $\mathcal{G}_a = (\mathcal{V}, W, P)$ is an *augmented graph*. A node i for which $p_{ii}(t) > 0$ is *pinned* at time t. We say that a component of the graph is *pinned* if it contains at least one pinned node. We also say that $\mathcal{G}_a$ is *pinned* if all of its components are pinned. The matrix $P(t)$ is the *pinning matrix* of $\mathcal{G}_a$. For any two positive definite and unity-norm matrices $C, K \in \mathbb{R}^{n \times n}$ and any two positive scalars $c, k > 0$, the matrix

$$L_a(t) = c(L(t) \otimes C) + k(P(t) \otimes K) \tag{1}$$

is called *augmented Laplacian* of $\mathcal{G}_a$. The smallest eigenvalue of the augmented Laplacian is called *augmented connectivity* of $\mathcal{G}_a$.

Note that by construction, the augmented Laplacian is positive semidefinite and, therefore, its augmented connectivity is non-negative. Since the augmented Laplacian is not necessarily positive definite, the augmented connectivity may be zero. The following lemma shows that positive definiteness of the augmented Laplacian is determined only by the structure of the augmented graph, and not by C, K or the scalars c, k.

Lemma 2.3: The augmented Laplacian L_a of the augmented graph $\mathcal{G}_a$ is positive definite at time t if and only if $\mathcal{G}_a$ is pinned at time t.

Proof: We are going to prove this property for $c = k = C = K = 1$, which means that $n = 1$ and $L_a(t) = L(t) + P(t)$. Thanks to Lemma 2.2, the property extends automatically to generic values of c, k, C, K. Moreover, since the statement can be applied at any generic time instant, in the proof, we are going to omit time-dependency.

Without loss of generality, suppose that the graph has $m \geq 1$ components and the nodes are ordered so that consecutive nodes belong to the same component. Namely, the first component contains nodes $n_0 = 1, \ldots, n_1$, the second component

contains nodes $n_1 + 1, \ldots, n_2$, and the last component contains nodes $n_{m-1} + 1, \ldots, n_m = N$. With such node ordering, the Laplacian is block-diagonal with m blocks $L_1, \ldots L_m$. Each block can be seen as the Laplacian of the corresponding component, which is connected by definition. Hence, each block has exactly one zero eigenvalue. The corresponding eigenvector is $1_{[\ell_i]}$, where $\ell_i := n_i - n_{i-1}$ is the dimension of the ith block or, equivalently, the number of nodes in the ith component. The pinning matrix P is diagonal by definition. Hence, the augmented Laplacian is itself block-diagonal with m blocks $L_1 + P_1, \ldots, L_m + P_m$ and, consequently, its eigenvalues are the union of the eigenvalues of these blocks. Consider the generic ith block $L_i + P_i$. Since $L_i, P_i \geq 0$, we have $x^\mathsf{T}(L_i + P_i)x = 0 \iff x^\mathsf{T} L_i x = 0$ and $x^\mathsf{T} P_i x = 0$. The first condition holds when x is a scalar multiple of $1_{[\ell_i]}$, while the second condition holds when x has zero entries whenever P_i has nonzero entries. Hence, both of them are satisfied at the same time by a nonzero x only if $P_i = 0$, which means that the ith component is not pinned. On the other hand, if the ith component is pinned, then the ith block of L_a is positive definite. We can conclude that L_a is positive definite if and only if $\mathcal{G}_a$ is pinned. ∎

III. PROBLEM STATEMENT

A. System Model and Control Objective

In this section, we define the multiagent system model, the control objective, and the event-triggered control law. We consider a network of N interconnected dynamical agents. The state of the ith agent is denoted as

$$x_i(t) := \left[x_i^{(1)}(t), \ldots, x_i^{(n)}(t) \right]^\mathsf{T} \in \mathbb{R}^n$$

and the control input applied to that agent is denoted as

$$u_i(t) := \left[u_i^{(1)}(t), \ldots, u_i^{(n)}(t) \right]^\mathsf{T} \in \mathbb{R}^n.$$

The state of each agent evolves according to the nonlinear control system

$$\dot{x}_i(t) = f(t, x_i(t)) + u_i(t), \quad x_i(0) = x_{i,0} \quad (2)$$

with $t \geq 0$. It is desired that the agents converge to the reference trajectory $r(t) \in \mathbb{R}^n$ defined by

$$\dot{r}(t) = f(t, r(t)), \quad r(0) = r_0 \quad (3)$$

with $t \geq 0$. We introduce the *tracking errors* $e_i(t) := r(t) - x_i(t)$ and the *mismatches* $e_{ij}(t) := x_j(t) - x_i(t) = e_i(t) - e_j(t)$. We also introduce the stack vectors

$$\begin{aligned} x(t) &:= \left[x_1(t)^\mathsf{T}, \ldots, x_N(t)^\mathsf{T} \right]^\mathsf{T} \\ e(t) &:= \left[e_1(t)^\mathsf{T}, \ldots, e_N(t)^\mathsf{T} \right]^\mathsf{T} \\ u(t) &:= \left[u_1(t)^\mathsf{T}, \ldots, u_N(t)^\mathsf{T} \right]^\mathsf{T} \\ r_{[N]}(t) &:= \left[r(t)^\mathsf{T}, \ldots, r(t)^\mathsf{T} \right]^\mathsf{T} \end{aligned}$$

all belonging to $\mathbb{R}^{Nn}$. Moreover, we define

$$F(t, x(t)) := \left[f(t, x_1(t))^\mathsf{T}, \ldots, f(t, x_N(t))^\mathsf{T} \right]^\mathsf{T} \in \mathbb{R}^{Nn}.$$

For convenience, we denote $\eta(t) := \|e(t)\|$. The control goal is to achieve convergence of the agents' states to the reference trajectory, in the sense that

$$\lim_{t \to +\infty} \eta(t) = 0.$$

B. Control Strategy

To solve the problem stated before, we propose the following piecewise-constant control signal for agent i in (2):

$$\begin{aligned} u_i(t) = c \sum_{j=1}^{N} w_{ij}\left(t_{k_i}^{(i)}\right) C e_{ij}\left(t_{k_i}^{(i)}\right) \\ + k p_{ii}\left(t_{k_i}^{(i)}\right) K e_i\left(t_{k_i}^{(i)}\right), \quad t \in \left[t_{k_i}^{(i)}, t_{k_i+1}^{(i)}\right) \quad (4) \end{aligned}$$

where the matrices $C, K > 0$ and scalars $c, k > 0$ are design parameters as in (1). Times $t_{k_i}^{(i)}$ when signal $u_i(t)$ changes value are *events* for agent i. Note that the control signal $u_i(t)$ is piecewise-constant, since it is held constant over each interval $[t_{k_i}^{(i)}, t_{k_i+1}^{(i)})$. Introduce the errors

$$\begin{aligned} \tilde{e}_{ij}(t) &:= e_{ij}\left(t_{k_i}^{(i)}\right) - e_{ij}(t) \\ \tilde{e}_i(t) &:= e_i\left(t_{k_i}^{(i)}\right) - e_i(t) \quad (5) \end{aligned}$$

for $t \in [t_{k_i}^{(i)}, t_{k_i+1}^{(i)})$. The sequence $\{t_{k_i}^{(i)}\}_{k_i=0}^{+\infty}$ is now defined recursively as follows:

$$\begin{aligned} t_{k_i+1}^{(i)} := \inf\Big\{ t > t_{k_i}^{(i)} : w_{ij}\left(t_{k_i}^{(i)}\right) \|\tilde{e}_{ij}(t)\| \geq \varsigma(t) \text{ for some } j \in \mathcal{V} \\ \text{or } w_{ij}(t) \neq w_{ij}\left(t_{k_i}^{(i)}\right) \text{ for some } j \in \mathcal{V} \text{ or} \\ p_{ii}\left(t_{k_i}^{(i)}\right) \|\tilde{e}_i(t)\| \geq \varsigma(t) \text{ or} \\ p_{ii}(t) \neq p_{ii}\left(t_{k_i}^{(i)}\right) \Big\} \quad (6) \end{aligned}$$

where the *threshold function* $\varsigma(t)$ is defined as

$$\varsigma(t) := \varsigma_0 e^{-\lambda_\varsigma t} \quad (7)$$

with ς_0 and λ_ς being given positive design parameters. All sequences are initialized at $t = 0$. Note that the events related to agent i include all of the instants when that agent establishes or loses a connection with another agent or with the reference. The control law (4) is now completely defined.

C. Control Implementation

Let us now discuss the implementation of the control law (4)–(7). We assume that each agent $i \in \mathcal{V}$ at each update time $t_{k_i}^{(i)}$ computes the new control input $u_i(t_{k_i}^{(i)})$ according to (4), given the values $w_{ij}(t_{k_i}^{(i)})$ for all $j \in \mathcal{V}$, $e_{ij}(t_{k_i}^{(i)})$ (with j being a neighbor of i), $p_{ii}(t_{k_i}^{(i)})$, and $e_i(t_{k_i}^{(i)})$ (if i is a pin). We also assume that each agent i is equipped with predictors that can locally estimate the dynamics (2) of the agents themselves. When two agents i, j connect, they exchange their current states x_i, x_j. With such information, they update their control inputs u_i, u_j. According to (6), acquiring a new connection triggers a control update. After the update, the agents broadcast their newly computed control inputs to their respective neighbors. Similarly, when the reference node r connects to agent i, it

sends its own current state to that agent, which updates its control input u_i, and broadcasts it to its neighbors. Hence, neighboring agents know each other's state and control input at each connection time. For each neighbor, j agent i runs a state prediction by integrating the equation

$$\dot{\hat{x}}_j^{(i)}(t) = f\left(\hat{x}_j^{(i)}(t), t\right) + u_j(t)$$

where $\hat{x}_j^{(i)}$ denotes the state of agent j predicted by agent i. A similar prediction is run by agent i for its own state and, if i is pinned, for the reference trajectory. Since the predictors are based on the exact knowledge of the agent dynamics, the predicted states $\hat{x}_j^{(i)}$ coincide with the real states $x_j(t)$. Consequently, agent i estimates $\tilde{e}_{ij}(t)$ according to (5) without communicating continuously with neighbors, but predicting $x_j(t)$ instead. Similarly, if agent i is pinned, it estimates $\tilde{e}_i(t)$ according to (5) without continuously querying the reference. Notice that having piecewise-constant control signals $u_i(t)$ implies that interagent communication is only necessary at update times, when a newly calculated control input is broadcast. Hence, the control law (4) can be implemented locally, since each agent relies only on information provided by neighboring agents. Similarly, each agent i does not need to be aware of all the events, such as topology switches, but only of those relative to $p_{ii}(t)$ and $w_{ij}(t)$, with j being its neighbor before or after the switch.

Remark 3.1: In the proposed implementation, we allow neighboring agents to have up-to-date estimates of each others' states. In principle, such estimates may be used to compute time-continuous control signals, resembling those that would be applied in traditional continuous-time consensus algorithms. However, depending on the application, there can be reasons to choose piecewise-constant control signals despite having continuous state estimates at disposal. For example, it is not uncommon that continuously varying control signals are avoided in order to reduce actuator wear, or even that the available actuators are technologically unable to exert a continuously varying control input. Moreover, if the agents were to use continuously vary control inputs, the predictors embedded in an agent would need to continuously receive information from the neighbors of that agent.

IV. MAIN RESULTS

Consider here a special class of augmented graphs, called *switching augmented graphs*.

Definition 4.1: The augmented graph $\mathcal{G}_a = \{\mathcal{V}, W, P\}$ is said to be *switching* if the matrices $W(t)$ and $P(t)$ are piecewise-constant, that is, $w_{ij}(t), p_{ii}(t) \geq 0$ are piecewise-constant for all $i, j \in \mathcal{V}$. A discontinuity point of $w_{ij}(t)$ is called a *switch* for the pair (i, j) and a discontinuity point of $p_{ii}(t)$ is called a *switch* for node i. A switching-augmented graph is said to have a *dwell time* $\tau_d > 0$ if two consecutive switches relative to the same pair or the same node are separated by a time greater than or equal to τ_d.

Note that in a switching augmented graph, the augmented Laplacian is a piecewise-constant matrix and the augmented connectivity is a piecewise-constant scalar.

We make the following assumptions.

Assumption 4.1: The function f in (2) and (3) is globally Lipschitz with Lipschitz constant $L_f > 0$.

Assumption 4.2: The interactions among agents (2) and (3) are described by a switching-augmented graph $\mathcal{G}_a = (\mathcal{V}, W, P)$ with dwell time $\tau_d > 0$. Moreover, there exists constant positive upper bounds $\bar{w}_{ij}, \bar{p}_{ii}$ such that

$$0 \leq w_{ij}(t) \leq \bar{w}_{ij}$$
$$0 \leq p_{ii}(t) \leq \bar{p}_{ii}$$

for all $t \geq 0$ and for all $i, j \in \mathcal{V}$.

Assumption 4.3: Let $\alpha(t) := \lambda_a(t) - L_f$, where $\lambda_a(t)$ is the augmented connectivity of the augmented graph $\mathcal{G}_a$ defined as in Assumption 4.2. There exist two positive constants T and ψ such that

$$0 < \lambda_\varsigma < \psi \leq \frac{1}{T} \int_t^{t+T} \alpha(\tau) d\tau.$$

Remark 4.1: To satisfy Assumption 4.3, it is not necessary for the augmented graph to be always pinned. However, it is necessary that there exists $T > 0$ such that within any finite time window $[t, t + T]$, the augmented graph is pinned for some nonzero time interval. The intuition behind Assumption 4.3 is that in order to guarantee convergence without inducing Zeno [31], the control parameters should be designed in such a way that the enforced convergence rate λ_ς is slower than the natural convergence rate of the network, which is quantified as ψ. If a faster convergence rate is enforced, Zeno behavior may occur.

Theorem 4.1: Consider the pinning control system defined by the dynamics (2), reference (3), and control law (4)–(7). If Assumptions 4.1, 4.2, and 4.3 hold, then the event sequences do not present accumulation points and the normed error $\eta(t)$ converges exponentially to zero.

Proof: We first prove that no accumulation points of events occur. To do so, we note that Assumption 4.2 excludes this possibility for events generated by switches. Still, we have to prove that there are no accumulation points of events generated by conditions $w_{ij}(t)\|\tilde{e}_{ij}(t)\| \geq \varsigma(t)$ or $p_{ii}(t)\|\tilde{e}_i(t)\| \geq \varsigma(t)$.

Consider the closed-loop dynamics of the error

$$\begin{aligned}
\dot{e}_i(t) &= \dot{r}(t) - \dot{x}_i(t) \\
&= f(t, r(t)) - f(t, x_i(t)) \\
&\quad - c \sum_{j=1}^{N} w_{ij}(t) C \left(e_{ij}(t) + \tilde{e}_{ij}(t)\right) \\
&\quad - k p_{ii}(t) K \left(e_i(t) + \tilde{e}_i(t)\right).
\end{aligned}$$

If we denote with $l_i(t)^\mathsf{T}$ and $p_i(t)^\mathsf{T}$, the ith row of the Laplacian and the pinning matrix, respectively, we can rewrite the last expression as

$$\begin{aligned}
\dot{e}_i(t) &= f(t, r(t)) - f(t, x_i(t)) \\
&\quad - \left[\left(c l_i(t)^\mathsf{T} \otimes C\right) + \left(k p_i(t)^\mathsf{T} \otimes K\right)\right] e(t) \\
&\quad - c \sum_{j=1}^{N} w_{ij}(t) C \tilde{e}_{ij}(t) - k p_{ii}(t) K \tilde{e}_i(t). \quad (8)
\end{aligned}$$

Denoting the sum of the last two terms of the above equation with $\xi_i(t)$, we have

$$
\begin{aligned}
\dot{e}_i(t) = {} & f\left(t, r(t)\right) - f\left(t, x_i(t)\right) \\
& - \left[\left(cl_i(t)^\mathsf{T} \otimes C + kp_i(t)^\mathsf{T} \otimes K\right)\right] e(t) - \xi_i(t).
\end{aligned}
$$

Denoting $\xi(t) := [\xi_1(t)^\mathsf{T}, \dots, \xi_N(t)^\mathsf{T}]^\mathsf{T}$, we can group the previous equations for $i = 1, \dots, N$ as

$$
\dot{e}(t) = F\left(t, r_{[N]}(t)\right) - F\left(t, x(t)\right) - L_a(t)e(t) - \xi(t)
$$

where $L_a(t)$ is the augmented Laplacian. From the triggering condition (6), we have $w_{ij}(t)\|\tilde{e}_{ij}(t)\| < \varsigma(t)$ and $p_{ii}(t)\|\tilde{e}_i(t)\| < \varsigma(t)$. Therefore, taking into account that the matrices C, K are unity-norm, we have

$$
\|\xi_i(t)\| \leq \left(cd_i(t) + kp_{ii}(t)\right)\varsigma(t)
$$

where $d_i(t)$ is the degree of node i. Considering this inequality for all $i \in \mathcal{V}$, we can write

$$
\|\xi(t)\| \leq \Delta(t)\varsigma(t) \tag{9}
$$

where

$$
\Delta(t) := \sqrt{\sum_{i=1}^N \left(cd_i(t) + kp_{ii}(t)\right)^2}.
$$

Then, we can write

$$
\begin{aligned}
e(t)^\mathsf{T}\dot{e}(t) = {} & e(t)^\mathsf{T}\left[F\left(t, r_{[N]}(t)\right) - F\left(t, x(t)\right)\right] \\
& - e(t)^\mathsf{T}L_a(t)e(t) - e(t)^\mathsf{T}\xi(t).
\end{aligned}
$$

Assumption 4.1 and the upper bound (9) yield

$$
e(t)^\mathsf{T}\dot{e}(t) \leq L_f\eta(t)^2 - \lambda_a(t)\eta(t)^2 + \eta(t)\Delta(t)\varsigma(t)
$$

which, if we introduce $\alpha(t)$ as in Assumption 4.3, can be rewritten as

$$
e(t)^\mathsf{T}\dot{e}(t) \leq -\alpha(t)\eta(t)^2 + \eta(t)\Delta(t)\varsigma(t).
$$

Hence

$$
\begin{aligned}
\dot{\eta}(t) = \frac{d}{dt}\|e(t)\| = \frac{e(t)^\mathsf{T}\dot{e}(t)}{\|e(t)\|} = \frac{e(t)^\mathsf{T}\dot{e}(t)}{\eta(t)} \\
\leq -\alpha(t)\eta(t) + \Delta(t)\varsigma(t). \tag{10}
\end{aligned}
$$

Applying the comparison lemma [33] to (10) over a time interval $[t, t+T)$ yields

$$
\begin{aligned}
\eta(t+T) \leq {} & e^{-\int_t^{t+T}\alpha(\tau)d\tau}\eta(t) \\
& + \int_t^{t+T} e^{-\int_\tau^{t+T}\alpha(\sigma)d\sigma}\Delta(\tau)\varsigma(\tau)d\tau.
\end{aligned}
$$

Under Assumption 4.2, we have

$$
d_i(t) \leq \bar{d}_i := \sum_{j=1}^N \bar{w}_{ij}
$$

and consequently

$$
\Delta(t) \leq \bar{\Delta} := \sqrt{\sum_{i=1}^N (c\bar{d}_i + k\bar{p}_{ii})^2}
$$

while under Assumption 4.3, we have

$$
\begin{aligned}
\int_\tau^{t+T}\alpha(\sigma)d\sigma = {} & \int_t^{t+T}\alpha(\sigma)d\sigma - \int_t^\tau\alpha(\sigma)d\sigma \\
& \geq \psi T - (\tau - t)\bar{\alpha}
\end{aligned}
$$

where

$$
\bar{\alpha} := \max_{\substack{0 \leq w_{ij}(t) \leq \bar{w}_{ij},\ \forall i,j \\ 0 \leq p_{ii}(t) \leq \bar{p}_{ii},\ \forall i}} \alpha(t).
$$

Therefore, we can bound $\eta(t+T)$ as

$$
\eta(t+T) \leq e^{-\psi T}\eta(t) + \bar{\Delta}e^{-\psi T}\int_t^{t+T} e^{(\tau-t)\bar{\alpha}}\varsigma(\tau)d\tau.
$$

Substituting $\varsigma(\tau)$ with its expression (7), we obtain

$$
\eta(t+T) \leq e^{-\psi T}\eta(t) + \frac{\bar{\Delta}e^{-\psi T}\left(e^{(\bar{\alpha}-\lambda_\varsigma)T} - 1\right)}{\bar{\alpha} - \lambda_\varsigma}\varsigma(t).
$$

Note that Assumptions 4.2 and 4.3 guarantee that $\bar{\alpha} - \lambda_\varsigma > 0$. For $t = kT$, we have

$$
\eta((k+1)T) \leq a\eta(kT) + b\varsigma(kT) \tag{11}
$$

where a and b are the positive constants

$$
a := e^{-\psi T},
$$

$$
b := \frac{\bar{\Delta}e^{-\psi T}\left(e^{(\bar{\alpha}-\lambda_\varsigma)T} - 1\right)}{\bar{\alpha} - \lambda_\varsigma}.
$$

From inequality (11), we can compute

$$
\eta(kT) \leq a^k\eta(0) + b\sum_{h=0}^{k-1}\varsigma(hT)a^{k-1-h}.
$$

Substituting the expressions of a and $\varsigma(kT)$, we obtain

$$
\eta(kT) \leq e^{-\psi kT}\eta(0) + b\varsigma_0 e^{-\psi(k-1)T}\sum_{h=0}^{k-1} e^{(\psi-\lambda_\varsigma)hT}. \tag{12}
$$

By explicitly computing the summation in (12), we obtain

$$
\begin{aligned}
\eta(kT) & \leq e^{-\psi kT}\eta(0) + b\varsigma_0 e^{-\psi(k-1)T}\frac{e^{(\psi-\lambda_\varsigma)kT} - 1}{e^{(\psi-\lambda_\varsigma)T} - 1} \\
& \leq e^{-\psi kT}\eta(0) + b\varsigma_0\frac{e^{\psi T}}{e^{(\psi-\lambda_\varsigma)T} - 1}e^{-\lambda_\varsigma kT}.
\end{aligned}
$$

Taking into account that $\lambda_\varsigma < \psi$ yields

$$
\begin{aligned}
\eta(kT) & \leq \left(\eta(0) + b\varsigma_0\frac{e^{\psi T}}{e^{(\psi-\lambda_\varsigma)T} - 1}\right)e^{-\lambda_\varsigma kT} \\
& = k'_\eta\varsigma(kT) \tag{13}
\end{aligned}
$$

where

$$k'_\eta := \frac{\eta(0)}{\varsigma_0} + b\frac{e^{\psi T}}{e^{(\psi-\lambda_\varsigma)T} - 1}.$$

Observing that $\alpha(t) = \lambda_a(t) - L_f$ is lower-bounded by $-L_f$, we can write

$$\dot{\eta}(t) \leq L_f\eta(t) + \bar{\Delta}\varsigma(t)$$

which integrates both sides over an interval $[kT, t]$ with $kT \leq t < (k+1)T$, giving

$$\begin{aligned}\eta(t) &\leq e^{L_f(t-kT)}\eta(kT) + \bar{\Delta}\varsigma_0\int_{kT}^t e^{L_f(t-\tau)}e^{\lambda_\varsigma\tau}d\tau\\ &\leq e^{L_f(t-kT)}\eta(kT)\\ &\quad + \bar{\Delta}\varsigma_0 e^{L_f t}\frac{e^{-(L_f+\lambda_\varsigma)kT} - e^{-(L_f+\lambda_\varsigma)t}}{L_f + \lambda_\varsigma}\\ &\leq e^{L_f T}\left(\eta(kT) + \frac{\bar{\Delta}}{L_f + \lambda_\varsigma}\varsigma(kT)\right).\end{aligned}$$

Together with (13), the previous inequality yields

$$\eta(t) \leq k''_\eta\varsigma(kT)$$

where

$$k''_\eta := e^{L_f T}\left(k'_\eta + \frac{\bar{\Delta}}{L_f + \lambda_\varsigma}\right).$$

As $kT \leq t < (k+1)T$, we have that $\varsigma(kT) = e^{\lambda_\varsigma T}\varsigma((k+1)T) \leq e^{\lambda_\varsigma T}\varsigma(t)$, which leads to

$$\eta(t) \leq k''_\eta e^{\lambda_\varsigma T}\varsigma(t). \tag{14}$$

The argument above is valid for all $k = 0, 1, \ldots$; therefore, inequality (14) is valid at all times $t \geq 0$. Consider now the dynamics of $\|\dot{e}_i(t)\|$. From (8), we apply the triangular inequality, which, considering Assumption 4.1 and that C, K are unity-norm, yields

$$\begin{aligned}\|\dot{e}_i(t)\| \leq (L_f \|e_i(t)\| + (c\|l_i(t)\| + kp_{ii}(t)) \|e(t)\|\\ + cd_i(t) + p_{ii}(t)k)\varsigma(t). \end{aligned}\tag{15}$$

Under Assumption 4.2, we have $p_{ii}(t) \leq \bar{p}_{ii}$, $d_i(t) \leq \bar{d}_i$, and

$$\|l_i(t)\| \leq \bar{l}_i := \sqrt{2\sum_{j=1}^N \bar{w}_{ij}^2}.$$

Substituting these bounds into (15) and noting from (14) that $\|e_i(t)\| \leq \|e(t)\| = \eta(t) \leq k''_\eta e^{\lambda_\varsigma T}\varsigma(t)$, we can write

$$\|\dot{e}_i(t)\| \leq \left[(L_f + c\bar{l}_i + k\bar{p}_{ii})k''_\eta e^{\lambda_\varsigma T} + c\bar{d}_i + k\bar{p}_{ii}\right]\varsigma(t) = \Omega_i\varsigma(t) \tag{16}$$

where

$$\Omega_i := (L_f + c\bar{l}_i + k\bar{p}_{ii})k''_\eta e^{\lambda_\varsigma T} + c\bar{d}_i + k\bar{p}_{ii}.$$

Now, observe that

$$\tilde{e}_i(t) = -\int_{t_k^{(i)}}^t \dot{e}_i(\sigma)d\sigma$$

and therefore

$$\|\tilde{e}_i(t)\| \leq \int_{t_k^{(i)}}^t \|\dot{e}_i(\sigma)\| d\sigma. \tag{17}$$

Substituting (16) into (17) yields

$$\|\tilde{e}_i(t)\| \leq \Omega_i\int_{t_{k_i}^{(i)}}^t \varsigma(\tau)d\tau \leq \Omega_i\varsigma\left(t_{k_i}^{(i)}\right)\left(t - t_{k_i}^{(i)}\right).$$

Hence, the inequality $p_{ii}(t_{k_i}^{(i)})\|\tilde{e}_i(t)\| \geq \varsigma(t)$ cannot be satisfied as long as

$$\Omega_i p_{ii}\left(t_{k_i}^{(i)}\right)\varsigma\left(t_{k_i}^{(i)}\right)\left(t - t_{k_i}^{(i)}\right) < \varsigma(t) = \varsigma\left(t_{k_i}^{(i)}\right)e^{-\lambda_\varsigma\left(t-t_{k_i}^{(i)}\right)}$$

that is

$$\Omega_i p_{ii}\left(t_{k_i}^{(i)}\right)\left(t - t_{k_i}^{(i)}\right) < e^{-\lambda_\varsigma\left(t-t_{k_i}^{(i)}\right)}.$$

The above inequality is guaranteed in a nonempty interval $[t_{k_i}^{(i)}, t_{k_i}^{(i)} + \tau]$, where $\tau > 0$ solves the equation $\Omega_i\bar{p}_{ii}\tau = e^{-\lambda_\varsigma\tau}$. Therefore, there exists a positive lower bound on the time needed to have $p_{ii}(t_{k_i}^{(i)})\|\tilde{e}_i(t)\| \geq \varsigma(t)$ after $t_{k_i}^{(i)}$.

In the same way, it is possible to prove that there exists a lower bound on the interevent time needed to have $w_{ij}(t_{k_i}^{(i)})\|\tilde{e}_{ij}(t)\| \geq \varsigma(t)$ after $t_{k_i}^{(i)}$, by considering

$$\tilde{e}_{ij}(t) = -\int_{t_{k_i}^{(i)}}^t \dot{e}_{ij}(\sigma)d\sigma = \int_{t_{k_i}^{(i)}}^t (\dot{e}_i(\sigma) - \dot{e}_j(\sigma))\,d\sigma$$

and

$$\|\tilde{e}_{ij}(t)\| \leq \int_{t_{k_i}^{(i)}}^t (\|\dot{e}_i(\sigma)\| + \|\dot{e}_j(\sigma)\|)\,d\sigma.$$

Therefore, we conclude that event sequences $\{t_{k_i}^{(i)}\}_{k_i=0}^{+\infty}$ present no accumulation points.

Exponential convergence of the error norm $\eta(t)$ follows from (14), and this concludes the proof. ∎

Remark 4.2: Since events can be generated by switches, which are exogenous with respect to the agents' dynamics, two consecutive updates of signal u_i—one caused by a switch and one caused by $p_{ii}(t_{k_i}^{(i)})\|\tilde{e}_i(t)\|$ or some $w_{ij}(t_{k_i}^{(i)})\|\tilde{e}_{ij}(t)\|$ meeting the threshold function, may be arbitrarily close in time. For this reason, although we proved that no accumulation points of events exist, our algorithm can still generate control updates that are close. However, this would not be a Zeno behavior.

Definition 4.2: An augmented graph $\mathcal{G}_a = (\mathcal{V}, W, P)$ is *static* if $W(t)$ and $P(t)$ are constant.

Note that in a static-augmented graph, all of the entries w_{ij} and p_{ii}, respectively, of W and P are constant scalars, and so is the degree d_i of node i for $i = 1, \ldots, N$; moreover, the augmented Laplacian is a constant matrix L_a and the augmented connectivity is a constant scalar λ_a. The following corollary descends directly from Theorem 4.1.

Corollary 4.1: Consider the pinning control system defined by the dynamics (2), reference (3), control law (4)–(7), and a static-augmented graph $\mathcal{G}_a$ with augmented connectivity λ_a. If
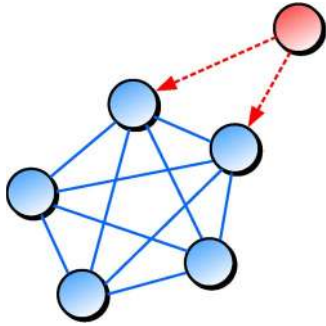
Fig. 1. Illustration of the augmented graph underlying the simulated network. The blue vertices represent the interconnected agents and the red vertex represents the reference. The blue edges represent interagent connections, while the red dashed edges represent the time-varying actions that the reference exerts on two of the agents.

Assumption 4.1 holds and $0 < \lambda_\varsigma < \alpha := \lambda_a - L_f$, then the interevent times $t^{(i)}_{k_i+1} - t^{(i)}_{k_i}$ are lower-bounded by a positive constant and the normed error $\eta(t)$ converges exponentially to zero.

Remark 4.3: When the graph is static, less conservative bounds can be derived for the normed error and the interevent times. Namely, we find

$$\eta(t) \leq k_\eta \varsigma(t)$$

with

$$k_\eta := \frac{\eta(0)}{\varsigma_0} + \frac{\Delta}{\alpha - \lambda_\varsigma}$$

and

$$\omega_i p_{ii} \left(t^{(i)}_{k_i+1} - t^{(i)}_{k_i} \right) \geq e^{-\lambda_\varsigma \left(t^{(i)}_{k_i+1} - t^{(i)}_{k_i} \right)}$$

with

$$\omega_i := \left(L_f + c \, \|l_i\| + k p_{ii} \right) k_\eta + c d_i + k p_{ii}.$$

Here, Δ, l_i, d_i, and p_{ii} are defined as for a switching graph, but they are all constant since the graph is static.

V. NUMERICAL EXAMPLES

In order to illustrate the effectiveness of the proposed control algorithm, we apply it to a simulated network of $N = 5$ identical Chua oscillators [34]. The individual dynamics of each oscillator is described by

$$f(x) = \begin{bmatrix} a \left(x_2 - x_1 - \phi(x_1) \right) \\ x_1 - x_2 + x_3 \\ -b x_2 \end{bmatrix} \quad (18)$$

where

$$\phi(y) := m_1 y + \frac{1}{2}(m_0 - m_1)\left(|y + 1| - |y - 1| \right) \ \forall y \in \mathbb{R}.$$

Choosing $a = b = 0.9$, $m_0 = -1.34$, $m_1 = -0.73$, the oscillators are globally Lipschitz with $L_f = 3.54$. See [27] for details. Let the controls be given by (4) with $C = K = I_3$, interaction gain $c = 5$, and control gain $k = 30$. All of the agents are connected to each other with interaction weight $w_{ij} = 1$.

Fig. 1 provides an illustration of the augmented graph underlying the simulated network. Our simulation is set on a time
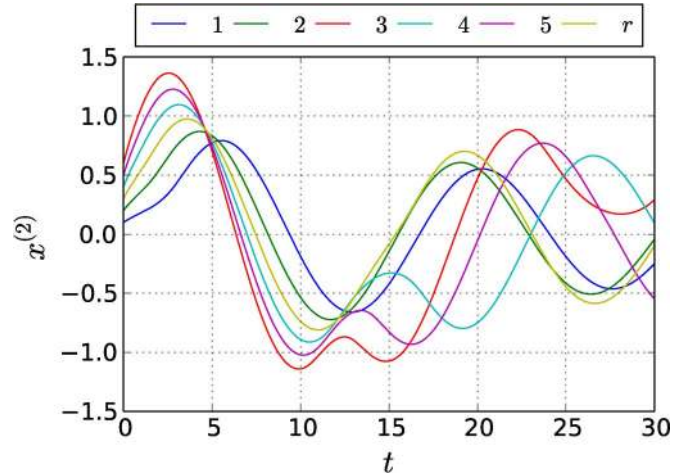

Fig. 2. Second state variable $x_i^{(2)}$ for all agents $i = 1, \ldots, 5$ and reference $r^{(2)}$, when no control input is applied. The state variables do not converge to the reference.
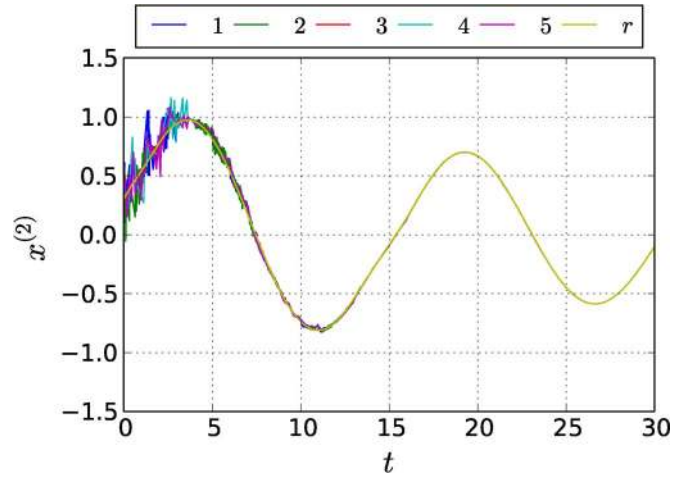

Fig. 3. Same variables as in Fig. 2, but with the proposed control algorithm applied. As predicted by our main result, all state variables converge to the reference.

interval [0, 30]s. At the beginning of the experiment, two agents are pinned with $p_{ii} = 1$, which yields $\lambda_a = 6.14$. At $t = 0.75$, one pin is removed, so that $\lambda_a = 2.88$. At $t = 0.90$ s, the two remaining pins are removed as well, which yields $\lambda_a = 0$. At $t = 1.0$, the original pinning scheme is restored and the cycle repeats itself every second. It is easy to see that Assumption 4.2 holds with $\tau_d = 1$. If we set $T = 1$, we can calculate

$$\psi = \frac{1}{T} \int_t^T \alpha(\tau) d\tau = 1.50.$$

For the threshold function, we pick $\varsigma_0 = 1.0$ and $\lambda_\varsigma = 0.30$, so that Assumption 4.3 holds. For all of the agents, the initial state values are chosen in the domain of attraction of an uncontrolled Chua's oscillator with the given parameters.

Fig. 2 shows the trend of the second state variable of all of the agents and the reference when no control input is applied. Fig. 3 shows the trend of the same state variables when the proposed control input is applied. Fig. 4 shows in detail the same state trajectories in the time interval [0.0, 1.0]. Fig. 5 shows the control updates for each of the agents during the
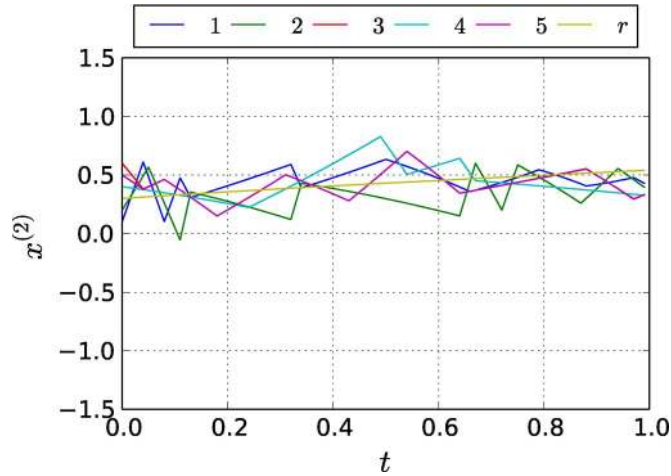
Fig. 4. Second state variable $x_i^{(2)}$ for all agents $i = 1, \ldots, 5$ and $r^{(2)}$ for the reference, in the time interval [0.0,1.0], when the proposed control algorithm is applied.
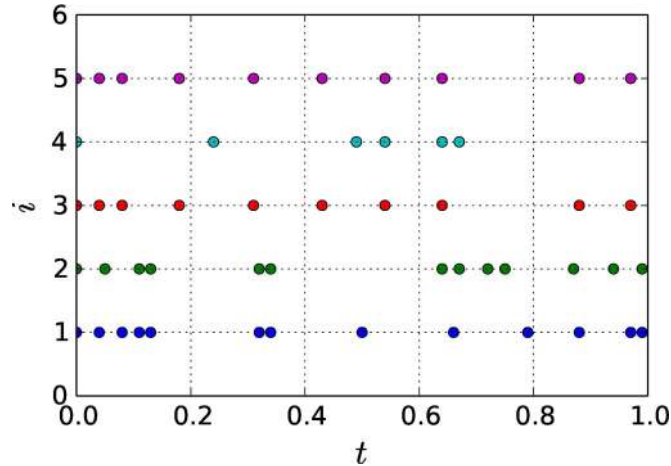

Fig. 5. Instants when a control update is triggered during the time interval [0.0, 1.0]. The vertical positions of the markers indicate which agent updates its control signal.

TABLE I
AVERAGE INTEREVENT TIME FOR EACH AGENT IN THE TIME INTERVAL [0.0, 30.0], WITH THE PROPOSED CONTROL ALGORITHM APPLIED

node	average $t_k^i - t_{k-1}^i$
1	0.061
2	0.054
3	0.115
4	0.123
5	0.115

first second of the simulation, while Table I shows the average interevent time exhibited by each agent during the simulation. It is possible to observe that the lowest of these values is above 0.05 s, which means that the agent that updates its control input more often performs less than 20 updates/s on average.

To show the advantages of using an event-triggered control law instead of a time-triggered control law, we also ran a parallel simulation with time-triggered control updates. We used the same network with the same initial conditions, but we excluded the connection failures that characterized the original simulation. We chose a fixed updating period for all of the nodes, equal to 0.04 s. Hence, all of the nodes update their
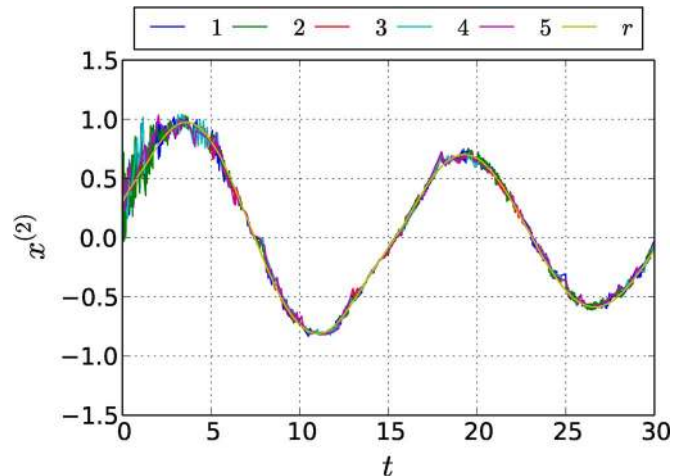

Fig. 6. Second state variable $x_i^{(2)}$ for all agents $i = 1, \ldots, 5$ and $r^{(2)}$ for the reference, when the control algorithm for robust bounded convergence is applied.

control input more often, on average, than in the original simulation. Despite more conservative settings, the closed-loop system turned out to be unstable. Simulations are omitted here for the sake of brevity.

VI. ROBUSTNESS

The previous sections focus on the scenario where exact knowledge of the agents' model is available. It was shown that Zeno behavior can be avoided even if perfect convergence is required. However, if disturbances or modelling errors are present, we can modify the algorithm so that *bounded* convergence can be achieved with the absence of Zeno behavior. We define bounded convergence as

$$\limsup_{t \to \infty} \eta(t) \leq \epsilon$$

for some $\epsilon > 0$. What needs to be changed in the algorithm is the expression of the threshold function $\varsigma(t)$. In the absence of disturbances, we set $\varsigma(t) = \varsigma_0 e^{-\lambda_\varsigma t}$ which forces all of the error signals to eventually shrink to zero. If disturbances are present, we can set $\varsigma(t) = \varsigma_{00} + \varsigma_0 e^{-\lambda_\varsigma t}$, which will force the global error $e(t)$ to converge to a ball of radius proportional to ς_0. Proof of a similar convergence result is given in [27].

To corroborate the considerations before, we reconsider the example proposed in Section V and assume that the predictors embedded in the agents' controllers rely on the model (18), with the parameters given in Section V. However, we assume that the real agents have parameter b equal to 0.84, 0.88, 0.92, and 0.96, respectively. The reference agent has $b = 0.90$. The threshold function is modified as $\varsigma(t) = \varsigma_{00} + \varsigma_0 e^{-\lambda_\varsigma t}$, with $\varsigma_{00} = 0.1$. Figs. 6–8 and Table II illustrate the results of the simulation. We can see that bounded convergence is achieved and Zeno behavior does not occur.

VII. CONCLUSION

We proposed an algorithm for event-triggered pinning synchronization of complex networks with possibly switching

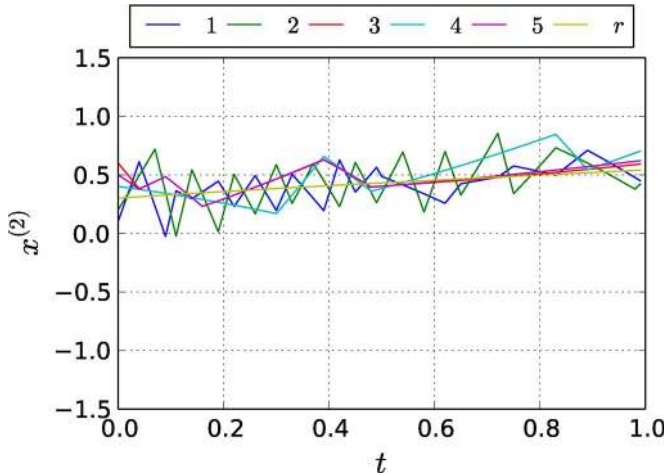Fig. 7.   Second state variable $x_i^{(2)}$ for all agents $i = 1, \ldots, 5$ and $r^{(2)}$ for the reference, in the time interval [0.0, 1.0], when the algorithm for robust bounded convergence is applied.
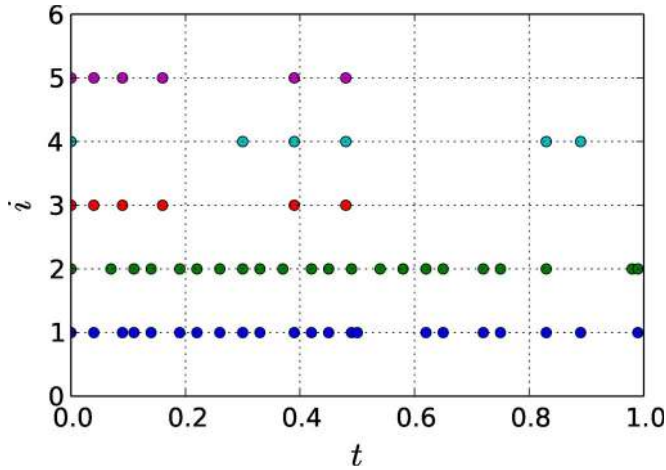

Fig. 8.   Instants when a control update is triggered in the control algorithm for robust bounded convergence, during the time interval [0.0, 1.0]. The vertical positions of the markers indicate which agent updates its control signal.

TABLE II
AVERAGE INTEREVENT TIME FOR EACH OF THE AGENTS IN THE
TIME INTERVAL [0.0, 30.0], WHEN THE ALGORITHM FOR
ROBUST BOUNDED CONVERGENCE IS APPLIED

node	average $t_k^i - t_{k-1}^i$
1	0.065
2	0.051
3	0.115
4	0.128
5	0.123

topologies. We found conditions for networked nonlinear systems with event-triggered controllers under which Zeno behavior is excluded and the norm of the error signal vanishes exponentially. A constant lower bound on the interevent times has also been provided for the case of static networks. Numerical examples have been presented to validate the theoretical results.

Some viable extensions of this work include the application of the proposed algorithm to more general classes of networks, such as networks with asymmetric couplings between the agents and networks where errors in the communication can occur, such as delays and packet drops.

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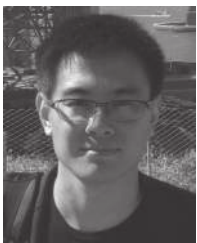
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