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## Evolution of Cosmic String Networks

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### Abstract

We summarize our new results on cosmic strings presented in [1]. These results include: (1) The application of non-equilibrium statistical mechanics to cosmic string evolution. (2) A simple "one scale" model for the long strings which has a great deal of predictive power. (3) Results from large scale numerical simulations. (4) A discussion of the observational consequences of our results. An upper bound on  $G\mu$  of approximately  $10^{-7}$  emerges from the millisecond pulsar gravity wave bound. We discuss how numerical uncertainties affect this. Any changes which weaken the bound would probably also give the long strings the dominant role in producing observational consequences.



We have recently concluded a major investigation of the evolution of cosmic strings using both analytical and numerical techniques. Our results are presented in detail in [1], and expand on work begun in [2]. The purpose of this letter is to present our most important results in a concise form.

We consider only the simplest, directional and local strings, formed as defects at a unification phase transition. These accurately follow the Nambu equations soon after they are formed. In the simplest scenario, where the strings accrete matter gravitationally the observable consequences depend only on  $\mu$ , the mass per unit length of the string[3]. This makes the theory highly predictive. As we shall explain, from improved numerical calculations and millisecond pulsar timing measurements,  $\mu$  is now very tightly constrained.

Vachaspati and Vilenkin[4] showed that the initial conditions for cosmic strings consist mainly of long strings, in the form of random walks. We take their initial configuration as our starting point.

Statistical mechanics calculations [5] show that the initial state for the strings is rapidly pushed far out of equilibrium by the expansion of the universe. As the density in strings goes down, phase space heavily favors the long string chopping up into loops. One thus expects string interactions to work in the direction of equilibration, acting to speed up the rate at which the long string density  $\rho_L$  falls.

This process is well described by a “one scale model”[6] in which the length scale  $\xi$ , defined by

$$\frac{\rho_L}{\mu} = \frac{1}{\xi^2} \quad (1)$$

is assumed to be the only relevant scale in the problem. The long strings are random walks of step size  $O(\xi)$  and mean separation  $O(\xi)$ . Equilibration proceeds through the chopping of small loops off the long string network.

On dimensional grounds, the rate for this must take the form

$$\dot{\rho}_L = -\frac{c}{\xi}\rho_L \quad (\text{due to chopping}) \quad (2)$$

Here  $\dot{\rho}_L \equiv \frac{d\rho}{dt}$  and we use units where the speed of light is unity. The ‘chopping efficiency’  $c$  is essentially a geometrical constant giving the rate, in units of  $\xi$ , that a random walk of step length  $\xi$  chops itself into loops. One expects  $c$  to be a fairly small fraction of unity. We have tested (2) in exact flat spacetime simulations and find it accurately obeyed, with  $c \approx 0.1$  [7].

In an expanding universe there are two other important effects. Hubble expansion decreases the string density and also stretches the long string. It is convenient to define a new variable

$$\gamma = \frac{R_H}{\xi} \quad (3)$$

being the ratio of the Hubble radius to the scale  $\xi$  on the string. The full equation for this is [1]

$$\dot{\gamma} = -\frac{1}{2R_H}(c\gamma^2 - (2R_H - 3)\gamma - 1) \quad (4)$$

The one-scale model has only one free parameter,  $c$ , and predicts the evolution of the long string density through the entire history of the universe. In the matter and radiation eras,  $\dot{R}_H = 3/2$  and  $2$  respectively, and (4) has the stable fixed points  $\gamma_m = \frac{1}{\sqrt{c}}$ ,  $\gamma_r \approx \frac{1}{c}$ , for  $c \ll 1$ . Whatever the initial string density,  $\gamma$  approaches the appropriate fixed point and the network is ‘scaling’ ( $\xi \propto R_H$ ). Unlike previous analytic approaches [6],[8], our discussion of the statistical mechanics fixes the *sign* of  $c$  and makes it clear that the scaling solution is inevitable[6],[9].

In [1] we made a significant advance in numerical technique over previous work [2], [10], through the use of new nonsingular evolution variables. Our new crossing detection scheme is also a significant improvement over our previous very crude one [2].

We have found that with  $c = .074$ , equation (4) gives excellent agreement with all our expanding universe simulations in the radiation, matter, and transition eras. The importance of the stable fixed point is that the state of the string network at late times depends very little on the precise initial conditions. This fact makes the resulting predictions more “fundamental”, and less susceptible to adjustments in the details of the theory. The fixed point behavior is also of practical importance. In the universe, the cosmic strings have to evolve for an expansion factor of more than  $10^{20}$  before matter accretion begins, whereas any present simulation can evolve them for a factor of 10 at best.

It is important for many predictions to understand the evolution of loops chopped off the long strings. A single isolated loop can only break up into a finite number of offspring[11]. Interactions between loops are necessary for equilibration into infinitesimal loops. However, the expansion of the universe increases the separation between loops and redshifts their velocities to zero, so all loops are eventually isolated and a distribution of non-intersecting loops remains.

We define  $\rho_{NI}(l)dl$  to be the energy density in loops with ‘length’  $l$  to  $l + dl$ , where  $l$  is the rest mass of the loop divided by  $\mu$ . The production of loops is described by

$$\dot{\rho}_{NI}(l) = \frac{\mu}{\xi^4} f_{NI}(l/\xi) \quad (5)$$

where  $f_{NI}$  is a dimensionless function, the ‘energy production function’. Following the one scale model, the assumption is that at any time and for any string density the production of loops looks the same when everything is scaled according to  $\xi$ . We then define

$$\mathcal{F}_n \equiv \int_0^\infty x^n f_{NI}(x) dx \quad (6)$$

to represent the different moments of  $f_{NI}(x)$ .

One point we stress in [1] is that different observational consequences depend on different  $\mathcal{F}_n$ ’s. For example, the zeroth moment  $\mathcal{F}_0$  is related to  $c$  as follows from (2) and (5). The reason they are not strictly equal is that there is, in addition to the long string, an ‘intermediate’ population of intersecting loops with density  $\rho_I \approx .5\rho_L$ . This results in  $\mathcal{F}_0 = c(\rho_L + \rho_I)/\rho_I > c$ .

The final non-intersecting loop distribution is given in the radiation era by

$$\rho_{NI}(l) = \frac{\mu\lambda_r}{R_H^{\frac{3}{2}}l^{\frac{3}{2}}} \quad (7)$$

$$\lambda_r = \frac{1}{2}\gamma_r^{5/2}\mathcal{F}_{1/2} \quad (8)$$

and in the matter era by

$$\rho_{NI}(l) = \frac{\mu\lambda_m}{R_H^2 l} \quad (9)$$

$$\lambda_m = \frac{2}{3}\gamma_m^3\mathcal{F}_0 \quad (10)$$

In [1] we suggest that the process of loop emission may be viewed as ‘thermal’ radiation from the long strings. Even though the system is far from equilibrium, as with radiation from a hot body the spectrum of loops may be ‘thermal’. Detailed balance considerations then indicate a simple form for the energy production function

$$f_{NI}(x) = Ax^{-\frac{1}{2}}e^{-Bx} \quad (11)$$

for  $x > 1$ .

This form is well fitted by our numerical results, right down to our resolution with  $A \approx .27, B \approx 2.6$ .

The agreement of our results with the ‘thermal’ prediction, is intriguing but certainly no more than suggestive at this stage. There are many aspects of it we do not understand. For example, detailed balance yields the equilibrium production of *all* loops, whereas our results are for *non-intersecting* loops.

We see good evidence that above our resolution the energy production function (5) is independent of time, as the one scale model predicts, and the various moments of  $f_{NI}(x)$  remain fairly constant throughout the simulation. However, in [1], we argue from the fact that chopping off must be favoured over reconnection that  $f_{NI}(x)$  *must* have an (integrable)  $x^{-\frac{1}{2}}$  divergence at small  $x$  in the radiation era scaling solution. Our results are consistent with this: we observe a buildup of  $f_{NI}(x)$  at small scales as the simulation proceeds and the resolution improves. This contributes to the small scale resolution dependence.

Since the various parameters defined above are all related to different moments of  $f_{NI}(x)$ , the uncertainty in  $f_{NI}$  gives us a rough measure of the uncertainties in the parameters. How large are these? From higher resolution runs in the radiation era, and from flat spacetime simulations we have recently performed, we estimate our value of  $c$  could be too low by as much as a factor of 2. Considering the dependence on  $f_{NI}$ , one finds from the above equations that  $\gamma_r$  could be too high by a factor of 2,  $\lambda_r$  too large by a factor of  $2^{\frac{3}{2}} \approx 3$ ,  $\gamma_m$  could be too high by  $2^{\frac{1}{2}}$  and  $\lambda_m$  by a factor of  $2^{\frac{1}{2}}$ . Here we have made the simplistic assumption that all  $\mathcal{F}_n$ ’s have the same uncertainty. In fact the higher moments depend less on small scales and should have smaller uncertainties. We caution that there could also be systematic uncertainties such as those we discuss below which would alter these estimates.

Recent results from the simulations by Bennett and Bouchet [12] differ qualitatively from ours in some respects. Their chopping efficiency is a factor of two larger, but that is consistent with our uncertainties. However, they see much more of their loop production all the way down at their scale of resolution. We believe the differences arise from the fact that our algorithm introduces numerical diffusion on the scale of our resolution ( $\ll \xi$ ) whereas the method of Bennett and Bouchet prevents the scale of kinks falling below a fixed minimal scale, of order their resolution. It is still not clear to us which method better approximates the continuum limit.

Now we turn to the observational consequences of cosmic strings. The observation most likely to rule out the simplest gravitational accretion scenario is the millisecond

pulsar timing constraint [13],[14]. String loops radiate energy at a rate  $\dot{E} = -\Gamma G\mu^2$  with  $\Gamma$  typically 40-70 for simple loop trajectories [15], [16], [17]. If gravity waves with periods of years today are redshifted back to early times, one finds the waves were emitted around  $10^6$  seconds, well before equal matter and radiation density. For these periods the density in gravity waves today may be calculated very simply from energy conservation and scaling. If  $\rho_g(\omega)d\omega$  is the energy density in waves of frequency  $\omega$  to  $\omega + d\omega$  and  $\rho_{rad}$  the density in background radiation (photons plus neutrinos) then we find [1]

$$\frac{\omega\rho_g(\omega)}{\rho_{rad}} = 2.2 \times 10^{-3} \lambda_r (G\mu_0)^{\frac{1}{2}} \Gamma_{50}^{-\frac{1}{2}} \quad (12)$$

where  $\lambda_r$  is given from (7),(8).  $G\mu_0 = G\mu \times 10^6$  and  $\Gamma_{50} = \Gamma/50$ .

From (12) we calculate the density in gravity waves divided by the density in the present microwave background  $\rho_\gamma = \rho_{rad}/(1 + .23N_\nu)$  where  $N_\nu$  is the number of neutrino species still relativistic when the gravity waves were emitted. We take  $N_\nu = 3$ . Unpublished results from millisecond pulsar timing indicate that today  $\omega\rho_g(\omega)/\rho_\gamma < 3.2 \times 10^{-3}$ [18], for waves with periods of 6 years, in the form of a stochastic background such as strings would produce. There are also weaker bounds at shorter periods - the maximum sensitivity is to waves whose period equals the total observation time. This gives the bound

$$\lambda_r (G\mu_0)^{\frac{1}{2}} \Gamma_{50}^{-\frac{1}{2}} < 0.9 \quad (13)$$

The observational limit is conservative: a full analysis has yet to be completed[18]. In our simulations,  $\lambda_r \approx 10$ , which rules out strings with  $G\mu > 10^{-6}$ . However as we emphasized in [1], there are still large uncertainties. With our uncertainty estimates given above, we still rule out  $G\mu > 10^{-7}$ . This value is the minimum necessary for significant gravitational accretion by strings to occur by today [1].

There are a few small effects that could weaken the bound. Late time splitting of loops or annihilation of string near cusps could conceivably reduce (12) by a factor of two. Therefore a reduction of the observational bound by a factor of 3 is still probably necessary to rule out the simplest cosmic string scenario. This should be possible soon[18].

Bennett and Bouchet have recently suggested a way around the problem [12] - from the fact that loop production evolves towards smaller and smaller scales in their latest simulations. They suggest that in the true scaling solution the long string may lose *most* of its energy to loops well below the resolution of current simulations. To see how small

these would have to be, one can show from the above that in the scaling solution  $\lambda_r$  is fixed by energy conservation to be

$$\lambda_r = \frac{\gamma_r^{\frac{3}{2}}}{2} \langle \left(\frac{l}{\xi}\right)^{\frac{1}{2}} \rangle \quad (14)$$

where  $\langle \rangle$  means the expectation value weighted with  $f_{NI}(x)$  defined above, and  $\lambda_r$  is defined in terms of loop energies rather than masses. The quantity  $\gamma_r$  is as defined in (1),(2), where we include intersecting loops in the long string density. Bennett and Bouchet[12], and Allen and Shellard [19] report in recent simulations that  $\gamma_r^2 \approx 50$ . Even using this value, (13) and (14) still imply that  $\langle (l/\xi)^{\frac{1}{2}} \rangle < .07$ . Thus to avoid the bound for  $G\mu_6 = 1$  typical loops must be produced with lengths of order  $5 \times 10^{-3}\xi$ . With redshifting and late time effects one would still have to argue that most of the energy in the long string goes into loops of length one or two percent of the scale  $\xi$ .

If this is the case, then it is clear that the dominant role in large scale structure formation would have to be played by the long strings. Detailed simulations of the accretion of matter around strings in the matter era are now underway [20]. It is already clear that long strings play a major role.

In some situations, the gravity wave bound does not apply. Strings may couple to other long range fields and emit electromagnetic or goldstone boson radiation. For example the global string scenario [3] with  $G\mu = 10^{-6}$  is still viable, though the dynamics of the network is much harder to calculate.

We have also considered the gravitational lensing produced by strings in the matter era, and conclude that the fraction  $F$  of sky lensed by strings out to a redshift  $Z$  would be

$$F \approx 10^{-4} Z^2 G\mu_6 (\lambda_m + \gamma_{m4}^2) \quad (15)$$

where, from our simulations we find  $\lambda_m = 2.1$ , and  $\gamma_{m4} = \gamma_m/4 = 1.0$ , with errors estimated above. The typical angle between lensed objects would be  $2.6 G\mu_6$  arc seconds[21]. With our results, there would be approximately one lensed galaxy per 2 square degrees of the sky. Most galaxies would be lensed by long loops, and since galaxy clustering is quite weak at high  $Z$ , the lensed galaxies would typically be separated by a degree or so on the sky. Thus the Cowie-Hu event[22], where four apparently lensed galaxies were found within a region 40 arc seconds square, cannot be claimed as a successful prediction of the cosmic string scenario.

To conclude, we have reported several analytical and numerical advances in the understanding of cosmic string networks. These advances have emerged from the application of statistical mechanics to cosmic strings, from the development of the one scale model, and from the use of our new non-singular variables in computations. We have also discussed some of the important observable consequences. The best chance of ruling out (or confirming!) the simplest gravitational accretion cosmic string scenario lies with the millisecond pulsar. The present constraint appears to threaten the survival of this scenario, but a better understanding of the string network on very small scales will be necessary before there is general agreement on this point. More complex cosmic string scenarios, however, are not as tightly constrained.

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