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# Evolution of Irregularities in a Chaotic Early Universe

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First the model of a dust-like universe in the anti-Newtonian approximation is derived, which describes the transition from an inhomogeneous universe to the Friedmann universe. Secondly in the post-anti-Newtonian approximation the perturbations to the anti-Newtonian model are treated for the radiation-dominated and dust-like cases, and it is shown that, if the linear dimension of irregularities is comparable with the radius of spatial curvature within them, the perturbations grow so that non-linearity plays a dominant role. Lastly a one-dimensional model for the anisotropic collapse to be followed after the above stage is derived and the energy release due to the shock-wave dissipation of the kinetic energy in this collapse is estimated.

## § 1. Introduction

Our observable universe in the present state seems to be homogeneous and isotropic, if we smooth-out local irregularities. The measurements for isotropy of cosmic microwave radiation show that the universe has been isotropic after the decoupling epoch, which corresponds to the redshift $z=10$ or 1500 according to whether or not the universe is filled with fully ionized intergalactic gas.[1] However, we have no direct observational evidence that the universe was homogeneous and isotropic at any epoch earlier than the decoupling epoch.

From the theoretical point of view it seems fairly reasonable that for the early universe we assume a general inhomogeneous model rather than a homogeneous and isotropic model which is remarkably different from general models. Accordingly it seems significant to develop a theory for the evolution of fully inhomogeneous (or chaotic) models and their reduction to a homogeneous and isotropic model.

In a previous paper[2] which will be referred to as [1], we have derived a general inhomogeneous model in the radiation-dominated case. There we have used the anti-Newtonian approximation, in which only the lowest-order terms with respect to $ct/L$ are taken into account, where $c, t, L$ are the light velocity, the cosmic time and the linear dimension of irregularities, respectively. On the basis of this model we have analyzed the element formation in a chaotic universe.[3] Moreover we have recently discussed in another paper[4] how to construct such a theory.

In this paper we shall first in § 2 apply the above approximation to the dust-like case. The reason why this case also is taken up here is that we have no direct evidence that the universe earlier than the decoupling epoch was filled with radiation more than ordinary matter, and is that the very early universe may have been cold and the cosmic radiation may have been derived later, as Rees[5] has indicated its possibility. Similarly to the radiation-dominated case, the derived model in the dust-like case describes the transition from anisotropic expansion to the isotropic Friedmann expansion.

In § 3 we shall treat the post-anti-Newtonian approximation by considering the next terms in the expansion with respect to $ct/L$. This approximation will be applied to both cases of the radiation-dominated one and the dust-like one. In this approximation the effect brought about by the full inhomogeneity of the spatial curvature will be considered and it will be shown that the density perturbations aroused within the irregularities grow to the epoch $ct/L=1$, when the expansion with respect to $ct/L$ is inapplicable. At this epoch the perturbations will essentially be nonlinear if the radius $r$ of the curvature is comparable with $L$. Since the evolution of these density perturbations is anisotropic in general, the evolution in the case $r/L\sim 1$ after the epoch $ct/L=1$ can ideally be described as a one-dimensional collapse, and will be followed by their shock-wave dissipation. On the other hand, if $r>L$, the above perturbations will remain to be small at the epoch $ct/L=1$, and thereafter in the radiation-dominated case they will propagate as sound waves while in the dust-like case they will continue to grow to the epoch $ct/r=1$.

In § 4 we shall treat a one-dimensional relativistic collapse of dust-like matter and estimate the thermal energy which will be released by the shock-wave dissipation. This treatment is analogous to Zeldovich's Newtonian one[6] at the later stage. It will be shown that this heating process can be expected as the origin of cosmic microwave radiation. In § 5 some concluding remarks will be given.

## § 2. Dust-like model in an anti-Newtonian approximation

In the dust-like case the Einstein equation in the synchronous coordinate condition[7] is written as

$$\frac{1}{2}\dot{\kappa}_\alpha{}^\alpha+\frac{1}{4}\kappa_\alpha{}^\beta\kappa_\beta{}^\alpha=\varepsilon\left(u_0u^0+\frac{1}{2}\right), \tag{2·1}$$

$$\frac{1}{2}(\kappa^\beta_{\beta;\alpha}-\kappa^\beta_{\alpha;\beta})=\varepsilon u_\alpha u^0\,, \tag{2·2}$$

$$\frac{1}{2\sqrt{-g}}(\sqrt{-g}\,\kappa_\alpha{}^\beta)^\cdot+P_\alpha{}^\beta=\varepsilon\left(u_\alpha u^\beta+\frac{1}{2}\delta_\alpha{}^\beta\right), \tag{2·3}$$

where the notations are the same as in [I]. In this section $P_\alpha{}^\beta$ and $u_\alpha u^\alpha$ are neglected, because they are assumed to be small compared with $(ct)^{-2}$, $c^2$, respectively. Accordingly Eqs. (2·1)$\sim$(2·3) are reduced to

$$\dot{\kappa}_\alpha{}^\alpha+\frac{1}{2}\kappa_\alpha{}^\beta\kappa_\beta{}^\alpha=-\varepsilon\,, \tag{2·4}$$

$$\frac{1}{2}(\kappa^\beta_{\beta;\alpha}-\kappa^\beta_{\alpha;\beta})=\varepsilon u_\alpha\,, \tag{2·5}$$

$$\frac{1}{\sqrt{-g}}(\sqrt{-g}\,\kappa_\alpha{}^\beta)^{\cdot}=\varepsilon\delta_\alpha{}^\beta\,. \tag{2·6}$$

From Eq. (2·6) and $\kappa_\mu{}^\mu=(\ln|g|)^{\cdot}$, the functional form of $\kappa_\alpha{}^\beta$ is found to be

$$\kappa_\alpha{}^\beta=(\dot{X}/X)\delta_\alpha{}^\beta+A_\alpha{}^\beta X^{-3/2},\ X\equiv(-g)^{1/3}, \tag{2·7}$$

where $A_\alpha{}^\beta$ is traceless functions of spatial variables with $g_{\alpha\mu}A_\beta{}^\mu=g_{\beta\mu}A_\alpha{}^\mu$. Substituting Eq. (2·7) into Eqs. (2·5) and (2·6), we obtain

$$X=[(t-T+\sqrt{9\xi/8})^2-9\xi/8]^{2/3}\,, \tag{2·8}$$

$$\varepsilon=\frac{4}{3}X^{-3/2}\,, \tag{2·9}$$

$$u_\alpha=-\frac{3}{2}(X_{,\alpha}/X)\sqrt{X^{3/2}+9\xi/8}+(\sqrt{X^{3/2}+9\xi/8})_{,\alpha}$$
$$-\frac{3}{8}\left(A^\beta_{\alpha,\beta}-\frac{1}{2}g^{\mu\nu}g_{\mu\lambda,\alpha}A_\nu{}^\lambda\right), \tag{2·10}$$

where $\xi\equiv A_\alpha{}^\beta A_\beta{}^\alpha/12$ and $T$ indicates the time dilatation. Moreover, if we define $\tau$ by

$$\tau\equiv\frac{1}{3}(2/\xi)^{1/2}\ln[(t-T)/(t+T+2\sqrt{9\xi/8})],$$

we obtain from the definition of $\kappa_\alpha{}^\beta$

$$d/d\tau(g_{\alpha\beta}/X)=A_\alpha{}^\gamma(g_{\beta\gamma}/X),$$

so that by the use of replacement of (3·11) in [I] all metric components can be derived in the same way as in [I]. If we write the eigenvectors and eigenvalues as $e_\alpha{}^a$, $r_a$ $(a=1, 2, 3)$ which correspond to $D_{\alpha a}$ and $r_a$ of Eq. (3·13) in [I], the line element can be expressed as

$$ds^2=c^2dt^2-\gamma_{ab}e_\alpha{}^a e_\beta{}^b dx^\alpha dx^\beta\,,$$
$$\gamma_{ab}=X\begin{pmatrix}e^{r_1\tau} & & \\ & e^{r_2\tau} & \\ & & e^{r_3\tau}\end{pmatrix}, \tag{2·11}$$

and $p_a=(1/3)(1+r_a/\sqrt{2\xi})$ satisfies the relations $\sum_a p_a=\sum_a p_a{}^2=1$. In the neighbourhood of $t-T=0$, this metric approaches the Kasner metric, because $Xe^{r_a\tau}\propto(t-T)^{2p_a}$, and, at the epoch $(t-T)/\sqrt{\xi}\gg1$, the metric approaches the Friedmann isotropic one.

## § 3. Post-anti-Newtonian approximation

Now let us take into consideration the terms of spatial curvature $P_\alpha{}^\beta$ which were neglected as being small in the previous treatments. Here for the metric $g_{\alpha\beta}$, $\kappa_\alpha{}^\beta$ and $P_\alpha{}^\beta$ we shall use their triad components $\gamma_{ab}$, $\kappa_a{}^b$ and $P_a{}^b$. As the triads we choose $e_\alpha{}^a$, $e_a{}^\alpha$ $(e_\alpha{}^a e_b{}^\alpha = \delta_b{}^a)$ which were derived in the anti-Nowtonian approximation. Then $\gamma_{ab}$ is not diagonal in general, and the Einstein equation is written as

$$\dot{\kappa}_c{}^c + \frac{1}{2}\kappa_a{}^b \kappa_b{}^a = -(\varepsilon + 3p), \tag{3·1}$$

$$\kappa^c_{c,\alpha} e_a{}^\alpha - \kappa^b_{a,\beta} e_b{}^\beta - \kappa_a{}^b (e^\beta_{b;\beta} + e_a{}^\alpha e^a_{\alpha;\beta} e_b{}^\beta) = 2(\varepsilon + p) u_a , \tag{3·2}$$

$$\frac{1}{\sqrt{-g}}(\sqrt{-g}\,\kappa_a{}^b)^\cdot + 2P_a{}^b = (\varepsilon - p)\,\delta_a{}^b , \tag{3·3}$$

where $p$ denotes the pressure and $u^\alpha u_\alpha / u^0 u_0$ is neglected compared with unity. By contracting Eq. (3·3), we have

$$\frac{1}{\sqrt{-g}}(\sqrt{-g}\,\kappa_c{}^c)^\cdot + 2P_c{}^c = 3(\varepsilon - p)$$

and by substituting this into Eq. (3·3), we obtain

$$\frac{1}{\sqrt{-g}}\left[\sqrt{-g}\left(\kappa_a{}^b - \frac{1}{3}\kappa_c{}^c \delta_a{}^b\right)\right]^\cdot + 2\left(P_a{}^b - \frac{1}{3}P_c{}^c \delta_a{}^b\right) = 0 .$$

Integrating this equation, we get

$$\kappa_a{}^b = \frac{1}{3}\kappa_c{}^c \delta_a{}^b - \frac{2}{\sqrt{-g}}\int^t \sqrt{-g}\left(P_a{}^b - \frac{1}{3}P_c{}^c \delta_a{}^b\right) dt ,$$

where $\kappa_c{}^c = (\ln \gamma)^\cdot$ and $\gamma \equiv \det(\gamma_{ab})$. If we express by $\delta\kappa_a{}^b$, $\delta\gamma$ the perturbations of $\kappa_a{}^b$, $\gamma$ due to $P_a{}^b$, we have

$$\delta\kappa_a{}^b = \frac{1}{3}(\delta \ln \gamma)^\cdot \delta_a{}^b - 2X^{-3/2}\int^t X^{3/2}\left(P_a{}^b - \frac{1}{3}P_c{}^c \delta_a{}^b\right) dt , \tag{3·4}$$

where $X$ is the unperturbed part of $(-g)^{1/3}$. From Eqs. (3·1) and (3·3), moreover, we get

$$\delta\dot{\kappa}_c{}^c + \kappa_b{}^a \delta\kappa_a{}^b + \frac{1+3p/\varepsilon}{3(1-p/\varepsilon)}\left[\delta\dot{\kappa}_c{}^c + \frac{3}{2}(\delta \ln X)^\cdot \kappa_c{}^c + 2P_c{}^c\right] = 0 ,$$

and , substituting Eqs. (3·4) and (2·6) into this equation, we obtain

$$(\delta \ln \gamma)^{\cdot\cdot} + \frac{3}{8}(3 + p/\varepsilon)(\dot{X}/X)(\delta \ln \gamma)^\cdot = -\frac{1}{2}(1 + 3p/\varepsilon) P_c{}^c$$

$$+ \frac{3}{2}(1 - p/\varepsilon) X^{-3} A_b{}^a \int^t X^{3/2} P_a{}^b dt \equiv Q(t, x^\alpha) .$$

Integrating this equation, we get

$$\delta \ln \gamma = \int^t dt' X(t', x^\alpha)^{-\mu} \int^{t'} dt'' X(t'', x^\alpha)^\mu Q(t'', x^\alpha), \tag{3·5}$$

where $\mu \equiv -(3/8)(3+p/\varepsilon)$ and $p/\varepsilon$ is assumed to be constant. From the definition of $\kappa_a{}^b$, moreover, we have

$$\begin{aligned}\delta\kappa_a{}^b &= \delta\gamma^{bc}\dot{\gamma}_{ca} + \gamma^{bc}\delta\dot{\gamma}_{ca} \\ &= -\gamma^{bd}\gamma^{ce}\dot{\gamma}_{ca}\delta\gamma_{de} + \gamma^{bc}\delta\dot{\gamma}_{ca},\end{aligned}$$

and, if we notice that $\gamma_{ab}$, $\gamma^{ab}$ have only diagonal components $(a_1^2, a_2^2, a_3^2)$, $(a_1^{-2}, a_2^{-2}, a_3^{-2})$, respectively, we can integrate the above equation as

$$\delta\gamma_{ab} = a_b^2 \int^t dt (a_b/a_a)^2 \delta\kappa_a{}^b, \tag{3·6}$$

where the summation with respect to suffices $a$, $b$ is not taken. By the use of Eqs. (3·4) and (3·5), $\delta\gamma_{ab}$ can be derived explicitly.

For the perturbation of $\varepsilon$ we get from Eqs. (2·6), (3·1), (3·4) and (3·5)

$$\begin{aligned}\delta\varepsilon &= -(\delta\kappa_c{}^c)^{\cdot} + \kappa_a{}^b\delta\kappa_b{}^a \\ &= \frac{1}{8}(\dot{X}/X)(\delta \ln \gamma)^{\cdot} + \frac{1}{2}P_c{}^c + \frac{1}{2}X^{-3}A_a{}^b \int X^{3/2} P_b{}^a dt,\end{aligned} \tag{3·7}$$

and for the perturbation of $u^a$ we get from Eq. (3·2)

$$\begin{aligned}\delta u^a = &-(\delta_b{}^a \delta\varepsilon/\varepsilon + \gamma^{ac}\delta\gamma_{cb}) u^b \\ &- \frac{1}{2}(\varepsilon+p)^{-1}\gamma^{ac}[\delta\kappa^b_{b,\alpha} e_c{}^\alpha - \delta\kappa^b_{c,\beta} e_b{}^\beta - \delta\kappa_c{}^b (e^\beta_{b;\beta} + e_d{}^\alpha e^d_{\alpha;\beta} e_b{}^\beta)],\end{aligned} \tag{3·8}$$

where $\delta\varepsilon$, $\delta\gamma_{cb}$, $\delta\kappa_c{}^b$ have already been derived in Eqs. (3·4)~(3·7).

So far we have not used any explicit expression for $P_a{}^b$ yet. In the present approximation, $P_a{}^b$ can be calculated in the diagonal metric $\gamma_{ab}$ and their expressions have already been derived in Appendix D of Ref. 8), where we have only to replace $X(e^{r_1 \tau}, e^{r_2 \tau}, e^{r_3 \tau})$ by $(a^2, b^2, c^2)$, and $e_\alpha{}^a, e_\alpha{}^b, e_\alpha{}^c$ by $l_\alpha, m_\alpha, n_\alpha$.

In the above expressions for the perturbations we find that the ratio of the terms explicitly containing $A_a{}^b$ to the terms not containing is of the order of $A_a{}^b t X^{-3/2}$ which decreases with time. Therefore the role of $A_a{}^b$ (i.e., rotational and gravitational wave modes) for inhomogenisation is weakened with time, just as in the unperturbed anti-Newtonian model. However, another inhomogenisation is aroused through the terms not containing $A_a{}^b$, as follows.

In order to simplify the analysis, let us consider such a stage that in the unperturbed state the expansion can be regarded as isotropic, i.e., $t \gg T$ so that $\gamma_{bc} \simeq X\delta_c{}^b$, $a_1 \simeq a_2 \simeq a_3 \simeq X^{1/2}$. In the following $X$ is written as $t^{2\nu}$, where $\nu = 1/2$ and $2/3$ for radiation-dominated and dust-like cases, respectively. Then the order of magnitude of $P_a{}^b$ is $\sim 1/(X r_c^2)$. Here $r_c$ is the minimum coordinate length for which the directions of the triads $e_a{}^\alpha$ change remarkably, and $r \equiv X^{1/2} r_c$ is the radius of curvature in each inhomogeneous region. This radius may be of the same order as the linear scale $L$ of irregularities, but here we shall distinguish $r$ and $L$, because $L$ is determined by the inhomogeneity of values of both $a_b$ and $e_a{}^\alpha$ and so $L \sim r$ or $L < r$ in general. If we pick up in Eqs. (3·4)$\sim$(3·8) the terms which grow most rapidly and become finally dominant, we have

$$\begin{aligned} &\delta\kappa_a{}^b \sim t^{1-2\nu}/r^2, \\ &X^{-1}\delta\gamma_{ab} \sim \delta \ln \gamma \sim t^{2(1-\nu)}/r_c^2 \sim (ct/r)^2, \\ &\delta\varepsilon/\varepsilon \sim (ct/r)^2, \\ &\delta v^a/c \equiv X^{1/2}\delta u^a/c \sim (ct/r)^3, \end{aligned} \tag{3·9}$$

where we have used the anti-Newtonian relations $\varepsilon \sim X^{-2}$ and $X^{-3/2}$, $u^b \equiv e_\alpha{}^b g^{\alpha\beta} u_\beta \sim X^{-1} u_\beta \sim (X^{-1/2}$ and $X^{-1})$ $A^\alpha_{\beta;\alpha}$ for radiation-dominated and dust-like cases.

Here we shall examine the applicability of this result. In the above we have used $P_a{}^b$ which is calculated in the unperturbed metric, but, as the perturbations grow, their influence on $P_a{}^b$ increases so as not to be negligible. The perturbation of $P_a{}^b$ can be estimated as

$$\delta P_a{}^b \sim r^{-1}(\delta \ln \gamma / L),$$

and so $P_a{}^b$ and $\delta P_a{}^b$ are comparable at the epoch $ct/L = 1$. At the epoch $ct/L > 1$, therefore, the result in Eq. (3·9) is not valid. However, if the perturbations are small, their behavior can be clarified by the use of the gravitational instability theory in the Friedmann universe,[8] which takes $\delta P_a{}^b$ into account. Accordingly we can have the following conclusion. If $r \sim L$, the perturbed quantities reach $\sim 1$ at the epoch $ct/L = 1$ and the nonlinear effect becomes essential. At this stage the fluidal motion is in general anisotropic and so the anisotropic collapse or compression will arise thereafter. If $r > L$, the perturbations grow to the epoch $ct/L = 1$, when they remain small. In the radiation-dominated case they propagate as sound waves thereafter. In the dust-like case they continue to grow to the epoch $\delta\varepsilon/\varepsilon \sim 1$ and thereafter collapse.

## § 4. One-dimensional relativistic collapse

General anisotropic collapse will be too complicated to be analyzed. In order to get some fundamental characters of the anisotropic collapse analytically, we shall impose here several conditions for simplification, similar to Zeldovich's Newtonian treatment.[6] First we shall assume that the fluidal motion is inhomogeneous only in one direction ($x$) and homogeneous in the other two directions ($y, z$), and further assume that it is irrotational and axially symmetric around the $x$ axis. Then the line-element can be expressed as

$$ds^2 = -dt^2 + A(t,x)\,dx^2 + B(t,x)\,(dy^2 + dz^2) \tag{4·1}$$

and the Einstein equation is given by

$$-T_1^{\ 1} = -\ddot{B}/B + \frac{1}{4}(\dot{B}/B)^2 + \frac{1}{4}(B'/B)^2/A\,, \tag{4·2}$$

$$-T_2^{\ 2} = -T_3^{\ 3} = \frac{1}{2}[B''/(AB) - \ddot{A}/A - \ddot{B}/B] - \frac{1}{4}[(B'/B)^2/A + (A'/A^2)(B'/B) - (\dot{A}/A)^2 - (\dot{B}/B)^2 + (\dot{A}\dot{B})/(AB)]\,, \tag{4·3}$$

$$-T_0^{\ 0} = B''/(AB) - \frac{1}{4}[(B'/B)^2/A + 2A'B'/(A^2B) + 2\dot{A}\dot{B}/(AB) + (\dot{B}/B)^2]\,, \tag{4·4}$$

$$-AT_0^{\ 1} = -\dot{B}'/B + \frac{1}{2}(B'/B)(\dot{B}/B + \dot{A}/A)\,, \tag{4·5}$$

where a prime denotes the derivative with respect to $x$.

Next we shall assume that the fluid is dust-like and inviscid. Then we can always choose the coordinates to be comoving, and in the comoving coordinates the energy-momentum tensor is expressed as $T_0^{\ 0} = -\varepsilon$ (the other components vanish).

From Eq. (4·5) we get

$$A = (B')^2/\{\alpha(x)B\} \tag{4·6}$$

and substituting this into Eq. (4·2) we have

$$4\ddot{B}/B - (\dot{B}/B)^2 - \alpha/B = 0\,.$$

The integration over this equation leads to

$$B = [(u^2 - 9\alpha/16)/\beta(x)]^{-2}\,, \tag{4·7}$$

$$\frac{3}{2}\alpha\beta^{-1}(x)[t - \gamma(x)] = u\Big/\left(u^2 - \frac{9}{16}\alpha\right) + \frac{2}{3}\alpha^{-1/2}\ln\left[\left(u - \frac{3}{4}\alpha^{1/2}\right)\Big/\left(u + \frac{3}{4}\alpha^{1/2}\right)\right], \tag{4·8}$$

where $\alpha(x)$, $\beta(x)$ are integration constants in time. If we differentiate $B$ in Eq. (4·7) with respect to $x$, we get

$$B'/\sqrt{B} = 2\beta(\beta'/\beta - \alpha'/\alpha)\Big/\left(u^2 - \frac{9}{16}\alpha\right) + \frac{8}{3}u[(3\alpha'/\alpha - 2\beta'/\beta)(t+\gamma) + 2\gamma']\,. \tag{4·9}$$

Therefore $A$ and $B$ are given by Eqs. (4·6)$\sim$(4·9) as functions of $t$ and $x$. Moreover, we obtain from Eq. (4·4)

$$\varepsilon = -\frac{8}{9}\beta'/(B^{1/2}B') = -\frac{8}{9}\beta'/(\alpha AB^2)^{1/2}\,. \tag{4·10}$$

The role of the time dilatation $\gamma$ is to make the expansion for small $t$ anisotropic, and the case $\alpha/u^2$ $(\propto(t-\gamma)^{2/3})\ll 1$ corresponds to the anti-Newtonian stage. In the case $\gamma=0$, $(A/B)^{1/2}=(2/3)\beta'/(\beta\sqrt{\alpha})$ and $A\propto t^{4/3}$ in the limit $\alpha/u^2\to 0$, that is to say, the expansion is isotropic. Here we shall impose $\sqrt{\alpha}=(2/3)\beta'/\beta$ so as to have $A/B=1$ in this limit.

If we expand the above formulae with respect to $\eta\equiv\alpha(4t/\beta)^{2/3}$ in the case $\gamma=0$, we have

$$u^2/\alpha=\eta^{-1}\left[1+\frac{9}{20}\eta+\frac{2}{35}\left(\frac{9}{16}\right)^2\eta^2+\cdots\right],$$

$$B=(\beta/\alpha)^2\eta^2\left[1+\frac{9}{40}\eta+\frac{1}{175}\left(\frac{9}{16}\right)^2\eta^2+\cdots\right],$$

$$A=(\beta/\alpha)^2\eta^2\left[1-\frac{9}{80}\eta+\frac{9}{175}\left(\frac{9}{16}\right)^2\eta^2+\cdots-\frac{27}{80}\lambda\eta\left\{1-\frac{27}{280}\eta+\cdots\right\}\right], \quad (4\cdot 11)$$

where $\lambda\equiv 2(1-\beta''\beta/\beta'^2)$. These expansions are convergent at least for $\eta\lesssim 5$, and they can be approximated by each first term for $\eta\lesssim 1$. From Eq. (4·11) we see that with time $A$ increases at first, stops the increase at the epoch $\lambda\eta\sim 1$ and decreases thereafter, while $B$ increases monotonically. Even if $\gamma\neq 0$, we can use these formulae for $t\gg|\gamma|$ and so we shall assume $\gamma=0$ in the following.

Moreover the spatial curvature $P_1{}^1$ in the $x$ direction is found to be

$$P_1{}^1=A^{-1}\left[-B''/B+\frac{1}{2}(B'/B)(A'/A+B'/B)\right]=\frac{3}{64}\lambda\eta t^{-2}(1+0(\eta)) \quad (4\cdot 12)$$

for $\eta\ll 1$ and so $\lambda\eta$ is related to $ct$ divided by the radius of curvature.

Now, in order to make the situation clearer, we shall take up an example specified by

$$\beta=1+\frac{1}{2}\cos x/l \quad (4\cdot 13)$$

so that $P_1{}^1$ may change the sign during each period of $2\pi l$. Then $\lambda=2(1+2\cos x/l)/\sin^2 x/l$ and the singular points $(A=0)$ are given by $\lambda\eta=80/27$ or

$$(4t)^{2/3}=\frac{40}{3}\left(1+\frac{1}{2}\cos x/l\right)^{8/3}/(1+2\cos x/l). \quad (4\cdot 14)$$

Realistically this singularity should be replaced by the high density region where the pressure gradient is not negligible. From Eq. (4·14) we find that the fluid in the point of $\cos x_0/l=0.4$ or $x_0/l\doteqdot 0.369\pi$ reaches a singularity for the first time, and that the fluid around this point approaches this point. Here we shall consider the velocity $V_0$ at the moment when the surrounding fluid collides with the fluid in the point $x_0$. For that purpose we shall define the proper distance from the point $x_0$ to any point:

$$X=\int_{x_0}^{x}A^{1/2}dx\,. \quad (4\cdot 15)$$

If we approximate $A^{1/2}$ by the first terms in $A$ of Eq. (4·11), i.e., $A^{1/2}=(\beta/\alpha)\cdot\eta(1-27/160\cdot\lambda\eta)$, we obtain

$$X=(4t)^{2/3}I_1-\frac{3}{40}(4t)^{4/3}I_2\,, \tag{4·16}$$

where

$$I_1\equiv\int_{x_0}^{x}\left(1+\frac{1}{2}\cos x/l\right)^{1/3}dx\,,$$

$$I_2\equiv\int_{x_0}^{x}(1+2\cos x/l)/\left(1+\frac{1}{2}\cos x/l\right)^{7/3}dx\,.$$

Except for the point of $x/l\simeq 2\pi/3$, the value of $\eta$ for $A=0$ are $\lesssim 1$, so that this approximation is accurate enough. The time derivation of Eq. (4·16) leads to

$$\dot{X}=\frac{8}{3}(4t)^{-1/3}I_1-\frac{2}{5}(4t)^{1/3}I_2\,. \tag{4·17}$$

The velocity $V_0$ is, therefore, given by

$$V_0\equiv\dot{X}(x=0)=\mp\frac{4}{\sqrt{30}}(I_1I_2)^{1/2}\,, \tag{4·18}$$

where the upper or lower sign corresponds to $x>$ or $<x_0$. The fluid which can collide with the fluid at the point $x_0$ is in the region $0\leq x<x_0$ and $x_0<x<(2\pi/3)l$. If $\pi l\geq x\geq(2\pi/3)l$, the fluid continues to expand and does not approach the point $x_0$. Therefore the maximum colliding velocities $V_{01}$ and $V_{02}$ in the region $x>$ and $<x_0$ are brought from the points $x=0$ and $(2\pi/3)l$, and their numerical values are 0.97 and $-0.62$, respectively. Because these velocities are relativistic, the energy released by the shock dissipation after their collisions is comparable with the rest mass energy, so that the energy density of radiation may be comparable with that of ordinary matter. Since Jeans' wavelength becomes $\sim ct$ in this situation, the small perturbations which remain after the shock dissipation will be bounced by the pressure gradient and propagate as sound waves. In order to clarify this situation qualitatively, the collapse in the realistic fluid with pressure and viscosity must be treated.

## § 5. Concluding remarks

In a fully inhomogeneous model an irregularity includes irregularities with smaller dimension within it, each of them includes irregularities smaller than them and so on. Smaller irregularities dissipate or become sound waves at earlier epochs. The upper limit of their linear dimension of irregularities is empirically estimated to be the size of a typical cluster of galaxies.[4]

In previous papers,[2),3)] we have not considered the influence of the inhomogeneity of the spatial curvature on the density. If $r/L \sim 1$ in most irregularities, the evolution of the early universe includes the successive dissipative process which brings about the release of vast energy comparable with the rest mass energy, so that the thermal and chemical history at the early stage depends on the evolution of irregularities.

At the above stage with frequent shock-wave dissipation, the conservation of circulation does not hold, and the angular momentum for rotational motion change with time.



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