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Evolution of neutral and charged droplets in an electric field

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We study the evolution of drops of a very viscous and conducting fluid under the influence of an external electric field. The drops may be neutral or may be charged with some amount of electric charge. If both the external electric field and total drop charge are sufficiently small, then prolate spherical shapes develop according to Taylor's observations. For sufficiently large charge and/or external field a self-similar conelike singularity develops in a mechanism different from Taylor's prediction. The opening semiangle of the cones both for uncharged and charged drops in a constant electric field is typically around 30° with a very slight dependence on the viscosity ratio and independence from both total charge and external field. We also discuss the structure of electric and velocity fields near the tip. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2980030]

I. INTRODUCTION

An important problem in fluid mechanics is the evolution and breakup of drops of a conducting fluid, both neutral and electrically charged, under the influence of an external electric field. It has been the subject of numerous studies since the pioneering works of Rayleigh,[1] Zeleny,[2] Wilson and Taylor,[3] and Taylor,[4] where the stability of spherical or spheroidal drops was analyzed and singular fluid interfaces in the form of cones were observed and discussed. The problem is knowing a renewed interest due to its relevance in connection with a variety of physical and technological situations, such as the breakup of water droplets in thunderstorms, electrospraying, nanoencapsulation, electrospinning, and electropainting,[5–7] as well as the development of colloid thrusters.[8] The process has also received attention in connection with new analytical methodologies for extracting and mass analyzing biomolecules from solution. If an analyte is dissolved in the droplets, then the developing jets are a source of gas-phase analyte ions for mass spectrometry.[9] The process is known as field induced droplet ionization. A general overview of the subject is given in a recent review by Fernández de la Mora.[10]

On the other hand, the development of experimental settings that are able to register the very fast events previous to the disintegration of charged drops in an electric field[5,9] has shown that such disintegration takes place after the development of conical tips from which ultrathin jets are emitted. In the case of isolated charged drops of a conducting fluid it was shown[11] that the evolution leads to the formation of a conelike singularity that develops in a self-similar manner. An important characteristic of such cones is that they have an opening semiangle much smaller than that of the classical stationary Taylor's cone. They result from a balance of viscous stresses and electrostatic forces instead of the static balance between capillary and electrostatic forces.

Numerical studies of the evolution of droplets of an inviscid fluid were performed in a series of papers by Basaran *et al.* (see Refs. 12 and 13). In Ref. 12 the static shapes of drops in an electric field were computed. The dynamic situation was studied in Ref. 13 in the absence of viscous stresses that, as we will see below, play a fundamental role in the formation of conical singularities and are certainly present in micrometric drops where Reynolds number is small.

In this paper we show that cones of the type described above develop when charged drops are immersed in a constant external electric field, such as the one created in a planar capacitor. Interestingly, when the imposed electric field is nonzero, the opening semiangle is essentially independent of both the external field and the net charge in the drop. Nevertheless, the opening semiangle is slightly different to one found (cf. Ref. 11) for charged drops in the absence of an external electric field. Our investigation relies in the use of a boundary integral formulation of the problem that we explain in Sec. II and its numerical implementation described in Sec. III. In Sec. IV we discuss the stability of drops and validate our code with theoretical estimates due to Taylor.[4] Section V is devoted to the description of singularities developing in the unstable situation, and Sec. VI to the analysis of such singularities. Finally, Sec. VII contains three-dimensional (3D) simulations intended to address stability in nonaxisymmetric situations.

II. MATHEMATICAL FORMULATION

The drop occupies a region $\Omega(t)$ and the liquid of the drop is a perfect conductor with infinite conductivity. Hence the electric potential V is constant inside and at the drop surface, and all the electric charge will be located at the boundary since the surrounding medium is a dielectric, the total charge Q remains constant. The electric field $\mathbf{E}$ outside the drop is given by $\mathbf{E}=-\nabla V$, where

$$\Delta V=0 \quad \text{outside } \Omega(t), \tag{1}$$