EVOLUTION OF PHASE TRANSITIONS

This work began with the authors' exploration of the applicability of the finite deformation theory of elasticity when various standard assumptions such as convexity of the energy or ellipticity of the field equations of equilibrium are relinquished. The finite deformation theory of elasticity turns out to be a natural vehicle for the study of phase transitions in solids where thermal effects can be neglected. This is a valuable work for those interested in the development and application of continuum-mechanical models that describe the macroscopic response of materials capable of undergoing stress- or temperature-induced transitions between two solid phases. The focus is on the evolution of phase transitions, which may be either dynamic or quasi-static, controlled by a kinetic relation that in the framework of classical thermomechanics represents information that is supplementary to the usual balance principles and constitutive laws of conventional theory. The book should be of interest to mechanicians, material scientists, geophysicists, and applied mathematicians.

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EVOLUTION OF PHASE TRANSITIONS

A Continuum Theory

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Contents

Preface

This monograph threads together a series of research studies carried out by the authors over a period of some fifteen years or so. It is concerned with the development and application of continuum-mechanical models that describe the macroscopic response of materials capable of undergoing stress- or temperature-induced transitions between two solid phases.

Roughly speaking, there are two types of physical settings that provide the motivation for this kind of modeling. One is that associated with slow mechanical or thermal loading of alloys such as nickel–titanium or copper–aluminum–nickel that exhibit the shape-memory effect. The second arises from high-speed impact experiments in which metallic or ceramic targets are struck by moving projectiles; the objective of such studies – often of interest in geophysics – is usually to determine the response of the impacted material to very high pressures. Phase transitions are an essential feature of the shape-memory effect, and they frequently occur in high-speed impact experiments on solids. Those aspects of the theory presented here that are purely phenomenological may well have broader relevance, in the sense that they may be applicable to materials that transform between two "states," for example, the ordered and disordered states of a polymer.

Our development focuses on the evolution of the phase transitions modeled here, which may be either dynamic or quasistatic. Such evolution is controlled by a "kinetic relation," which, in the framework of classical thermomechanics, represents information supplementary to the usual balance principles and constitutive laws of conventional theory. We elucidate the rather remarkable way in which the classical theory "calls for" this kind of supplementary information when the material is capable of changing phase, though such additional information is *not* called for – indeed, cannot be imposed – in the case of a single-phase material.

The simplest context in which to illustrate the need for kinetic relations and the role they play is that furnished by the purely mechanical theory of one-dimensional nonlinear elasticity, with thermal effects suppressed. After the Introduction, which comprises Part I of the monograph, we pursue the subject in this context in Part II. Even this simplest version of the theory to be set out here has some utility, as we show in Chapters 3 and 4. Part III presents the full three-dimensional theory, taking both mechanical and thermal effects into account. We specialize this theory to one space dimension in Part IV, where we are able to make some comparisons with experiments. In Part V, we discuss some three-dimensional problems.

The material presented here is drawn primarily from our own research over the period from the late 1980s forward. We came to this subject as practitioners of solid mechanics interested in exploring the range of applicability of the finite deformation theory of elasticity when various standard assumptions such as convexity of various energies or ellipticity of the field equations of equilibrium were relinquished. When broadened in this way, finite elasticity is a natural vehicle for the study of those aspects of phase transitions in solids that can be discussed with thermal effects neglected. Nonlinear *thermoelasticity*, similarly unencumbered by conventional restrictions, provides the natural framework for the study of mechanical and thermal effects together.

Our hope is that this book will be of interest to materials scientists, engineers and geophysicists as well as to mechanicians and applied mathematicians. The perfectly prepared reader would be acquainted with continuum mechanics at the level of Chadwick's *Continuum Mechanics*, Wiley, New York, 1976; with thermodynamics as treated, for example, in J. L. Ericksen's *Introduction to the Thermodynamics of Solids*, Chapman and Hall, New York, 1991; with material behavior as described by T. H. Courtney in *Mechanical Behavior of Materials*, McGraw-Hill, New York, 1990; with partial differential equations at the level of J. D. Logan's *An Introduction to Nonlinear Partial Differential Equations*, Wiley-Interscience, New York, 1994; and with the elements of Cartesian tensors as discussed, for example, in *Linear Vector Spaces and Cartesian Tensors*, Oxford, New York, 1998, by J. K. Knowles. However, expecting many potential readers to be less than perfectly prepared, we have tried to make the presentation as self-contained as is practicable, citing appropriate sources for those results that are used but not derived.

Although the book deals almost entirely with our own work, we have nevertheless had the enormous benefit of interactions with many others, and it is a pleasure to acknowledge them *all* with gratitude. We would be remiss not to mention the particular influence that Tom Ahrens, Kaushik Bhattacharya, Mort Gurtin, Rick James, Stelios Kyriakides, Jim Rice, the late Eli Sternberg, Lev Truskinovsky, and our former doctoral students, especially Phoebus Rosakis and Stewart Silling, have had on our learning of this subject.

Some of the fruitful interactions alluded to above took place in small, informal summer gatherings held at MIT's Talbot House in South Pomfret, Vermont. We are indebted to MIT for the use of this wonderful place, which – alas – is no longer owned by MIT.

Special thanks go to Debbie Blanchard, who drew the figures in the early part of the book, and then taught us how to draw the rest.

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