STATISTICA, anno LXX, n. 4, 2010

EVOLUTION TOWARDS A SYNTHESIS[1]

Bruno De Finetti (1960)

Since the present event motivates us to look back to the past and to consider the developments of disciplines within and around statistics during the last decades, we would certainly be able to point out many relevant novelties, each of us choosing according to his or her orientation the salient facts in various directions and interpreting them according to the personal viewpoint. The field is vast, the different specialists' interests are being divided amongst numerous aspects, and the brewing of ideas is always exciting, which springs from exchanges and contrasts among different conceptions, schools and mentalities. From my specific visual angle, the aspect which appears dominant to me in this vast panorama and of which, hence, I choose to speak about on this present occasion is the evolution that I believe is taking place towards a synthesis, that is towards a unitary vision which is gaining hold· and in which it appears that a place and a link are found for theories and application fields, which were previously unconnected and were in the developing stage, as parts which are now recomposed in an organic way.

And allow me, before I enter into the merit of this specific subject, to say why generally, according to me, such a type of synthesis has by itself the maximum value and interest, whatever is the implied field of concepts, and, that is, independent of the fact that the case in which we will be dealing is a topic of my own interest, and a confirmation of a line I support. In general there is a wide-spread tendency to break knowledge up into claustrophobic and self-contained compartments, an attitude which I consider to be deplorable and destructive. Temporarily such a separation might be, and always is, useful or necessary in order to explain and, for mnemonic facility or otherwise, in order to set the ideas by purposely leaving aside the aspects which would take us further away from the problem under consideration; but this process would be useful only provided we do not forget the fact that we are facing a momentary separation, only tolerable as such, and not an amputation which might be demanded as stable. If detached from the whole, no portion of thought is alive; instead it often appears that the cultivators of any small garden have a wrong supreme ambition of autarchic isolation (and even our ineffable, bureaucratic norms, talking about "didactic auton-

[1] Long abstrac of the essay published in *Vent'anni di vita (1941-1960)*, Special Issue for the 20th of "Statistica", October-December, 1960, pp. 527-538.

omy and scientific dignity", seem to be inspired to such distortion). It is from such a world that books are published with a blind attitude to prepare students who are allowed to know everything without understanding anything! What a difference, between the feeling of sickness one has when reading such type of publications, and the relief of joy and admiration one receives from authors who come alive through their writings and who show how they understand all the implications and meanings of all the examples, from the more banal to the more fundamental ones, experienced by them in the practical life! Such a difference, between the publications aiming at presenting sterile, pedantic or rhetoric doctrinaire thoughts and, instead, the agile instruments aiming at refining the intellect so that it can attempt to penetrate into the problems, is, as far as I am concerned, the difference between the school, unfortunately as it is, and the one which those who deplore such backward attitude and who fight for changes, would like to have.

In order not to be vague and to emphasize the contrast with the tendency towards synthesis that I want to stress later on, it is worth mentioning some of those damaging compartments one often comes across in the specific areas we are dealing with. Financial mathematics is often presented as completely detached from economic motivations which give meaning and foundation to it, thus reducing it to a mere formality of practical rules and dull attempts of ennobling them by pseudo-theoretical improvements. Actuarial mathematics (besides being independent of economics, which itself should be included, for the same reasons of the previous case and also for the "risk" aspect) is sometimes even independent of probability theory (by interpreting the elimination table as the preset plan of a massacre, and without realizing that even so the conclusions remain based on thin air). One often wants to disconnect the theory of probability from the intuitive notion of probability, trying, rather than defining it, to replace it with something extremely limited and totally insufficient (as the notions connecting it to "equiprobable cases" or to "statistical frequency", without realizing that, by refusing the general concept, it is just impossible, among other things, to attribute validity to what is valid in such considerations. Statistics (in its role fitting in this picture) often attempts to give autonomous decisional criteria, that is, independent of either the value (economic or otherwise) of consequences, or the theory of probability (except that sham or deceptive probability theory which is originated from its own bosom and anyway without the Bayesian principle). Again, to avoid such a type of foundations, the decision theory formulates for many cases criteria based on schemes whose formal complications hide the non-existence of a meaning which might correspond to them. For such a reason, the particular case of decision, considered by the game theory, appears a much more special matter than it really is, thus obscuring, to say the least, the meaning of certain interpretations. And not only for more or less vast theories, as in the cases so far mentioned, but even for very particular applications such as the theory of the observation errors, it has sometimes been attempted to create an autonomous construction (where the notion of "precision" was introduced as a primitive concept, and its properties were considered as axioms).

On the other hand – and thus the cycle closes – economics neglects to mention among the presuppositions of its layouts many analyses of essential questions belonging to the other fields which have been mentioned, for which reason economics repeats and absorbs them in an extremely rudimental form (without mentioning other reasons for verification, and thus it often ends up with advocating the right to judge comparatively among different economic systems or regimes, starting from assumptions which theorize or at least presume the characteristics of only one). One could go on and on, broadening the vision into other areas; but the example is already sufficient and, besides, I prefer to avoid expressing my opinion in a slightly imprudent way regarding subjects too far from my competence.

Where does it come' from and what is it, really, the movement which made the possibility of a synthesis almost as achieved and in the process to gain an even growing consent? As an old saying goes "the roads to God are inscrutable", it would not be a risky paraphrase to state that the roads to progress are inscrutable, especially the ones towards the scientific progress. History of ideas, inventions and discoveries could supply ample proof material for those who wish to get deeper into the subject; here I will limit myself to recalling such a statement to draw attention to the elements which will be useful for strengthening in the brief context of this relation and to apologize for omitting such a lot more that would be necessary to include if one wanted to enlighten all the aspects and their interdependence.

This "synthesis" is not something new, grand or unpredictable (as in other fields relativity theory or quantum physics are): it is, above all, going back to the straight understanding of facts and problems with the naturally penetrating use of methods and concepts which, instead, in some excessively technical distortions, caused the fragmentations of the topics in parts devoid of content. Sometimes I feel I can identify the 1600-1700 scientific explosion with the golden age of a synthetic intelligent vision, and the following 1800 and beginning of the 1900s work with the overtaking of more limited and involved visions, as an explainable counter-balance to the extraordinary crops of discoveries from deep and necessarily specialized researches, to which the geniality of so many scientists must have mainly applied, and to the indispensable refinement of the critical spirit, which unfortunately sometimes leads to formal and dry arguments in order to avoid (instead of penetrating into and solving) the obscure points of previous intuitions. Regarding the area we are referring to, I feel there must be something true in such a description, although too many objections and denials do not allow us to fully support such a simplistic thesis, in general as well as in the limited field we are interested in. Besides, general theses are always too biased and debatable; I mention them to underline some aspects of the subjects, where the re-connection to viewpoints of two centuries ago will appear characteristic, viewpoints which remained misunderstood and not well-known for a long-time.

The essence of such a viewpoint can be said to have fed and kept alive a critical attitude from the outside towards the ruling conceptions; such attitude, insuf-

ficient alone to prevail, became prolific when criticism inspired by it appeared as answering to the needs of the criticism arisen within the prevailing conceptions, in order to eliminate inadequacies and contradictions and moreover when applications and considerations, happily concurring, arisen from the field of practical reality, strongly and spontaneously repurposed concepts and criteria which conformed to the hushed-up requirements of synthesis.

The practical needs, which have more authoritatively dominated the renovation of the viewpoints and their becoming more suitable for actual applications, are probably those we encountered within the problems now called operational research: name and type of problems at first linked to choice of decisions or strategies in the battlefields and, therefore, to vaster similar topics of economic nature, particularly of the business-economic type. Initially, it was the case of a number of applications, more or less distant from each other and of a very limited conceptual weight (considering each one on its own). But from their togetherness, necessity arose of a clear and general thinking about various aspects, common to their layout: the determination of the objectives, of the preferential criteria related to them, of the decision making methods to be adopted, with special attention to uncertain situations of different nature and to the possibility and convenience of modifying them.

Uncertainties of different nature may appear due to a lack of information, to the dependence of certain facts on aleatory results in the sense usually attributed to "statistical phenomena", or on other types called "absolutely unpredictable": they are cases often labeled with different denominations such as situations of "ignorance", "risk" and "uncertainty". To these the uncertainty of "competitive" type must be added, that is, the one deriving from somebody else's decision, presumably chosen to counteract ours from the person who has opposite interests and from more or less analogous situations. The convenience of modifying the uncertain situations means the convenience of gathering certain information (among other things, for instance, carrying out investigations, tests or surveys) before taking a final decision, which will thus be dependent on the newly acquired knowledge.

In many situations it did occur that those who aimed at answering particular and practical problems of the kind with a pure and unprejudiced eye, suddenly, without even realizing it, jumped over the bulks of doubts and obscure points which had accumulated in decades of suffocating attempts in those already blemished stagnant compartments. For example, I experienced this feeling of miraculous freshness and immunity from distortions while reading the very interesting, however elementary, book by Robert Schlaifer, *Probability and Statistics for Business Decision* (McGraw-Hill, New York, 1959).

For what concerns the aspect of "uncertainty" that was described above in a more ample way, we must deal with it again, on purpose, because it is the aspect that intimately touches statistics and its more closely related areas. But in the meantime let us consider, even if superficially, the other aspect worth mentioning. The habit of a conscious format of the problems of choice, the experience of the

complexity of the analyses required for answering it, even on the simple business level, naturally predispose to conceiving the problem of very good planning with an equal wide and independent viewpoint in each field, even wider, as the general economic for developing planning on inter-communal or regional or national scale. So it will be possible to analyze the problems of economics under the so-called welfare economics viewpoint. This is – or could be – independent from every institutional specificity and aprioristic superstition regarding spontaneous acceptable equilibria. For every conception of this type, the scheme is always the Pareto optimum, but reduced to its essential form, free from spurious influences. Incidentally, in such a picture it is possible to insert and take into proper consideration the worthwhile weight of those factors of a different nature, from those strictly economic ones and therefore not to ignore the sociological and similar aspects, if you wanted to do so. As an example of a treatment in compliance with such a wide layout, one can quote Kenneth J. Arrow, *Social Choice and Individual Value* (Wiley, New York, 1951); it is worth mentioning the start book *Introduction to Econometrics* (Pergamon, London, 1959) by Oskar Lange, a work by a famous scientist who is one of the principal contemporary Polish exponents, which, while dealing with other more technical subjects (although very briefly), shows the truly universal and institutionally agnostic, and hence genuinely scientific, character of the doctrines explained there.

Remaining in the economic field, but with reference to more specific problems, one can find interesting examples on the simplicity of the modem mathematical formulations, for instance in J. Lesourne, *Technique économique et géstion industrielle* (Dunod, Paris, 1959): particularly, I would like to point out the considerations on investments and depreciations (mainly by M. Boiteux), because of their contrast with the above deprecated formal spirit with which such subjects are usually dealt with in the financial mathematics.

The result of the critical developments produced within the particular theories, flowing in perfect harmony into those revealed by the practical requirements we just mentioned, have happily provided ready statements which are at least sufficient to give a basis with a broader scope to the hoped-for synthesis.

Above all, it was a question of formulating a coherent and complete theory of decision, one with a particular significance in its section concerning uncertain conditions. Here one met too many fragments, albeit isolated and disjointed, of specific statistical theories: but already some internal criticisms had developed, some of which with awareness and some without, which had enlightened the incompatibility among criteria suggested on the basis of an arbitrarily assumed level, and thus, for this very reason, inconsistent. Arguments among supporters of different variants can be considered as unwitting criticisms, because from their comparison the faults of each were implicitly evident, as well as those overcomings which really occurred, such as with Abraham Wald's formulation, which aimed only to perfect and not to destroy the already existing edifice. Responsible criticisms are those by authors pointing out faults and deficiencies with the precise aim of sorting them out through a renewal in the formulation, as done for

instance by Dennis V. Lindley and mainly by Leonard J Savage, whose fundamental work *The Foundations of Statistics* (Wiley, New York, 1954), must be mentioned, but it must be noticed that this author's viewpoint has further perfected, as evident in his latest publications, among which there is a very short one in Italian: the conferences on *La probabilita soggettiva nei problemi pratici della statistica*, C.I.M.E. course: "Induzione e statistica" (Varenna, 1959, Ed. Cremonese, Roma, 1960).

The fundamental conclusion, partly implicit in Wald's results but always more and more improved and rich with developments which were aimed at achieving that auspicious synthesis with the: successive build-ups, is to realize that there is a complete and reversible equivalence between the assumption of a coherent preference as a basis for decisions, and the choice of a coherent evaluation for probability and utility. That is, by assuming to assign certain probabilities to various events and certain utilities to different situations, not only a decisional criterion is univocally determined: but also vice versa. Hence, it does not make sense to refuse to evaluate the probabilities of any event (for instance, on the pretext that they are subjective) without admitting that we are incapable of choosing whichever decision. Better still: every argument regarding decisional criteria has to be judged to be vain, if one does not want to admit to base it on a probability evaluation, because, by itself, the decisional criterion one admits implies an evaluation of probability (and utility).

Thus the justification of any distinction of principle among "ignorance", "risk" and "uncertainty" fails (for instance as supported by Knight, on the basis of the possibility, or not, of bringing a probability evaluation back to the special statistical or classical definitions), and by doing so there is an immense gain in the meaning and simplicity of the conclusions. Mistaken ideas are avoided, such as the attempts to speak of a decision under uncertainty conditions, forcing everything to intervene except determinant factors, that is to say the evaluation of probability as a result of such uncertainty by the individual deciding it.

But there is more to it: the way in which one must take further information and particularly statistical results into account in order to make decisions; the method to be preferred in order to select the information to be considered, or in particular the "design" of the statistical experiment one has to follow to reach such an end; all this and all the possible details and aspects of these matters get unitarily included and resolved in the decision theory. The first question leads us to say that statistical induction becomes the application of the Bayesian principle, or, if one wants, the likelihood ratio. The second one simply indicates that the usefulness of a decision conditioned by further information is the utility, in the usual sense, of the decision which consists in taking that information, and afterwards proceeding, for each answer, in the estimated way. So, naturally, the value of the information becomes expressed by the difference between the usefulness of the optimal decision conditioned on it, and the possible optimal decision without resorting to other information; but it is also necessary to consider the information costs (for instance, testing or surveying) for the optimal choice: it will consists of taking the most convenient information, that is the one that maximizes the difference between value and cost. This way, all statistics (in the Anglo-Saxon sense), from the testing hypotheses the-

ory and parameter estimation to the design of experiments, is nothing other than the theoretical concept informing operational research.

This does not mean that all past work becomes useless: many of the aspects which were investigated remain interesting, although the viewpoint inspiring that work becomes superseded. However, the acceptability of those instruments and the limits of such acceptability will derive from comparisons with the correct formulations, and no longer from a more or less rigid list of formal and arbitrary *desiderata*, relative to the indexes or to the methods to be adopted.

In turn, the results of criticism developed from within the specific conceptions have, after all, merged with the currents which, on the outside, had kept the conceptual opposition alive. We have mentioned the principle of maximum utility and the inductive reasoning of Bayesian kind as new achievements: and in some ways they are so, but they constitute return to concepts of the 1700s (to Daniele Bernoulli's "moral expectation" and to Thomas Bayes' principles), concepts which were abandoned and rejected while them only required be deepening and amending. This viewpoint (and more generally the need for an ample conception of the probability theory, and not one artificially restricted) had been supported in the last decades in the most radical way by Frank P. Ramsey, B. de Finite, B. O. Koopmans (and I hope I will be forgiven for quoting myself, which I did to explain the special satisfaction I found in recording the improved perspectives of what I thought to be a hopeless battle for a truth against which extremely well-guarded walls of incomprehension had been erected).

Partially in a concordant sense, similar ideas remained alive in different fields: in the good intentions of intellectuals close to practical problems (like the Americans Fry and Molina), in the logical spirit, probably of Hume derivation, of some English (like Keynes and Jeffrey's), in the brilliant intuition and finesse of the French (which surfaces in Borel and Paul Levy, not to mention Poincare).

In this group of considerations, the appearance of the theory of games can also find a place, although the vast crop of new theoretical and practical results, and the link with the mentioned evolution of statistical criteria by Wald, would rather justify its place in the two previous sections. But, in this overall summary, rather than the technical aspect, one should point out some historical reference and conceptual meaning. It must then be said that the fundamental concept relative to strategies of the most simple problem, i.e. is that of the *minimax* solution (by J. von Neumann), had been reached, (although in a simplified case) since 1712. Particular reasons prevented the appreciation of the solution at that time, and the mentality of the following period went so far from this order of ideas, that nobody mentioned that early concept which today makes us marvel: not even Todhunter who reports many insignificant details from the works by Montfort and other older authors. Bertrand touches the subjects suspiciously; Borel reconsiders it, von Neumann introduces the most decisive contributions and (in collaboration with O. Morgenstern) writes the famous book *Theory of Games and Economic Behaviour* (Princeton, 1944); for a more concise and updated general description it is worth resorting to R.D. Luce and H. Raiffa, *Games and Decisions* (Wiley, New York, 1957).

As a major proof of the interconnection existing among the various evolution

factors, however incidentally, we also note that the first modem axiomatic definition of the probabilistic notion of utility is to be found, for the needs of their exposition, in appendix to the book by von Neumann and Morgenstern; there the modem reappearance of the ancient "moral expectation" by Daniel Bernoulli (apart from F.P. Ramsey's precursor hint) occurs and was followed by a rapid spread of publications by a number of authors.

To mention another aspect, which will probably need a further investigation, we indicate a difficulty related to the separate definition of probability and utility on the basis of decisions. The often mentioned difficulty is that if I prefer to receive 1000 lire, in case E rather than E occurs, this might mean that I consider E to be more probable, but it could also mean that in the case of E occurring (although I consider it less probable) the 1000 lire would be very handy to me. This fact (to say it with P. Samuelson's words on commenting Ramsey) "would violate the implicit independence assumption we make in separating out a man's probability beliefs from his evaluation of outcomes". The various authors' attempts to define such independence would have been vain if, as Drèze maintains, the problem does not exist due to lack of "identifiability".

The evolution is still going on, not only in the sense that the outlined viewpoints are still strenuously opening up a road towards a more general acceptance, but even in the deeper sense that concerns its development: there is a lot more to do to reconstruct old doctrines, or part of such doctrines, and build up new ones according to the ideas informing the new synthesis.

This is a particularly favorable opportunity for the young Italian researchers, because the handicap of an insufficient connection with many of the doctrines to be reformed could perhaps turn into an advantage, due to the consequent higher sense of freedom from prejudice they might experience in starting their work. This has been pointed out by Savage, for example, in his conclusive words at the above mentioned seminar in Varenna.

For this purpose, those who would like less summarized information regarding some of the subjects here mentioned, might find ample enough material in the proceedings of the same seminar C.I.M.E. in Varenna, in Savage's lessons and in another course of mine, as well as in other conferences at the seminar. Here and in the quoted books one can find rich and appropriate references.

At last, to point out how the opportunity of new researches or of other revivals appears, I wish to remind that in the already quoted book by Arrow and elsewhere, one can find again the problems concerning voting and similar issues, problems which already bothered Condorcet, and which are related to the problems of probabilities of judicial errors and analogous ones, which have been in disrepute for a long time (they were even defined *le scandale des mathematiques*!). I believe it is time we seriously reconsider examining these subjects, because it always appears to be of vital and very passionate interest to see clearly into the paradoxical aspects they present and into the problems of practical opportunity they propose.

BRUNO DE FINETTI