

# **Research Article**

# **Evolutionary Game Dynamics of the Competitive Information Propagation on Social Networks**

# Xue Yang,<sup>1,2</sup> Zhiliang Zhu<sup>(b)</sup>,<sup>1</sup> Hai Yu<sup>(b)</sup>,<sup>1</sup> Yuli Zhao,<sup>1</sup> and Li Guo<sup>3</sup>

<sup>1</sup>Software College, Northeastern University, Shenyang 110819, China

<sup>2</sup>College of Mathematics and Information Science, Anshan Normal University, Anshan 114005, China <sup>3</sup>School of Computer Science and Engineering, Northeastern University, Shenyang 110819, China

Correspondence should be addressed to Zhiliang Zhu; zhuzl\_neu@163.com

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To better control the scope of information propagation and understand its dynamic characteristics, we propose an information propagation model based on evolutionary game theory. The model can simulate an individual's strategy selection in social networks when facing two pieces of competitive information, whereby "competitive information" is defined as two pieces of information which have the opposite meaning. First, a reasonable payoff function is designed for individuals based on pairwise interaction. Second, each individual selects a friend it trusts. Third, a probability value is used to indicate whether an individual imitates the strategy of the selected friend. In the model, we consider not only the heterogeneous influence of friends' strategies on individual decision-making in the process of communication but also the attenuation of individuals' attention to information when information about friends is received repeatedly. The simulation results show that our model can accurately simulate the propagation of two pieces of competitive information. Furthermore, we find that the basic payoff that accrues to individuals as a result of spreading their information and the network topology are two factors that significantly influence the propagation result. The results provide effective insights into how to better control and guide public opinion.

## 1. Introduction

In recent years, face-to-face conversation has gradually been replaced by online communication using online social software such as Facebook, WeChat, Twitter, and so on. The timeliness and convenience of online social software has enabled the propagation of information at an unprecedented speed [1]. The wide use of social software virtually speeds up the formation of public opinion. Public opinion has a significant effect on society. It is difficult for people to avoid being affected by public opinion in some way. Positive public opinion can potentially make society more stable. However, negative public opinion can be fake, anonymous, and unofficial [2, 3] and can be used to cause social unrest, trigger large-scale social upheaval, as well as to shape financial markets [4–6]. It is difficult to estimate the magnitude of the impact of negative public opinion on the social trust system. The essence of positive and negative public opinion is a kind of collective behavior that is characterized by information propagation. The low cost and lack of punitive consequences can lead to a plethora of negative opinions being propagated. Thus, in order to control and guide public opinion and thereby reduce potentially negative impacts, it is necessary to analyze the inner mechanism of information propagation and research its rules and characteristics [7].

Establishing a reasonable social network model through complex network theory to quantitatively research the information propagation process has become a prevailing point of interest for current research [8–13]. The most common approach is to build an undirected graph, where each node represents a user or an individual and edges express the relationships between individuals. Then, some nodes are selected as the source nodes to begin the propagation through the networks. In the process of propagation, whether an individual will transmit the information it receives is related to the social influence of the individual who transmits the information. This measure of social influence will affect individuals' thought and behavior. There are two kinds of influence propagation models that can be used to capture this phenomenon: the linear threshold model and the independent cascade model [14]. Most previous works have focused on the analysis of influence propagation on the static network structure and the discovery of the subset of the most influential individuals. However, these studies are limited in that they have not clarified the impact of the information content on the propagation results.

Strategy selection based on game theory can be applied to address this gap. Specifically, it can be used to consider the influence of different friends on individuals and to fully consider the perceived accuracy of the information based on the recognition degree of the information. In a game theory approach, each node has to select a strategy at a given step, whereby "strategy selection" means that the individual chooses whether to spread the information. If an individual approves the information based on their interaction with their friends, they choose to spread it. Otherwise, the individual stays silent. It is obvious that the individual's strategy is influenced by their interaction with their friends [15]. Thus, the result of the information propagation is determined by the strategy selected by the individuals in the network.

Strategy selection is the main research focus of game theory. Some research studies show that game theory provides a mathematical model for analyzing the strategic interaction between rational decision-makers [16-19]. Sun et al. [20] studied a knowledge propagation model by game theory. In game theory, individuals are typically presumed to exhibit complete rationality and have complete information [17]. Nowak and May [21] analyzed the two-dimensional square lattice-repeated prisoner's dilemma game and thereby introduced the spatial structure evolution game for the first time. Evolutionary game theory combines game theory with a dynamic evolution process and focuses on how individuals who exhibit bounded rationality optimize their payoff through adaptive learning over time during the repeated game [22]. Riehl and Cao [23] investigated the control of evolutionary games on networks in which each edge represents a two-player repeated game between neighboring agents. In essence, the process of information propagation based on complex networks is represented by the process of the game for each payoff recipient [24–26]. In contrast to other dissemination behavior, whether the information is spread is determined by the game players. In this analysis, the information spreaders and receivers are the players in the game. As time evolves, each player gambles with their friends and updates their strategy to maximize their payoff.

Evolutionary game theory provides an effective framework for simulating the process of information propagation in this context. The literature has considered how complex network knowledge can be integrated with game theory, and certain methods to simulate the real network information propagation process have been proposed. Jiang et al. [27] proposed an evolutionary game theoretical framework to model the dynamic information diffusion process in social networks. Li et al. [28] investigated the rumor diffusion process according to the evolutionary game framework. The author shows that punishing the higher degree nodes is the most effective measure to reduce the spread of a rumor. Jonnalagadda et al. [29] proposed a method of community discovery based on evolutionary game theory. Kermani et al. [30] used evolutionary game theory to identify the most influential node in information propagation. Etesami et al. [31] studied a class of games known as diffusion games that model the competitive behavior of a set of social actors on an undirected, connected social network.

Most existing works on information propagation that apply evolutionary game theory focus on single-information scenarios. In the real world, the phenomenon of multiinformation spreading is highly prevalent and much more complicated [32]. Many works contributed to competing pathogens. Beutel et al. [33] defined and studied the problem of partial competition, where two viruses/products provide partial immunity against each other. Chen et al. [34] proposed a model where two strains compete with each other at the expense of common susceptible individuals on heterogeneous networks by using pairwise approximation closed by the probability-generating function (PGF). Similarly, when a rumor appears, corresponding information refuting the rumor might be released by authoritative organizations. In this case, two pieces of information about a topic are spread in the network at the same time and they compete for believers. These two pieces of information are referred to as "competitive information" throughout this paper. Given the societal importance of this topic, more research is needed to effectively simulate the propagation of competitive information. To this end, it is important to consider that an individual's friends have different levels of ability in terms of spreading competitive information. Past research has considered the heterogeneous influences of various friends on information spreading [35]. However, in most previous works, the ability of different friends to spread information is stable throughout the whole evolution process. To more accurately depict real-life processes, it is necessary to incorporate dynamic changes in heterogeneous influences as time evolves.

In this study, a propagation model that analyzes two pieces of competitive information is proposed based on evolutionary game theory. The time step of propagation is considered as one round in an evolutionary game. In each round, each individual selects one of their friends based on the dynamic spreading ability of their friends. Specifically, the individual decides whether to imitate the strategy that the selected friend has adopted. In the specific context of information propagation in social networks, an individual's attention to the information that the selected friend believes has a significant effect on whether it chooses to update their current strategy. The number of rounds for which an individual receives a piece of information but does not spread it can be used to express the attention to an individual toward the information. In this study, the influence of an individual's attention to the information that the selected friend believes is considered in the model design. The results of the modeling exercise suggest that evolutionary game

theory is very well suited for modeling the competitive information propagation. The proposed propagation model is deemed to be effective and practical; namely, the simulation results show that the average degree and average clustering coefficient have a significant influence on the propagation results. In addition, it is exemplified that the propagation result can be controlled by adjusting the value of each parameter.

The remainder of this paper is organized as follows. In Section 2, the model for the propagation of two pieces of competitive information is introduced in detail. Section 3 presents and discusses the simulation results. The paper is concluded in Section 4.

## 2. Modeling the Propagation of Two Pieces of Competitive Information in a Social Network

In this section, the propagation model for two pieces of competitive information using evolutionary game theory is presented. We first analyze the player's payoffs, utility function, and the game rules, and then updated rules are designed according to evolutionary game theory. Table 1 clarifies the main symbols used in the model developed in this paper and their specific meanings.

We consider a finite set of individuals as a social network where individuals are nodes and contacts between individuals are edges. The network is modeled as an undirected graph, denoted by G. G = (V, E), where V is the set of nodes and *E* is the set of edges. *N* denotes the number of nodes in the set V, and M denotes the number of edges in the set E. We assume that the graph is simple; that is, no vertex is connected to itself, and there are no parallel edges. Two nodes of G are said to be friends if there is an edge between them. In the process of information propagation, individuals exist in the network in three possible states: ignorant, believers of the first piece of information, and believers of the second piece of information. They could be represented as I,  $B_1$ , and  $B_2$ , respectively. I means that the individual has never heard any information or has heard a certain piece of information but has not spread it. An individual in state  $B_1$  $(B_2)$  has chosen to spread the first (second) piece of information. However, if the same individual receives the other piece of information and chooses to spread it at a later time, this individual changes their state to  $B_2$  ( $B_1$ ). We assume that the information is propagated by direct contacts of spreaders with others. The state transition diagram is shown in Figure 1.

2.1. The Players' Payoffs. Initially, some individuals are selected to be in state  $B_1$  or  $B_2$  at random. All other individuals are in state I. Then, the information may be propagated over the network depending on other individuals' spreading actions; individuals choose whether to spread the information. For each individual, the decision to spread the information is determined by many factors, including the individual's own interest in this information, actions of their friends, the degree to which they believe their friends, and so on. Suppose that individuals in the network are only

TABLE 1: Model notation and definitions.

Notation	Definition	
$\delta_i(t)$	The strategy set of $i$ at time $t$	
$g_{ij}(t)$	The payoff of $i$ for one pairwise interaction with friend $j$ at time $t$	
$u_1$	The basic payoff of individuals who choose strategy $A$	
<i>u</i> <sub>2</sub>	The basic payoff of individuals who choose strategy B	
α <sub>1</sub>	The bonus attained by an individual who believes one piece of information when their friend chooses the same strategy	
α <sub>2</sub>	The punishment accrued by an individual who believes either piece of information when their friend chooses to believe the other competing piece of information	
$U_i(t)$ $N_i$	The total payoff of individual <i>i</i> at time <i>t</i> The friends set of individual <i>i</i>	
k,	The number of friends connected to individual <i>i</i>	
$k_{i1}(t)$	The number of friends connected to individual $i$ at time $t$ who adopt strategy $A$	
$k_{i2}(t)$	The number of friends connected to individual $i$ at time $t$ who adopt strategy $B$	
$r_i^*(t)$	The friend of individual <i>i</i> who attains the minimum payoff	
$\mathcal{Q}_u$ $p^s_{i\longrightarrow j}(t)$	The sensitivity to payoffs of individuals The probability that individual <i>i</i> believes its friend <i>j</i> at time <i>t</i>	
ε	The environment noise factors, which reflect the individual uncertainty at the time when strategy updates occur	
$w_{ij^{*}}(t)$	The number of times individual <i>i</i> has received the information that is believed by their friend $j^*$ before time <i>t</i>	
$s_{ij^*}(t)$	The number of times individual $i$ and $j^*$ have adopted different strategies when $i$ receives the information believed by $j^*$ before time $t$	
$p_{i\longrightarrow j^*}(t+1)$	The probability that individual $i$ will adopt the strategy of friend $j$ at time $t + 1$	

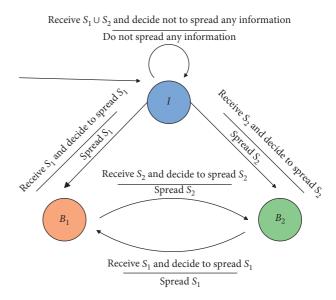


FIGURE 1: The state transition diagram of the propagation process for two pieces of competitive information.

permitted to interact with their friends and they repeatedly interact with their friends. Thus, friends' actions can influence the individuals' decisions on information propagation. As discussed above, the propagation of two pieces of competitive information is analyzed based on evolutionary game theory in this paper.

Information propagation can be modeled as evolutionary games in networks as follows. The players in the game are the N individuals. Three strategies can be defined to correspond to each individual's state. Accordingly, the strategy of individual *i* can be denoted as  $\delta_i$ ,  $\delta_i \in \{A, B, C\}$ , where A and B indicate whether individual i is in state  $B_1$  or  $B_2$ . If the individual *i* does not believe any information, they choose strategy C. Different payoffs can be achieved by each individual according to the different strategies. In this scenario, payoff represents an individual's prestige in the eyes of their friends. The payoff is used to evaluate the impact of individuals' strategy choices. Spreading behavior that occurs without any purpose, which thereby does not lead to any payoff for the individual, is not considered in this model. Therefore, the individual has no benefit from choosing strategy C, and the corresponding payoff is zero. An individual gains a basic payoff when it chooses either strategy A or B and the corresponding friend chooses strategy C. The spreading of information can change an individual's prestige among their friends; that is, the spreading of correct information will improve prestige, while the spreading of wrong information will reduce prestige. It is assumed that individuals spread information in the hope of getting more endorsements from friends. In general, the more endorsements an individual has, the more likely their message is to be correct. Thus, if an individual adopts strategy A or B, they obtain a bonus payoff if their friend chooses the same strategy and a punishment payoff if their friend chooses to believe the competitive information. The payoffs of an individual depending on the strategy a friend chooses are depicted in Table 2.

As mentioned above,  $u_1(u_2)$  represents the basic payoff of individuals who choose the strategy A(B). It is clear that  $u_1 > 0$  and  $u_2 > 0$ . Parameter  $\alpha_1$  represents the bonus attained by an individual who believes one piece of information at the time when their friend chooses the same strategy. Parameter  $\alpha_2$  denotes the punishment that accrues to an individual who

TABLE 2: The payoff of an individual, depending on their strategy and their friend's strategy.

The strategy of $i$	The strategy of friend $j$	The payoff of $i$
Α	Α	$(1 + \alpha_1) \cdot u_1$
Α	В	$(1-\alpha_2)\cdot u_1$
Α	С	$u_1$
В	A	$(1-\alpha_2)\cdot u_2$
В	В	$(1 + \alpha_1) \cdot u_2$
В	С	$u_2$
С	A	0
С	В	0
С	С	0

believes either piece of information when their friend chooses to believe the competitive one.

The most fundamental issue in our model is that, for each individual, which strategy should be chosen among the previously described? The answer depends not only on the payoff functions of strategies but also on the interaction type of the network. The pairwise interaction enables us to consider the strategy selection based on a single node in a social network, whereby individuals interact in pairs to acquire their payoffs. The time-related interaction is normally bursty. Zhang et al. [36] proposed a gaming temporal network model, which reproduces the burstiness of human activities solely by a well-known strategy "Generous Tit-for-Tat." Our approach takes the burstiness feature into consideration by imitating the more efficient strategies of friends, where the more efficient strategies refer to the strategies of gaining more payoff. In general, a network game can be defined as  $\Gamma_g = (G, \{\delta_i(t) | i \in V\}, \{g_{ij}(t) | i \in V\})$  $V, j \in N_i$ }), where  $g_{ij}(t)$  is the payoff of *i* for one pairwise interaction with its friend j at time t and  $N_i$  is the friend set of individual *i*. Thus, the total payoff of individual *i* equals the sum of payoffs between the *i*-th individual and their friends, which is represented by the following equation:

$$U_i(t) = \sum_{j \in N_i} g_{ij}(t), \quad i \in V.$$
<sup>(1)</sup>

More specifically, the total payoff of individual i equals the sum of payoffs among the i-th individual and their friends. This is calculated as follows:

$$U_{i}(t) = \sum_{j \in N_{i}} g_{ij}(t) = \begin{cases} (k_{i} + k_{i1}(t) \times \alpha_{1} - k_{i2}(t) \times \alpha_{2}) \times u_{1}, & \delta_{i} = A, \\ (k_{i} + k_{i2}(t) \times \alpha_{1} - k_{i1}(t) \times \alpha_{2}) \times u_{2}, & \delta_{i} = B, \\ 0, & \delta_{i} = C, \end{cases}$$
(2)

where  $k_i$  is the number of friends connected to the individual *i*. In addition, individual *i* has  $k_{i1}$  (*t*) friends who adopt strategy *A* and  $k_{i2}$  (*t*) friends who adopt strategy *B* at time *t*.  $\delta_i$  is the strategy of individual *i*.

*2.2. The Update Rules.* In the beginning, all individuals adopt strategy *C*. Then, some nodes are randomly selected to define

their strategy as either A or B. Suppose at time t, an individual receives a piece of information that it does not believe. Then, at time t + 1, it needs to decide whether to believe the newly received information, and this is determined by two steps. First, the individual selects a friend according to the different spreading ability of friends. Second, it decides whether to imitate the strategy that the selected friend adopts.

In real social networks, different friends have a different level of spreading ability with respect to any one friend [35]. We use the payoffs to represent the level of spreading ability of different friends. Thus, the friend that has a higher payoff is more likely to be selected. The probability that the *j*-th friend is selected is given by

$$p_{i \longrightarrow j}^{s}(t) = \frac{\left(U_{j}(t) - U_{r_{i}^{*}(t)}(t) + (1/q_{u})\right)}{\sum_{r \in N_{i}} \left(U_{j}(t) - U_{r_{i}^{*}(t)}(t) + (1/q_{u})\right)},$$
(3)

where  $r_i^*(t)$  is the friend of individual *i* who has the minimum payoff. This variable is expressed as

$$r_{i}^{*}(t) = \left\{ r_{i}(t) \mid \min_{r \in N_{i}} U_{r}(t) \right\}.$$
(4)

Considering the sensitivity to the payoffs of individuals, a parameter  $q_u$  ( $q_u > 0$ ) is included. This parameter amplifies or lessens the heterogeneity in the influence ability of the different friends of any individual. If  $q_u$  is large,  $p_{ij}^s$  (t) is sensitive to  $U_j$  (t), which means that a small perturbation in  $U_j$  (t) can lead to a big change of  $p_{ij}^s$  (t). On the other hand, if  $q_u$  is small,  $p_{ij}^s$  (t) is insensitive to  $U_j$  (t), which means that a big perturbation in  $U_j$  (t) can lead to a small change in  $p_{ij}^s$  (t).

Then, the individuals will decide whether to imitate the strategy of the selected friend. Specifically, we consider that when an individual repeatedly receives a piece of information, the degree of individual's attention to the information is proportional to the number of times that the individual received the information but did not spread it. Combining the attention degree with the payoffs gap between the individual and the selected friend, we express the probability of individuals imitating the strategy of the selected friend as follows:

$$p_{i \longrightarrow j^*}(t+1) = \frac{\exp\left(-s_{ij^*}^2(t)/w_{ij^*}(t)\right)}{1 + \exp\left[\left(U_i(t) - U_{j^*}(t)\right)/\varepsilon\right]}.$$
 (5)

In (5), the denominator is based on the Fermi function [37], which states that the individual *i* is more likely to adopt the strategy of friend  $j^*$  in t + 1 round if  $j^*$  has a higher payoff than i in the round at time t. Otherwise, the individual i is more likely to maintain their original strategy. Parameter  $\varepsilon$ describes the noise factors, which reflect an individual's uncertainty when they have to update their strategy.  $\varepsilon \longrightarrow 0$ represents complete rationality;  $\varepsilon \longrightarrow +\infty$  infers complete randomness of a strategy decision. The numerator is an exponential (exp) function, and it denotes the attenuation degree of individual *i* attention to the information friend  $j^*$  believed. Let  $w_{ii^*}(t)$  denote the number of times individual *i* receives the information that is believed by their friend  $j^*$  before time *t*.  $s_{ij^*}(t)$  denotes the number of times individuals *i* and *j*<sup>\*</sup> adopt different strategies when *i* receives the information believed by *j*<sup>\*</sup> before time *t*. The exp function is an incremental function. With the increase in  $s_{ij^*}(t)$ , the value of  $(-s_{ij^*}^2(t)/w_{ij^*}(t))$ decreases and exp function decreases, which ultimately leads to the decrease in the whole probability function. Thus, equation (5) can exactly express the probability of individuals imitating the strategy of the selected friend.

Evolutionary game theory studies how a group of players converge to a stable equilibrium after a period of strategic interactions. The final equilibrium state is defined as the evolutionary stable state (ESS). Evolutionary stability strategy refers to when an individual conducting any small number of mutation strategies cannot invade the whole population, which means that the whole population does not change in the long run. In the scenario of competitive information propagation, we obtain the ESS through iterative experimental simulation. When the number of nodes in each state fluctuates very little, the evolution is considered to be relatively stable.

#### 3. Experimental Data and Analysis of Results

To support our analysis of the propagation of the two pieces of competitive information in the evolutionary game model, we conduct an information propagation experiment using real-world social networks. We use data from four different regions, supplied by the website http://networkrepository. com/. For simplicity, we only select the maximal connected subgraphs. The network properties are listed in Table 3, whereby  $\langle k \rangle = 1/N \sum_{i=1}^{N} k_i$  is the average degree,  $\langle c \rangle = 1/N \sum_{i=1}^{N-1} 2 \times m_i/k_i \times (k_i - 1)$  is the average clustering coefficient, and  $m_i$  denotes the number of links between *i*'s friends.

In this section, we randomly select 0.1% nodes as the propagation sources of the competitive information. This means the initial fractions of individuals in the states of  $B_1$  and  $B_2$  are 0.1%. Then, two pieces of competitive information begin to spread over the network. Let  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  denote the final average fractions of individuals in the states of I,  $B_1$ , and  $B_2$ , respectively. As time goes on,  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  reach a steady state. The numerical results are averaged over 500 independent runs performed using parameters modeled off four real social networks.

The values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  for the different basic payoffs  $u_1$  and  $u_2$  are shown in Figure 2. The other three parameters are set to be  $\alpha_1 = 2$ ,  $\alpha_2 = 2$ , and  $q_{\mu} = 0.1$ . From top to bottom, the rows correspond to the four social networks: Hamsterster, Simmons81, Oberlin44, and Bowdoin47. It can be observed from Figures 2(a), 2(d), 2(g), and 2(j) that  $\rho_I$ decreases when  $u_1$  or  $u_2$  increases. Simulation results in Figures 2(b), 2(e), 2(h), and 2(k) show that  $\rho_{B1}$  increases when  $u_1$  increases and  $u_2$  does not increase. From Figures 2(c), 2(f), 2(i), and 2(l), we observe  $\rho_{B2}$  increases when  $u_2$  increases and  $u_1$  does not increase. In addition,  $\rho_{B1} > \rho_{B2}$  when  $u_1$  is significantly bigger than  $u_2$ . On the contrary,  $\rho_{B1} > \rho_{B2}$  when  $u_1$  is significantly smaller than  $u_2$ . Thus, in the competitive environment, the value of  $u_1/u_2$  is a key factor in determining the values of  $\rho_{B1}$  and  $\rho_{B2}$ . As a result of the characteristics of the four networks, under the same level of payoffs, there are less  $\rho_I$  in the network with a higher average degree. This means that, under the same level of ratio of the payoffs for the two pieces of competitive information, there are more believers in the network with a higher average degree. The reason for this is that, when the degree of an individual becomes larger, it has more opportunities to receive information.

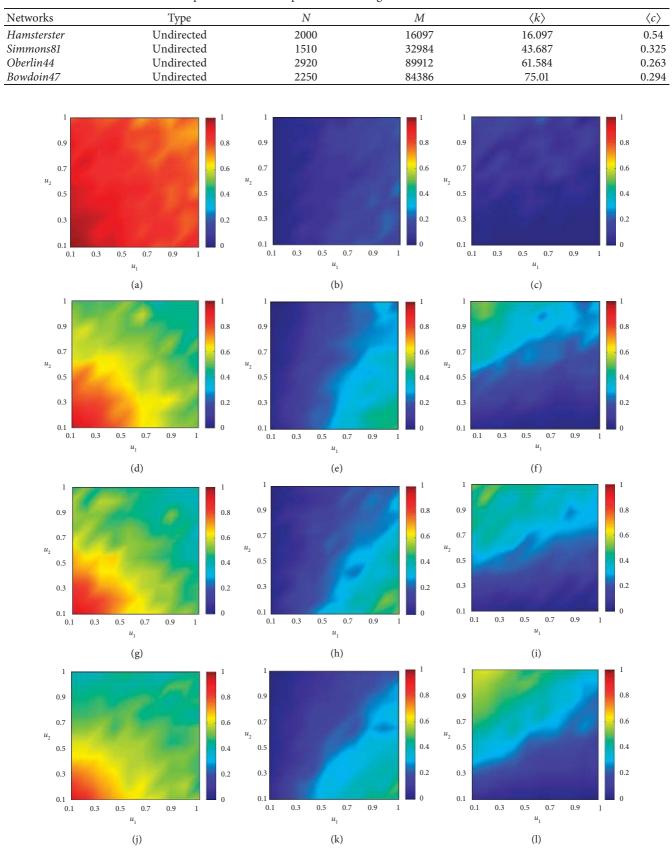


TABLE 3: Comparison of network parameters among four real-world network datasets.

FIGURE 2: The values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  for different  $u_1$  and  $u_2$  payoffs in the four networks. Parameters are set as  $\alpha_1 = 2$ ,  $\alpha_2 = 2$ ,  $q_u = 0.1$ , and  $\varepsilon = 0.1$ . (a)  $\rho_I$  of *Hamsterster*; (b)  $\rho_{B1}$  of *Hamsterster*; (c)  $\rho_{B2}$  of *Hamsterster*; (d)  $\rho_I$  of *Simmons* $\delta_I$ ; (e)  $\rho_{B1}$  of *Simmons* $\delta_I$ ; (f)  $\rho_{B2}$  of *Simmons* $\delta_I$ ; (g)  $\rho_I$  of *Oberlin44*; (h)  $\rho_{B1}$  of *Oberlin44*; (i)  $\rho_{B2}$  of *Oberlin44*; (j)  $\rho_I$  of *Bowdoin47*; (k)  $\rho_{B1}$  of *Bowdoin47*.

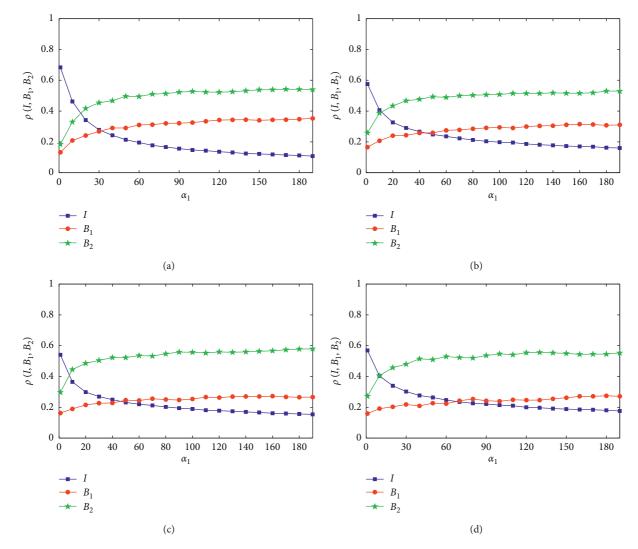


FIGURE 3: The values of  $\rho_1$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  for different  $\alpha_1$  payoffs in the four networks. Parameters are set as  $u_1 = 0.4$ ,  $u_2 = 0.5$ ,  $\alpha_2 = 3$ ,  $q_u = 0.1$ , and  $\varepsilon = 0.1$ . (a) *Hamsterster*; (b) *Simmons*81; (c) *Oberlin*44; (d) *Bowdoin*47.

Figure 3 shows the values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  for the different levels of bonus  $\alpha_1$  in the four networks. The other parameters are fixed with  $u_1 = 0.4$ ,  $u_2 = 0.5$ ,  $\alpha_2 = 3$ , and  $q_{\mu} = 0.1$ . When the evolution is stable, the values of  $\rho_{I}$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  are about 0.115, 0.345, and 0.54 in *Hamsterster*, 0.16, 0.31, and 0.53 in Simmons81, 0.16, 0.27, and 0.57 in Oberlin44, and 0.18, 0.275, and 0.545 in Bowdoin47. As the bonus parameter  $\alpha_1$  increases,  $\rho_I$  decreases, while  $\rho_{B1}$  and  $\rho_{B2}$  increase. The effects on the result tend to be stable when  $\alpha_1$  reaches a fairly high level. Moreover,  $\rho_{B2}$  is always larger than  $\rho_{B1}$ . This is because  $u_2 > u_1$ , which means that the ignorant individuals are more likely to believe the strategy that has a higher payoff. Comparing the data of the four networks, it can be observed that  $\rho_I$  falls faster in the network that has a higher average clustering coefficient and a lower average degree. Furthermore, the difference between  $\rho_{B1}$  and  $\rho_{B2}$  in the network with a lower average clustering coefficient exceeds that of the network with a higher average clustering coefficient.

Figure 4 shows the values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  versus different punishment levels, represented by  $\alpha_2$ , in the four

networks when  $\alpha_1 = 3$ . When the evolution is stable, the values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  are about 0.93, 0.025, and 0.045 in Hamsterster, 0.94, 0.022, and 0.038 in Simmons81, 0.954, 0.018, and 0.028 in Oberlin44, and 0.966, 0.015, and 0.019 in Bowdoin47. As  $\alpha_2$  increases, it can be observed that  $\rho_1$ increases, while  $\rho_{B1}$  and  $\rho_{B2}$  decrease. This illustrates that increasing the value of  $\alpha_2$  inhibits the spread of the two pieces of information. The inhibition caused by  $\alpha_2$  for the second piece of information is more obvious than for the first piece of information, and the change rate obviously speeds up at the beginning and smoothens toward the end of the timeframe of repeated interactions. Moreover, it can be seen that  $\rho_I$  increases faster in the network that has a lower average clustering coefficient and higher average degree. This is because the individual in state I is more likely to believe the strategy that has a lower number of opponents.

Figure 5 shows the values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  versus different  $q_u$ . When  $q_u$  increases to a certain value, the number of individuals seems to become constant. When the

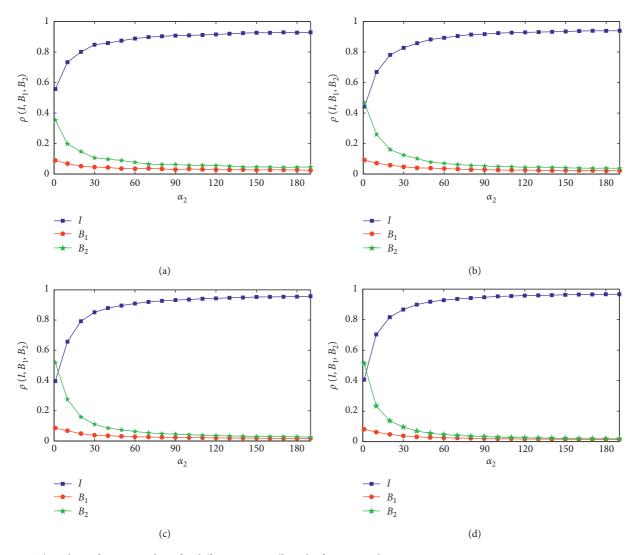


FIGURE 4: The values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  for different  $\alpha_2$  payoffs in the four networks. Parameters are set as  $u_1 = 0.4$ ,  $u_2 = 0.5$ ,  $\alpha_1 = 3$ ,  $q_u = 0.1$ , and  $\varepsilon = 0.1$ . (a) *Hamsterster*; (b) *Simmons81*; (c) *Oberlin44*; (d) *Bowdoin47*.

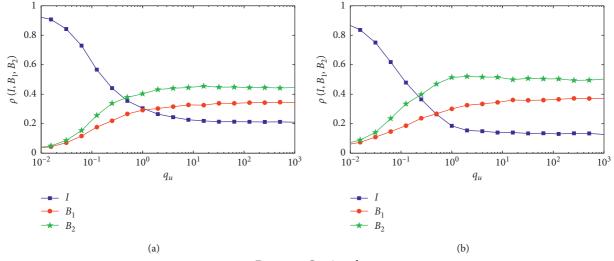


FIGURE 5: Continued.

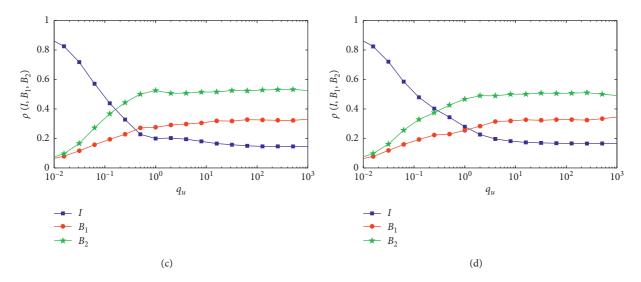


FIGURE 5: The values of  $\rho_1$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  versus different payoffs  $q_u$  in the four networks. Parameters are set as  $u_1 = 0.4$ ,  $u_2 = 0.5$ ,  $\alpha_1 = 3$ ,  $\alpha_2 = 3$ , and  $\varepsilon = 0.1$ . (a) *Hamsterster*; (b) *Simmons81*; (c) *Oberlin44*; (d) *Bowdoin47*.

evolution is stable, the values of  $\rho_I$ ,  $\rho_{B1}$ , and  $\rho_{B2}$  are about 0.21, 0.33, and 0.46 in *Hamsterster*, 0.135, 0.37, and 0.495 in *Simmons81*, 0.145, 0.325, and 0.53 in *Oberlin44*, and 0.16, 0.33, and 0.51 in *Bowdoin47*. Figure 5 clearly shows that, as  $q_u$  increases, the values of  $\rho_I$  decrease, while  $\rho_{B1}$  and  $\rho_{B2}$  increase. This means that if the heterogeneity is further amplified, the number of individuals spreading the two pieces of information increase. The comparative experimental data results based on the different network parameters show that the difference between  $\rho_{B1}$  and  $\rho_{B2}$  in the network that has a lower average clustering coefficient and a higher average degree is larger than in the networks that have a higher average clustering coefficient and a lower average degree.

Figures 3–5 illustrate that parameters  $\alpha_1$ ,  $\alpha_2$ , and  $q_u$ effectively control the information propagation range. In particular, they accurately control the information that is spread by the individuals if they receive a high basic payoff from spreading the information. Moreover, this piece of information gains more believers than the other, competitive piece of information. Thus, this piece of information can be recognized as the winner of the competitive game. Furthermore, when two competitive messages travel in a network with a lower average clustering coefficient and a larger average degree, the number of individuals spreading the information that prevails in the competition is relatively large. This is because individuals will face a less heterogeneous environment (in terms of the strategies of their friends) when making decisions in the networks that have a higher average clustering coefficient. As a result, individuals are unlikely to change their strategy once they believe either of the pieces of information.

#### 4. Conclusions

In this paper, we adopt evolutionary game theory to analyze the propagation of two pieces of competitive information throughout a social network. By using the payoff matrix to

abstract the individuals' decision process, we propose a propagation model for the two pieces of competitive information. Simulation results indicate that, when the system reaches stability, a certain piece of information will win the game if the individuals attain a larger basic payoff after spreading it, whereby the higher payoff leads more individuals to prefer to spread this piece of information. Moreover, the topology of networks also influences the pairwise interaction of two pieces of competitive information. Using the same parameters, the results show that the network with a higher average degree has a lower number of ignorant individuals. If the networks possess a lower average clustering coefficient and a larger average degree, the gap in the numbers of individuals trusting different pieces of information grows. This means that the number of individuals spreading the information that wins the game significantly surpasses the number of individuals believing the other piece of information. Overall, the results show that the proposed method is effective and practical for modeling the propagation of two pieces of competitive information and provide effective insights into how to better control and guide public opinion.

Future work could examine how to incorporate the nonoptimal or bounded rationality of players into the model, how to incorporate the heterogeneous rationality of players, and how to extend the model to more than two pieces of competitive information. Furthermore, in this paper, we assume that the individuals received the information from their friends only. However, people could also acquire the information in other ways (such as via the Internet or from a newspaper). Thus, we advocate the development of a more precise model to investigate more complex propagation processes.

#### **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

## **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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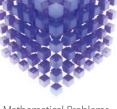
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