Evolutionary games on scale-free networks with a preferential selection mechanism

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ABSTRACT

Considering the heterogeneity of individuals' influence in the real world, we introduce a preferential selection mechanism to evolutionary games (the Prisoner's Dilemma Game and the Snowdrift Game) on scale-free networks and focus on the cooperative behavior of the system. In every step, each agent chooses an individual from all its neighbors with a probability proportional to k^α indicating the influence of the neighbor, where k is the degree. Simulation results show that the cooperation level has a non-trivial dependence on α. To understand the effect of preferential selection mechanism on the evolution of the system, we investigate the time series of the cooperator frequency in detail. It is found that the cooperator frequency is greatly influenced by the initial strategy of hub nodes when $\alpha > 0$. This observation is confirmed by investigating the system behavior when some hub nodes' strategies are fixed.

1. Introduction

Ranging from biological to social systems, from economic to political activities, cooperative behavior is ubiquitous in the real world consisting of selfish individuals. Since the emergence of cooperation apparently contradicts Darwinian evolution theory, many scientists from different fields often resort to game theory, especially that of the Prisoner's Dilemma Game (PDG) and the Snowdrift Game (SG) together with an evolutionary context [1–4], as a common mathematical framework for investigating the dilemma of cooperation between individuals.

In the original PDG, two players have to decide simultaneously on whether they want to cooperate or defect. For mutual cooperation both players receive the rewards R, but only the punishment P for mutual defection. A defector exploiting a cooperator gets an amount T (the temptation to defect) and the exploited cooperator receives S (the sucker's payoff). These elements satisfy the following two conditions: $T > R > P > S$ and $2R > T + S$. Consequently, the cooperation dilemma emerges: although mutual cooperation yields the highest collective payoff, defectors will do better no matter what strategy the opponent chooses. Thus the selfish players will both decide to defect, whereby each of them gets the second worst payoff. As a reformation of PDG, the SG which is believed more favorable to sustain cooperation is later proposed. The

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SG is just different from the PDG in the order of P and S, as $T > R > S > P$. Thus, the best action depends now on the opponent: to defect if the other cooperates, but to cooperate if the other defects. However, there still exists unstable cooperative behavior which is contrary to the ubiquitous cooperation in the real world. In the last few years, many efforts have been made to understand the conditions that can enable cooperative behavior to emerge and persist [5]. It is found that several strategies with memory, such as "tit-for-tat" [6] and "win stay and lose shift" [7], can enhance the cooperation. Recently, an interesting work by Nowak and May showed that the evolutionary PDG on a simple spatial lattice can induce the persistence of cooperation and even for using unconditional defection or cooperation [8]. In contrast, the work of Hauert and Doebeli demonstrates that the spatial structure often prohibits cooperation in the SG [9]. With inspiration provided by the spatial extensions, much attention has been given to the evolutionary games on different population structures, including on regular networks [10–18] and on complex networks [14,19–26].

However, previous works mainly ignore the heterogeneity of individuals' influence. The agents are selected with equal probability for strategy learning during the evolution, which apparently contradicts the real situation. It is natural to consider that different individuals may have different influence in the society. The rational agents may not randomly select a neighbor when updating their strategies. Recently, Szolnoki et al. and Guan et al. have investigated the effect of heterogeneous influence in the evolutionary networked games, including inhomogeneous teaching activity [27–29], the diversity of reproduction rate [30], and non-linear selection [31,32]. A variety of empirical researches have revealed that social networks are actually associated with the small-world property and a scale-free, power-law degree distribution [33, 34]. In this paper, we investigate the evolutionary games on the well known BA scale-free network considering the influence heterogeneity induced by individuals' degree and focus on the effects of preferential selection mechanisms on the system's cooperation level.

The paper is organized as follows. In the next section, we describe the models of evolutionary games and the learning rule used in this work. The simulation results and discussion are given in Section 3. Finally the paper is concluded by the last section.

2. The model

We consider the evolutionary PDG and SG with players located on the BA scale-free network which is generated by the rules of "growth" and "preferential attachment" [35]: starting from m_0 fully connected nodes, at every step a new node is added with m ($m \leqslant m_0$) edges that link to m different existing nodes in such a way that the probability of being connected to the existing node i is proportional to its degree

$$p_i = \frac{k_i}{\sum_l k_l}, \tag{1}$$

where k_i is the degree of node i and l runs over all the existing nodes. The BA scale-free network is considered as a suitable model for representing the social structure.

For the original PDG, we can simplify the payoff matrix in accordance with common practice: $T = b, R = 1$ and $P = S = 0$, where b represents the advantage of defectors over cooperators [8]. Generally, we can set $1 \leqslant b \leqslant 2$. For the SG, we can simplify the model in the following way: $R = 1, S = 1 - r, T = 1 + r$ and $P = 0$, where $0 \leqslant r \leqslant 1$ indicates the rate of labor cost [9].

Initially, the strategy of C or D is randomly distributed on the nodes in the BA network. In each time step, the agents play PDG or SG with their direct neighbors and get payoffs depending on the payoff matrix. The total payoff of player i for the step is stored as U_i. When player i is updated, it firstly chooses a neighbor j with a probability

$$P_{ij} = \frac{k_j^{\alpha}}{\sum_l k_l^{\alpha}}, \tag{2}$$

where P_{ij} denotes the influence of the neighbor j, k_j is the degree of neighbor j, l runs over all neighbors of player i, and α is a tunable parameter describing the extent of the preferential effect. If $\alpha = 0$, player i will ignore the neighbors' degree information and randomly select a neighbor to learn from, so the rule is reduced to the standard PDG. If $\alpha = 1$, it leads to the proportional selection rule [36]. If $\alpha \to \infty$, the neighbor with the largest connectivity is selected, which resembles the deterministic selection rule. For other positive values of α, the influence of the neighbors is a non-linear function of their degrees, which is in accordance with the fact that people with more interactions usually attract more attention in the society. To be more general, we also considered negative values of α, where the non-influential neighbors are more likely to be selected.

Hereafter, the player i will adopt the strategy of j with a probability [10]

$$H_{i \to j} = \frac{1}{1 + \exp[(U_i - U_j)/\kappa]}, \tag{3}$$

where $0 \leq \kappa \leq \infty$ characterizes the environmental noise representing the irrationality of people, including bounded rationality, individual trials, errors in decision, etc. $\kappa = 0$ denotes the completely deterministic adoption and $\kappa = \infty$
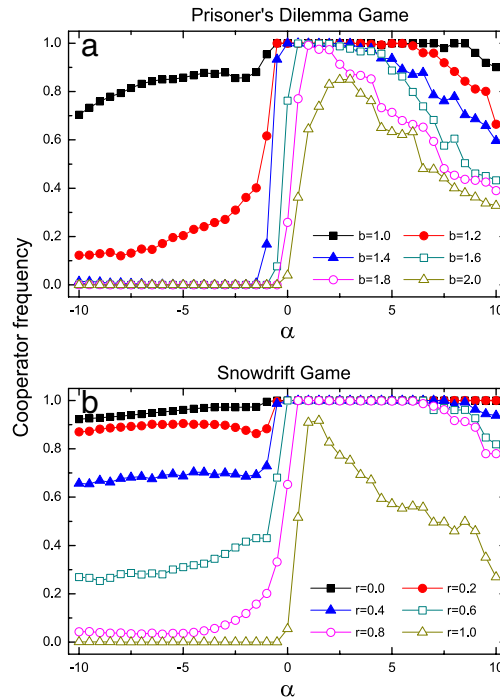
Fig. 1. (Color online.) The cooperator frequency *vs* α for different values of b (PDG) and r (SG).

denotes the completely random adoption. The effect of κ has been well studied on lattices and regular graphs [37]. In this paper, κ is set to 0.1 as in many other works. And the strategies of all agents are updated in parallel.

3. Simulation results and discussion

All the simulations below are carried out on BA scale-free networks with network size $N = 1000$ and $m_0 = m = 4$. Initially, strategies (C and D) are randomly distributed among the population. Equilibrium frequencies of cooperators are obtained by averaging over 3000 generations after a transient time of 10 000 generations. The results are averaged over 50 individual runs.

We first investigate the cooperator frequency of the system, which is the most important quantity for characterizing the cooperative behavior. It is defined as the fraction of cooperators in the population: $f_C = \frac{N_C}{N_C + N_D}$, where N_C and N_D are the numbers of cooperative and defecting agents in the system. Fig. 1 shows the cooperator frequency as a function of α for different values of b (PDG) and r (SG). The cooperation level has a non-trivial dependence on α and monotonically decreases with the incrementation of b or r.

For $\alpha = 0$ the preferential selection is switched off. In this case the agents have equal influence and the system is equivalent to those studied by Santos and Pacheco [24]. For $\alpha > 0$, players are more likely to learn from the influential hub nodes. From Fig. 1, One can see that the cooperator frequency is evidently promoted and there is a maximum, indicating that too strongly depending on hub nodes will harm the maintenance of cooperation. Besides, the critical point of α_c corresponding to the maximal cooperation level will increase with the incrementation of b and r. For $\alpha < 0$, players are more likely to learn from non-hub nodes. Consequently, the cooperator frequency is highly restrained. Moreover, the system soon falls to a pure D state for $b > 1.2$ for PDG when $\alpha < 0$. But the pure D state emerges only when $r = 1.0$ for SG. This is consistent with the previous conclusion: SG is more favorable for sustaining cooperation than PDG.

To investigate the effect of the preferential learning mechanism in more detail, it is necessary to study the evolution process. Fig. 2 shows the time series of cooperation frequency for the system. One can see that: (i) if $\alpha = 0$, the cooperator frequency fluctuates around a certain value (Fig. 2(c), (d)); (ii) if $\alpha < 0$, the cooperator frequency is low and the time series are quite smooth (Fig. 2(a), (b)); (iii) if $\alpha > 0$, the time series present as four types: (1) keep a high cooperation level, (2) keep a low cooperation level, (3) jump from a low cooperation to a high cooperation level, (4) fall from a high cooperation state to a low cooperation state (Fig. 2(e), (f)). Here we give a brief explanation for the different behaviors of the time series. When $\alpha = 0$, the connected hub nodes play an important role during the evolution and the system's cooperator frequency fluctuates with the hub nodes' choice. When $\alpha < 0$, the evolution is no longer driven by the hub nodes because their influences are restrained. Thus the time series become smooth. When α is slightly larger than zero, the hub nodes are moderately enhanced. Since individuals prefer to follow the hub nodes' strategy, the cooperation level is promoted. However, the positive values of α are not always beneficial to the persistence of cooperative behavior. When α is very large,
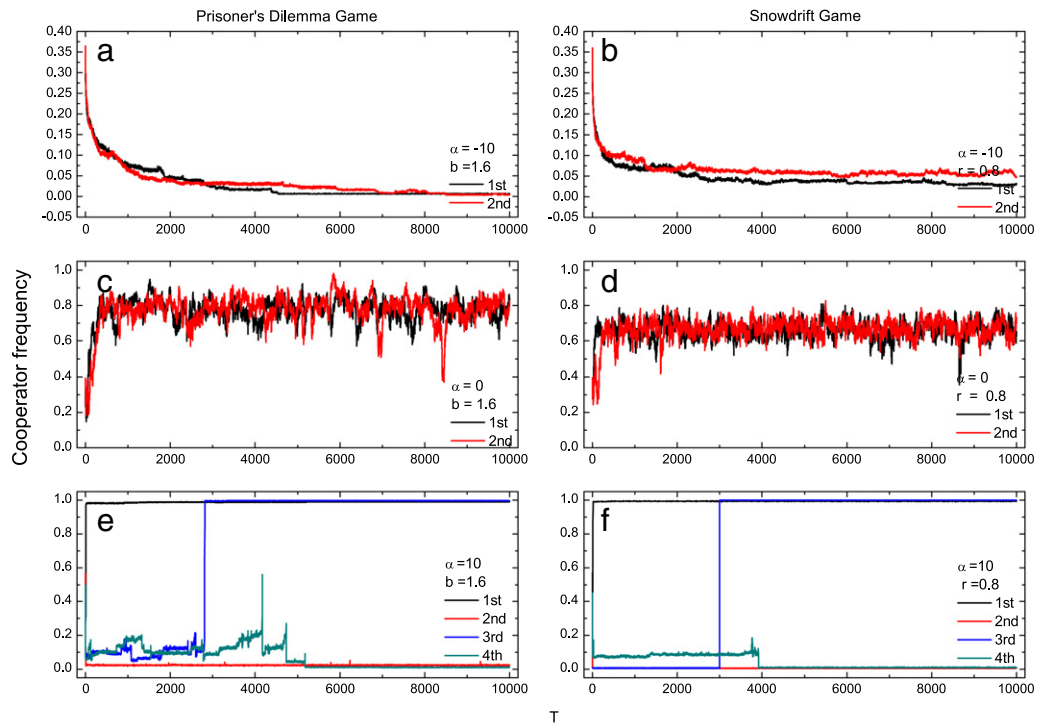
Fig. 2. (Color online.) Time series for different values of α, b or r.

e.g. $\alpha = 10$, according to the neighbor-selecting rule (Eq. (2)) and strategy-updating rule (Eq. (3)), the entire system behavior is dominated by the connected hub nodes of the BA network: if the hub nodes choose to cooperate (defect), the cooperator frequency is promoted (restrained) to 1 (0)—the first and second time series in Fig. 2(e), (f)); if the hub nodes reverse their strategy, there will be a sudden jump in the cooperation level—the third and fourth time series in Fig. 2(e), (f)).

Previous works have shown that the hub nodes of the BA network play a key role in the evolution of cooperation [38]. In Ref. [38], they find an overwhelming dominance of cooperation whenever graphs are generated through the mechanisms of growth and preferential attachment. And cooperation dominates from large population sizes down to even nearly 100 individuals. To further demonstrate the effects of hub nodes' influence, we investigate the cooperation level of the system by pinning the strategies of some hub nodes. We select five top nodes with the largest degree out of the 1000 nodes, and fix their strategies to either C (Fig. 3(a), (b)) or D (Fig. 3(c), (d)). One can see that for both PDG and SG, when $\alpha > 0$, the cooperator frequency strongly depends on the hub nodes' choices. When the hub nodes are cooperative, the cooperator frequency is soon promoted to 1.0, while it takes a very low value when the hub nodes defect. This is consistent with our previous discussion: the hub nodes' influence is magnified when $\alpha > 0$. In contrast, when $\alpha < 0$, the cooperator frequency does not change remarkably with the strategy of hub nodes. Interestingly, we can observe that the cooperator frequency for $\alpha < 0$ is apparently larger than for $\alpha > 0$. When $\alpha < 0$, all nodes prefer to learn from non-hub nodes; thus the influence of the five hub nodes is restrained. Therefore, the cooperation level of the system will not change much no matter whether the five hub nodes are cooperators or defectors (Figs. 1 and 3). However, when $\alpha > 0$, nodes are more likely to learn from hub nodes, and the system's behavior will be dominated by the hub nodes. If the top five hub nodes are fixed as cooperators, the cooperation level will be remarkably promoted. If they are fixed as defectors, the cooperation level is highly restrained. Therefore, the system is more robust to resist the invasion of hub defectors when $\alpha < 0$. Besides, it is also worth noting that compared with PDG, SG is more preferential for cooperating when the hub nodes are defectors.

Moreover, previous researches have proposed a normalized learning strategy to avoid additional bias caused by the heterogeneity of the node degree:

$$H_{i \to j} = \frac{1}{1 + \exp[(U_i/k_i - U_j/k_j)/\kappa]}, \tag{4}$$

where the ratio of the total payoff and its degree U_i/k_i is defined as the normalized payoff [36]. Recently, Szolnoki et al. have studied a system providing a continuous transition between the unnormalized and normalized cases [39]. Fig. 4 shows the cooperator frequency as a function of α with the normalized learning rule. For the PDG, compared with the former results of Eq. (3), the cooperation level is evidently depressed. For all values of b, the cooperator frequency is zero when $\alpha = 0$. Interestingly, the cooperator frequency of $b = 1.0$ can be promoted by not only positive values of α but also negative values. When α is a little larger than zero, the cooperator frequency suddenly jumps from 0 to 1. For the SG, the cooperator
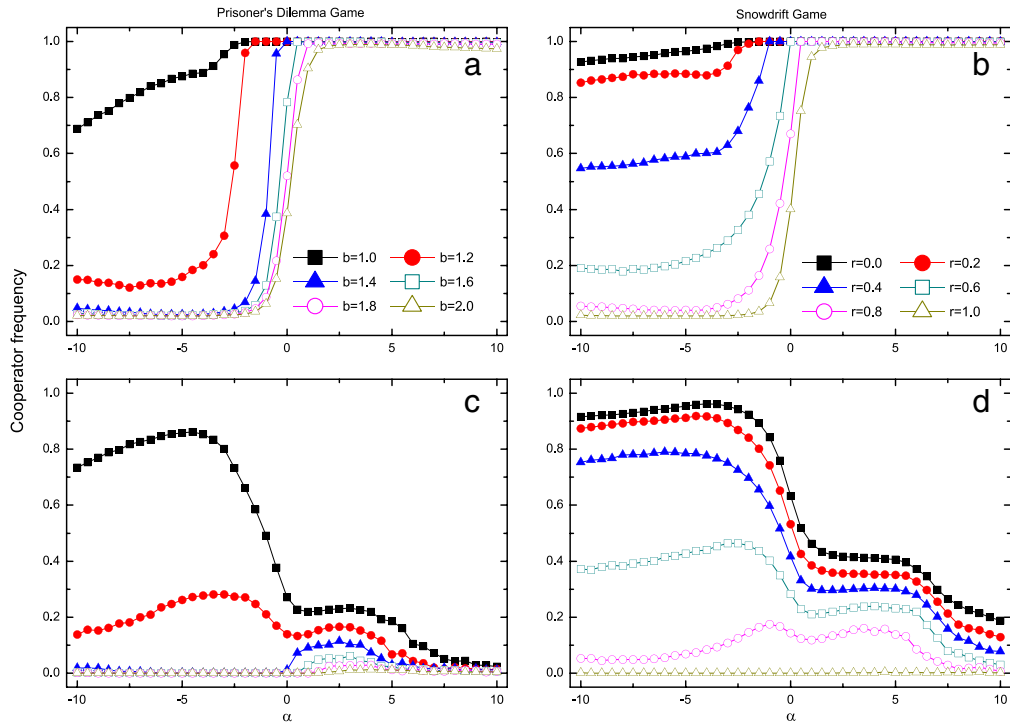
Fig. 3. (Color online.) The cooperator frequency *vs* α for different values of *b* (PDG) and *r* (SG) with pinning five hub nodes to the C strategy ((a), (b)) and D strategy ((c), (d)).
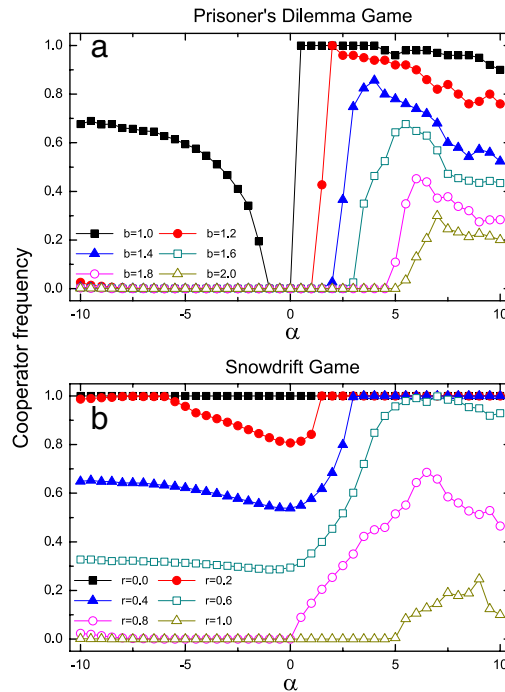

Fig. 4. (Color online.) The cooperator frequency *vs* α for different values of *b* (PDG) and *r* (SG) with a normalized learning strategy.

frequency is also depressed by the normalized learning strategy except for $r = 1.0$, where the cooperator frequency stays at 1 for the whole range of α. This reveals that the preferential selection mechanism can still remarkably affect the cooperator frequency without the diversity in node degrees.

4. Conclusion

In summary, we have investigated the cooperative behavior in the context of evolutionary games on scale-free networks with a preferential selection mechanism. An individual's influence is estimated as k_i^α. Compared with the homogeneous influence mechanism, e.g. $\alpha = 0$, the cooperation level can be promoted over a large range of α and there exists an optimal value of α_c ($\alpha_c \geq 0$) resulting in the highest cooperation level. Although the cooperation can be promoted when the influences of hub nodes are moderately enhanced, excessive influence of them will make the system defect.

This work reveals that the introduction of the preferential selection mechanism plays an important role in the emergence and maintenance of cooperation in evolutionary networked systems.

Acknowledgements

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