Evolving Systems: An Outcome of Fondest Hopes and Wildest Dreams

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New theory is presented for evolving systems, which are autonomously controlled subsystems that self-assemble into a new evolved system with a higher purpose. Evolving systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process. This is the concept of Adaptive Key Component Control that operates through one specific component to maintain stability during the evolution. In addition, this control must often overcome persistent disturbances that occur while the evolution is in progress. Theoretical results will be presented for Adaptive Key Component control for persistent disturbance rejection. An illustrative example will demonstrate the Adaptive Key Component controller on a system composed of rigid body and flexible body modes.

I. Introduction

EVOLVING systems are autonomously controlled subsystems which self-assemble into a new evolved system with a higher purpose[1-2]. Evolving systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process[3-5]. An adaptive key component controller has been shown to restore stability in evolving systems that would otherwise lose stability during evolution[6-8]. The adaptive key component controller uses a direct adaptation control law to restore stability to the evolving system through a subset of the input and output ports on one key component of the evolving system. The reader is directed to refs. 7-8 for a more detailed description of evolving systems. In this paper, we address the situation where persistent disturbances can appear in some components and must be mitigated by the adaptive key component controller. Such disturbances will often be attendant in actively controlled rendezvous and docking or component capture applications.

The control laws used by the adaptive key component controller to restore stability in an evolving system are guaranteed to have bounded gains and asymptotic tracking if the evolved system is almost strictly dissipative. Hence, it is desirable to know when the dissipativity traits of the subsystem components, including the key component, are inherited in an evolving system. We present results describing when an evolving system will inherit the almost strict passivity traits of its subsystem components. Then we will present an adaptive key component controller that restores asymptotic stability with bounded adaptive gains and mitigates the effect of persistent disturbances during evolution. The paper is organized as follows: the next section describes the framework for evolving systems, then the conditions under which dissipativity traits are inherited in evolving systems will be presented. Next, the design of adaptive key component disturbance accommodating control will be described. Finally, an illustrative example is presented to demonstrate the concepts presented earlier.

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II.   Evolving Systems Framework

A. Mathematical Formulation of Evolving Systems

A mathematical formulation of a nonlinear time-invariant evolving system is given here. Consider a system of L components of individually, actively controlled subsystems which can be described by the following equations for the ith component:

$$\begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = g_i(x_i, u_i) \end{cases} \tag{1}$$

where $i = 1, 2, ..., L$. The ith component has a Performance Cost Function J_i and a Lyapunov Function V_i. These are the building blocks of the evolving System. When these individual components are joined to form an evolved system, the new entity becomes:

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases} \tag{2}$$

with $x = [x_1 ... x_L]^T$, $y = [y_1 ... y_L]^T$, Performance Cost Function J, and Lyapunov Function V. The ith component in the above evolved system is given by:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j=1}^{L} \varepsilon_{ij} f_{ij}(x_i, x_j, u_j) \tag{3}$$

where $0 \le \varepsilon_{ij} \le 1$ and $f_{ij}(x_i, x_j, u_j)$ represents the interconnections between the ith and j^{th} components. Note that when $\varepsilon_{ij} = 0$, the system is in component form and when $\varepsilon_{ij} = 1$, the system is fully evolved. As the system evolves, or joins together, the ε_{ij}'s evolve from 0 to 1.

The components of the evolving system are actively controlled by means of local control. Local control means dependence only on local state or local output information, i.e., $u_i = h_i(x_i)$ or $u_i = h_i(y_i)$. In general, the local controller on the ith component would have the form:

$$\begin{cases} u_i = h_i(y_i, z_i) \\ \dot{z}_i = l_i(y_i, z_i) \end{cases} \tag{4}$$

where z_i is the dynamical part of the control law. Local control will be used to keep the components stable and to meet the individual component performance requirements, J_i.

Once the system is fully evolved, the ith component in the fully evolved system becomes:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j=1}^{L} f_{ij}(x_i, x_j, u_j) \tag{5}$$

A state space version of the ith individual component of an evolving system where the components are connected through the states can be represented as:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B_i(x_i)u_i + \sum_{j=1}^{L} \varepsilon_{ij} A_{ij}(x_i, x_j); \ x_i(0) \equiv x_{0_i} \\ y_i = C_i(x_i) \end{cases} \qquad (6)$$

where $i = 1, 2, \ldots L$, $x_i \equiv [x_1^i \ldots x_{n_i}^i]^{\mathrm{T}}$ is the component state vector, $u_i \equiv [u_1^i \ldots u_{m_i}^i]^{\mathrm{T}}$ is the control input vector, $y_i \equiv [y_1^i \ldots y_{p_i}^i]^{\mathrm{T}}$ is the sensor output vector, $(A_i(x_i), B_i(x_i), C_i(x_i))$ are vector fields of dimension $n_i \times n_i, n_i \times m_i$, and $p_i \times n_i$, respectively, and the connection forces between components are represented in the $n_i \times n_j$ connection matrix, $A_{ij}(x_i, x_j)$ with $\varepsilon_{ji} = \varepsilon_{ij}$. The state space representation of the evolved system then becomes:
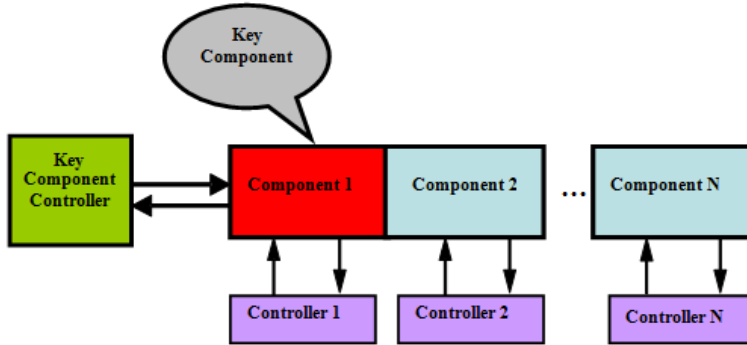
$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \qquad (7)$$

which will also be written as $(A(x), B(x), C(x))$.

B. Characteristics and control of evolving systems

We say a subsystem trait, such as stability, is inherited when the evolved system retains the characteristic of the trait from the subsystem. Previous papers have examined the inheritance of stability and shown that stability is not a generally inherited trait in evolving systems[3-5]. Control can be used in evolving systems to maintain stability in a system that would fail to inherit stability during evolution.

Key component control has been proposed to restore stability in evolving systems that would otherwise lose stability during evolution. The approach used for key component control design is for the control and sensing of the components to remain local and unaltered except in the case of one key component which has additional local control added to stabilize the system during evolution. The key component controller operates solely through a single set of input-output ports on the key component, see Figure 1.

Only the key component of the evolving system needs modification to restore the inheritance of stability. A clear advantage of the key component design is that components can be reused in many different configurations of evolving systems without the need for component redesign. The reuse of components which are space-qualified, or at least previously designed and unit tested, could reduce the overall system development and testing time and should result in a higher quality system with potentially significant cost savings and risk mitigation.

Fig. 1. Evolving system with key component controller.

In many aerospace environments and applications, the parameters of a system are poorly known and difficult to obtain. Adaptive key component controllers, which make use of a direct adaptation control law, are a good design choice for restoring stability in evolving systems where access to precisely known parametric values is limited. The sufficient condition for a nonlinear evolving system with an adaptive key component controller to be guaranteed to have bounded gains and to have asymptotic output tracking is that the system be almost strictly dissipative. So, we are interested in the conditions under which the inheritance of almost strict dissipativity can be guaranteed in evolving systems. The next section gives definitions and theorems related to dissipativity.

III. Inheritance of Almost Strict Dissipativity in Evolving Systems

Inheritance of almost strict dissipativity of subsystems is desirable in evolving systems that use an adaptive key component controller to restore stability. Consider a nonlinear system given by Eq. 7. We say this system is *Strictly Dissipative* when there exists $V(x) > 0 \; \forall x \neq 0$ such that the Lie derivatives satisfy:

$$\begin{cases} L_A V \equiv \nabla V A(x) \leq -S(x) \; \forall x \\ L_B V \equiv \nabla V B(x) = C^T(x) \end{cases} \tag{8}$$

where $\nabla V \equiv gradient V$.

The function $V(x(t))$ is called the *Storage Function* for Eq. 7, and the above says that the storage rate is always less than the external power. This can be seen by taking the time derivative of $V(x(t))$ to obtain:

$$\begin{aligned} \dot{V} &\equiv \nabla V[A(x) + B(x)u] \\ &\leq -S(x) + C^T(x)u \\ &= -S(x) + \langle y, u \rangle \end{aligned} \tag{9}$$

Taking $u \equiv 0$, it is easy to see that Eq. 9 implies the first equation in Eq. 8 but not necessarily the second equation; so Eq. 8 implies Eq. 9 but not conversely. They are only equivalent if the first equation in Eq. 8 is an equality. (When equality holds in Eqs. 8 and 9, the property is known as *Strict Passivity*.)

We will say a system (u, y) is *Almost Strictly Dissipative (ASD)* when there is some output feedback, $u = G_* y + u_r$, that results in the following being strictly dissipative:

$$\begin{cases} \dot{x} = A_C(x) + B(x)u_r \\ A_C(x) \equiv A(x) + B(x)G_*C(x) \\ y = C(x) \end{cases} \tag{10}$$

Now if each component is ASD, then we have

$$\begin{cases} \nabla V_i[A_i(x_i) + B_i(x_i)G_iC_i(x_i)] \leq -S_i(x_i) + \sum_{j=1}^{L} \varepsilon_{ij} \nabla V_i A_{ij}(x_j, u_j) \\ \nabla V_i B_i(x_i) = C_i^T(x_i) \end{cases} \tag{11}$$

where $\nabla V_i \equiv gradient(V_i)$. Due to the interconnection terms, Eq. 11 is not necessarily Strictly Dissipative. However, in some circumstances, the interconnection terms have a special form and ASD is inherited when the system evolves.

Suppose we have a pair of subsystems of the form:

$$\begin{cases} \dot{x}_i = A_i(x_i) + \varepsilon B_i(x_i)u_i + B_i^A(x_i)u_i^A \\ y_i = C_i(x_i) \\ y_i^A = C_i^A(x_i) \end{cases} \tag{12}$$

where $i = 1, 2$ and both subsystems $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix} \right)$ and $\left(\begin{bmatrix} u_2 \\ u_2^A \end{bmatrix}, \begin{bmatrix} y_2 \\ y_2^A \end{bmatrix} \right)$ have storage functions V_i. We have the following result:
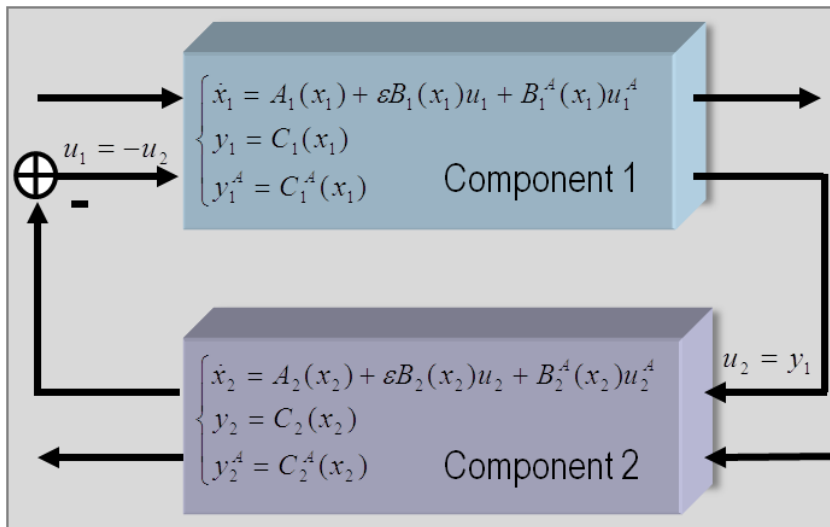
Theorem 1: If the subsystems (u_1^A, y_1^A) and (u_2^A, y_2^A) are ASD and

$$\nabla V_i B_i(x_i) = C_i^T(x_i); \, i = 1, 2 \qquad (13)$$

then the resulting feedback connection, $y_1 = u_2$ and $u_1 = -y_2$, will leave the composite system

$\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} \right)$ almost strictly passive.

Proof: See appendix.

The physical connection of components in evolving systems is equivalent to the feedback connection of the admittance of one component to the impedance of the other component[3,5]. See fig. 2 for a schematic of the admittance-impedance connection between two evolving system components. The A superscript on the input and output variables is used to indicate that the component is in admittance-impedance form. For a nonlinear evolving system with components (u_1, y_1) and (u_2, y_2) connected in admittance-impedance form, Theorem 1 shows that ASD is an inherited property.

Fig. 2. Admittance-Impedance feedback connection of evolving systems.

IV. Adaptive Key Component Control with Persistent Disturbance Mitigation

This section describes the adaptive key component control with persistent disturbance mitigation. The key component is chosen to be *component #1* and will be modeled by the following *nonlinear evolving system component* with an external persistent disturbance:

$$\begin{cases} \dot{x}_1 = A_1(x_1) + \varepsilon B_1(x_1)u_1 + B_1^A(x_1)u_1^A + \Gamma_1(x_1)u_D \\ y_1 = C_1(x_1) \\ y_1^A = C_1^A(x_1) \end{cases} \qquad (14)$$

which will be respresented by the triple $(A_1(x_1), B_1(x_1), B_1^A(x_1), \Gamma_1(x_1), C_1^A(x_1))$. All vector fields in Eq. 14 will have the appropriate compatible dimensions and be smooth in their arguments with a single equilibrium point at 0 in

a neighborhood U. The persistent disturbance input vector $u_D(t)$ is N_D-dimensional and will be thought to come from the following *Disturbance Generator:*

$$\begin{cases} u_D = \Theta\, z_D \\ \dot{z}_D = F\, z_D\,; z_D(0) = z_0 \end{cases} \tag{15}$$

where the disturbance state $z_D(t)$ is N_D-dimensional. Such descriptions of persistent disturbances were first used to describe signals of known form but unknown amplitude[10]. For example, step disturbances yield $\Theta = 1$ and $F=0$ while sinusoidal disturbances can be described by $\Theta = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $F = \begin{bmatrix} 0 & 1 \\ -\omega_D^2 & 0 \end{bmatrix}$ where the frequency ω_D is known but the amplitudes are not known.

Assume that the disturbance generator parameter F is known. In many cases this is not a severe restriction, e.g. step disturbances and sinusoidal disturbances are common forms of persistent disturbances. It turns out that it is better to rewrite the above in the following equivalent form:

$$\begin{cases} u_D = \Theta\, z_D \\ z_D = L\varphi_D \end{cases} \tag{16}$$

where φ_D is a vector composed of the known basis functions for the solutions of Eq. 15 and (L, Θ) need not be known. The following example illustrates the basis function form for the disturbance generator:

$$\begin{aligned} z_D(t) &= e^{Ft} z_D(0) \\ &= \begin{bmatrix} \varphi_1(t), \varphi_2(t), ..., \varphi_{N_D}(t) \end{bmatrix} z_D(0) \\ &= \sum_{i=1}^{N_D} z_D^i \varphi_i(t) = L\varphi_D \end{aligned}$$

Note that L is directly related to F via its columns but not to θ . Some rearrangement of the entries in the columns of F is needed to create φ_D. A simple example of the above is given by the following:

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_D \\ u_D = a\sin(\omega_D t + b) \\ \quad = a_1 \sin(\omega_D t) + a_2 \cos(\omega_D t) \end{cases} \tag{17}$$

Assume component #1 $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix} \right)$ is ASD. Also let the following *matching condition* be true:

$$R(\Gamma_1(x_1)) \subseteq R(B_1^A(x_1)) \tag{18}$$

The matching condition means that there exists H_* such that $B_1^A(x_1)H_* = \Gamma_1(x_1)$. Let *component #2* represent the remainder of the evolving system and assume it is strictly dissipative by choice of local controllers, that is:

$$\begin{cases} \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2)u_2 \\ y_2 = C_2(x_2) \end{cases} \tag{19}$$

The components are in admittance-impedance form so when they are joined $u_1 = -y_2$ and $u_2 = y_1$. The *Adaptive Key Component Controller with Disturbance Mitigation* works through the control input-output ports (u_1^A, y_1^A) of component #1 and is given by:

$$\begin{cases} u_1^A = G_e y_1^A + G_D \varphi_D \\ \dot{G}_e = -y_1^A (y_1^A)^T \gamma_e; \gamma_e > 0 \\ \dot{G}_D = -y_1^A (\varphi_D)^T \gamma_D; \gamma_D > 0 \end{cases} \tag{20}$$

This produces $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t \to \infty]{} 0$ with bounded adaptive gains (G_e, G_D) as the following convergence theorem shows:

Theorem 2: Assume that V_1 and V_2 are positive $\forall x \neq 0$ and radially unbounded, and $\big(A(x), B(x), C(x)\big)$ are continuous functions of x and $S(x)$, above, is positive $\forall x \neq 0$ and has continuous partial derivatives in x. Furthermore, assume:

1) The conditions of Theo.1 are satisfied; so that $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix} \right)$ is Almost Strictly Dissipative (ASD)

2) The following matching condition is met: $R(\Gamma_1(x_1)) \subseteq R(B_1^A(x_1))$

3) φ_D is bounded (or equivalently, F has only simple imaginary poles and no right half-plane poles)

Then the adaptive Controller given by Eq. 20 produces $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t \to \infty]{} 0$ with bounded adaptive gains (G_e, G_D) when Component 1 is joined with Component 2 into an evolved system and the outputs are $y_i = C_i(x_i) \xrightarrow[t \to \infty]{} 0$.

Proof: See Appendix.

It should be noted that the above results might only hold on a neighborhood $N_i(0, r_i) \equiv \{x_i / \|x_i\| < r_i\}$. Then the stability in Theo. 2 is only locally asymptotic to the origin.

In the next section, an illustrative example will be given.

V.   Illustrative Example

Example 1 is a two component linear flexible structure evolving system, see fig. 3. The components of Example 1 are stable when they are unconnected components, but the evolving system fails to inherit the stability of the components. This example will be used to demonstrate the inheritance and lack of inheritance of almost strict dissipativity in evolving systems.
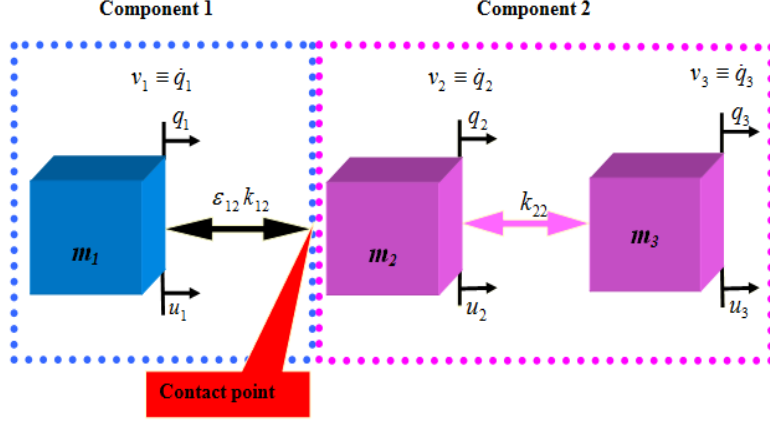

Fig. 3. A two component flexible structure Evolving System.

The dynamical equations for the components of Example 1 are:

$$\begin{cases} m_1\ddot{q}_1 = u_1 - \varepsilon_{12}k_{12}(q_1 - q_2) \\ y_1 = [q_1, \dot{q}_1]^{\mathrm{T}} \end{cases}$$

$$\begin{cases} m_2\ddot{q}_2 = u_2 - \varepsilon_{12}k_{12}(q_2 - q_1) \\ \quad - k_{22}(q_2 - q_3) \\ m_3\ddot{q}_3 = u_3 - k_{22}(q_3 - q_2) \\ y_2 = [q_2, \dot{q}_2]^{\mathrm{T}} \\ y_3 = [q_3, \dot{q}_3]^{\mathrm{T}} \end{cases} \tag{21}$$

with $m_1 = 30$, $m_2 = 1$, $m_3 = 1$, $k_{12} = 4$, and $k_{22} = 1$. Example 1 has the following controllers:

$$\begin{cases} u_1 = -(0.9s + 0.1)q_1 \\ u_2 = -\left(\dfrac{0.1}{s} + 0.2s + 0.5\right)q_2 \\ u_3 = -(0.6s + 1)q_3 \end{cases} \tag{22}$$

When two components join to form an Evolved System, at their point of contact, their velocities are equal and the forces exerted are equal and opposite. If the two components are given by (f_1, v_1) and (f_2, v_2), then the contact dynamics of the Evolved System can be represented by:

$$\begin{cases} f_1 = -f_2 \\ v_1 \equiv \dot{q}_1 = v_2 \equiv \dot{q}_2 \end{cases} \tag{23}$$

This connection is modeled as the admittance of one component connected in feedback with the impedance of the other component, as described in section II. When we use this idea of the joining of two components of an Evolving System as the feedback connection of their admittance and impedance, we can apply Theorem 1 from above to determine whether almost strict dissipativity is inherited by the Evolved System. The subsystem components from Example 1 are stable in closed-loop form when they are unconnected, i.e., $\varepsilon_{12} = 0$. When $\varepsilon_{12} = 1$, the system is fully evolved and it has a closed-loop eigenvalue at 0.17, resulting in an unstable Evolved System.

Fig. 4. Adaptive key component on mass 1.
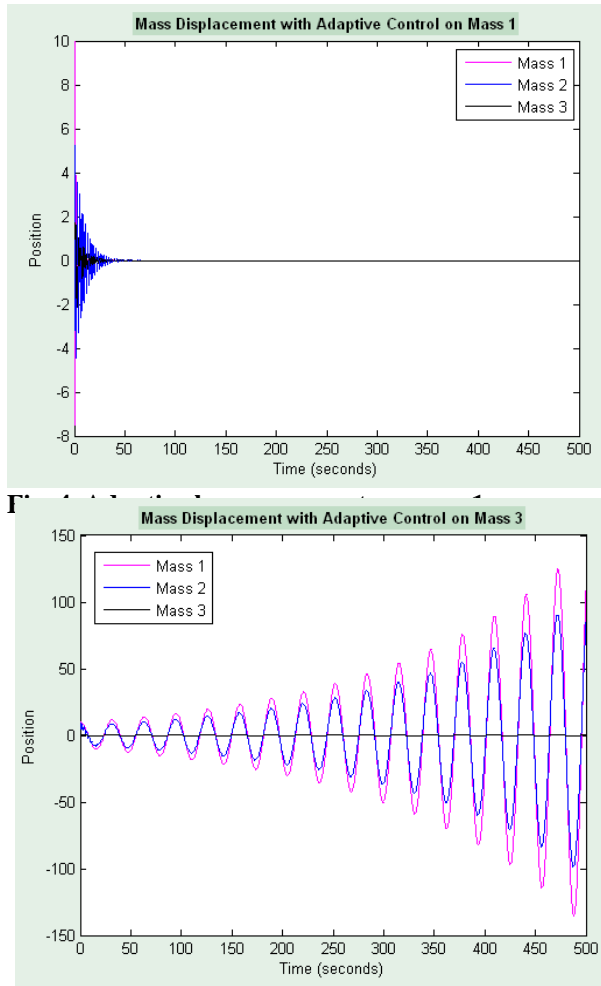
Fig. 5. Adaptive key component on mass 3.

A Simulink model was created to implement an adaptive key component controller for Example 1 as described in the previous section. Simulations were run in which the connection parameter, ε_{12}, ranged from 0 to 1, allowing the system to go from unconnected components to a fully Evolved System. The key component controller was able to maintain system stability during the entire evolution process when it used the input-output ports on mass 1 of component 1, see fig. 4. When component 1 was the key component, $\left(\bar{A}, \bar{B}, \bar{C}\right)$ is ASPR.

When the key component controller was located on component 2 and used the input-output ports on mass 3, stability was not maintained, see Figure 5. The adaptive key component controller was not able to restore stability on mass 3 because that system was not ASPR, i.e., it had nonminimum phase zeros at 0.00515±0.2009i.

A second study was performed where a proportional integral (PI) controller is used to control a rigid body. The plant has additional flexible body modes. The perturbation parameter ε is used to connect or disconnect the flexible modes. In this way, a system with flexible modes that vary depending on the operating condition can be modeled. For the example described here, the system needs the adaptive key component control to maintain stability of the rigid and flex modes. Finally, one flex mode is defined that causes a non-minimum phase zero to appear in the plant when the feedback control is used. A residual mode filter is designed for this mode and successfully applied to remove the system that is causing the dissipativity condition to not be satisfied. The final paper will provide details of these models and results of the simulation.

VI. Conclusion

We have presented a result (Theorem 1) describing when an evolving system will inherit the almost strict dissipativity traits of its subsystem components. An example was given of successful inheritance of almost strict dissipativity and failed inheritance of almost strict dissipativity. This result allows a control system designer to determine a sufficient condition for an evolving system to use an adaptive key component controller to restore stability. We also presented a convergence result (Theorem 2) for an adaptive key component controller to restore stability during evolution and mitigate persistent disturbances. An illustrative example of a system with a rigid mode and flexible modes is used to demonstrate these ideas.

Appendix

Proof of Theorem 1:

Let $\left(u_i^A, y_i^A\right)$ be ASD. From (9) and (11), there exists G_i^* such that

$$
\begin{aligned}
\nabla V_i A_i^C(x_i) &\equiv \nabla V_i \left[A_i(x_i) + B_i^A(x_i) G_i^* C_i^A(x_i) \right] \\
&\leq -S_i(x_i) + \varepsilon_{ij} \nabla V_i A_{ij}(x_i, u_i, u_i^A) \\
\nabla V_i B_i^A(x_i) &= C_i^A(x_i)^{\mathrm{T}}
\end{aligned}
\tag{40}
$$

If we connect (u_1, y_1) in feedback with (u_2, y_2), then $y_1 = u_2$ and $u_1 = -y_2$ and, use (12) and (13), then we have $\nabla V_1 A_{12}(x_1, u_1, u_1^A) = \nabla V_1 B_1(x_1) u_1 = C_1^T(x_1)[-y_2] = -y_1^T y_2$ and similarly, $\nabla V_2 A_{21}(x_2, u_2, u_2^A) = y_2^T y_1$.

Let $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and, from (12),

$$
\begin{cases}
\dot{x} = A(x) + B(x)u \\
\quad = \begin{bmatrix} A_1^C(x_1) + \varepsilon_{12} A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon_{21} A_{21}(x_1) \end{bmatrix} \\
\quad + \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix} \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \\
y = \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} = C(x) = \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix}
\end{cases}
\tag{41}
$$

with $V = V_1 + V_2$, using (13) and $\varepsilon_{ji} = \varepsilon_{ij} \equiv \varepsilon$ from (3),

$$
\nabla V A(x) = \begin{bmatrix} \nabla V_1 & \nabla V_2 \end{bmatrix} \begin{bmatrix} A_1^C(x_1) + \varepsilon A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon A_{21}(x_1) \end{bmatrix}
$$

$$
= \nabla V_1 A_1(x_1) + \varepsilon(-y_1^T y_2) + \nabla V_2 A_2(x_2) + \varepsilon(y_2^T y_1)
$$

$$
\leq -[S_1(x_1) + S_2(x_2)] + \varepsilon(-y_1^T y_2) + \varepsilon(y_2^T y_1)
$$

$$
= -S(x)
$$

and

$$
\nabla V B(x) = [\nabla V_1 \ \nabla V_2] \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix}^T = C^T(x)
$$

Therefore $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} \right)$ is ASD with output feedback $\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \equiv \begin{bmatrix} G_1^* & 0 \\ 0 & G_2^* \end{bmatrix} \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} + \begin{bmatrix} u_1^{Ar} \\ u_2^{Ar} \end{bmatrix}$ as desired. End of proof.

Proof of Theorem 2:
Since the physical connection of Component 1 to Component 2 is equivalent to the feedback connection

$u_1 = -y_2$ and $u_2 = y_2$.

By Theo.1 we have that the closed-loop system (u_1^A, y_1^A) below is ASD:

$$\begin{cases} \dot{x}_1 = A_1(x_1) - \varepsilon B_1(x_1)C_2(x_2) + B_1^A(x_1)u_1^A \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2)C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases}$$

(42)

Rewrite (20), to obtain

$$\begin{cases} u_1^A = G_e y_1^A + G_D \varphi_D = G_e^* y_1^A + G_D^* \varphi_D + \Delta G \eta; \\ \Delta G \equiv G - G^* = \begin{bmatrix} \Delta G_e & \Delta G_D \end{bmatrix}; \eta \equiv \begin{bmatrix} y_1^A \\ \varphi_D \end{bmatrix}^w \\ \Delta \dot{G} = \dot{G} = -y_1^A (y_1^A)^T \gamma; \gamma \equiv \begin{bmatrix} \gamma_e & 0 \\ 0 & \gamma_D \end{bmatrix} > 0 \end{cases}$$

(43)

Combining (21) and (22) yields:

$$\begin{cases} \dot{x}_1 = A_1^C(x_1) - \varepsilon B_1(x_1)C_2(x_2) \\ \quad + [B_1^A(x_1)G_D^* + \Gamma_1(x_1)\theta L]\varphi_D + B_1^A(x_1)w \\ \qquad \text{with } w \equiv \Delta G \eta \\ \quad \text{and } G_D^* \equiv -H_*\theta L \text{ from}(19) \\ \qquad \text{and } A_1^C(x_1) \equiv A_i(x_i) + B_i^A(x_i)G_i^* C_i^A(x_i) \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2)C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases}$$

(44)

Let $V = V_1 + V_2$ and we have:

$$\dot{V} = -S(x) + \left\langle y_1^A, w \right\rangle$$

(45)

Form $V_G \equiv \dfrac{1}{2} tr(\Delta G \gamma^{-1} \Delta G^T)$ and obtain from (22):

$$\begin{aligned} \dot{V}_G &\equiv tr(\Delta \dot{G} \gamma^{-1} \Delta G^T) \\ &= -tr(y_1^A (y_1^A)^T \Delta G^T) \\ &= -tr(y_1^A (w)^T) \\ &= -\left\langle y_1^A, w \right\rangle \end{aligned}$$

(46)

Define: $V(x, \Delta G) \equiv V(x) + V_G(\Delta G)$ and, from (24) and (25), we have:

$$\dot{V}(x, \Delta G) \equiv \dot{V}(x) + \dot{V}_G(\Delta G)$$
$$= -S(x) + \left\langle y_1^A, w \right\rangle - \left\langle y_1^A, w \right\rangle$$
$$= -S(x) \leq 0 \tag{47}$$

This guarantees that all trajectories $(x, \Delta G)$ are bounded. If $\dot{V}(x, \Delta G)$ is uniformly continuous or $\ddot{V}(x, \Delta G)$ is bounded, then Barbalat's Lemma [10] yields:

$$S(x) \xrightarrow[t \to \infty]{} 0$$

and the positivity and continuity of $S(x)$ imply that $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t \to \infty]{} 0$.

Consider

$$\ddot{V}(x, \Delta G) = -\dot{S}(x)$$
$$\leq \left| \dot{S}(x) \right|$$
$$= \left| \frac{\partial S(x)}{\partial x} \dot{x} \right|$$
$$\leq \left\| \frac{\partial S(x)}{\partial x} \right\| \|\dot{x}\|$$
$$\leq \left\| \frac{\partial S(x)}{\partial x} \right\| \left[\|A(x)\| + \|B(x)\| \|w_1^A\| \right]$$
$$\leq \left\| \frac{\partial S(x)}{\partial x} \right\| \left[\|A(x)\| + \|B(x)\| \|\Delta G\| \|C_1^A(x_1)\| \right]$$

which is bounded because $(x, \Delta G)$ is bounded, $S(x)$ has continuous partial derivatives and $(A(x), B(x), C(x))$ are continuous, and a continuous function of bounded $x(t)$ is also bounded in t.

So, $y_i = C_i(x_i) \xrightarrow[t \to \infty]{} 0$ because $C_i(x_i)$ is continuous. End of Proof.

Acknowledgments

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