An Analysis of Three-Phase Rectifiers with Constant Voltage Loads

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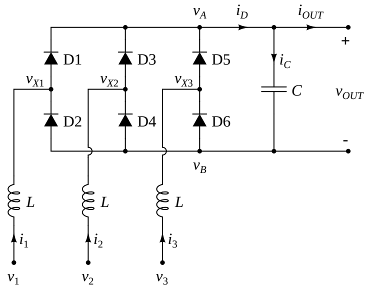
The rectifier



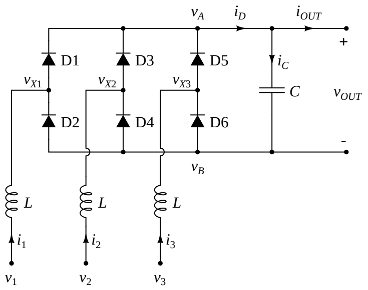
The rectifier, properties

- simple
- inexpensive
- robust
- AC side parameters?
- AC side compliance with regulations?
- DC side parameters, dependence of V_{OUT} on I_{OUT}?

The rectifier, available information

- old rectifier, well known?
- V. Caliskan, D. J. Perreault, T. M. Jahns, and J. G. Kassakian, "Analysis of three-phase rectifiers with constant-voltage loads," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 50, no. 9, pp. 1220–1226, Sep. 2003.
- P. Pejović, J. W. Kolar, "Exact analysis of three-phase rectifiers with constant voltage loads," *IEEE Trans. Circuits Syst. II, Express Briefs*, vol. 55, no. 8, pp. 743–747, Aug. 2008.
- continuous conduction mode (CCM) covered, three diodes conduct
- what about the discontinuous conduction mode(s) (DCM)?

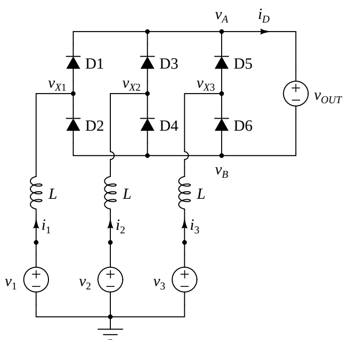
The rectifier to be analyzed



Assumptions

- output ripple neglected, $v_{OUT} = V_{OUT}$
- symmetrical undistorted three-phase system
- $v_k = V_m \cos\left(\omega t - (k-1)\,\frac{2\pi}{3}\right)$, for $k \in \{1, 2, 3\}$
- resistance neglected
- line inductance can be included in the model
- ideal diodes assumed, V_D could be included

The rectifier model



Normalization

- $v_k = V_m \cos\left(\omega t - (k-1)\,\frac{2\pi}{3}\right)$, for $k \in \{1, 2, 3\}$
- $m = \frac{v}{V_m}$
- $j = \frac{\omega L}{V_m}\, i$
- $\varphi = \omega t$
- $L \frac{di_k}{dt} = v_k - v_{Xk}$
- $\frac{dj_k}{d\varphi} = m_k - m_{Xk}$

An analysis of combinations of diode states

- ideal diodes assumed
- one bit sufficient to code diode state, either on or off
- 6 diodes, $2^6 = 64$ combinations
- some combinations forbidden
- $V_{OUT} > 0$, diodes in pairs (D1, D2), (D3, D4), and (D5, D6) cannot conduct simultaneously
- pair coded as +1, 0, or −1, reduction to $3^3 = 27$ states
- $i_1 + i_2 + i_3 = 0$, combinations like $(+1, +1, +1)$, $(-1, 0, -1)$, or $(0, 0, +1)$ cannot occur, 14 of them
- final reduction to 13 combinations (out of 64)

Combinations of diode states, listed

combination	phase leg state 1	2	3
0	0	0	0
1	+1	−1	0
2	+1	0	−1
3	−1	+1	0
4	0	+1	−1
5	−1	0	+1
6	0	−1	+1
7	+1	+1	−1
8	+1	−1	+1
9	−1	+1	+1
10	+1	−1	−1
11	−1	+1	−1
12	−1	−1	+1

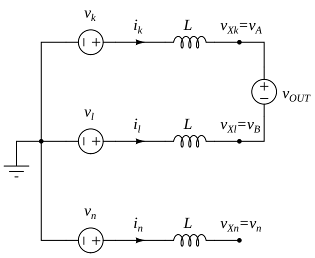
Circuit description

- equations over inductor currents
- equations for the output terminal voltages
- boundary conditions, theoretically 6 of them
- combination transition rules

Equations for state 0, without conducting diodes

- $j_k = j_l = j_n = 0$
- $m_A - m_B = M_{OUT}$
- $m_{kl} < M_{OUT}$
- the system order is zero

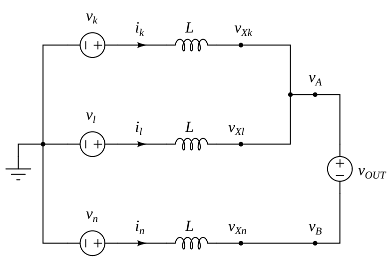
Equivalent circuit for two conducting diodes



Equations for two conducting diodes

- state(k) = +1, state(l) = −1, state(n) = 0
- $\dfrac{dj_k}{d\varphi} = \dfrac{1}{2}(m_k - m_l - M_{OUT})$, $j_l = -j_k$, $j_n = 0$
- $m_A = \dfrac{1}{2}(M_{OUT} - m_n)$, $m_B = \dfrac{1}{2}(M_{OUT} + m_n)$
- $j_k > 0$, to combination 0 if violated
- $m_n < \dfrac{1}{3} M_{OUT}$ leg n to state +1 if violated
- $m_n > -\dfrac{1}{3} M_{OUT}$ leg n to state −1if violated
- the circuit is of the first order

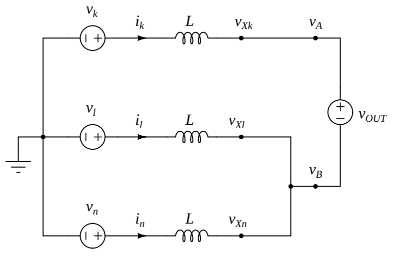
Equivalent circuit for three conducting diodes, two to the positive output terminal



Equations for three conducting diodes, two to the positive output terminal

- $\dfrac{dj_k}{d\varphi} = m_k - \dfrac{1}{3} M_{OUT}$
- $\dfrac{dj_l}{d\varphi} = m_l - \dfrac{1}{3} M_{OUT}$
- $j_n = -j_k - j_l$
- $m_A = \dfrac{1}{3} M_{OUT}$, $m_B = -\dfrac{2}{3} M_{OUT}$
- $j_k > 0$, $j_l > 0$, $j_n < 0$
- possible instantaneous combination transitions, additional inequalities
- the system is of the second order

Equivalent circuit for three conducting diodes, one to the positive output terminal



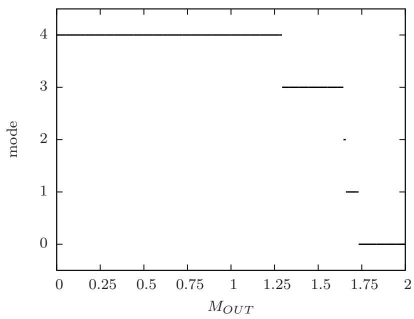
Equations for three conducting diodes, one to the positive output terminal

- $\dfrac{dj_k}{d\varphi} = m_k - \dfrac{2}{3}M_{OUT}$
- $\dfrac{dj_l}{d\varphi} = m_l + \dfrac{1}{3}M_{OUT}$
- $j_n = -j_k - j_l$
- $m_A = \dfrac{2}{3}M_{OUT},\ m_B = -\dfrac{1}{3}M_{OUT}$
- $j_k > 0,\ j_l < 0,\ j_n < 0$
- possible instantaneous combination transitions, additional inequalities
- the system is of the second order

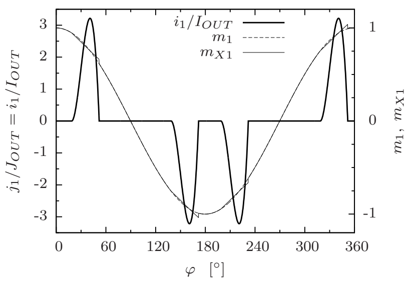
Modes

- mode 0, only combination 0
- mode 1, combinations with 0 and 2 conducting diodes
- mode 2, combinations with 0, 2, and 3 conducting diodes
- mode 3, combinations with 2, and 3 conducting diodes
- mode 4, only combinations with 3 conducting diodes, CCM

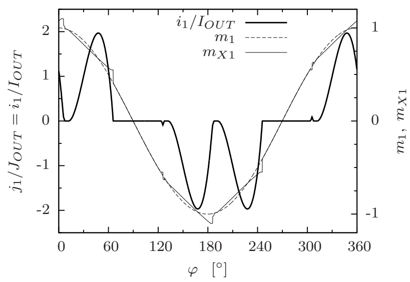
Dependence of the operating mode on M_{OUT}



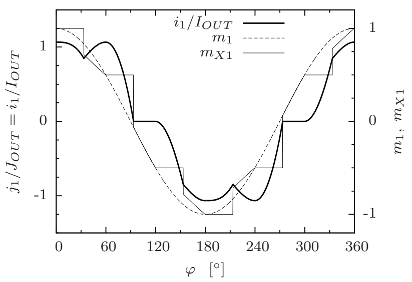
Mode 1, waveforms of i_1, v_1, and v_{X1}, $M_{OUT} = 1.7$



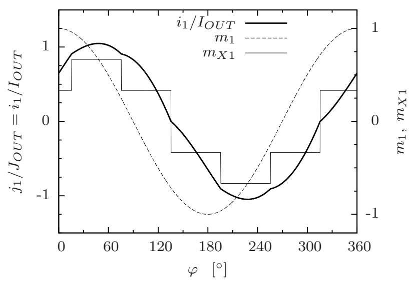
Mode 2, waveforms of i_1, v_1, and v_{X1}, $M_{OUT} = 1.6475$



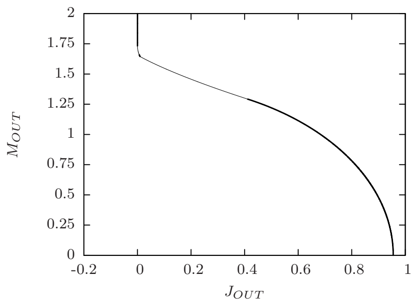
Mode 3, waveforms of i_1, v_1, and v_{X1}, $M_{OUT} = 1.5$



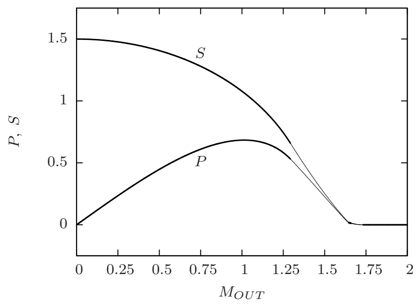
Mode 4, waveforms of i_1, v_1, and v_{X1}, $M_{OUT} = 1$



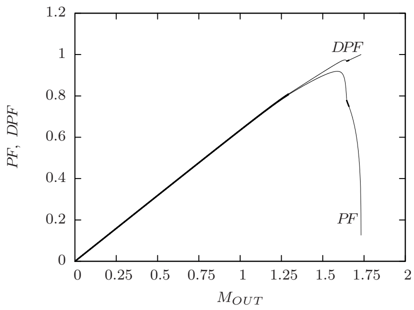
Dependence of M_{OUT} on J_{OUT}



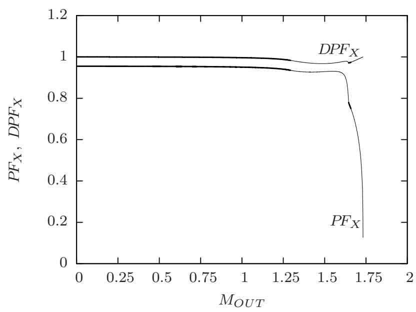
Dependence of the rectifier power and apparent power on M_{OUT}



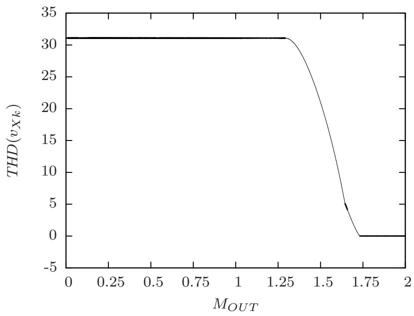
Dependence of the rectifier power factor and the displacement power factor on M_{OUT}.



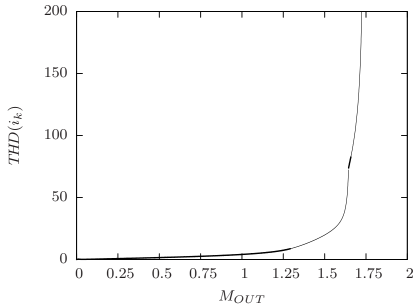
Dependence of PF_X and DPF_X on M_{OUT}



Dependence of $THD(v_{Xk})$ on M_{OUT}



Dependence of $THD(i_k)$ on M_{OUT}



Conclusions 1

- numerical analysis of a three-phase voltage loaded rectifier
- analysis performed on the equation system level, normalization
- insight in the rectifier operation, identification of the operating modes
- combinations of diode states, combinatorial approach
- out of $2^6 = 64$ combinations of diode states only 13 might occur
- for all 13 circuit equations are derived
- circuit order might be zero, one, or two, depending on the diode state combination

Conclusions 2

- boundary inequalities are derived, combination transition rules
- instantaneous combination transitions
- modes of the circuit operation are defined
- simulation, M_{OUT} from 2 to 0 in steps of 0.0005
- dependence of M_{OUT} on J_{OUT} is presented
- dependence on M_{OUT} of P, S, PF, PF_X, DPF, DPF_X, $THD(i_k)$, $THD(v_{Xk})$
- obtained diagrams quick reference guide for the rectifier design
- some interest in education