Exact BER Analysis for M-QAM Modulation with Transmit Beamforming under Channel Prediction Errors

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Abstract—Significant throughput improvements can be obtained in multiple-input multiple-output (MIMO) fading channels by merging beamforming at the transmitter and maximal ratio combining (MRC) at the receiver. In general, accurate channel state information (CSI) is required to achieve these performance gains. In this paper, we analyze the impact of channel prediction error on the bit error rate (BER) of combined beamforming and MRC in slow Rayleigh fading channels. Exact closed-form BER expressions are obtained in terms of elementary functions. Numerical results show that imperfect CSI causes little BER degradation using channel prediction of moderate complexity.

Index Terms—Multiple input multiple output (MIMO) systems, beamforming, maximal ratio combining (MRC), imperfect channel state information (ICSI), channel prediction, bit error rate (BER).

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) systems can considerably increase data rates through spatial multiplexing and significantly improve robustness and coverage through beamforming and diversity combining [1]. The capacity and performance of MIMO systems with multiplexing, beamforming and diversity depends on the availability and accuracy of the channel state information (CSI) at both the transmitter and receiver. The impact of imperfect CSI has been the subject of much recent investigation (see e.g. the special issues [2] and [3]).

This paper focuses on the impact of imperfect CSI on MIMO beamforming, which has been recently addressed in [4], [5], [6], [7]. In particular, [7] determined the pdf, cdf, and moment-generating function (MGF) of the output SNR in transmit beamforming under imperfect CSI. The system model assumed in [7] considers the same CSI to perform both beamforming at the transmitter and MRC at the receiver. However, in this paper we focus on a different system model which considers a more accurate CSI for MRC than for beamforming, as was also considered in [4], [5], [6]. These papers investigated the impact of imperfect CSI on transmit beamforming combined with MRC at the receiver and adaptive modulation. These works do not attempt to obtain exact closed-form BER expressions but rather use approximations, typically based on exponential bounds (e.g. [8, eq. 17]).

In contrast, our analysis focuses on obtaining exact BER expressions.

The BER for transmit beamforming will depend on the effective channel gain. If the CSI available to perform beamforming at the transmitter is different from the CSI to perform MRC at the receiver, the pdf of the effective channel gain can seldom be obtained in closed-form. In fact, the well-known pdf/MGF approach [9] cannot be extended to combined beamforming and MRC analyzed in this paper, and thus alternative analysis techniques are needed.

Under imperfect CSI caused by channel prediction errors, we obtain exact closed-form BER expressions for transmit beamforming using a different approach. Specifically, for a system with fixed power and M-QAM constellation, we compute the exact BER by first calculating the conditional BER, conditioned on the predicted channel, using Proakis' analysis of complex Gaussian quadratic forms [10, Appendix B][11] and then averaging the conditional BER over the distribution of the predicted channel.

The remainder of this paper is organized as follows. Section II describes the system model. In Section III the BER expressions are derived. Section IV presents numerical results which exploit the analytical expressions derived in previous sections. Finally, conclusions are provided in Section V.

II. SYSTEM MODEL

The system model for MIMO beamforming with MRC is briefly described in this section. Further details on the adopted system model can be found in [4]. We consider N_T transmit antennas and N_R receive antennas, and the channel is modeled by an $N_R \times N_T$ complex matrix $\mathbf{H}$, so that each entry $\mathbf{H}_{i,j}$ is the channel coefficient between the jth transmit and the ith receive antennae. These channel coefficients exhibit frequency-flat slowly time-varying fading. The entries $\mathbf{H}_{i,j}$ are assumed independent identically-distributed (i.i.d) complex circularly symmetric normal random variables (RVs), with zero-mean and unity-variance, i.e. $\mathbf{H}_{i,j} \sim \mathcal{CN}(0,1)$. Noise is modeled by an additive N_R-dimensional vector $\mathbf{n}$, whose entries n_k are i.i.d. complex circularly symmetric normal RVs $\sim \mathcal{CN}(0,N_0)$. The received signal can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{w} + \mathbf{n}, \qquad (1)$$

where $\mathbf{y}$ is the received N_R dimensional complex vector and $\mathbf{w}$ is the transmitted N_T dimensional complex vector.

In our receiver model, as in [4], [5], [6], we assume an imperfect channel prediction to obtain the predicted beam-steering vector which must be fed back to the transmitter. The

predicted channel $\hat{\mathbf{H}}$ can be expressed as follows

$$\hat{\mathbf{H}} = \mathbf{H} - \hat{\boldsymbol{\Xi}}, \quad \text{with}$$

$$\begin{cases} \hat{\mathbf{H}}_{i,j} & \text{i.i.d. RVs} \sim \mathcal{CN}(0, 1 - \chi) \\ \hat{\boldsymbol{\Xi}}_{i,j} & \text{i.i.d. RVs} \sim \mathcal{CN}(0, \chi) \end{cases}, \quad (2)$$

where $\hat{\boldsymbol{\Xi}}$ is the prediction error matrix and χ the mean square error. We assume the entries of the predicted channel matrix $\hat{\mathbf{H}}_{i,j}$ and the entries of the prediction error matrix $\hat{\boldsymbol{\Xi}}_{i,j}$ are orthogonal. As in [4], our analysis adopts a slowly time-varying channel model in which the channel response remains invariant along the frame interval.

Using the predicted channel $\hat{\mathbf{H}}$, the optimal beam-steering vector $\hat{\mathbf{v}}$ is the N_T-dimensional eigenvector corresponding to the largest eigenvalue $\hat{\lambda}$ of matrix $\hat{\mathbf{H}}^{\mathcal{H}} \hat{\mathbf{H}}$ [12], which is given by $\hat{\lambda} = \hat{\mathbf{v}}^{\mathcal{H}} \hat{\mathbf{H}}^{\mathcal{H}} \hat{\mathbf{H}} \hat{\mathbf{v}}$. On each frame, the receiver feeds vector $\hat{\mathbf{v}}$ back to the transmitter to perform beamforming, so that the transmitted vector becomes $\mathbf{w} = \hat{\mathbf{v}} z$, where z is the complex transmitted symbol. The effective channel is an N_R-dimensional vector defined as $\mathbf{h} \triangleq \mathbf{H} \hat{\mathbf{v}}$ and the predicted effective channel is the vector $\hat{\mathbf{h}} \triangleq \hat{\mathbf{H}} \hat{\mathbf{v}}$, whose square Euclidean norm is $||\hat{\mathbf{h}}||^2 = \hat{\lambda}$. The effective channel can also be expressed as $\mathbf{h} \triangleq \mathbf{H} \hat{\mathbf{v}} = (\hat{\mathbf{H}} + \hat{\boldsymbol{\Xi}}) \hat{\mathbf{v}} = \hat{\mathbf{h}} + \boldsymbol{\Psi}$, where $\boldsymbol{\Psi}$ is a complex normal N_R-dimension vector whose entries Ψ_k, assuming that $\hat{\mathbf{v}}$ is a unitary vector, are i.i.d. RVs $\sim \mathcal{CN}(0, \chi)$ [13, p. 26].

At the receiver, the effective channel vector $\mathbf{h} = \mathbf{H} \hat{\mathbf{v}}$ is assumed perfectly estimated to perform MRC. This assumption is reasonable since the received signals can be stored so that non-causal channel estimation (smoothing) with high accuracy can be performed [14]. The symbol r which results from applying MRC to received vector $\mathbf{y}$ is given by

$$r \triangleq \frac{\mathbf{h}^{\mathcal{H}} \mathbf{y}}{||\mathbf{h}||^2} = z + n', \quad \text{with } n' \triangleq \frac{(\hat{\mathbf{h}} + \boldsymbol{\Psi})^{\mathcal{H}} \mathbf{n}}{||\hat{\mathbf{h}} + \boldsymbol{\Psi}||^2}, \quad (3)$$

where n' is the resultant noise after MRC. As the entries of noise vector $\mathbf{n}$ are i.i.d circularly symmetric Gaussian variables and $\mathbf{h}$ and $\mathbf{n}$ are mutually independent, it is straightforward to show that n' is circularly symmetric too.

III. BER ANALYSIS

One approach to calculate the BER from (3) could be to average the conditional BER over the effective SISO channel gain $||\mathbf{h}||^2$. Unfortunately, the pdf of the effective SISO channel gain is unknown to the best of the authors' knowledge. For this reason we use an alternative analysis method. First, we compute the BER conditioned on the predicted effective channel vector $\hat{\mathbf{h}}$, using the Proakis' analysis of Gaussian quadratic forms [10, Appendix B][11]. As is shown in this section, this conditional BER expression only depends on $\hat{\lambda} = ||\hat{\mathbf{h}}||^2$, whose pdf is directly related to the well-known pdf of λ.

The BER analysis presented in this section considers L-PAM or square M-QAM ($M = L^2$) with independent bit-mapping for in-phase and quadrature components, e.g, Gray mapping. Under these assumptions and reminding that the resultant noise n' is circularly symmetric, the BER can be

expressed, in a similar way to [15][16], as

$$BER = \sum_{n=1}^{L-1} \omega(n) \mathcal{I}(n), \quad (4)$$

where $\mathcal{I}(n)$ are called components of error probability (CEP) and the $\omega(n)$ are coefficients dependent on the constellation mapping. The CEPs are defined as $\mathcal{I}(n) = \Pr\{\Re\{n'\} > (2n - 1)d\}$, where d is the minimum distance between the symbol and the decision boundary and can be expressed as a function of the constellation energy E_s as $d = \sqrt{\frac{6E_s}{M-1}}$ for M-QAM constellation and $d = \sqrt{\frac{12E_s}{L^2-1}}$ for L-PAM constellation. The coefficients $\omega(n)$ can be obtained using the method described in [16] or directly computed for Gray mapping as in [15].

In Section III-A we derive the CEP conditioned on the predicted effective channel vector $\hat{\mathbf{h}}$. In Section III-B, using the previous results, we obtain both the conditional BER and the BER averaged over $\hat{\lambda}$.

A. Conditional CEP

In this section we derive, for our system model, the CEP conditioned on a predicted channel state (CCEP), that can be expressed as

$$\mathcal{I}(n; \hat{\mathbf{h}}) = \Pr\left\{ \Re\{n'\} > (2n - 1)d \mid \hat{\mathbf{h}} \right\}. \quad (5)$$

In order to obtain the CCEP we use Proakis' analysis of complex Gaussian quadratic forms [10, Appendix B], more specifically, we adopt the compact expressions presented in [11] and restated in Table I.

According to our system model, when the symbol $z = s$ is transmitted, each entry of the received signal y_k and each entry of the effective channel h_k are

$$y_k = h_k s + n_k = (\hat{h}_k + \Psi_k)s + n_k, \quad (6)$$

$$h_k = \hat{h}_k + \Psi_k, \quad (7)$$

where both y_k and h_k are complex normal variables conditioned on the predicted effective $\hat{h}_k$. Note that $\hat{h}_k$ is assumed orthogonal to the channel error Ψ_k (e.g. $\hat{h}_k$ is predicted using a FIR Wiener filter) and therefore, the probability distribution of Ψ_k is independent of the $\hat{h}_k$ value. Defining the sum of quadratic forms $D = \sum_{k=1}^{N_R} \mathbf{x}_k^{\mathcal{H}} \mathbf{Q} \mathbf{x}_k$, where

$$\mathbf{x}_k = \begin{bmatrix} y_k \\ h_k \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 0 & -1/2 \\ -1/2 & \Re\{s\} + (2n-1)d \end{bmatrix}, \quad (8)$$

we have

$$\Pr\{D < 0\} =$$
$$\Pr\{-\Re\{\mathbf{h}^{\mathcal{H}} \mathbf{y}\} + (\Re\{s\} + (2n-1)d)||\mathbf{h}||^2 < 0 \mid \hat{\mathbf{h}}\} = \quad (9)$$
$$\Pr\{\Re\{n'\} > (2n-1)d \mid \hat{\mathbf{h}}\} = \mathcal{I}(n; \hat{\mathbf{h}}).$$

In Table I we present the expressions to obtain this probability. Using the quadratic form matrix $\mathbf{Q}$, the mean vector $\mathbf{m}_k$ and the covariance matrix $\mathbf{R}$ of vector $\mathbf{x}_k$, we can calculate the parameters a, b, η and C_l necessary to obtain this probability. From (6) and (7), we obtain the mean and the covariance

TABLE I
PROBABILITY COMPUTATION OF THE GAUSSIAN QUADRATIC FORM.

FUNCTIONS AND PARAMETERS	DEFINITIONS
D	$\sum_{k=1}^{L} \mathbf{x}_k^{\mathcal{H}} \mathbf{Q} \mathbf{x}_k$
$\mathbf{m}_k$	$\mathrm{E}\{\mathbf{x}_k\}$
$\mathbf{R}$	$\mathrm{E}\{(\mathbf{x}_k - \mathbf{m}_k)(\mathbf{x}_k^{\mathcal{H}} - \mathbf{m}_k^{\mathcal{H}})\}$
$\{\delta_i\}_{i=1,2}$	$\frac{1}{2}\mathrm{tr}(\mathbf{RQ}) \pm \sqrt{\left(\frac{1}{2}\mathrm{tr}(\mathbf{RQ})\right)^2 - \det(\mathbf{RQ})}$
η	$\left|\dfrac{\delta_1}{\delta_2}\right|$
a	$\sqrt{\dfrac{2\delta_2\left(\Sigma_{k=1}^{L}\mathbf{m}_k^{\mathcal{H}}\left[\mathbf{Q} - \delta_1\mathbf{R}^{-1}\right]\mathbf{m}_k\right)}{(\delta_1 - \delta_2)^2}}$
b	$\sqrt{\dfrac{2\delta_1\left(\Sigma_{k=1}^{L}\mathbf{m}_k^{\mathcal{H}}\left[\mathbf{Q} - \delta_2\mathbf{R}^{-1}\right]\mathbf{m}_k\right)}{(\delta_1 - \delta_2)^2}}$
$C_l(a,b,\eta)$	$\begin{cases} -1 + \dfrac{1}{(1+\eta)^{2L-1}} \sum_{k=0}^{L-1} \binom{2L-1}{k} \eta^k, & l = 0 \\[2mm] \dfrac{1}{(1+\eta)^{2L-1}} \sum_{k=0}^{L-1-l} \binom{2L-1}{k} \left[\left(\dfrac{b}{a}\right)^l \eta^k - \left(\dfrac{a}{b}\right)^l \eta^{2L-1-k}\right], & l \neq 0 \end{cases}$
PROBABILITY	EXPRESSION
$\Pr\{D < 0\}$	$\mathrm{Q}_1(a,b) + \sum_{l=0}^{L-1} C_l(a,b,\eta)\, \mathrm{I}_l(ab)\exp\left\{-\frac{1}{2}(a^2+b^2)\right\}$

matrix of the vector $\mathbf{x}_k$ as

$$\mathbf{m}_k = \begin{bmatrix} \hat{h}_k s \\ \hat{h}_k \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} \chi|s|^2 + N_0 & \chi s \\ \chi s^* & \chi \end{bmatrix}. \quad (10)$$

Using Table I expressions, equations (8) and (10), and taking into account $\sum_{k=1}^{N_R}|\hat{h}_k|^2 = \hat{\lambda}$, the constants a, b and η are given by

$$a = a_n\sqrt{\hat{\lambda}}, \text{ with}$$
$$a_n = \sqrt{\frac{1 + 2\kappa_n\bar{\gamma}\chi - 2\sqrt{\kappa_n\bar{\gamma}\chi\,(\kappa_n\bar{\gamma}\chi + 1)}}{2\,(\kappa_n\bar{\gamma}\chi + 1)\chi}},$$
$$b = b_n\sqrt{\hat{\lambda}}, \text{ with} \quad (11)$$
$$b_n = \sqrt{\frac{1 + 2\kappa_n\bar{\gamma}\chi + 2\sqrt{\kappa_n\bar{\gamma}\chi\,(\kappa_n\bar{\gamma}\chi + 1)}}{2\,(\kappa_n\bar{\gamma}\chi + 1)\chi}},$$
$$\eta = \eta_n = 1 + 2\kappa_n\bar{\gamma}\chi + 2\sqrt{\kappa_n\bar{\gamma}\chi\,(\kappa_n\bar{\gamma}\chi + 1)},$$

where the average SNR is defined as $\bar{\gamma} \triangleq \frac{E_s}{N_0}$, and the constant $\kappa_n \triangleq \frac{3(2n-1)^2}{2(M-1)}$ for M-QAM and $\kappa_n \triangleq \frac{3(2n-1)^2}{(L^2-1)}$ for L-PAM. Note that, after some algebra manipulations, it can be shown that $\eta_n = b_n/a_n$.

Substituting the parameters a, b, and C_l in the expression of $\mathrm{P}\{D < 0\}$ that appears in Table I, the CCEP is expressed as

$$\mathcal{I}(n;\hat{\mathbf{h}}) = \mathcal{I}(n;\hat{\lambda}) = \mathrm{Q}_1(a_n\sqrt{\hat{\lambda}}, b_n\sqrt{\hat{\lambda}}) +$$
$$\sum_{l=0}^{N_R-1} C_l(a_n,b_n)\,\mathrm{I}_l(a_n b_n\hat{\lambda})\exp\left[-\frac{\hat{\lambda}}{2}(a_n^2 + b_n^2)\right], \quad (12)$$

where Q_1 is the Marcum-Q function, and I_l is the l-th order modified Bessel function of the first kind.

In the absence of channel prediction error, i.e. $\Psi_k = 0$, the resultant noise after MRC n', whose expression is given in (3), is a Gaussian variable conditioned on $\hat{h}$ whose variance is $N_0/\hat{\lambda}$. Thus, the calculation of CCEP is equivalent to the BER calculation for a standard Gaussian noise channel, resulting in

$$\mathcal{I}(n;\hat{\lambda}) = \mathrm{Q}\left(\sqrt{2\hat{\lambda}\kappa_n\bar{\gamma}}\right), \quad (13)$$

where Q is the Gaussian Q-function.

B. Conditional and average BER expressions

The CCEP calculated in the previous section allows us to obtain the conditional BER (CBER). This probability represents the BER conditioned on the predicted effective channel gain $\hat{\lambda}$, i.e, the BER under imperfect channel state information (CSI). Introducing (12) in (4) yields

$$CBER(\hat{\lambda}) = \sum_{n=1}^{L-1} \omega(n)\mathcal{I}(n;\hat{\lambda}). \quad (14)$$

$$BER = \sum_{n=1}^{L-1} \sum_{k=1}^{N_1} \sum_{r=N_2-N_1}^{(N_2+N_1-2k)k} \frac{\omega(n)B_{k,r}}{k^{r+1}} \times$$

$$\left\{ \frac{r!}{k^{r+1}} \left\{ 1 + \frac{b_n^2}{s_{n,k}} \sum_{m=0}^{r} (m+1) \left(\frac{2k}{s_{n,k}} \right)^m \left[\frac{a_n^2}{s_{n,k}} {}_2F_1 \left(\frac{m+2}{2}, \frac{m+2}{2} + \frac{1}{2}; 2; \frac{4a_n^2 b_n^2}{s_{n,k}^2} \right) - \right. \right. \right.$$

$$\left. \left. \left. \frac{1}{1+m} {}_2F_1 \left(\frac{m+1}{2}, \frac{m+1}{2} + \frac{1}{2}; 1; \frac{4a_n^2 b_n^2}{s_{n,k}^2} \right) \right] \right\} + \sum_{l=0}^{N_R-1} \frac{(r+l)!}{(a_n b_n)^{r+1}} \frac{1}{l!} \left(\frac{w_{n,k}-1}{w_{n,k}+1} \right)^{l/2} \left(\frac{w_{n,k}+1}{2} \right)^r \times \right. \tag{17}$$

$$\left. {}_2F_1 \left(-r, -r+l; l+1; \frac{w_{n,k}-1}{w_{n,k}+1} \right) \sqrt{\left(w_{n,k}^2 - 1 \right)^{r+1}} \right\},$$

The BER is obtained by averaging the CBER over the predicted eigenvalue $\hat{\lambda}$ as

$$BER = \sum_{n=1}^{L-1} \omega(n) \int_0^\infty \mathcal{I}(n; \hat{\lambda}) p(\hat{\lambda}) d\hat{\lambda}. \tag{15}$$

Using the fact that the pdf of the largest eigenvalue of complex Wishart matrices can be expressed as a weighted sum of elementary Gamma pdfs [17], the pdf of $\hat{\lambda}$ is given by

$$p(\hat{\lambda}) = \sum_{k=1}^{N_1} \sum_{r=N_2-N_1}^{(N_2+N_1-2k)k} B_{k,r} \frac{\hat{\lambda}^r}{(1-\chi)^{r+1}} \exp \left(\frac{-k\hat{\lambda}}{1-\chi} \right), \tag{16}$$

where $N_1 \triangleq \min\{N_T, N_R\}$, $N_2 \triangleq \max\{N_T, N_R\}$ and the coefficients $B_{k,r}$ are easily deducted from the expression [17, eq.10].

Substituting equations (12) and (16) in (15), using the expression [18, eq.8.772-3] and the integrals given in [18, eq.8.914-1] and [19], we can obtain, after some algebraic manipulation, the closed-form BER expression (17) at the top of the page, where ${}_2F_1$ is the Gauss hypergeometric function, $s_{n,k} = a_n^2 + b_n^2 + 2k$ and $w_{n,k} = \left(1 - \frac{4a_n^2 b_n^2}{s_{n,k}^2} \right)^{-1/2}$. Note that function ${}_2F_1$ in (17) can be expressed as a finite sum of elementary functions for the values of its arguments, although it is not shown here for compactness reasons.

For the perfect CSI case, substituting expression (13) and (16) in (15) the BER under perfect CSI is obtained as

$$BER = \sum_{n=1}^{L-1} \sum_{k=1}^{N_1} \sum_{r=N_2-N_1}^{(N_2+N_1-2k)k} \frac{\omega(n)B_{k,r}r!}{2k^{r+1}} \times$$
$$\left[1 - \sqrt{\frac{\kappa_n \bar{\gamma}}{k + \kappa_n \bar{\gamma}}} \sum_{l=0}^{r} \binom{2l}{l} \left(\frac{k}{4(k + \kappa_n \bar{\gamma})} \right)^l \right], \tag{18}$$

where we use the fact that the required integral over $\hat{\lambda}$ is formally identical to [9, Eq. 5.18].

IV. NUMERICAL RESULTS

The numerical results are obtained from analytical expressions and simulations assuming a prediction subsystem similar to that described in [4]. We consider Jakes' model for the channel time-correlation and Wienner filtering to predict the CSI. The numerical values for the system parameters are: carrier frequency $f_c = 3$ GHz, mobile speed $v=36$ km/h, feedback delay $\tau=1.28$ ms and frame interval $T=0.64$ ms, which
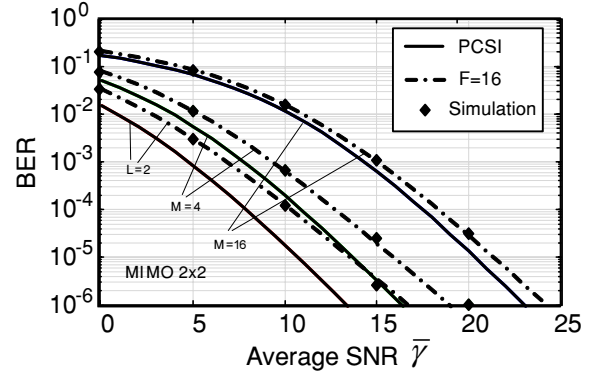

Fig. 1. BER as a function of average SNR $\bar{\gamma}$ for different constellations under perfect CSI (PCSI) and imperfect prediction (F=16 taps).

e.g. could correspond to a system with a symbol frequency f_S=100 kHz and 64 symbols per frame.

Figure 1 shows the BER for a 2x2 MIMO system as a function of the average SNR $\bar{\gamma}$ for some L-PAM and M-QAM constellations in two cases: *perfect CSI* using equation (18) and *imperfect CSI* using (17) with a 16-tap FIR Wiener prediction filter. In the last case, we assume channel estimation based on pilot symbol assisted modulation (PSAM) and the mean square error χ is computed as described in [4, section II-B]. As a double check, simulation results for the imperfect CSI case are also superimposed in the figure. There is an unavoidable channel prediction error floor independent of the number of filter taps and, thus, the BER for perfect CSI cannot be achieved. Note that at around BER $\approx 10^{-3}$ the relative SNR losses due to the channel prediction error for the considered system parameters are about 2.5 dB for BPSK, 1.5 dB for 4-QAM and less than 1 dB for 16-QAM.

In Figure 2 the influence of antenna configuration on the BER is depicted for 16-QAM. Obviously in the SISO case the channel prediction error does not affect the BER performance. Moreover, the 2x4 and 4x2 MIMO systems show the same BER for perfect CSI. This is because, if perfect CSI is assumed, the BER performance for different antenna configurations only depends on the pdf of $\hat{\lambda}$, which is the same for the 2x4 and 4x2 MIMO systems. However, for the imperfect CSI case the BER performance is better for 2x4 MIMO systems.
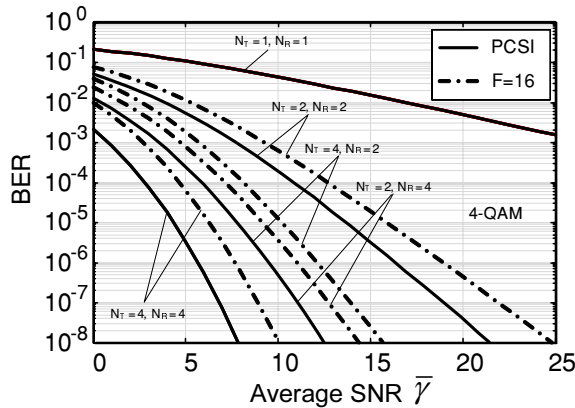
Fig. 2. BER as a function of average SNR $\overline{\gamma}$ for different antenna configurations under perfect CSI (PCSI) and imperfect prediction (F=16 taps).

V. CONCLUSIONS

Exact closed-form BER expressions for MIMO beamforming with MRC systems under channel prediction errors have been derived. These results allow us to analyze the performance of L-PAM and square M-QAM using practical estimation methods under Rayleigh fading. The system performance has been analyzed for different scenarios with different values of the mobile speed, number of prediction filter taps, adaptation delay, number of transmitter and receiver antennas, constellation size and average SNR. Our results indicate that practical constraints imposed by channel prediction do not significantly degrade BER performance.

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