Exact Bremsstrahlung Function in $\mathcal{N}=2$ Superconformal Field Theories

Bartomeu Fiol,[1,*] Efrat Gerchkovitz,[2,†] and Zohar Komargodski[2,‡]
[1]*Departament de Física Fonamental, Institut de Ciències del Cosmos (ICCUB), Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain*
[2]*Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 76100, Israel*
(Received 29 November 2015; published 25 February 2016)

We propose an exact formula for the energy radiated by an accelerating quark in $\mathcal{N}=2$ superconformal theories in four dimensions. This formula reproduces the known bremsstrahlung function for $\mathcal{N}=4$ theories and provides a prediction for all the perturbative and instanton corrections in $\mathcal{N}=2$ theories. We perform a perturbative check of our proposal up to three loops.

DOI: 10.1103/PhysRevLett.116.081601

Introduction and review.—Many interesting questions in quantum field theory revolve around the behavior of external probes coupled to the theory. In particular, if a heavy particle moves with some proper acceleration a in the vacuum of a gauge theory, it radiates energy proportional to the proper acceleration squared:

$$E = 2\pi B \int dt a^2. \tag{1}$$

The well-known result (Larmor's formula) for a particle of charge e in Maxwell's theory is

$$B = \frac{e^2}{12\pi^2}. \tag{2}$$

A convenient way to describe a charged heavy probe is by a Wilson operator. It is labeled by the representation $\mathcal{R}$ of the gauge group and worldline C. To discuss energy loss, start with a probe at rest that receives a sudden kick, continuing thereafter at a constant speed. The worldline thus has a cusp, and the vacuum expectation value of the Wilson operator develops a logarithmic divergence that depends on the boost parameter φ:

$$\langle W_\varphi \rangle \sim e^{-\Gamma_{\mathrm{cusp}}(\varphi)\log(\Lambda_{\mathrm{UV}}/\Lambda_{\mathrm{IR}})}, \tag{3}$$

where Λ_{UV} and Λ_{IR} represent UV and IR cutoff scales, respectively [1]. The quantity $\Gamma_{\mathrm{cusp}}(\varphi)$ is the cusp anomalous dimension, and it enters a number of questions, like the IR divergences in the scattering of massive particles. It has been computed to three loops in QCD [2] and in $\mathcal{N}=4$ super Yang-Mills (SYM) theory [3] and to four loops in planar $\mathcal{N}=4$ [4].

While obtaining the full expression for $\Gamma_{\mathrm{cusp}}(\varphi)$ in any interacting gauge theory appears to be a daunting task, various limits of this function are more accessible and already encode interesting physics. In what follows, we will limit the discussion to conformal field theories, although some of the results are more general. In the limit of very large boosts, $\Gamma_{\mathrm{cusp}}(\varphi)$ is linear in the boost parameter [5,6]:

$$\Gamma_{\mathrm{cusp}}(\varphi) \sim \Gamma^{\infty}_{\mathrm{cusp}}\varphi \tag{4}$$

and characterizes the IR divergences of massless particles. On the other hand, in the limit of very small boosts we have

$$\Gamma_{\mathrm{cusp}}(\varphi) = B\varphi^2 + \mathcal{O}(\varphi^4). \tag{5}$$

The coefficient B was dubbed the bremsstrahlung function in Ref. [7]. For conformal field theories it determines the energy radiated by an accelerating quark [7], as in (1), and its momentum diffusion coefficient [8].

Let us now discuss the Wilson line corresponding to a probe moving at constant proper acceleration. We can measure the energy density by studying the two-point function of the stress-energy tensor and this Wilson line. In conformal field theories, this is related by a conformal transformation to the two-point function of the stress-energy tensor and a straight Wilson line:

$$\langle T_{\mu\nu}(x)\rangle_W \equiv \frac{\langle W T_{\mu\nu}(x)\rangle}{\langle W\rangle}. \tag{6}$$

Its x dependence is determined by conformal invariance, up to a single coefficient h_W [9–11]:

$$\langle T_{00}(x)\rangle_W = \frac{h_W}{r^4}, \tag{7}$$

where r is the distance from the line. There is no simple general relation between B and h_W [12].

The main subject of this Letter is the computation of B in $\mathcal{N}=2$ superconformal field theories (SCFTs). We first review the case of the maximally supersymmetric $\mathcal{N}=4$ SCFT.

Review of $\mathcal{N}=4$.—The massive probe is described by the Wilson loop in a representation $\mathcal{R}$ of the gauge group

$$W_{\mathcal{R}} = \frac{1}{\dim \mathcal{R}} \mathrm{tr}_{\mathcal{R}} \mathcal{P} \exp\left(i \int (A_\mu dx^\mu + i\Phi_i \theta^i ds)\right). \tag{8}$$

Here, A_μ and Φ^i, $i = 1, \ldots, 6$, are the gauge fields and scalars, respectively, of the $\mathcal{N}=4$ vector multiplet, θ^i is some unit vector in $\mathbb{R}^6$, and $\mathcal{P}$ is the path ordering operator. When the contour is a straight line and θ^i is constant, $W_{\mathcal{R}}$ is 1/2 Bogomol'nyi-Prasad-Sommerfeld (BPS). Another 1/2 BPS configuration is given by a circular Wilson loop with constant θ^i. The two configurations are formally related by a conformal transformation.

For the straight line we have $\langle W \rangle = 1$. The transformation that relates the straight and circular Wilson loops turns out to be anomalous [13]. It was conjectured in [13,14] and later proved in [15] that the expectation value of the circular Wilson loop is given by a Gaussian matrix integral over the Lie algebra

$$\langle W \rangle = \frac{\int da \mathrm{Tr} e^{-2\pi a} e^{-(8\pi^2 N/\lambda)\mathrm{Tr}(a^2)}}{\int da e^{-(8\pi^2 N/\lambda)\mathrm{Tr}(a^2)}}, \tag{9}$$

where $\lambda = g^2 N$ is the 't Hooft coupling, with g the usual Yang-Mills coupling.

According to Ref. [7], for $\mathcal{N} = 4$ $U(N)$ SYM theory this vacuum expectation value determines the bremsstrahlung function through

$$B = \frac{1}{2\pi^2} \lambda \partial_\lambda \ln\langle W \rangle. \tag{10}$$

In the 't Hooft limit and at large λ, this agrees with the replacement rule $e^2/3 \leftrightarrow \sqrt{\lambda}$ found via the AdS/CFT correspondence [16,17].

On the other hand, the coefficient $h_W(\lambda)$ in (7) was computed in [18], obtaining a result proportional to B. This relation was clarified in [12], who argued for $\mathcal{N} = 4$ theories that

$$B = 3 h_W. \tag{11}$$

The argument relies on the existence of a dimension-two scalar operator in the supermultiplet of the energy-momentum tensor.

Some basics of $\mathcal{N} = 2$.—Let us now consider $\mathcal{N} = 2$ SCFTs in four dimensions. We can define the following Wilson loop:

$$W_{\mathcal{R}} = \frac{1}{\dim \mathcal{R}} \mathrm{tr}_{\mathcal{R}} \mathcal{P} \exp\left(i \oint (A_\mu dx^\mu + i \Phi ds) \right), \tag{12}$$

with Φ one of the scalars in the $\mathcal{N} = 2$ vector multiplet. As before, if the contour is straight or circular, the Wilson loop is 1/2 BPS. If we introduce a cusp, then we can infer the bremsstrahlung coefficient according to (5).

The expectation value of the circular Wilson loop in $\mathcal{N} = 2$ SCFTs can be obtained via localization [15] on $\mathbb{S}^4$. It is also useful to review what happens when the Wilson loop is placed on the ellipsoid:

$$\frac{x_0^2}{r^2} + \frac{x_1^2 + x_2^2}{\ell^2} + \frac{x_3^2 + x_4^2}{\tilde{\ell}^2} = 1. \tag{13}$$

In SCFTs, the expectation value of the Wilson loop is a function of the dimensionless squashing parameter

$$b \equiv \left(\frac{\ell}{\tilde{\ell}} \right)^{1/2}. \tag{14}$$

There are two supersymmetric Wilson loops on the ellipsoid. They transform into each other under $\ell \leftrightarrow \tilde{\ell}$ and approach the 1/2 BPS Wilson loop considered by Pestun in the round $\mathbb{S}^4$ limit $\ell = \tilde{\ell} = r$. According to Ref. [19] (see also [20,21]), the vacuum expectation value of one of them is

$$\langle W_b \rangle = \frac{\int da \mathrm{Tr} e^{-2\pi b a} e^{-(8\pi^2/g^2)\mathrm{Tr}(a^2)} Z_{\text{1-loop}}(a,b) |Z_{\text{inst}}(a,b)|^2}{\int da e^{-(8\pi^2/g^2)\mathrm{Tr}(a^2)} Z_{\text{1-loop}}(a,b) |Z_{\text{inst}}(a,b)|^2}, \tag{15}$$

while the second Wilson loop is obtained by replacing $\mathrm{Tr} e^{-2\pi b a}$ by $\mathrm{Tr} e^{-2\pi b^{-1} a}$. The integration in (15) is over the Lie algebra. Z_{inst} is Nekrasov's instanton partition function [22], with the equivariant parameters identified as

$$\ell = \epsilon_1^{-1}, \qquad \tilde{\ell} = \epsilon_2^{-1}; \tag{16}$$

thus, $b \equiv (\epsilon_2/\epsilon_1)^{1/2}$. The expression for the one-loop determinant, $Z_{\text{1-loop}}$, can be found in Ref. [19] (see also [23]).

Consider now the normalized two-point function of the stress-energy tensor with a straight Wilson line (6) in an $\mathcal{N} = 2$ SCFT. It is some function of the marginal coupling constants (7), $h_W(g^i)$. The stress-energy tensor belongs to a short representation of the $\mathcal{N} = 2$ superconformal group [24] that always contains a scalar of dimension two, O_2 [24]. Because the Wilson loop is BPS, there is a relation between $\langle W T_{\mu\nu}(x) \rangle$ and $\langle W O_2(x) \rangle$. If we define $\langle O_2(x) \rangle_W = (C/r^2)$, then $h_W(g^i) = \frac{8}{3} C(g^i)$. The derivation of this relation follows the same steps as in $\mathcal{N} = 4$ theories [10].

Two conjectures.—Because the relation between $\langle W T_{\mu\nu}(x) \rangle$ and $\langle W O_2(x) \rangle$ exists in any $\mathcal{N} = 2$ theory, one can imagine, as in [12], improving the energy-momentum tensor in such a way that the leading singularity near the Wilson line is removed.

Therefore, we suggest that the bremsstrahlung coefficient in $\mathcal{N} = 2$ theories can be inferred from h_W as in (11):

$$B = 3 h_W. \tag{17}$$

In general, $\mathcal{N} = 2$ theories contain many exactly marginal operators, and one should not expect a formula analogous to (10), because these exactly marginal operators are unrelated to insertions of the energy-momentum tensor. Instead, we conjecture that the coefficient h_W and therefore the bremsstrahlung function for $\mathcal{N} = 2$ SCFTs is given by

$$B = 3 h_W = \frac{1}{4\pi^2} \partial_b \ln\langle W_b \rangle|_{b=1}. \tag{18}$$

The proposal $h_W = (1/12\pi^2) \partial_b \ln\langle W_b \rangle|_{b=1}$ is motivated by the fact that an infinitesimal equivariant deformation of

$\mathbb{S}^4$ corresponds to an insertion of an integrated energy-momentum supermultiplet [25].

In the absence of the Wilson loop, the background (13) is invariant under $\epsilon_1 \leftrightarrow \epsilon_2$, and therefore the Wilson loop insertion $\mathrm{Tr}e^{-2\pi ba}$ in Eq. (15) is the only factor in the integrand that contains a term linear in $b-1$. Therefore, $\langle W_b \rangle$ in (18) can be computed using the one-loop determinant and instanton factors of the round $\mathbb{S}^4$ matrix model.

It is worth pointing out that, for planar $\mathcal{N}=2$ superconformal gauge theories, there is an interesting proposal [27,28] to obtain $\Gamma^{\infty}_{\mathrm{cusp}}$ from the corresponding quantity in planar $\mathcal{N}=4$ SYM theory, by applying a substitution rule for the coupling. It would be interesting to see if that procedure also generalizes for the coefficient B.

Tests of the conjectures.—In the rest of the Letter, we provide some checks of the conjecture (18). For $\mathcal{N}=4$ theories, we show that (18) is equivalent to (10). For $\mathcal{N}=2$ SCFTs, (18) predicts a deviation from the $\mathcal{N}=4$ result starting at the g^6 order in perturbation theory. Indeed, we find that conformal invariance ensures that the one- and two-loop contributions to h_W and Γ_{cusp} are independent of the matter content. For $SU(2)$ with four fundamental hypermultiplets, we compute the g^6 correction to Γ_{cusp} and we find agreement with (18). In addition, we show [for $SU(N)$ with $2N$ fundamental hypermultiplets] that the right-hand side of (18) is positive, as required by the interpretation of B as the energy radiated by a quark. For $\mathcal{N}=4$ SYM theory, B and h_W can be computed holographically [16,17,29], and the explicit leading-order results satisfy the conjecture (17). These holographic computations immediately extend to $\mathcal{N}=2$ SCFTs that are orbifolds of $\mathcal{N}=4$ SYM theory, providing additional evidence in favor of the conjecture.

$\mathcal{N}=4$.—For $\mathcal{N}=4$ U(N) SYM theory, it was proven in Ref. [7] that

$$B = \frac{1}{2\pi^2}\lambda\partial_\lambda \ln\langle W\rangle. \tag{19}$$

Let us check that this is in agreement with our conjecture (18). The localization formula gives

$$\langle W_b\rangle = \frac{\int da \mathrm{Tr}e^{-2\pi ba}e^{-(8\pi^2N/\lambda)\mathrm{Tr}(a^2)}}{\int da e^{-(8\pi^2N/\lambda)\mathrm{Tr}(a^2)}} + \mathcal{O}((b-1)^2). \tag{20}$$

The rescaling of the integration variable $a=\sqrt{\lambda}\tilde{a}$ makes it manifest that $\langle W_b\rangle$ is a function of a single variable $b\sqrt{\lambda}$:

$$\langle W_b\rangle = \frac{\int d\tilde{a}\mathrm{Tr}e^{-2\pi b\sqrt{\lambda}\tilde{a}}e^{-8\pi^2N\mathrm{Tr}(\tilde{a}^2)}}{\int d\tilde{a}e^{-8\pi^2N\mathrm{Tr}(\tilde{a}^2)}} + \mathcal{O}((b-1)^2). \tag{21}$$

Thus, the conjectured formula (18) follows.

Free $\mathcal{N}=2$ $U(1)$ theory.—The simplest $\mathcal{N}=2$ SCFT is the free Abelian $\mathcal{N}=2$ gauge theory. From the field theory side, the value of h_W is the same as for the free Abelian $\mathcal{N}=4$ SYM theory. In the matrix model computation, the instanton contribution is now different from the identity [15], but, since it is moduli independent, it pulls out of the integrals and cancels out. Therefore, our conjecture (18) applies.

B and h_W to two-loop order.—We now study nontrivial, perturbative, $\mathcal{N}=2$ SCFTs. The vanishing of the β function implies that if we have $n_{\mathcal{R}}$ hypermultiplets in the representation $\mathcal{R}$ of the gauge group, then

$$C(\mathrm{Adj}) = \sum_{\mathcal{R}} n_{\mathcal{R}} C(\mathcal{R}). \tag{22}$$

As already noted in Ref. [15], this implies that the one-loop determinant in (15) has no $\mathcal{O}(a^2)$ term:

$$Z_{1\text{-loop}}(a) = 1 + \mathcal{O}(a^4). \tag{23}$$

As a consequence, for $\mathcal{N}=2$ SCFTs, the perturbative expansion of $\langle W\rangle$ starts depending on the matter content of the theory at the order of g^6. If the conjectured formula (18) is correct, the same thus must be true for the coefficients B and h_W.

We begin by considering h_W, which is given by $\langle O_2(x)W\rangle$, where $O_2(x)$ is the superconformal primary in the supermultiplet of $T_{\mu\nu}(x)$. The strategy, as in Ref. [30], is to focus on the diagrams where the hypermultiplets enter and argue that by virtue of (22) the result does not depend on the matter content. At the order of g^2 the hypermultiplets do not enter the computation, so the claim readily follows. At the order of g^4, hypermultiplets appear only in the diagrams shown in Fig. 1. For each one of these diagrams, the dependence on $\{n_{\mathcal{R}}\}$ is through the combination $\sum_{\mathcal{R}} n_{\mathcal{R}} C(\mathcal{R})$. Because of (22), this is independent of the matter content.

For the diagrams contributing to the cusped Wilson line Γ_{cusp} up to the order of g^4 (see also [31]), we find the following: At order g^2, the diagrams that contribute do not involve the hypermultiplets [Fig. 2(a)]. At the order of g^4, hypermultiplets enter in the one-loop correction of the vector

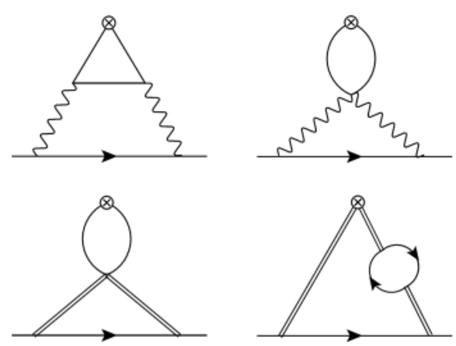

FIG. 1. Contributions to $\langle O_2(x)W\rangle$ that include hypermultiplet fields. Gauge fields are denoted with a wiggly line, vector multiplet scalars are denoted with a double line, and hypermultiplet fields are denoted with a plain line (with an arrow for fermions and without an arrow for scalars).

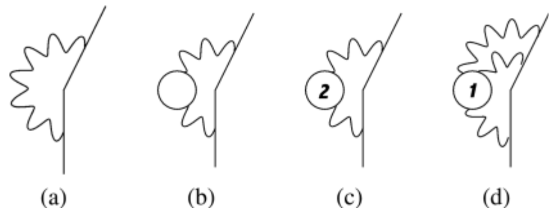

FIG. 2. Some of the Feynman diagrams that contribute to $\langle W_\varphi \rangle$. A wiggly line denotes vector multiplet fields (scalars or vectors), and a plain line denotes hypermultiplet fields (scalars or fermions): (a) One-loop diagrams. (b) Two-loop diagrams that involve hypermultiplet fields. (c),(d) The three-loop diagrams that involve hypermultiplet fields include the two-loop correction to the propagator of the vector multiplet bosonic fields and the one-loop correction to the vertex of three vector multiplet bosonic fields.

multiplet scalar and vector field propagators [Fig. 2(b)] through the factor $\sum_{\mathcal{R}} n_{\mathcal{R}} C(\mathcal{R})$, which is equal to $C(\mathrm{Adj})$ for conformal theories. Thus, for $\mathcal{N}=2$ SCFTs, B does not depend on the matter content up to the order of g^4.

Since our proposal (18) gives the correct result for $\mathcal{N}=4$, it follows that the conjectured formula (18) is correct up to the order of g^4 in all $\mathcal{N}=2$ SCFTs.

Cusp anomalous dimension to three-loop order.—At the order of g^6, hypermultiplets appear in diagrams of two types: two-loop correction to the scalar and gauge field propagator [Fig. 2(c)] and one-loop correction to the vertex of three bosonic fields from the vector multiplet [Fig. 2(d)]. We will restrict to the case of the $SU(2)$ gauge group, and we will compare the theory with four fundamental hypermultiplets ($\mathcal{N}=2$ SQCD) to the one with one adjoint hypermultipet ($\mathcal{N}=4$). The one-loop correction for the vertex is the same for the two theories [30]. The diagrammatic differences between the two-loop correction to the propagators in the two theories were calculated in Ref. [30]. The two-loop propagator $D^{(2)}(x,y)$ of the gauge field or vector multiplet scalar satisfies

$$D^{(2)}(x,y)_{\mathcal{N}=4} - D^{(2)}(x,y)_{\mathcal{N}=2} = \frac{15}{64\pi^4}\zeta(3) g^4 D^{(0)}(x,y). \tag{24}$$

This leads to

$$\langle W_\varphi \rangle_{\mathcal{N}=4} - \langle W_\varphi \rangle_{\mathcal{N}=2} = \frac{15}{64\pi^4}\zeta(3) g^4 \langle W_\varphi \rangle_{\mathcal{N}=4} + \mathcal{O}(g^8). \tag{25}$$

Thus,

$$\begin{aligned} B_{\mathcal{N}=4} - B_{\mathcal{N}=2} &= \frac{15}{64\pi^4}\zeta(3) g^4 B_{\mathcal{N}=4} + \mathcal{O}(g^8) \\ &= \frac{45}{2048\pi^6}\zeta(3) g^6 + \mathcal{O}(g^8), \end{aligned} \tag{26}$$

where we have used $B_{\mathcal{N}=4} = (3/32\pi^2) g^2 + \mathcal{O}(g^4)$ for a probe in the fundamental representation.

To compare this with our conjecture, we use the localization result for the expectation value of a Wilson loop (on the ellipsoid) in the fundamental representation:

$$\langle W_b \rangle_{\mathcal{N}=4} - \langle W_b \rangle_{\mathcal{N}=2} = \frac{45}{1024\pi^4}\zeta(3) g^6 b^2 + \mathcal{O}(g^8). \tag{27}$$

Thus, according to our conjecture,

$$\begin{aligned} B_{\mathcal{N}=4} - B_{\mathcal{N}=2} &= \frac{1}{4\pi^2}\partial_b(\langle W_b \rangle_{\mathcal{N}=4} - \langle W_b \rangle_{\mathcal{N}=2})|_{b=1} + \mathcal{O}(g^8) \\ &= \frac{45}{2048\pi^6}\zeta(3) g^6 + \mathcal{O}(g^8). \end{aligned} \tag{28}$$

This agreement is encouraging. Note that Eq. (18) contains two independently motivated conjectures, relating h_W to two *a priori* different quantities. We regard the order of g^6 match of these two quantities as evidence in favor of both conjectures, since the chain of reasoning relating them goes through h_W. It would be nice to perform explicit higher-order computations of h_W and test the two conjectures directly.

Positivity.—Since B is by definition positive, the consistency of our proposal requires that the right-hand side of (18) be positive.

Let us check this claim for the case of $SU(N)$ with $N_f = 2N$. The derivative of the Wilson loop insertion,

$$f(b;a) \equiv \partial_b \mathrm{Tr} e^{-2\pi b a} = \mathrm{Tr}((-2\pi a) e^{-2\pi a b}), \tag{29}$$

is positive at $b=1$, since

$$\partial_b f(b;a) = \mathrm{Tr}((-2\pi a)^2 e^{-2\pi a b}) > 0 \tag{30}$$

due to the Hermiticity of a. Therefore,

$$f(1;a) > f(0;a) = -2\pi \mathrm{Tr}(a) = 0. \tag{31}$$

Since the classical, one-loop, and instanton contributions are also positive,

$$\partial_b \ln\langle W_b \rangle|_{b=1} > 0. \tag{32}$$

Additional implications and open questions.—We end this Letter by pointing out two additional implications of the formula we have conjectured (18), and we suggest some open questions.

The first implication concerns the entanglement entropy due to a probe. For any 4d CFT in its vacuum state, the additional entanglement entropy of a spherical region due to the presence of a heavy probe located at its center is given by [12]

$$S = \log\langle W \rangle - 8\pi^2 h_W. \tag{33}$$

Our conjecture (18) then implies that the additional entanglement due to a heavy quark in $\mathcal{N}=2$ SCFTs is [32]

$$S = \left(1 - \frac{2}{3}\partial_b\right) \log\langle W_b\rangle|_{b=1}. \qquad (34)$$

The second implication concerns transcendentality in the perturbative expansion of B for $\mathcal{N}=2$ SCFTs. In $\mathcal{N}=4$ SYM theory, $\Gamma^{\infty}_{\text{cusp}}$ satisfies the rule of maximal transcendentality [33]: When expanded in powers of g/π, the coefficient of $(g/\pi)^{2n}$ has transcendentality $2n-2$. It follows from (10) that $B_{\mathcal{N}=4}$ satisfies the same rule [7]. Additionally, for $\mathcal{N}=2$ SCFTs, the conjecture (18) implies that, to each order in perturbation theory, the leading transcendentality terms in the bremsstrahlung function are given by the $\mathcal{N}=4$ result.

Finally, let us mention three open questions. One obvious question is further perturbative checks of (18): It would be nice to consider order of g^6 computations for general gauge groups. A second question is to find a nontrivial check for the nonperturbative corrections to $\Gamma_{\text{cusp}}(\varphi)$ entailed by (18). An additional question is to understand better the relation between derivatives with respect to the equivariant parameters and insertions of the energy-momentum supermultiplet. The relation may be nontrivial, for instance, due to the anomaly discussed in Ref. [34].

We thank O. Aharony, J. Gomis, J. Henn, A. Lewkowycz, J. Maldacena, V. Mitev, and E. Pomoni for discussions. B. F. thanks the Weizmann Institute of Science for hospitality. E. G. and Z. K. are supported by the ERC STG Grant No. 335182, by the Israel Science Foundation under Grant No. 884/11, by the United States–Israel Bi-national Science Foundation (BSF) under Grant No. 2010/629, by the Israel Science Foundation center for excellence grant (Grant No. 1989/14), and also by the I-CORE Program of the Planning and Budgeting Committee. B. F. is partially funded by the Spanish MINECO under Projects No. FPA2013-46570-C2-2-P and No. MDM-2014-0369 of ICCUB (Unidad de Excelencia "María de Maeztu") and by AGAUR, Grant No. 2014-SGR-1474.

*bfiol@ub.edu
†efrat.gerchkovitz@weizmann.ac.il
‡zohar.komargodski@weizmann.ac.il

[1] A. M. Polyakov, Gauge Fields as Rings of Glue, Nucl. Phys. **B164**, 171 (1980).
[2] A. Grozin, J. M. Henn, G. P. Korchemsky, and P. Marquard, Three Loop Cusp Anomalous Dimension in QCD, Phys. Rev. Lett. **114**, 062006 (2015).
[3] D. Correa, J. Henn, J. Maldacena, and A. Sever, The cusp anomalous dimension at three loops and beyond, J. High Energy Phys. 05 (2012) 098.
[4] J. M. Henn and T. Huber, The four-loop cusp anomalous dimension in $\mathcal{N}=4$ super Yang-Mills and analytic integration techniques for Wilson line integrals, J. High Energy Phys. 09 (2013) 147.
[5] G. P. Korchemsky and A. V. Radyushkin, Renormalization of the Wilson Loops Beyond the Leading Order, Nucl. Phys. **B283**, 342 (1987).
[6] G. P. Korchemsky and A. V. Radyushkin, Infrared factorization, Wilson lines and the heavy quark limit, Phys. Lett. B **279**, 359 (1992).
[7] D. Correa, J. Henn, J. Maldacena, and A. Sever, An exact formula for the radiation of a moving quark in N = 4 super Yang Mills, J. High Energy Phys. 06 (2012) 048.
[8] B. Fiol, B. Garolera, and G. Torrents, Exact momentum fluctuations of an accelerated quark in N = 4 super Yang-Mills, J. High Energy Phys. 06 (2013) 011.
[9] A. Kapustin, Wilson-'t Hooft operators in four-dimensional gauge theories and S-duality, Phys. Rev. D **74**, 025005 (2006).
[10] J. Gomis, S. Matsuura, T. Okuda, and D. Trancanelli, Wilson loop correlators at strong coupling: From matrices to bubbling geometries, J. High Energy Phys. 08 (2008) 068.
[11] J. Gomis and T. Okuda, S-duality, 't Hooft operators and the operator product expansion, J. High Energy Phys. 09 (2009) 072.
[12] A. Lewkowycz and J. Maldacena, Exact results for the entanglement entropy and the energy radiated by a quark, J. High Energy Phys. 05 (2014) 025.
[13] N. Drukker and D. J. Gross, An exact prediction of N = 4 SUSYM theory for string theory, J. Math. Phys. (N.Y.) **42**, 2896 (2001).
[14] J. K. Erickson, G. W. Semenoff, and K. Zarembo, Wilson loops in N = 4 supersymmetric Yang-Mills theory, Nucl. Phys. **B582**, 155 (2000).
[15] V. Pestun, Localization of gauge theory on a four-sphere and supersymmetric Wilson loops, Commun. Math. Phys. **313**, 71 (2012).
[16] M. Kruczenski, A note on twist two operators in N = 4 SYM and Wilson loops in Minkowski signature, J. High Energy Phys. 12 (2002) 024.
[17] A. Mikhailov, Nonlinear waves in AdS/CFT correspondence, arXiv:hep-th/0305196.
[18] B. Fiol, B. Garolera, and A. Lewkowycz, Exact results for static and radiative fields of a quark in N = 4 super Yang-Mills, J. High Energy Phys. 05 (2012) 093.
[19] N. Hama and K. Hosomichi, Seiberg-Witten theories on ellipsoids, J. High Energy Phys. 09 (2012) 033; 10 (2012) 051.
[20] L. F. Alday, D. Gaiotto, S. Gukov, Y. Tachikawa, and H. Verlinde, Loop and surface operators in N = 2 gauge theory and Liouville modular geometry, J. High Energy Phys. 01 (2010) 113.
[21] F. Fucito, J. F. Morales, and R. Poghossian, Wilson loops and chiral correlators on squashed sphere, J. High Energy Phys. 11 (2015) 064.
[22] N. A. Nekrasov, Seiberg-Witten prepotential from instanton counting, Adv. Theor. Math. Phys. **7**, 831 (2003).
[23] V. Pestun, Localization for $\mathcal{N}=2$ Supersymmetric Gauge Theories in Four Dimensions, arXiv:1412.7134.
[24] F. A. Dolan and H. Osborn, On short and semi-short representations for four-dimensional superconformal symmetry, Ann. Phys. (Amsterdam) **307**, 41 (2003).
[25] Analogous ideas have appeared, for example, in Refs. [12,26].

[26] C. Closset, T. T. Dumitrescu, G. Festuccia, and Z. Komargodski, Supersymmetric field theories on three-manifolds, J. High Energy Phys. 05 (2013) 017.

[27] E. Pomoni, Integrability in N = 2 superconformal gauge theories, Nucl. Phys. **B893**, 21 (2015).

[28] V. Mitev and E. Pomoni, The Exact Effective Couplings of 4D N=2 gauge theories, Phys. Rev. D **92**, 125034 (2015).

[29] J. J. Friess, S. S. Gubser, G. Michalogiorgakis, and S. S. Pufu, The stress tensor of a quark moving through N = 4 thermal plasma, Phys. Rev. D **75**, 106003 (2007).

[30] R. Andree and D. Young, Wilson loops in N = 2 superconformal Yang-Mills theory, J. High Energy Phys. 09 (2010) 095.

[31] Y. Makeenko, P. Olesen, and G. W. Semenoff, Cusped SYM Wilson loop at two loops and beyond, Nucl. Phys. **B748**, 170 (2006).

[32] Written in this way, the formula is reminiscent of the formula in 3d CFTs of [12]:

$$S = \left(1 - \frac{1}{2}\partial_b\right) \log\langle W_b\rangle|_{b=1}, \qquad 3d\mathcal{N} = 2 \text{ SCFT}. \tag{35}$$

A tempting guess is that for some class of d-dimensional CFTs $S = \{1 - [(d-2)/(d-1)]\partial_b\} \log\langle W_b\rangle|_{b=1}$.

[33] A. V. Kotikov and L. N. Lipatov, DGLAP and BFKL equations in the N = 4 supersymmetric gauge theory, Nucl. Phys. **B661**, 19 (2003); **685**, 405 (2004).

[34] J. Gomis, Z. Komargodski, P. S. Hsin, A. Schwimmer, N. Seiberg, and S. Theisen, Anomalies, Conformal Manifolds, and Spheres, arXiv:1509.08511.