Exact Evaluation of Maximal-Ratio and Equal-Gain Diversity Receivers for M-ary QAM on Nakagami Fading Channels

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***Abstract*—Exact integral expressions are derived for calculating the symbol-error rate (SER) of multilevel quadrature amplitude modulation (MQAM) in conjunction with L-fold antenna diversity on arbitrary Nakagami fading channel. Both maximal-ratio combining (MRC) (in independent and correlated fading) and equal-gain combining (EGC) predetection (in independent fading) diversity techniques have been considered. Exact closed-form SER expressions for two restricted Nakagami fading cases (MRC reception) are also derived. An exact analysis of EGC for MQAM has not been reported previously, despite its practical interest. Remarkably, the exact SER integrals can also be replaced by a finite-series approximation formula. A useful procedure for computing the confluent hypergeometric series is also presented.**

***Index Terms*— Diversity methods, Gauss–Chebychev quadrature, Nakagami fading channels, quadrature amplitude modulation.**

I. Introduction

RECENTLY, multilevel quadrature amplitude modulation (MQAM) format has been considered for high-rate data transmission over wireless links [1]. Antenna diversity is usually employed to mitigate the effects of deep fades and cochannel interference. Maximal-ratio combining (MRC), which provides the highest average output signal-to-noise ratio (SNR), is difficult to implement in practice. Equal-gain combining (EGC), however, is easier to implement but incurs a performance penalty. While the literature has thoroughly treated the performance of many modulation schemes in various fading channels, which generally involves averaging $\mathrm{erfc}(\sqrt{\gamma})$ over the fading distribution, where γ is the SNR, the MQAM problem presents a new wrinkle: averaging $\mathrm{erfc}^2(\sqrt{\gamma})$. Scanning the literature, one can identify two distinct approaches: the direct and the moment-generating function (MGF) approach. In the direct approach, one first derives the probability density function (pdf) of γ and performs the averaging. In the MGF approach, one uses the MGF of γ (often readily available) and some integral representation of $\mathrm{erfc}(\sqrt{\gamma})$ and $\mathrm{erfc}^2(\sqrt{\gamma})$ to perform the averaging.

Previous work includes the following. The symbol error rate (SER) of MQAM in the additive white Gaussian noise (AWGN) is furnished in [2, eq. (5-2-79)]. The performance of MQAM and MRC diversity in Rayleigh fading is given in [3]–[5]. In [6], we derive a simple expression (involving finite summations of the MGF) for the SER of MQAM with MRC diversity over a Nakagami fading channel with arbitrary parameters. More recently, Alouini and Goldsmith [7] presented a performance analysis for MQAM using the MGF approach [8]–[11]. By contrast, in this paper, we enhance our previous work by deriving several simple analytical expressions for MQAM with L-fold MRC diversity on a Nakagami fading channel. Further, exact closed-form SER expressions are derived for two special cases.

The direct method for the performance analysis of EGC is limited to Rayleigh fading and second-order diversity [12] because a closed-form expression for the pdf of a sum of Nakagami-distributed (or even Rayleigh-distributed) random variables (RV's) does not exist for $L > 2$. For higher order of diversity, the direct approach can be applied using a small argument approximation [13], [14]. Recently, Beaulieu [15] has devised an approximate infinite series for the pdf of a sum of independent Rayleigh RV's. Applying this series and the direct approach, Beaulieu and Abu-Dayya [16] present a comprehensive study of EGC for coherent and differential binary signaling schemes. Two previous papers [17], [18] make use of characteristic functions (CHF's) for analyzing the EGC performance. In [17], Zhang derived some closed-form solutions for binary signaling formats with second- and third-order diversity systems directly from the CHF of Rayleigh fading amplitudes. In [18], the authors derive an approximate solution for a binary case. Here, we derive the exact performance of EGC diversity systems for both binary and M-ary modulation formats. We apply Parseval's theorem to transform the error integral into the frequency-domain so that the average SER is expressed using the CHF of the EGC output. The resulting finite-range integral can be estimated very accurately with only a few CHF samples using the Gauss–Chebychev quadrature (GCQ) formula. The generality and computational efficiency of our new expressions render themselves a powerful tool for SER analysis under a myriad of fading scenarios.

Paper approved by N. C. Beaulieu, the Editor for Wireless Communication Theory of the IEEE Communications Society. Manuscript received October 26, 1998; revised February 22, 1999. This work was supported in part by a Strategic Project Grant from the Natural Sciences and Engineering Research Council (NSERC) of Canada and in part by BC Tel Mobility Cellular. This paper was presented in part at the IEEE International Conference on Communications, Vancouver, BC, June 1999.

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Publisher Item Identifier S 0090-6778(99)07460-7.

This paper has the following organization. The derivation of average SER of MQAM using an L-fold MRC space diversity on a Nakagami fading channel is outlined in Section II. Section III details the error performance of MQAM with predetection EGC. Subsequently in Section IV, selected numerical results are presented. Finally, the main points are summarized in Section V.

II. SER of MQAM with Maximal-Ratio Diversity Receiver

In MQAM, a symbol is generated according to $\log_2 M$ data bits, and each symbol in a quadrant has different SER. Among the various known signal constellations, rectangular QAM signal is the most frequently used in practice because [2]: 1) its signal constellation is easily generated as two PAM signals impressed on phase-quadrature carriers; 2) the task of signal demodulated can be performed without much difficulty; and 3) the average transmitted power required to attain a given minimum distance with rectangular QAM is only slightly higher than that of the best MQAM signal constellation. When $\log_2 M$ is even (i.e., square QAM), the exact SER for MQAM in the AWGN channel is given by [2]

$$P_s^{(E)}(\epsilon \mid \gamma_b) = 2q\,\mathrm{erfc}(\sqrt{p\gamma_b}) - q^2\,\mathrm{erfc}^2(\sqrt{p\gamma_b}) \quad (1)$$

where $q = 1 - 1/\sqrt{M}$, $p = 1.5\log_2 M/(M-1)$, and γ_b is the average received SNR per bit. On the other hand, when $\log_2 M$ is odd, there is no equivalent $\sqrt{M}$-ary PAM system. In this case, the symbol-error probability is tightly upper bounded by [2, eq. (5-2-80)]

$$P_s^{(O)}(\epsilon \mid \gamma_b) \le 2q\,\mathrm{erfc}(\sqrt{p\gamma_b}) - q^2\,\mathrm{erfc}^2(\sqrt{p\gamma_b}) \quad (2)$$

if the detector bases its decisions on the optimum distance metric (maximum-likelihood criterion).

A. Independent Fading

In this section, we outline several methods for computing the SER of MQAM with MRC diversity reception on a Nakagami fading environment. Each method is unique, interesting, and novel in its own right. Hence, we are presenting them in the hope of stimulating further applications.

1) Computation of SER Using pdf of γ_b: As in [5], we assume matched filter detection and perfect channel estimation is available at the receiver. Then, the average symbol-error probabilities in a slow and flat Nakagami fading channel may be derived by averaging the error rates for the AWGN channel over the pdf of the SNR in Nakagami fading

$$P_S^{(E)}(\epsilon) = \int_0^\infty P_s^{(E)}(\epsilon \mid \gamma_b) p_{\gamma_b}(\gamma_b)\, d\gamma_b = I_1 - I_2 \quad (3)$$

where

$$I_1 = \int_0^\infty 2q\,\mathrm{erfc}(\sqrt{p\gamma_b}) p_{\gamma_b}(\gamma_b)\, d\gamma_b \quad (4)$$

$$I_2 = \int_0^\infty q^2\,\mathrm{erfc}(\sqrt{p\gamma_b}) p_{\gamma_b}(\gamma_b)\, d\gamma_b \quad (5)$$

and $\gamma_b = (E_b/N_0)\sum_{l=1}^L \alpha_l^2 = \sum_{l=1}^L \gamma_l$ is the instantaneous SNR per bit with L-fold MRC diversity, where α denotes the Nakagami-distributed RV. The pdf of γ_b is readily obtained by invoking the basic Fourier inversion theorem

$$p_{\gamma_b}(\gamma_b) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{\gamma_b}(t)\exp(-jt\gamma_b)\, dt \quad (6)$$

where $\phi_{\gamma_b}(t)$ denotes the CHF of γ_b [2]

$$\phi_{\gamma_b}(t) = \prod_{l=1}^L \int_0^\infty \exp(jt\gamma_l) p_\gamma(\gamma_l)\, d\gamma_l = \prod_{l=1}^L \left[\frac{\lambda_l}{\lambda_l - jt}\right]^{m_l} \quad (7)$$

with the assumption that the fading statistics across the L antennas are uncorrelated (achieved through sufficient antenna separation). The notation $p_\gamma(\gamma)$ in (7) corresponds to the pdf of the received SNR of a single diversity branch in Nakagami fading environment, which has the chi-square pdf given in [2, eq. (14-3-14)]. Since $p_{\gamma_b}(\gamma_b)$ is real and the real part of the integrand is symmetric about $t = 0$, we get

$$\begin{aligned} p_{\gamma_b}(\gamma_b) &= \frac{1}{\pi}\int_0^\infty \frac{\cos\left[\sum_{l=1}^L m_l \tan^{-1}(t/\lambda_l) - t\gamma_b\right]}{\prod_{l=1}^L [1 + (t/\lambda_l)^2]^{m_l/2}}\, dt \\ &= \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Re}\{\exp[-jt\gamma_b + j\theta(t)]\}}{\zeta(t)}\, dt \end{aligned} \quad (8)$$

where $\zeta(t) = \prod_{l=1}^L [1 + (t/\lambda_l)^2]^{m_l/2}$ and $\theta(t) = \sum_{l=1}^L m_l \tan^{-1}(t/\lambda_l)$. The parameter m_l in (8) denotes the fading figure of the lth diversity branch (i.e., antenna) and $\lambda_l = m_l/\bar{\gamma}_l$, where $\bar{\gamma}_l = (E_b/N_0)E[\alpha_l^2] = (E_b/N_0)\Omega_l$ corresponds to the average received SNR of the lth antenna.

Now, consider the Fourier transform (FT)

$$G_1(\omega) = \int_0^\infty \exp(-j\omega t)\,\mathrm{erfc}(\sqrt{pt})\, dt = \frac{1}{j\omega}\left[1 - \frac{\sqrt{p}}{\sqrt{p + j\omega}}\right] \quad (9)$$

which is obtained using identity [20, eq. (6.283)]. Substituting (8) into (4) and recognizing that the integration with respect to γ_b is the FT shown in (9), I_1 can be manipulated into the form

$$\begin{aligned} I_1 = \frac{2q}{\pi}\int_0^\infty \frac{1}{\zeta(t)}\Bigg\{&\sin[\theta(t)]\left[1 - \frac{\sqrt{p}\cos[0.5\tan^{-1}(t/p)]}{(p^2+t^2)^{1/4}}\right] \\ &+ \cos[\theta(t)]\frac{\sqrt{p}\sin[0.5\tan^{-1}(t/p)]}{(p^2+t^2)^{1/4}}\Bigg\}\frac{dt}{t} \end{aligned} \quad (10)$$

by expressing $\sqrt{p+jt}$ in the polar form, i.e., $\sqrt{p+jt} = (p^2+t^2)^{1/4}\exp[j0.5\tan^{-1}(t/p)]$. By manipulating [20, eq. (8.258)], we get an FT identity

$$\begin{aligned} G_2(\omega) &= \int_0^\infty \exp(-j\omega t)\,\mathrm{erfc}^2(\sqrt{pt})\, dt \\ &= \frac{1}{j\omega}\left[1 - \frac{4}{\pi}\frac{\tan^{-1}\sqrt{1 + j\omega/p}}{\sqrt{1 + j\omega/p}}\right]. \end{aligned} \quad (11)$$

By substituting (8) in (5), changing the order of integration in (5), and using the transformation formula (11), I_2 can be restated as

$$I_2 = \frac{q^2}{\pi}\int_0^\infty \frac{1}{\zeta(t)}\mathrm{Re}\{G_2(t)\exp[j\theta(t)]\}\, dt. \quad (12)$$

Hence, substituting (10) and (12) into (3), we arrive to an exact analytical expression for SER of MQAM in Nakagami fading channel with arbitrary parameters. This one-dimension integral can be computed numerically (e.g., trapezoidal integration rule). As before, an upper bound for the SER of rectangular QAM with odd $\log_2 M$ may be computed using

$$P_s^{(O)}(\epsilon) \le P_s^{(E)}(\epsilon)\big|_{q=1}. \tag{13}$$

In some previous work (e.g., [1], [3], [19]), the authors have used an approximate SER formula for MQAM in fading channel by ignoring the second integral in (3), since $\mathrm{erfc}^2(\sqrt{\gamma_b}) \ll \mathrm{erfc}(\sqrt{\gamma_b})$ as $\gamma_b \to \infty$ (or for relatively large SNR per bit). However, the discrepancy between the exact SER and that of calculated SER via the coarse approximation described above can be quite large even for moderate values of $\bar{\gamma}_b$ [6].

2) Computation of SER Using pdf of γ_b and the GCQ Formula: Our second approach for calculating accurate SER for MQAM in conjunction with MRC diversity is based on knowledge of two FT identities [(9), (11)] and the application of the GCQ formula [21, eq. (25.4.38)]. Combining (10) and (12), we can write

$$P_s^{(E)}(\epsilon) = I_1 - I_2 = \frac{q}{\pi}\int_0^\infty \frac{\Psi(t)}{t\zeta(t)}\,dt \tag{14}$$

where

$$\Psi(t) = \mathrm{Re}\left\{e^{j\theta(t)-\pi/2}\left[2-q-\frac{2}{\kappa}+\frac{4q}{\pi}\frac{\tan^{-1}(\kappa)}{\kappa}\right]\right\}$$

and $\kappa = \sqrt{1+jt/p}$. Using variable substitution $t^2+1/2 = 1/(1+x)$ in (14), the integration limits become -1 to $+1$. Now, by applying the GCQ formula of the first kind to the transformed integral, we have a series expression for the SER of MQAM with MRC diversity on Nakagami fading channel

$$P_s^{(E)}(\epsilon) = \frac{q}{n}\sum_{k=1}^{n}\xi\left[\frac{1}{\sqrt{2}}\tan(\theta_k)\right]\Big/\sin(2\theta_k) + R_n \tag{15}$$

where $\theta_k = (\pi(2k-1))/4n$ and $\xi(x) = \Psi\{((1-x)/(2(1+x)))^{1/2}\}/\zeta\{((1-x)/(2(1+x)))^{1/2}\}$. Since the remainder term R_n vanishes quickly as n increases, (15) is a rapidly converging series.

3) Computation of SER Using MGF of γ_b and the GCQ Formula: Different from the conventional method for computing SER [i.e., direct evaluation of (3)], our third approach relies upon the knowledge of the MGF of γ_b, the use of an alternative exponential forms for one-dimension and two-dimension complementary error functions, as well as the application of the GCQ rule [9], [22], [23]. The MGF technique has been applied successfully in [6], but I_2 was evaluated with the aid of a two-dimension GCQ formula, i.e.,

$$P_s^{(E)}(\epsilon) = \frac{2q}{n}\sum_{i=1}^{n}\left\{\prod_{l=1}^{L}\left[1+\frac{\sec^2(\theta_i)}{\lambda_l/p}\right]^{-m_l} - \frac{q}{2n} \cdot\sum_{j=1}^{n}\prod_{l=1}^{L}\left[1+\frac{\sec^2(\theta_i)+\sec^2(\theta_j)}{\lambda_l/p}\right]^{-m_l}\right\} + R_n \tag{16}$$

where n is a small positive integer, $\theta_j = (2j-1)\pi/4n$, and $\theta_i = (2i-1)\pi/4n$. In the following, we derive a much simpler SER formula for MQAM modulation scheme on Nakagami fading channels. The new expression reduces the number of MGF samples required to achieve a specified accuracy from n^2 [in the case of (16)] to n. This is mainly attributed to the alternative exponential representation of the $\mathrm{erfc}^2(\cdot)$.

The MGF of γ_b is related to the CHF shown in (7) via relationship $\phi(s) = \phi_{\gamma_b}(js)$. Next, by exploiting the results from the definite integral [20, eq. (7.4.11)], the complementary error function can be represented via an alternative exponential form as

$$\mathrm{erfc}(\sqrt{p\gamma}) = \frac{2\sqrt{p}}{\pi}\int_0^\infty \frac{\exp\left[-\gamma\left(t^2+p\right)\right]}{t^2+p}\,dt, \qquad p>0, \quad \gamma>0. \tag{17}$$

From Appendix A, we have an alternative exponential form for the $\mathrm{erfc}^2(\cdot)$

$$\mathrm{erfc}^2(\sqrt{p\gamma}) = \frac{4}{\pi}\int_0^{\pi/4}\exp\left(-\gamma p\csc^2(\Theta)\right)d\Theta. \tag{18}$$

It is noted that (18) may also be derived directly using the results from definite integrals [20, eq. (7.4.12)] and (17) with some algebraic manipulations. Substituting (17) and (18) into (3), and recognizing $\int_0^\infty \exp(-\gamma_b s)p_{\gamma_b}(\gamma_b)\,d\gamma_b = \phi(s)$, we get

$$P_s^{(E)}(\epsilon) = \frac{4q\sqrt{p}}{\pi}\int_0^\infty \frac{\phi\left(t^2+p\right)}{t^2+p}\,dt - \frac{4q^2}{\pi}\int_0^{\pi/4}\phi\left(p\csc^2(\Theta)\right)d\Theta. \tag{19}$$

Equation (19) can be manipulated into a desired form (so that one can apply GCQ formulas directly) using variable transformations $t^2+p = 2p/(x+1)$ and $y = \cos 4\Theta$

$$P_s^{(E)}(\epsilon) = \frac{2q}{\pi}\int_{-1}^{1}\frac{\phi(2p/(x+1))}{\sqrt{1-x^2}}\,dx - \frac{q^2}{\pi}\int_{-1}^{1}\frac{\phi\left[p\csc^2\left(0.25\cos^{-1}(y)\right)\right]}{\sqrt{1-y^2}}\,dy. \tag{20}$$

Then, using the GCQ approximation [21, eq. (25.4.38)] leads (20) directly to a simple expression for the average SER of MQAM in a slow and flat Nakagami fading channel

$$P_s^{(E)}(\epsilon) = \frac{2q}{n}\sum_{i=1}^{n}\left\{\prod_{l=1}^{L}\left[1+\frac{\sec^2(\theta_i)}{\lambda_l/p}\right]^{-m_l} - \frac{q}{2}\prod_{l=1}^{L}\left[1+\frac{\csc^2(\theta_i/2)}{\lambda_l/p}\right]^{-m_l}\right\} + R_n \tag{21}$$

where $\theta_i = (2i-1)\pi/4n$. The remainder term R_n can be bounded using the results of [10, Appendix A] and/or [23]. However, this is not necessary in practice, since one simply computes (21) for several increasing values of n and stops when the result converges to a prescribed accuracy. Note the implications of (21): we are simply sampling the MGF at n points. So as long as the MGF exists and computable, this method can work very effectively. In fact, its accuracy will be high if the high-order derivatives of the MGF vanish rapidly. In Appendix A, we also present another method for computing I_2. The Gauss–Lobatto quadrature (GLQ) integration method also requires significantly fewer samples of MGF to evaluate the SER than the two-dimension GCQ technique [i.e., (16)] developed in [6].

Furthermore, using variable substitution $t = \sqrt{p}\tan(\Theta)$ in (19), we get a simple exact analytical expression for the SER of MQAM with MRC diversity receiver on generalized fading channels

$$P_s^{(E)}(\epsilon) = \frac{4q}{\pi}\int_0^{\pi/2} \phi\big(p\csc^2(\Theta)\big)\, d\Theta - \frac{q^2}{\pi}\int_0^{\pi/4} \phi\big(p\csc^2(\Theta)\big)\, d\Theta \quad (22)$$

which is identical to the results presented in [7]. This form is both easily evaluated and well suited to numerical integration since the integrand is well behaved over the finite-range of the integration limits.

4) Computation of SER Using Parseval's Theorem and the GCQ Formula: Our fourth technique for evaluating the SER of MQAM with MRC diversity relies on knowledge of two FT's: the application of Parseval's theorem and the GCQ formula. By applying Parseval's theorem in (3), we get

$$\begin{aligned} P_s^{(E)}(\epsilon) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{FT}\Big[P_s^{(E)}(\epsilon \mid \gamma_b)\Big]\phi_{\gamma_b}(\omega)\, d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \big[2qG_1(\omega) - q^2 G_2(\omega)\big]\phi_{\gamma_b}(\omega)\, d\omega \quad (23) \end{aligned}$$

where $\phi_{\gamma_b}(\omega)$, $G_1(\omega)$, and $G_2(\omega)$ are defined in (7), (9), and (11), respectively. Notice that our methods II-A.1 and II-A.4 are essentially the same. But the development of (23) is interesting because it lends itself into a unified form of SER for MQAM with MRC diversity on arbitrary fading environments (not restricted to only Nakagami fading). Now, using variable substitution $\omega = \tan\theta$ in (23) and then applying the GCQ formula, we get

$$\begin{aligned} P_s^{(E)}(\epsilon) &= \frac{1}{\pi}\int_0^{\pi/2} \upsilon(\tan\theta)\sec^2\theta\, d\theta \\ &= \frac{q}{2n}\sum_{k=1}^{n} \upsilon[\tan(\theta_k)]\sec^2(\theta_k) + R_n \quad (24) \end{aligned}$$

where $\upsilon(\omega) = \mathrm{Re}\{[2G_1(\omega) - qG_2(\omega)]\phi_{\gamma_b}(\omega)\}$ and $\theta_k = \pi(2k-1)/4n$.

5) Exact Closed-Form Formulas for SER of MQAM with MRC Diversity: Next, we will present exact closed-form SER formulas of MQAM with MRC diversity for two special cases of Nakagami fading: 1) identical $\lambda_l = m_l/\bar{\gamma}$ across the diversity branches and $\sum m_l$ is a positive integer; 2) distinct diversity branches and integer m_l's for $l = 1, \cdots, L$.

Case (a): Let us assume $\lambda_l = \lambda$ for $l = 1, \cdots, L$ and $\sum m_l = D$ is a positive integer. In this case, the RV γ_b has a gamma pdf [obtained by inverting (7)]

$$p_{\gamma_b}(\gamma) = \frac{\lambda^D}{(D-1)!}\gamma^{D-1}\exp(-\lambda\gamma). \quad (25)$$

Then, using identity [2, eq. (14-4-15)]

$$\begin{aligned} I_1 &= 4q\int_0^{\infty} \frac{1}{2}\,\mathrm{erfc}(\sqrt{p\gamma})p_{\gamma_b}(\gamma)\, d\gamma \\ &= 4q\left[\frac{1}{2}(1-\hat{\lambda})\right]^D \sum_{k=0}^{D-1}\binom{D-1+k}{k}\left[\frac{1}{2}(1+\hat{\lambda})\right]^k \quad (26) \end{aligned}$$

where $\hat{\lambda} = \sqrt{p/(p+\lambda)}$, and

$$\begin{aligned} I_2 &= \frac{q^2\lambda^D}{(D-1)!}\int_0^{\infty} \gamma^{D-1}\exp(-\lambda\gamma)\,\mathrm{erfc}^2(\sqrt{p\gamma})\, d\gamma \\ &= \frac{q^2\lambda^D(-1)^{D-1}}{(D-1)!} \\ &\quad\cdot\left\{\frac{d^{D-1}}{ds^{D-1}}\left[\frac{1}{s} - \frac{4}{\pi}\frac{\tan^{-1}\sqrt{1+s/p}}{s\sqrt{1+s/p}}\right]\Bigg|_{s=\lambda}\right\} \quad (27) \end{aligned}$$

by exploiting the FT identity in (11). For small values of D, the $(D-1)$th order differentiation in (27) can be computed by hand. For instance

$$\begin{aligned} I_2 &= q^2\left[1 - 4\mathcal{J} + 2\kappa\lambda - \frac{2\mathcal{J}\lambda}{p+\lambda}\right], \quad \text{when} \quad D=2 \\ I_2 &= q^2\left[1 - 4\mathcal{J} + 2\kappa\lambda - \frac{2\mathcal{J}\lambda}{p+\lambda} + \frac{3\kappa\lambda^2}{2(p+\lambda)} + \frac{\kappa\lambda^2}{2p+\lambda} - \frac{3\mathcal{J}\lambda^2}{2(p+\lambda)^2}\right], \quad \text{if} \quad D=3 \end{aligned}$$

where $\mathcal{J} = \tan^{-1}(\sqrt{1+\lambda/p})/(\pi\sqrt{1+\lambda/p})$ and $\kappa = p/(\pi(p+\lambda)(2p+\lambda))$. If D is large, then this differentiation may be performed with the aid of common mathematical software packages, such as the Maple, because the number of terms grows exponentially. Alternatively, by substituting $\tan^{-1}(z)/z = {}_2F_1(1/2, 1; 3/2; -z^2)$ [20, eq. (9.121.27)] in (27) and then invoking Leibnitz' rule (i.e., nth derivative of a product of two functions) [20, eq. (0.42)], and after simplifications, we get

$$I_2 = q^2\left[1 - \frac{4}{\pi}\sum_{k=0}^{D-1}\frac{\lambda^k p^{-k}}{(2k+1)}{}_2F_1\left(\frac{1}{2}+k, 1+k; \frac{3}{2}+k; -1-\frac{\lambda}{p}\right)\right]. \quad (28)$$

In this case, the final SER expression can be computed recursively in terms of Gauss hypergeometric series. For the particular case of $m = 1$, (28) reduces to the results given in [5].

Case (b): If the diversity branches are distinct and m_l's assume integer values, we obtain, upon performing the inverse Laplace transform of (7)

$$p_{\gamma_b}(\gamma) = \sum_{l=1}^{L}\sum_{k=1}^{m_l}\eta_l^{(m_l-k)}\frac{\gamma^{(k-1)}\lambda_l^k\exp(-\lambda_l\gamma)}{(k-1)!} \quad (29)$$

where

$$\eta_l^{(m_l-k)} = \frac{1}{(m_l-k)!\lambda_l^k}\left[\prod_{i=1}^{L}\lambda_i^{m_i}\right]\frac{d^{m_l-k}}{ds^{m_l-k}}\left[\prod_{i\neq l}^{L}(s+\lambda_i)^{-m_l}\right]\Bigg|_{s=-\lambda_l}. \quad (30)$$

Then, the corresponding exact SER may be evaluated as

$$P_S^{(E)}(\epsilon) = 4q\sum_{l=1}^{L}\sum_{k=1}^{m_l}\eta_l^{(m_l-k)}\left[\frac{1}{2}\left(1-\sqrt{\frac{p}{p+\lambda_l}}\right)\right]^k \cdot\sum_{i=0}^{k-1}\binom{k-1+i}{i}\left[\frac{1}{2}\left(1+\sqrt{\frac{p}{p+\lambda_l}}\right)\right]^i - p_0$$

$$p_0 = q^2\sum_{l=1}^{L}\sum_{k=1}^{m_l}\eta_l^{(m_l-k)}\frac{\lambda_l^k(-1)^{k-1}}{(k-1)!} \cdot\left\{\frac{d^{k-1}}{ds^{k-1}}\left[\frac{1}{s}-\frac{4}{\pi}\frac{\tan^{-1}\sqrt{1+s/p}}{s\sqrt{1+s/p}}\right]\Bigg|_{s=\lambda_l}\right\}. \quad (31)$$

Once again the $(k-1)$th-order derivative term may be replaced by an equivalent expression similar to (28). If the fading severity index is common to all diversity branches and $m=1$, (31) reduces to the SER formula for square MQAM on Rayleigh fading channel derived in [5]. To the best of the authors' knowledge, all the exact closed-form expressions for MQAM on Nakagami fading channel presented in this section are new.

B. Correlated Fading

When the diversity branches are correlated, the analysis proceeds in a similar manner as the independent fading scenario. But we need to find the corresponding CHF or MGF of the SNR at the output of the combiner. For the arbitrarily correlated Nakagami fading environment, the joint CHF of the instantaneous SNR may be written in the form [24]

$$\phi_\gamma(t_1,\cdots,t_L) = \det(I - jTR\Lambda)^{-m} \quad (32)$$

where I is the $L\times L$ identity matrix, Λ is a positive definite matrix of dimension L (determined by the branch covariance matrix), T and R are two diagonal matrices defined as $T=\operatorname{diag}(t_1,\cdots,t_L)$ and $R=\operatorname{diag}(\bar{\gamma}_1/m,\cdots,\bar{\gamma}_l/m)$, respectively, and m is the fading parameter. Then, the CHF of γ_b can be obtained from (32) by setting $t_k = t$ $(k=1,\cdots,L)$, i.e.,

$$\phi_{\gamma_b}(t) = \det(I - jtR\Lambda)^{-m} = \prod_{k=1}^{L}(1-j\lambda_k t)^{-m} \quad (33)$$

where λ_k are the eigenvalues of matrix $R\Lambda$. Thus, we can readily evaluate the exact SER performance of MQAM with MRC diversity by substituting (33) into (23) or (24).

For special cases of constant and exponential correlation [24] models (and with the assumption of identical fading severity index and signal strength across the diversity branches), the corresponding CHF's can be easily shown to be (34) and (35), respectively

$$\phi_{\gamma_b}(t) = \left(\frac{m^L[jt\bar{\gamma}(\rho-1)+m]^{1-L}}{[jt\bar{\gamma}(\rho-1-L\rho)+m]}\right)^m \quad (34)$$

$$\phi_{\gamma_b}(t) \approx \left(\frac{mL}{mL-jt\bar{\gamma}\tau}\right)^{mL^2/\tau} \quad (35)$$

where $\tau = L+(2\rho/(1-\rho))\,(L-(1-\rho^L)/(1-\rho))$, ρ is the correlation coefficient and L is the diversity order.

III. SER OF MQAM WITH EQUAL-GAIN DIVERSITY RECEIVER

In an EGC, the output of different diversity branches are first co-phased, equally weighted, and then summed to give the resultant output. The instantaneous SNR at the output of the EGC combiner is $\gamma_b = x^2$, where x is defined as

$$x = \sqrt{\frac{E}{LN_0}}\sum_{l=1}^{L}\alpha_l \quad (36)$$

where α_l is a Nakagami RV with the statistical parameters m_l and Ω_l as defined in Section II. Let $\bar{\gamma}_k = \Omega_k(E_b/N_0)$ denote the average SNR for the kth branch, which is consistent with our definition for the MRC case. The CHF of x (the sum of Nakagami RV's) in this case is simply the product of the individual CHF's, i.e.,

$$\phi_x(\omega) = \prod_{k=1}^{L}\int_0^\infty \frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\bar{\gamma}_k}\right)^{m_k}\alpha^{2m_k-1} \cdot\exp\left(\frac{-m_k\alpha^2}{\bar{\gamma}_k}+\frac{j\omega\alpha}{\sqrt{L}}\right)d\alpha. \quad (37)$$

Recognizing that the definite integral in (37) can be expressed in terms of parabolic cylinder function using identity [20, eq. (3.462)], we get

$$\phi_x(\omega) = \prod_{k=1}^{L}\frac{\Gamma(m_k+1/2)}{2^{-m_k}\sqrt{\pi}}e^{-w^2/(8\lambda_k L)}D_{-2m_k}\left(\frac{-j\omega}{\sqrt{2L\lambda_k}}\right) \quad (38)$$

where $\lambda_k = m_k/\bar{\gamma}_k$ and $D_{-\nu}(z)$ and is the parabolic cylinder function of order ν and argument z. Using identity [20, eq. (9.240)], (38) may be restated in terms of the more familiar confluent hypergeometric function of the first kind $\Phi(a,b;c)$

$$\phi_x(\omega) = \prod_{k=1}^{L}\left\{\Phi\left(m_k,\frac{1}{2};\frac{-\omega^2}{4L\lambda_k}\right)+\frac{j\Gamma(m_k+1/2)\omega}{\Gamma(m_k)\sqrt{L\lambda_k}} \cdot\Phi\left(m_k+\frac{1}{2},\frac{3}{2};-\frac{\omega^2}{4L\lambda_k}\right)\right\}. \quad (39)$$

The confluent hypergeometric function may be computed efficiently using a convergent series for small arguments and via a divergent expansion for large arguments (see Appendix B). From (1), the conditional error probability for square MQAM is

$$P_S^{(E)}(\epsilon\mid x) = 2q\operatorname{erfc}(\sqrt{p}x) - q^2\operatorname{erfc}^2(\sqrt{p}x) \quad (40)$$

and we are interested in calculating its average over the Nakagami pdf

$$P_S^{(E)}(\epsilon) = \int_0^\infty P_S^{(E)}(\epsilon \mid x) p_x(x)\, dx \tag{41}$$

where $p_x(x)$ is the pdf of the sum of L Nakagami RV's. In general, it is difficult (or impossible) to invert (39) to get a closed-form expression for the pdf of x. Therefore, a Fourier series approach has previously been used [16]. Since we already have the FT of the pdf, by transforming the product integral in (41) to the frequency-domain using Parseval's theorem, the need to find the pdf is circumvented. But we then also need the FT of $P_S^{(E)}(\epsilon \mid x)$, which surprisingly turns out to be very easily computed. Hence, for our subsequent development, the following two FT's are needed:

$$\begin{aligned}\psi(\omega) &= \omega \int_0^\infty \operatorname{erfc}(x) e^{j\omega x}\, dx \\ &= \frac{2}{\sqrt{\pi}} F\left(\frac{\omega}{2}\right) + j\left[1 - \exp\left(-\frac{\omega^2}{4}\right)\right]\end{aligned} \tag{42}$$

$$\begin{aligned}\varphi(\omega) &= \omega \int_0^\infty \operatorname{erfc}^2(x) e^{j\omega x}\, dx \\ &= j - j\left[\exp\left(-\frac{\omega^2}{4}\right) + \frac{4j}{\sqrt{\pi}} F\left(\frac{\omega}{2}\right) - \frac{4j}{\sqrt{\pi}} F\left(\frac{\omega}{2\sqrt{2}}\right)\right. \\ &\qquad \left. \cdot \exp\left(-\frac{\omega^2}{8}\right) + \frac{4}{\pi} F^2\left(\frac{\omega}{2\sqrt{2}}\right)\right]\end{aligned} \tag{43}$$

which were obtained using integration by parts, and $F(\cdot)$ denotes the Dawson integral

$$F(x) = e^{-x^2} \int_0^x e^{t^2}\, dt. \tag{44}$$

There are at least two methods for computing $F(x)$. First, it has the series representation

$$F(x) = x\Phi\left(1, \frac{3}{2}; -x^2\right). \tag{45}$$

Therefore, $F(x)$ can be computed using the procedure outlined in Appendix B [i.e., by evaluating sufficient number of terms in the series of $\Phi(a, c; x)$], which can handle any real a and c. It turns out that when $a = 1$ and $c = 3/2$, there is a much more efficient direct method to compute $\Phi(1, 3/2; -x^2)$. In this paper, we use this second approach due to Rybicki [25]. That is why (42) (which can also be expressed in terms of the confluent series) and (43) are expressed in terms of this function. Now applying Parseval's theorem in (41), we get

$$\begin{aligned}P_s^{(E)}(\epsilon) &= \frac{1}{2\pi} \int_{-\infty}^\infty FT\left[P_s^{(E)}(\epsilon \mid x)\right] \phi_x(\omega)\, d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^\infty \frac{1}{\omega}\left[2q\psi(\omega) - q^2\varphi(\omega)\right] \phi_x^*(\sqrt{p}\omega)\, d\omega.\end{aligned} \tag{46}$$

Since the imaginary part of this integral is zero, we may rewrite (46) as

$$P_S^{(E)}(\epsilon) = \frac{q}{\pi} \int_0^\infty \frac{\beta(\omega)}{\omega}\, d\omega \tag{47}$$

where

$$\beta(\omega) = \operatorname{Real}\{[2\psi(\omega) - q\varphi(\omega)]\phi_x^*(\sqrt{p}\omega)\}. \tag{48}$$

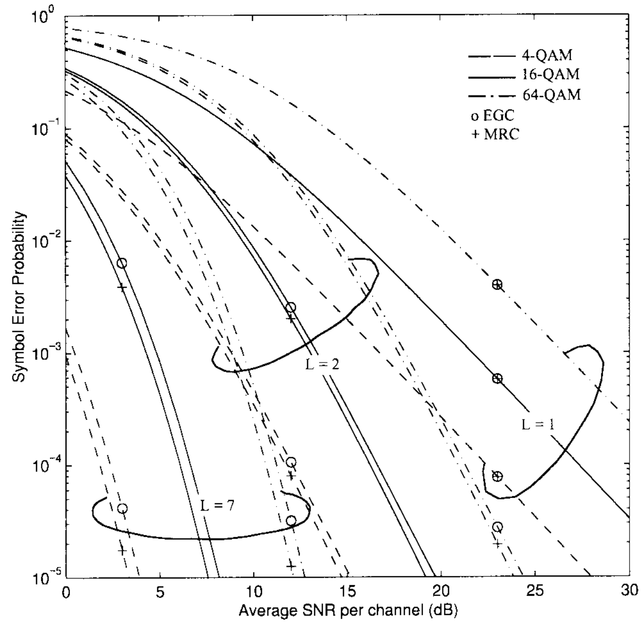

Fig. 1. Symbol-error probability for MQAM with MRC and EGC diversity receivers on Nakagami fading with fading figure $m = 1.8$.

Using variable substitution $\omega = \tan\theta$ in (47), we can express this integral in a more desirable form (i.e., suitable for numerical integration)

$$P_S^{(E)}(\epsilon) = \frac{2q}{\pi} \int_0^{\pi/2} \frac{\beta(\tan\theta)}{\sin(2\theta)}\, d\theta. \tag{49}$$

Note that (47) and (49) are exact analytical solutions for MQAM with EGC diversity. Yet making another variable substitution $\omega^2 + 1/2 = 1/(1+x)$ in (47) and then applying the GCQ formula, we obtain a rapidly converging series representation for the EGC performance on Nakagami fading channel

$$P_s^{(E)}(\epsilon) = \frac{q}{n} \sum_{k=1}^n \beta\left[\frac{1}{\sqrt{2}} \tan(\theta_k)\right] \Big/ \sin(2\theta_k) + R_n \tag{50}$$

where $\theta_k = \pi(2k-1)/4n$. It is also interesting to note that (15) and (50) are in similar forms.

IV. Numerical Results

In this section, we present selected numerical results to show the efficacy of MRC and EGC diversity receivers on a Nakagami fading channel with arbitrary fading parameters. When using the GCQ sum, we have used $n = 64$ and $n = 512$ for the MRC and EGC results, respectively. Note that these numbers were conservatively chosen to be large. In fact, as few as eight samples can be sufficient in some cases. Fig. 1 depicts the SER performance curves of 4-QAM, 16-QAM and 64-QAM with the assumption that all the MRC or EGC space diversity branches undergo identical Nakagami fading with $m = 1.8$. This fading severity index corresponds to a Rician channel with Rice factor $K = 2$. From this figure, it is apparent that diversity reception is an effective technique for combatting the detrimental effects of deep fades experienced in wireless channels. It is also observed that the penalty in SNR to achieve a given SER of MQAM system with a larger

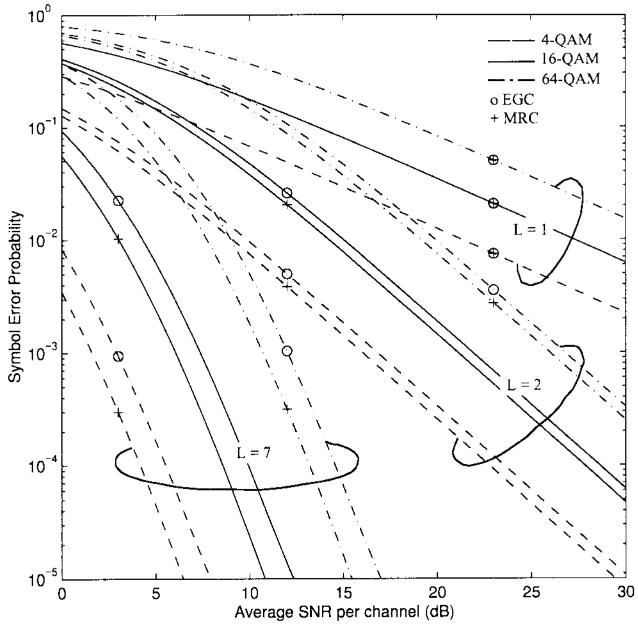

Fig. 2. Symbol-error probability for MQAM with MRC and EGC diversity receivers on Nakagami fading with fading figure $m = 0.75$.

signal constellation size declines more rapidly than that of a smaller signal set, as the diversity order increases. This is true for both MRC and EGC diversity systems. In other words, the diversity improvement is greater as the constellation size M increases. When $L = 1$, the performance curves evaluated using (15) and/or (21) coincide with that evaluated via (47), as anticipated (i.e., corresponds to the nondiversity case). As well, the penalty in SNR for the EGC diversity receiver to achieve the same level of performance with the optimum diversity receiver is quite minimal. For instance, the difference for 4-QAM at $P_s = 10^{-4}$ is only about 0.4 and 0.6 dB for $L = 2$ and $L = 7$, respectively.

In Fig. 2, we plot the SER curves for different QAM systems, with and without diversity reception, in Nakagami fading environment with $m = 0.75$. Comparison between Figs. 1 and 2 reveals that the relative diversity advantage is more pronounced in a poorer channel condition. This is intuitively satisfying, since the difference between the instantaneous received SNR on various diversity branches will be less as m increases. However, the SER performance is always better in a channel where a strong line-of-sight path exists for a specified average received SNR per branch $\bar{\gamma}_c$ and diversity order. We also observe that the discrepancy between the EGC and MRC diversity performance curves gets larger as the fading becomes more severe (i.e., smaller m).

Fig. 3 compares the exact SER with MRC [computed using (14) or (22)] with the approximate SER [which may be calculated via (10) or more efficiently by evaluating only the first term in (21)] for the system parameters considered in Figs. 1 and 2. The percentage of approximation error is defined as $100I_2/(I_1 - I_2)$. Notice that the approximate SER for a dual-diversity 16-QAM is more than 10% higher than the true SER even at $\bar{\gamma}_c = 10$ dB when $m = 0.75$. The discrepancy between the approximate and the exact SER diminishes as the average received SNR per branch increases or for higher order of diversity. On the other hand, their difference becomes more apparent if the channel condition degrades (i.e., smaller m) or for a larger signal constellation size.

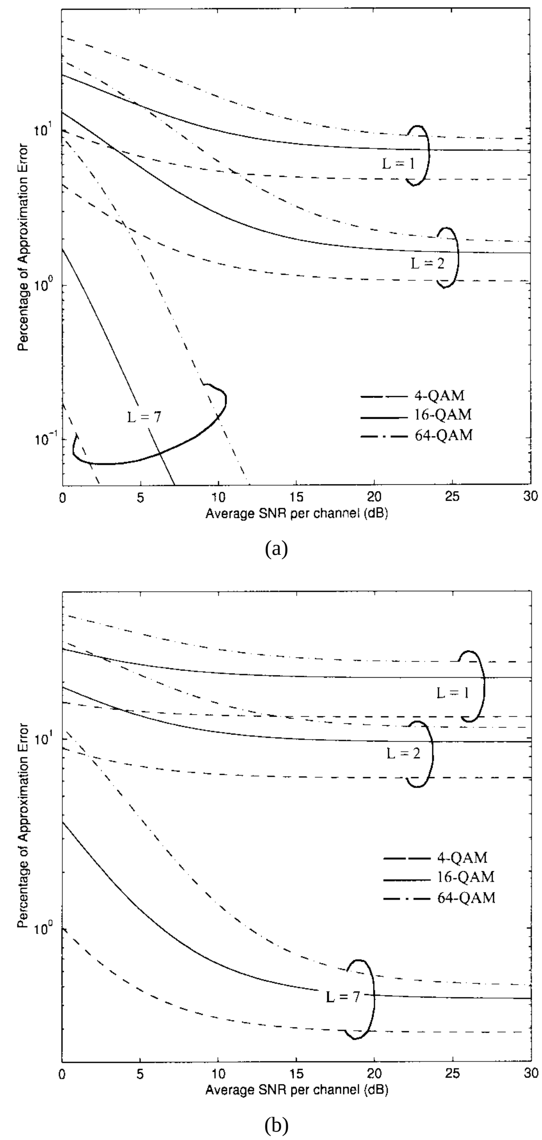

Fig. 3. Comparison between the exact and approximate SER of MQAM with MRC space diversity in different fading environments and for different diversity orders: (a) $m = 1.8$ and (b) $m = 0.75$.

Next in Fig. 4, the SER performance of 64-QAM system is plotted against the order of diversity for several fading severity indexes. All the diversity branches are assumed to have identical fading statistics, and the received SNR per branch is assumed to be $\bar{\gamma}_c = 10$ dB. The larger the number of diversity branches, the smaller the chance of the combined signal going into fade. However, the effective improvement in SNR for a fixed error performance does not improve in proportion to increasing L (see Figs. 1 and 2). The greatest improvement step occurs in going from a single-branch receiver to a two-branch receiver. The results in Fig. 4 indicate that the discrepancy between the error performance of MRC and EGC diversity receivers becomes more apparent as the diversity order grows. This may be attributed to the fact that MRC yields better statistical reduction of deep fades, as well as

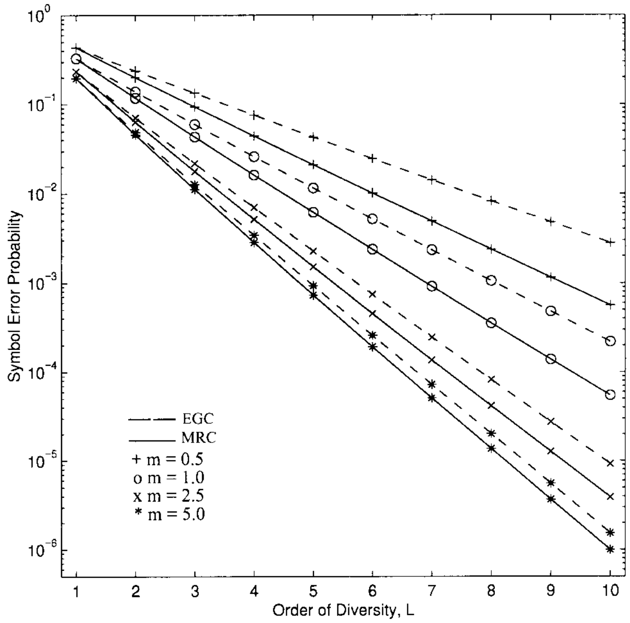

Fig. 4. Symbol-error probability versus order of diversity for 64-QAM with MRC and EGC diversity receivers.

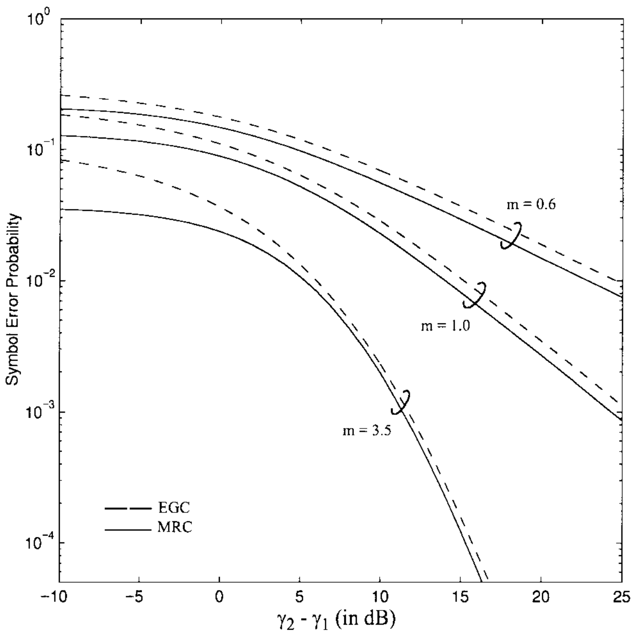

Fig. 5. Effect of unbalance mean signal strength ($\bar{\gamma}_1$ is fixed to 10 dB) on the SER performance of dual-diversity 16-QAM systems in different fading conditions.

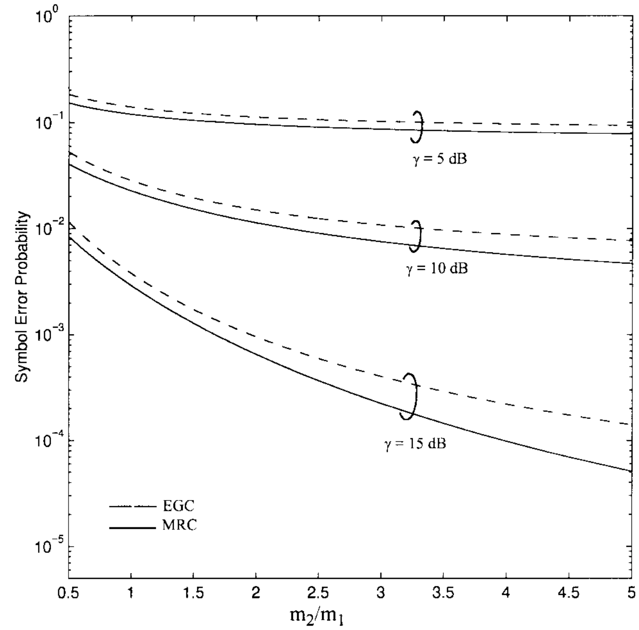

Fig. 6. Sensitivity of SER for 16-MQAM with dual-diversity MRC or EGC diversity receiver on Nakagami fading channels due to dissimilar fading severity index (m_1 is fixed to 1).

provides the higher average output SNR of the combined signal than EGC. Since the deviation between the EGC and MRC curves decline rapidly as m increases, we can conclude that the ability to mitigate the deep fades is the main factor that has contributed the difference in the performance of the two receiver structures.

Figs. 5 and 6 examine the sensitivity of the error probability for 16-QAM system with MRC or EGC diversity receivers in the presence of dissimilar mean signal strength and unequal fading parameters. It is clear that departure of the EGC performance curve from the MRC case is not very significant if the ratio $\bar{\gamma}_2/\bar{\gamma}_1$ is not excessively small and/or if the ratio m_2/m_1 is not too large. From Fig. 5, it can be concluded that MRC makes much more effective use in diversity of relatively weak signals than can the EGC. Besides, equal noise levels in all branches is crucial to proper operation of EGC, since otherwise those branches with large noise levels would dominate the output SNR even if the branch itself were weak in signal level. This, in turn, suggests that a very weak signal should not be combined in the equal-gain diversity-receiver configuration because it may cause a considerable degradation in the mean SNR (due to combination losses). Alternatively, one should equalize the noise levels across the diversity branches by introducing different gains in these branches, prior to the combiner. One way to explain the larger difference between the EGC and MRC performance curves as the ratio m_2/m_1 increases is by noting that fading severity index m has the diversity-like effect. Hence, the ability to mitigate the deep fades and average output SNR of the EGC combiner is inferior to the optimum MRC, specifically when the order of diversity increases (see Fig. 4).

V. Conclusion

Exact symbol-error probability expressions have been derived for coherent MQAM systems employing MRC and EGC antenna diversity in a Nakagami fading environment with an arbitrary fading severity index and/or dissimilar signal strength. The SER formula is exact for square QAM. A tight bound for the rectangular signal constellations was also presented. In particular, the SER formulas based on the GCQ approximation can be easily programmed and evaluated efficiently. Our results are sufficiently general to allow for arbitrary fading parameters, as well as dissimilar mean signal strengths across the diversity branches. The generality and computational efficiency of the new results presented in this paper render themselves as powerful means for both theoretical analysis and practical applications.

APPENDIX A

In this appendix, we present an alternative technique for evaluating the term involving $\mathrm{erfc}^2(\cdot)$ in (3) instead of the two-dimension GCQ method illustrated in [6]. By definition

$$\mathrm{erfc}^2(z) = \frac{4}{\pi}\int_z^\infty \int_z^\infty \exp[-(t^2+s^2)]\,dt\,ds. \tag{A.1}$$

By making variable substitution $t = v\cos\theta$ and $s = v\sin\theta$ in (A.1) and using integration by parts, we can show that the double integral in (A.1) reduces to

$$\begin{aligned}\mathrm{erfc}^2(z) &= \frac{4}{\pi}\int_{\sqrt{2}z}^\infty ve^{-v^2}[\cos^{-1}(z/v) - \sin^{-1}(z/v)]\,dv\\ &= \frac{4}{\pi}\int_1^\infty \frac{\exp-[z^2(1+x^2)]}{1+x^2}\,dx.\end{aligned} \tag{A.2}$$

Then, I_2 can be restated as

$$I_2 = \frac{4q^2}{\pi}\int_1^\infty \frac{\phi[p(1+x^2)]}{1+x^2}\,dx = \frac{2q^2}{\pi}\int_0^1 \frac{\phi[2p/(1-z)]}{\sqrt{1-z^2}}\,dz \tag{A.3}$$

where the second integral in (A.3) is obtained via variable transformation $x^2+1 = 2/(1-z)$. Now by applying the GLQ formula [21, eq. (25.4.36)], we arrive to a simple expression for evaluating (5)

$$I_2 = \frac{2q^2}{\pi}\sum_{i=1}^n w_i\left[\frac{\phi[2p/(1-z_i)]}{\sqrt{1+z_i}}\right] + R_n. \tag{A.4}$$

The abscissas and weights are given by $z_i = 1-\xi_i^2$ and $w_i = 2w_i^{(2n)}$, respectively, where ξ_i is the ith positive zero of Legendre function $P_{2n}(x)$, and $w_i^{(2n)}$ are the Gaussian weights of order $2n$. It is evident that this alternative method requires fewer samples of MGF (i.e., $2n \ll n^2$ samples for practical values of n) compared to the two-dimension GCQ formula. This is because the numerical approximation is performed over a single integral in GLQ instead of the double integral in the latter approach. Yet making another variable substitution $x = \cot\Theta$ in (A.2), we get

$$\mathrm{erfc}^2(z) = \frac{4}{\pi}\int_0^{\pi/4} \exp\left[\frac{-z^2}{\sin^2\Theta}\right]d\Theta \tag{A.5}$$

which is an alternative representation for the $\mathrm{erfc}^2(z)$. It is noted that this new form is essentially the same as the results presented recently by Simon and Divsalar in [11].

APPENDIX B

In this appendix, we present three series that are used in the calculations involving the confluent hypergeometric function of the first kind. The confluent series is defined as

$$\Phi(a,c;x) = \sum_{r=0}^\infty \frac{(a)_r x^r}{(c)_r r!} \tag{B.1}$$

where the polychamer symbol $(b)_r = b(b+1)\cdots(b+r-1)$. Note that if $-a$ is a positive integer, then the series is a finite polynomial of x, i.e.,

$$\Phi(a,c;x) = \sum_{r=0}^{-a} \frac{(a)_r x^r}{(c)_r r!}. \tag{B.2}$$

In the mathematical sense, the series (B.1) converges everywhere (i.e., the radius convergence is infinite). However, for large $|x|$, the series does not converge until $r \gg |x|$ by which time overflow problems may have occurred. Therefore, (B.1) is not computationally useful when $|x|$ is large. Note that when the series (B.1) reduces to (B.2), the convergence problem does not occur.

For EGC performance evaluation, both $\Phi(m+1/2, 3/2; -x^2)$ and $\Phi(m+1/2, 3/2; -x^2)$ are needed, where m can be real or integer. Beaulieu and Abu-Dayya provide a method to compute $\Phi(m+(1/2),(3/2);-x^2)$ for positive integer m [16, Appendix A], and their finite series simply follows from (B.3) and (B.2). They also provide a recursion to compute $\Phi(m, 1/2; -x^2)$ [16, Appendix B], which again holds for positive integer m only. In contrast, the following procedure handles both real and integer m.

We now consider the calculation of $\Phi(a,c;-x)$ for $a, c > 0$, and x is a positive real number. For this case, it is better to apply Kummer's transformation formula

$$\Phi(a,c;-x) = e^{-x}\Phi(c-a,c;x). \tag{B.3}$$

The advantage of this transformation is that if m or $m+1/2$ is an integer, then the series required in (39) is a finite polynomial. Therefore, no convergence problems are encountered. For $x < 200$, we use

$$\Phi(a,c;-x) = e^{-x}\sum_{r=0}^\infty \frac{(c-a)_r x^r}{(c)_r r!} \tag{B.4}$$

which can be computed via standard series evaluation techniques. For $x \geq 200$, we use the divergent series [26, p. 278]

$$\Phi(a,c;-x) = \frac{x^{-a}\Gamma(c)}{\Gamma(c-a)}[1 + O(|x|^{-1})]. \tag{B.5}$$

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers and the Editor for their helpful comments and constructive suggestions.

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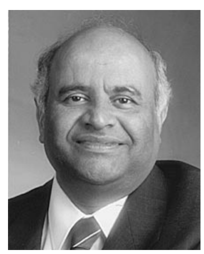
C. Tellambura, for photograph and biography, see p. 238 of the February 1999 issue of this TRANSACTIONS.



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