Exact Graph Search Algorithms for Generalized Traveling Salesman Path Problems

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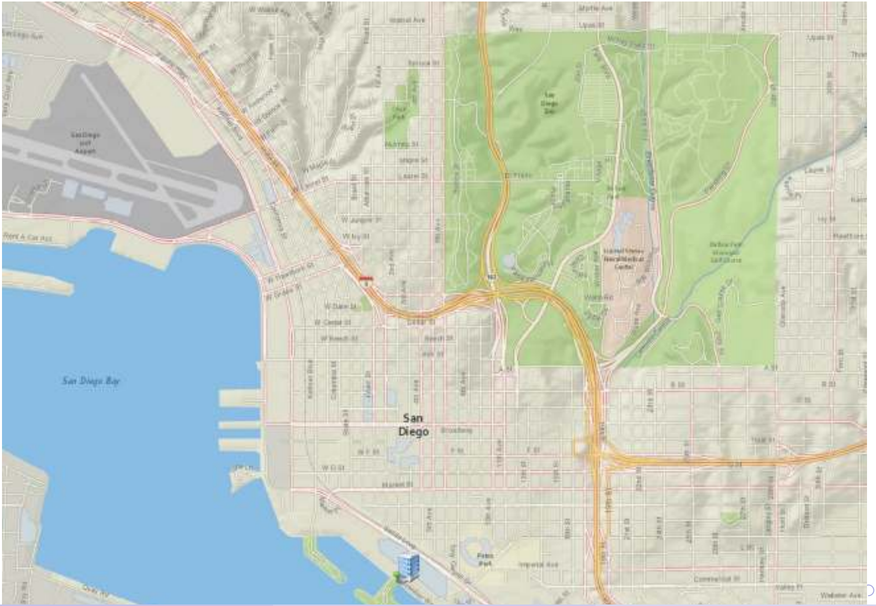
11th International Symposium on Experimental Algorithms

Motivation



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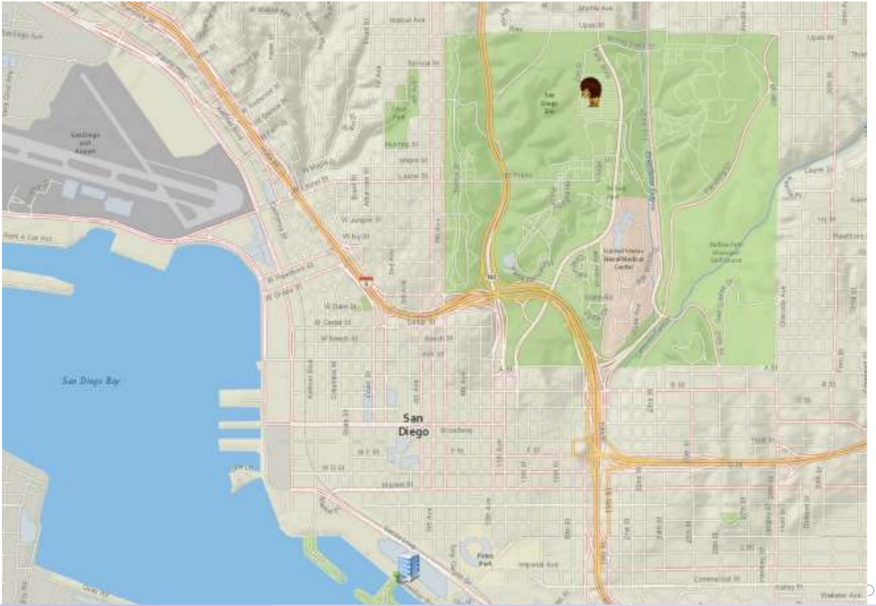

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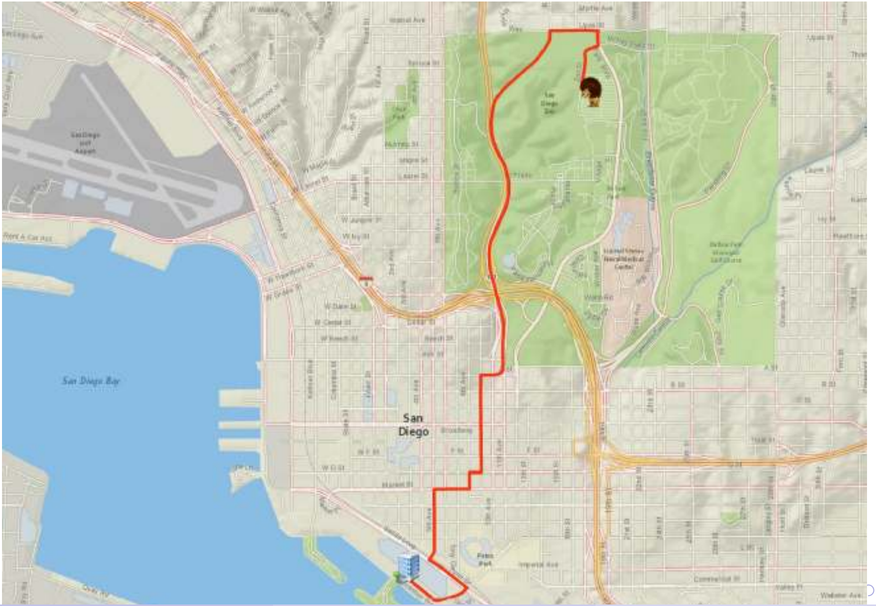

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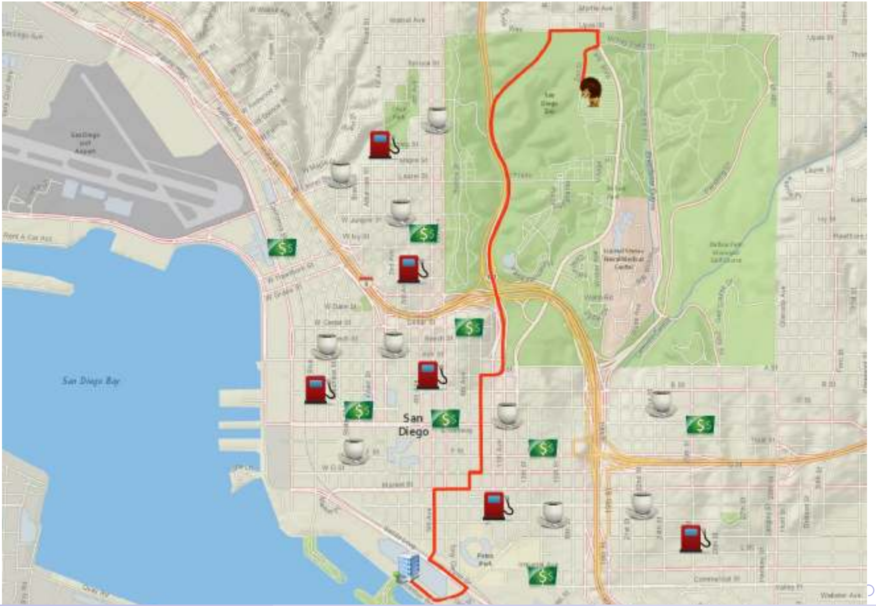

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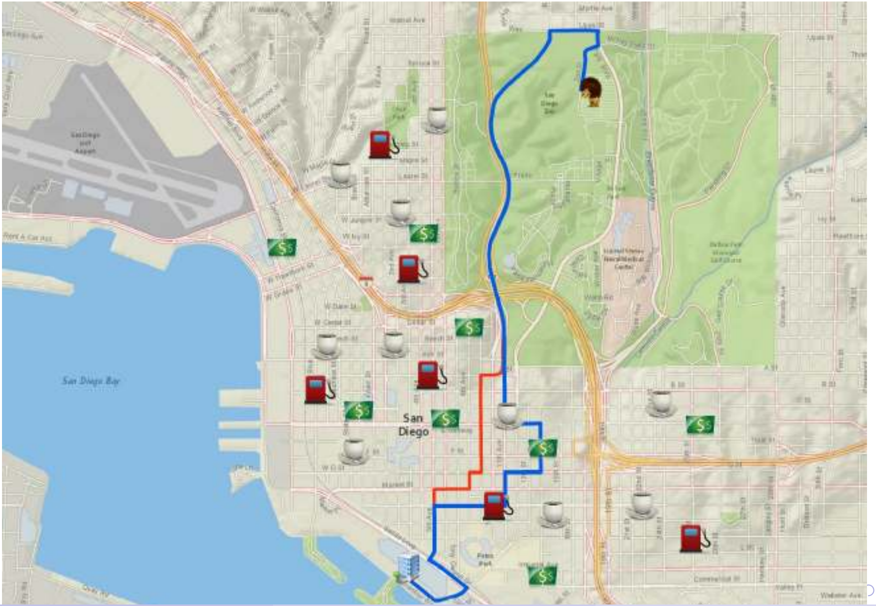

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A path, P, **satisfies** a category set C if, for $1 \leq i \leq k$, $P \cap C_i \neq \emptyset$.

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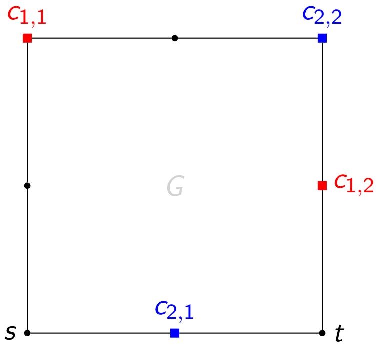
Generalized Traveling Salesman Path Problem (GTSPP)

- Instance: $\langle s, t, C \rangle$, for $s, t \in V$ and category set C
- Solution: Minimum-weight satisfying path from s to t

GTSPP Example



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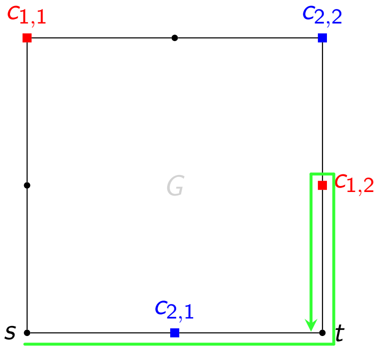

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- Solution: $P_{s,t}$ is optimal

Background and Related Work

- GTSP introduced in 1960s
- NP-hard generalization of the classical TSP
- Goes by many names...
  - Errand Scheduling
  - Group TSP
  - Set TSP
  - One-of-a-Set TSP
  - Multiple-Choice TSP
  - TSP with Neighborhoods
  - ...
- Many exact, approximate, and heuristic approaches exist, **but**. . .

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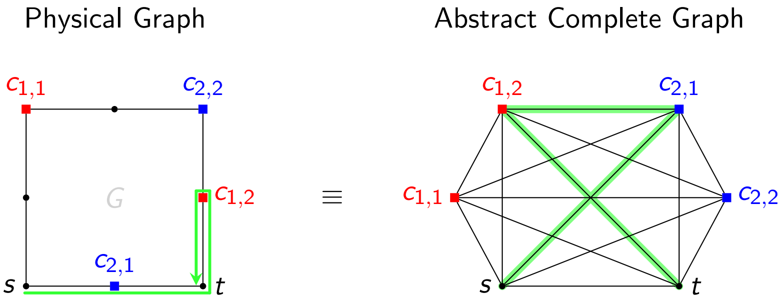

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Canonical Example

- $k = 5$, $g = 10,000$
- Constructing complete graph would require ≈ 1 minute preparation
- We solve it optimally in < 2 seconds!

Product Graph Framework

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Covering Graph

- For $C = \{C_1, C_2, \ldots, C_k\}$, let $G(\mathcal{B}_k) = (\mathcal{P}(C), E(\mathcal{B}_k))$
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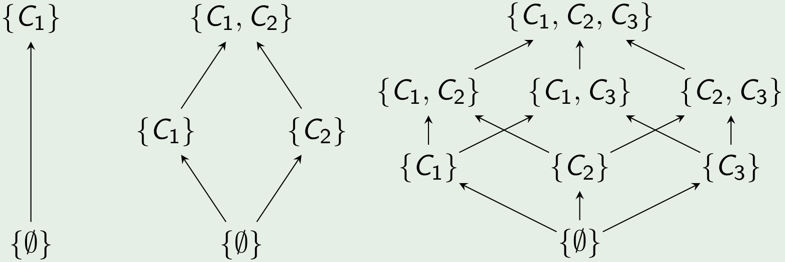

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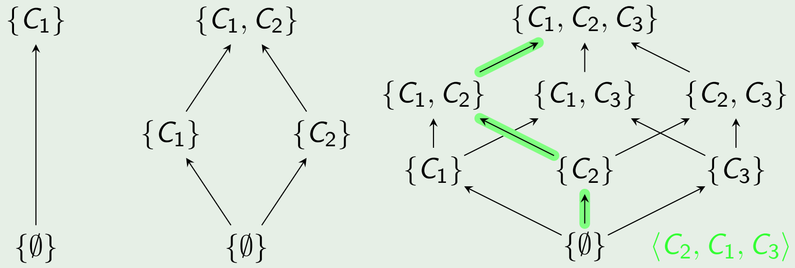

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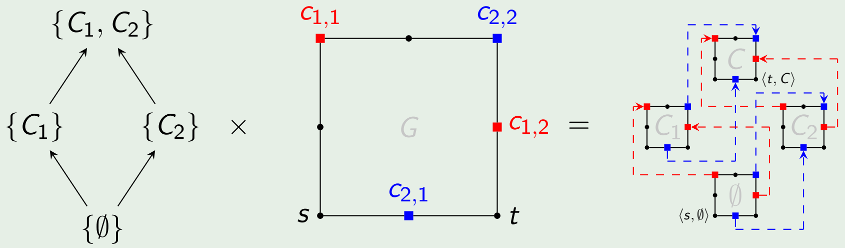

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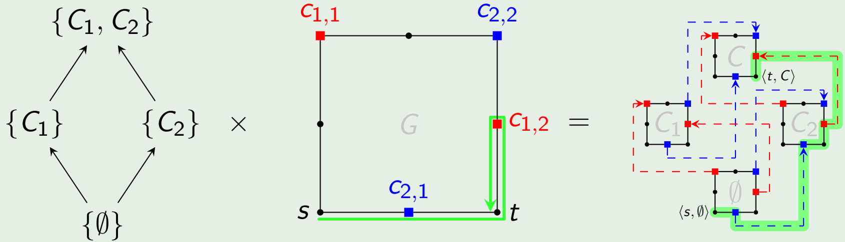

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Optimization

- Do not explicitly construct the product graph
- Materialize the graph implicitly as needed

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- We take advantage of two key aspects:
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- Extend product graph search to incorporate the state-of-the-art Contraction Hierarchies technique

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CH Preprocessing

Establish strict total ordering of nodes (i.e., the "hierarchy"), and "contract" nodes in this order.

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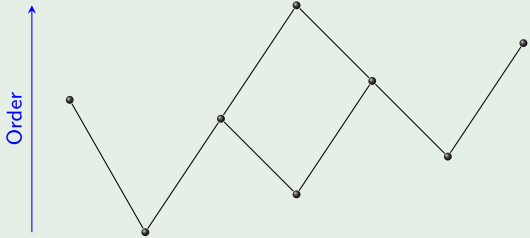

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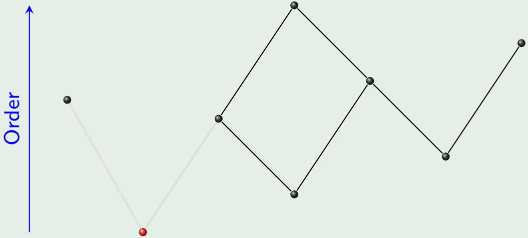

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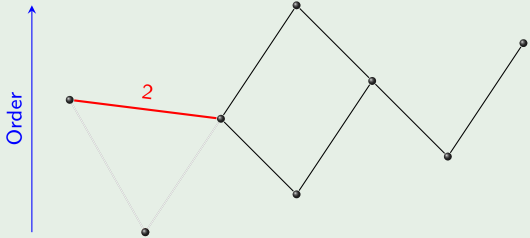

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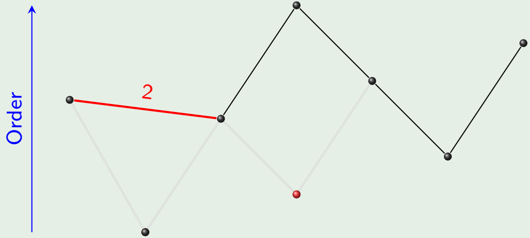

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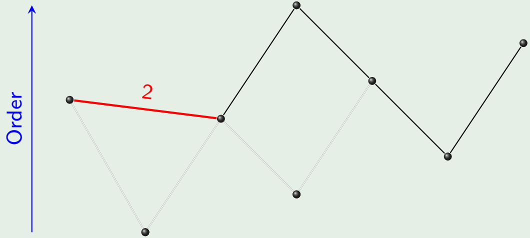

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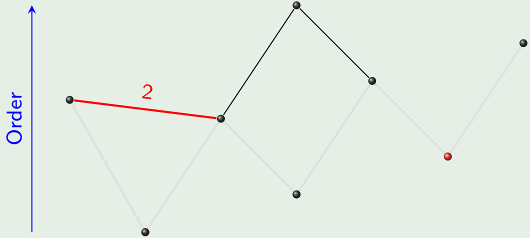

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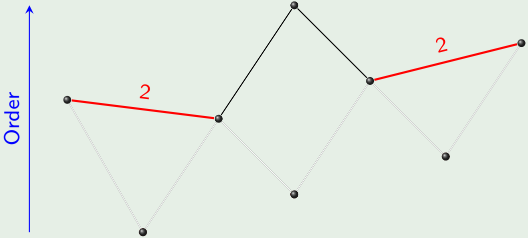

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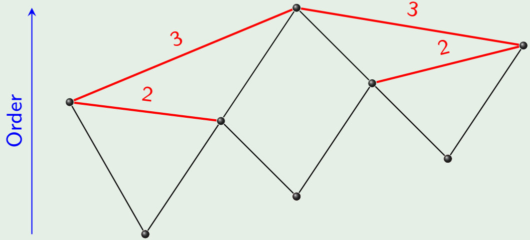

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Bidirectional-Dijkstra search (forward from s, backward from t), relaxing only "upward-leading" edges.

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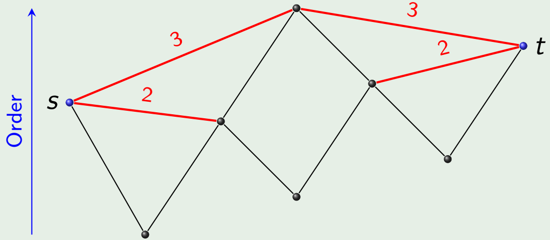

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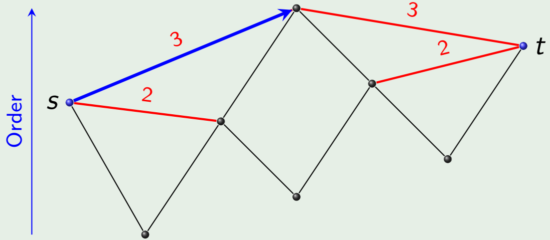

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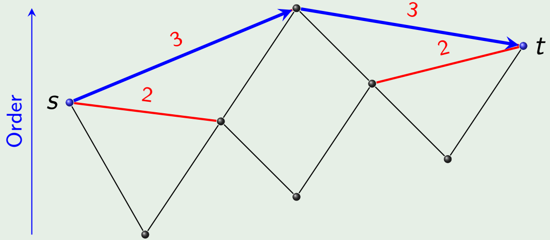

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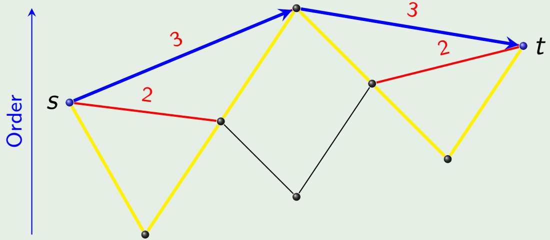

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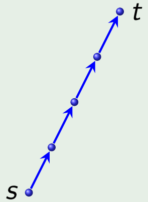

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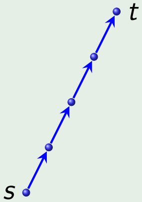

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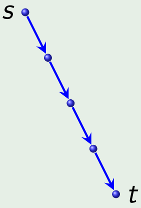

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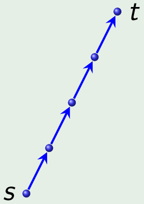

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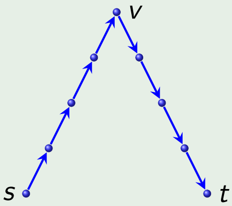

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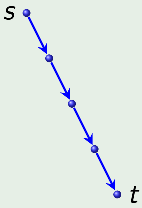

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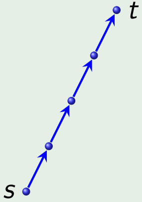

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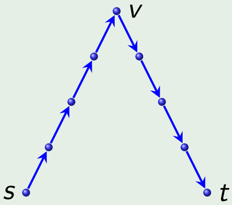

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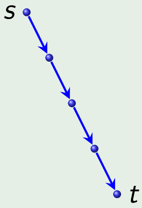

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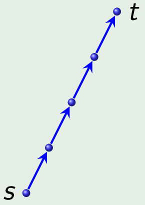

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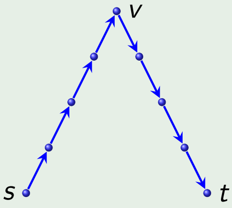

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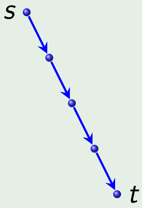

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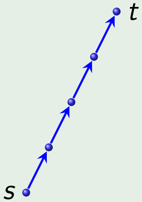

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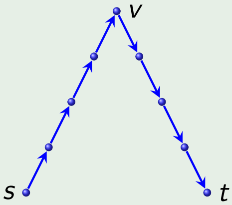

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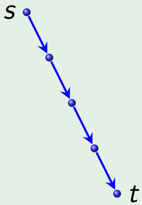

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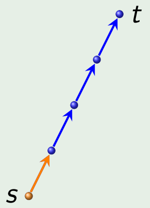

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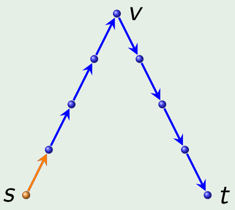

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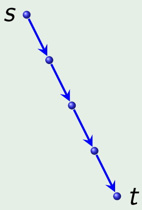

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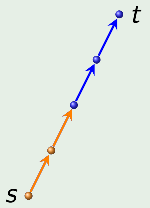
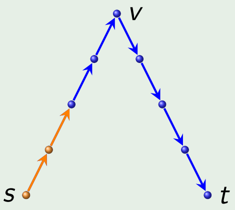


Path Type #2: Bitonic Rank



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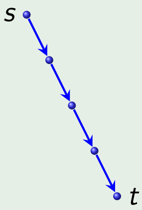

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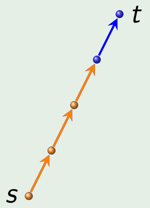

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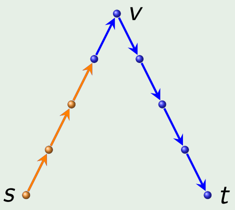

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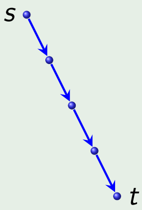

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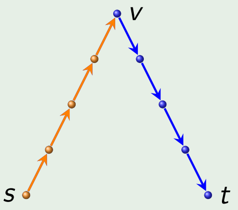

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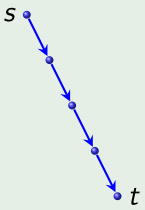

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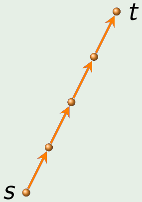

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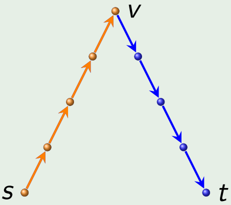

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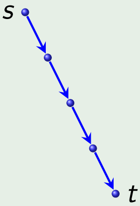

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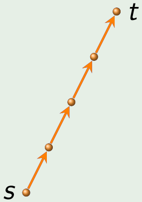

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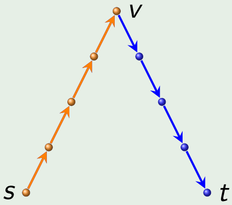

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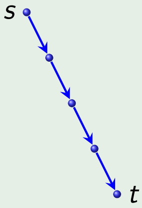

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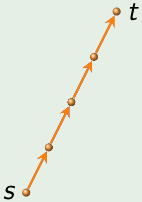

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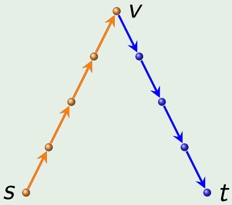

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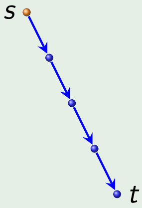

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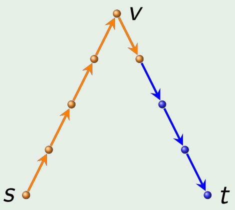

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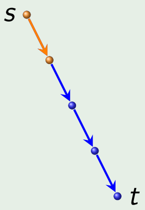

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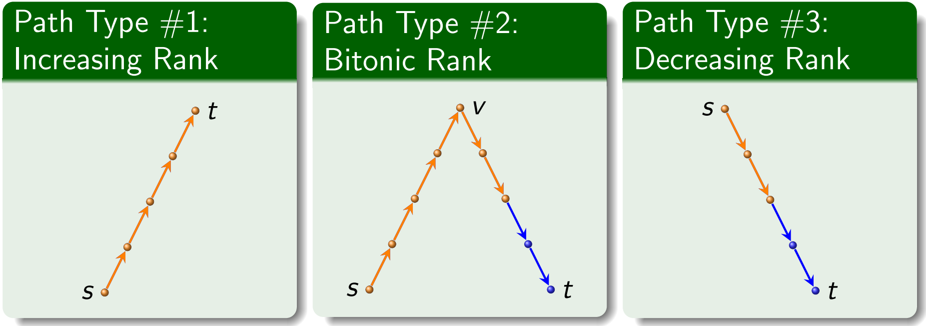

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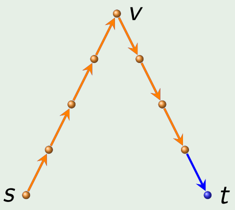

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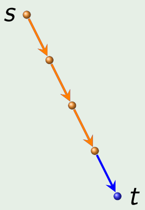

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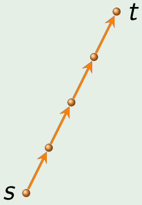

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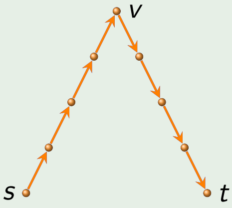

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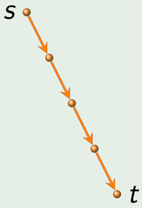

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Many (S)ources/(T)argets

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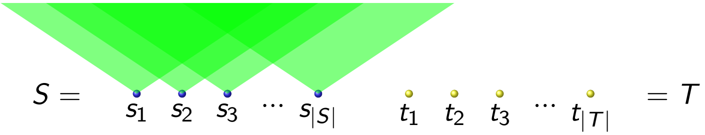

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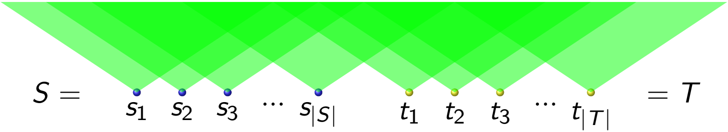

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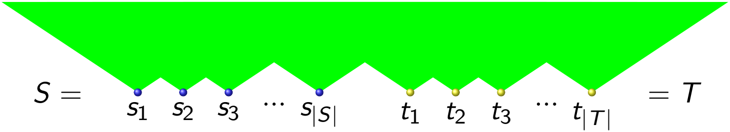

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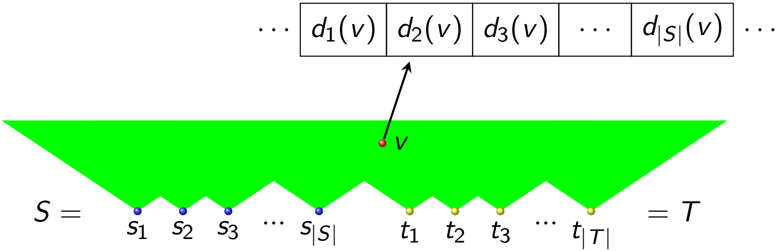
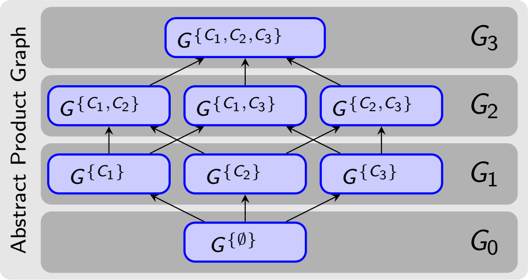


Utilizing Structural Properties of the Product Graph



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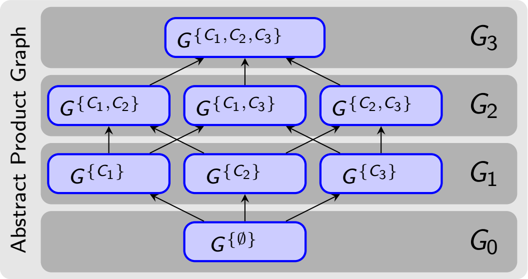

Level-Sweeping Search (LESS) Algorithm

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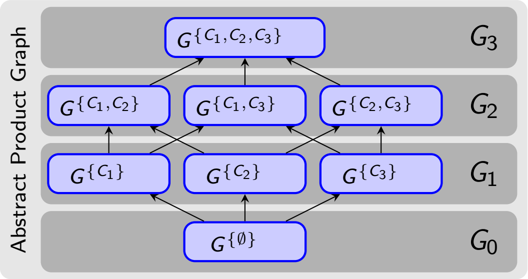

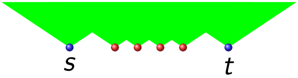

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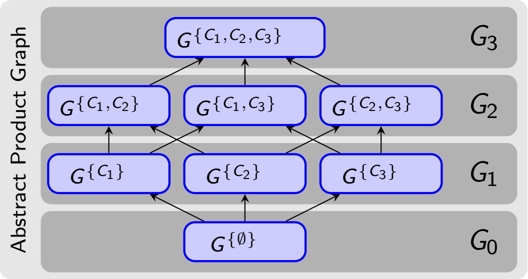

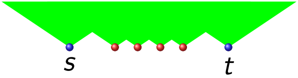

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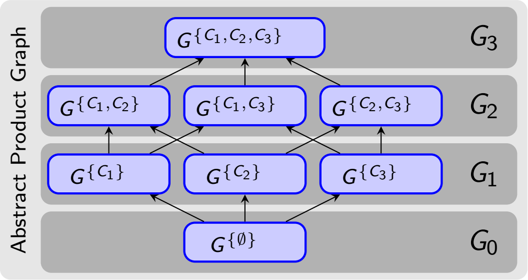

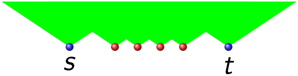

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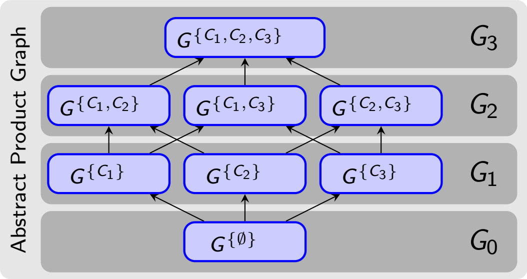

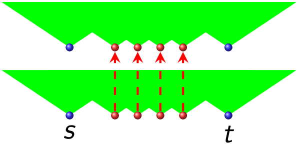

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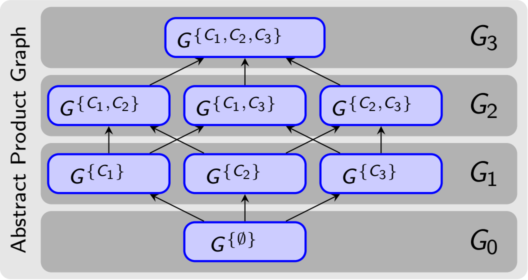

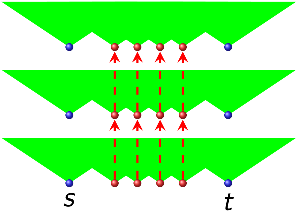

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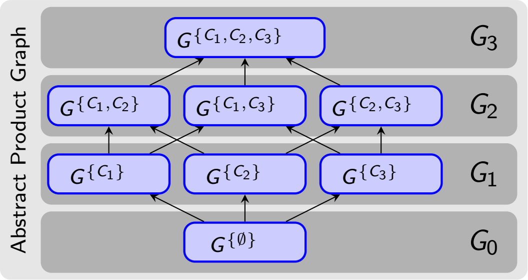

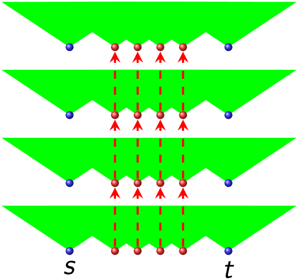

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    1. Prune $c_{i,j}$ if $\mu < h(s, c_{i,j}) + h(c_{i,j}, t)$

Pruning

Theorem

LESS runs in $O(2^k(m' + nk))$ time, where $m' = |E \cup E'|$.

- In practice, its runtime is proportional to the size of the unioned search space (influenced by g)
- **Question:** can we reduce the size of the search space?
- If we can identify suboptimal category nodes, we can remove them

Pruning (requires an admissible heuristic function $h : V \times V \to \mathbb{R}_{\geq 0}$)

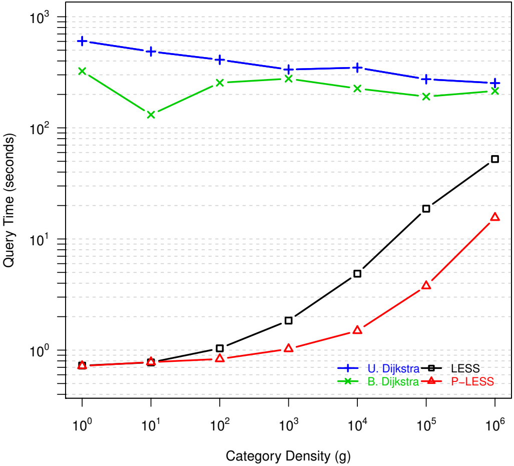
1. Establish upper bound on optimal solution:
   1. Construct satisfying path P' via greedy, nearest-neighbor strategy
   2. $\mu = w(P')$
2. For $1 \leq i \leq k$, for all $c_{i,j} \in C_i$:
   1. Prune $c_{i,j}$ if $\mu < h(s, c_{i,j}) + h(c_{i,j}, t)$

- After pruning, carry out LESS search, as before

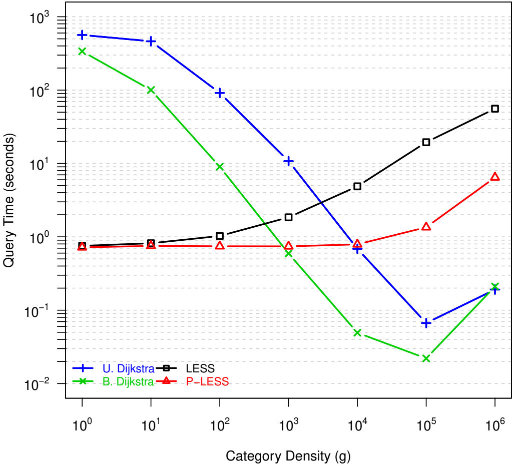
Experiments

- Dataset:
  - Road network of US/Canada with $|V| = 21M$ and $|E| = 52M$
- Environment:
  - Server: 2.53GHz CPU, 18GB RAM
  - Language: C++
- Preprocessing:
  - CH: 18 minutes preprocessing time
  - Pre-Computed Cluster Distances (PCD): 7 minutes (using CH)
- Algorithms:
  - Unidirectional Dijkstra (U. Dijkstra)
  - Bidirectional Dijkstra (B. Dijkstra)
  - Level-Sweeping Search (LESS)
  - LESS + Pruning (P-LESS)
- Queries:
  - Non-Local Queries: cases where $s \neq t$
  - Local Queries: cases where $s = t$

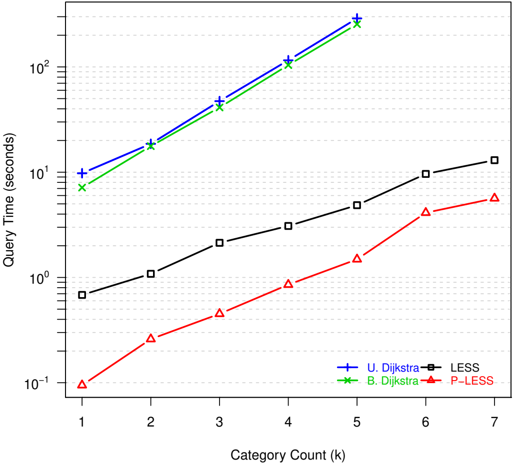
Category Density Experiments: Non-Local Queries ($s \neq t, k = 5$)



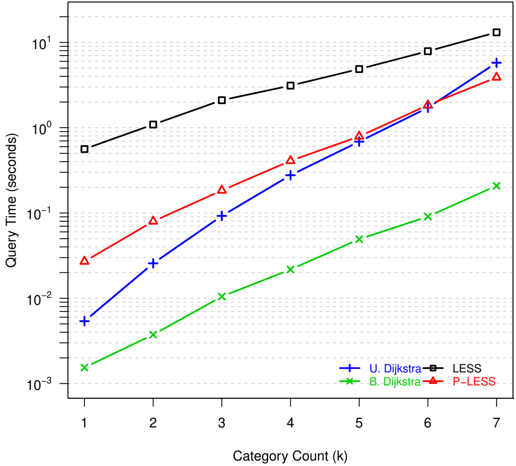
Category Density Experiments: Local Queries ($s = t, k = 5$)



Category Count Experiments: Non-Local Queries ($s \neq t, g = 10,000$)



Category Count Experiments: Local Queries ($s = t, g = 10,000$)



Summary

- New product graph framework for efficient graph search
- Can solve real-world GTSPP instances to optimality in seconds!
- Two competitive algorithms with performance tradeoffs:
  - Dijkstra: good for highly-local, very-dense queries (no pre-processing required)
  - LESS (with pruning): more consistent performance across various sizes and localities

Future Work

- Better space utilization (e.g., reduced memory overhead, better cache locality)
- More aggressive pruning strategies
- Incorporate goal-direction (e.g., A^*)
- Parallelization (exploiting subgraph independence)
- Approximation algorithms

Questions?