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EXACT HEDONIC PRICE INDEXES

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ABSTRACT

The purpose of this paper is to identify conditions under which hedonic price indexes provide an exact measure of consumer welfare. Our results provide a rationale for existing practices in the case where prices equal marginal costs. In that case, both the marginal value of characteristics and a fixed-weight price index can be estimated from a hedonic regression. Using the marginal value of characteristics, we show how to construct bounds on the exact hedonic price index. When prices are above marginal costs then our bounds still apply, but the value of characteristics cannot be measured so easily. Since the price-cost markups are an omitted variable in the hedonic regression, they will bias the coefficients obtained. For a special class of utility functions, we argue that a linear regression will still provide a measure of the marginal value of characteristics, but a log-linear regression will *overstate* these values.

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1. INTRODUCTION

While hedonic regressions, and associated hedonic price indexes, are used in many applications,[1] the theory underlying these methods is not well developed. The model of Rosen (1974) is often used to theoretically justify the relation between market prices and the characteristics of goods, though it is not obvious what this market relation implies about the welfare from purchasing goods embodying the characteristics. Thus, Triplett (1990) refers to the existing practice of hedonic price indexes as an "approximation" to the true measure of consumer welfare. A similar view is expressed by Griliches (1990, p. 189):

> What is being estimated [by the hedonic regression] is actually the locus of intersections of the demand curves of different consumers with varying tastes and the supply functions of different firms with possibly varying technologies of production. One is unlikely, therefore, to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

The interpretation of hedonic methods is even more difficult in the (realistic) situation where there are only a discrete number of goods, so that consumers are not optimizing in a marginal fashion over their choice of characteristics. In that case, firms will be selecting the characteristics bundle of each good.

The purpose of this paper is to identify the "special circumstances" under which hedonic price indexes provide an exact measure of consumer welfare, so that the welfare effects of quality change can be inferred. We shall focus on the case where consumers choose over a discrete number of products, whose characteristics are optimally selected by firms.[2] We begin in section 2 by specifying the utility function of individuals, which are aggregated to obtain a social welfare function. We review certain results of Anderson, de Palma and Thisse (1992), who model individuals as

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[1] The earliest applications are discussed in Griliches (1990), and a comprehensive application to various durable goods is Gordon (1990). Hedonic methods have been officially adopted in the U.S. to measure the change in computer prices (Cole *etal*, 1986, Cartwright, 1986).

[2] The case where consumers choose characteristics directly is considered in Feenstra (1993).

making discrete choices over products, subject to an additive random term in utility. They identify several cases where there exists an aggregate utility function - also interpreted as a utilitarian social welfare function - such that a representative consumer with this utility function would have the same demands as the aggregate of the individuals. Using a theorem due to McFadden (1978, 1983), we shall introduce a broad family of aggregate, indirect utility functions satisfying this criterion. This family includes the indirect utility functions considered by Spence (1975), where product price and quality do not necessarily enter as a ratio.

Firms choose the price and characteristics for each product to maximize profits, as described in section 3. We derive a simple equilibrium condition linking the marginal cost and marginal value of each characteristic. If competition forces firms to price at the marginal cost of production, then the marginal cost and value of characteristics are equalized, but more generally, they will differ by an amount that depends on the price-cost markups and on the precise form of the utility function. A comparison of the Nash equilibrium and social optimum is provided.

In section 4 we consider the construction of exact hedonic price indexes, that accurately reflect the cost-of-living for the aggregate consumer. We first consider several cases where the aggregate utility, or expenditure, function allows an exact index to be constructed if product characteristics are held constant. When these characteristics change, we show that the marginal values can be used to construct bounds on the exact hedonic index. This result is a natural analog to the usual construction of an exact index over prices of goods, and justifies what Triplett (1986, p. 39) refers to as an "explicit quality adjustment" in the price index. We demonstrate that a similar result holds for any aggregate expenditure function satisfying certain concavity conditions.

Hedonic regressions are considered in section 5. When price equals marginal cost for all products, then both the marginal value of characteristics and a fixed-weight price index can be estimated from a hedonic regression. When prices are above marginal costs, however, then the value of characteristics cannot be measured so easily.

Since the price-cost markups are an omitted variable in the hedonic regression, they will bias the coefficients obtained. For a special class of utility functions, we argue that a linear regression will still provide a measure of the marginal value of characteristics, but a log-linear regression will *overstate* these values. Conclusions are given in section 6, where we discuss the practice of imputing "missing" prices from the hedonic regression, and the proofs of Propositions are gathered in the Appendix.

2. CONSUMER CHOICE AND SOCIAL WELFARE

In the economic approach to index numbers, a price index is defined as *exact* if it equals the cost of obtaining a given level of utility for a representative consumer (Diewert, 1976). However, in models where goods are differentiated by their characteristics, we typically think of individuals as being heterogeneous in their preferences. In order to apply the concept of an exact price index, we first need to determine whether the individual behavior can be aggregated to some representative consumer. We will suppose that there are M consumers, choosing over a fixed number of product varieties $i=1,\ldots,N$, in addition to a numeraire good. The prices of the products in terms of the numeraire are p_i, and the income of each consumer is $y \geq p_i$, $i=1,\ldots,N$. Each product is described in terms of K observable characteristics $z_i \varepsilon R_+^K$, and we will sometimes let $a_i=g_i(z_i)$ denote a scalar measure of product quality, with $\partial g_i/\partial z_i \geq 0$.

2.1 Linear Random Utility Model

In the first case considered by Anderson, de Palma and Thisse (1992, chap. 3), the utility obtained by an individual consuming one unit of product i, while spending the remaining income on the numeraire good, is:

$$V_i = y - p_i + a_i + \varepsilon_i, \qquad i=1,\ldots,N, \tag{1}$$

where product quality $a_i=g_i(z_i)$ is identical across consumers, and ε_i is a random variable that reflects unmeasured features of the products or the utility function. Because the random term, price and income all enter (1) linearly, this is referred to as the linear random utility model (LRUM). The consumers are identical but statistically independent, so that each receives a different draw of $\varepsilon=(\varepsilon_1,\ldots,\varepsilon_N)$.[3] These random terms are distributed across product varieties according to the joint distribution function $F(\varepsilon)$, and each consumer selects the product with the highest value of $(a_i - p_i + \varepsilon_i)$. Letting P_i denote the probability that any consumer selects variety i, expected demand is simply $X_i(p,Y)=MP_i$, where $p=(p_1,\ldots,p_N)$ is the price vector and $Y=My$ denotes total income.

Anderson, de Palma and Thisse establish a general correspondence between the distribution function $F(\varepsilon)$ and the utilitarian social welfare function. As a leading example, suppose that ε_i are independent and identically distributed (i.i.d.) over product varieties according to the double-exponential distribution:

$$F_i(\varepsilon_i) = \exp\left\{-\exp\left[-\left(\frac{\varepsilon_i}{\mu} + \gamma\right)\right]\right\}, \qquad i=1,\ldots,N, \tag{2}$$

where γ is chosen so that the mean of ε_i is zero ($\gamma \approx 0.5772$), while its standard deviation is proportional to $\mu > 0$. Then it is shown that the expected demand functions take on the multinomial logit (MNL) form,

$$X_i(p,Y) = M\left\{\frac{\exp[(a_i-p_i)/\mu]}{\sum_{j=1}^{N}\exp[(a_j-p_j)/\mu]}\right\}. \tag{3}$$

In addition, the sum of the expected, maximized utilities over individuals is,

$$V = Y + M\mu \ln\left[\sum_{i=1}^{N}\exp\left(\frac{a_i - p_i}{\mu}\right)\right]. \tag{4}$$

The demand functions in (3) can be obtained by applying Roy's Identity to the

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3 Another interpretation of the model to to suppose that there are a continuum of consumers, each with a different value of ε, that are distributed according to $MF(\varepsilon)$. This interpretation is discussed more fully by Anderson, de Palma and Thisse (1992, chap. 3).

utilitarian social welfare function in (4), so that (4) represents the indirect utility for a representative consumer choosing the quantities X_i subject to the budget constraint $X_0 + \sum_{i=1}^{N} p_i X_i = Y$, where X_0 is the aggregate consumption of the numeraire good. For later reference, we note that the direct utility function corresponding to (4) is given by [Anderson, de Palma and Thisse (192, Prop. 3.7)]:

$$U(X_0, X, a) = \begin{cases} X_0 + \sum_{i=1}^{N} a_i X_i - \mu \sum_{i=1}^{N} X_i \ln(X_i/M) & \text{if } \sum_{i=1}^{N} X_i = M \\ -\infty \quad \text{otherwise,} & \end{cases} \tag{5}$$

where $X=(X_1,\ldots,X_N)$. The condition $\sum_{i=1}^{N} X_i = M$ is required because each individual consumes just one unit, so that M units are demanded in total. Any other demands from the representative consumer are inconsistent with individual optimization, so aggregate utility is set at $-\infty$.

2.2 CES Model

The second functional form considered by Anderson, de Palma and Thisse arises when individuals can choose a variable number of units x_i of product i. Conditional on choosing this product, individual utility is specified as:

$$U_i = \alpha \ln x_0 + \ln(x_i a_i) + \varepsilon_i, \qquad i=1,\ldots,N, \tag{6}$$

where $a_i = g_i(z_i) > 0$ again represents the scalar quality of each product, and x_0 is the consumption of the numeraire commodity.[4] For each i, consumers maximize (6) subject to the budget constraint $x_0 + p_i x_i = y$. This implies they will spend $\alpha/(1+\alpha)$ of their income on the numeraire good, and the remaining income on product i, which yields the indirect utility:

$$V_i = (1+\alpha)\ln y - \ln(p_i/a_i) + [\alpha \ln\alpha - (1+\alpha)\ln(1+\alpha)] + \varepsilon_i, \qquad i=1,\ldots,N. \tag{7}$$

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[4] Anderson, de Palma and Thisse do not introduce a_i into the individual utility function (6), but adding this term is a straightforward extension of their analysis.

Consumers then choose the product variety that gives the highest utility, which depends on the values of ε_i. In this case, expected demand for each product over all consumers is given by $X_i = Mx_iP_i$. Assuming that ε_i are i.i.d. according to the double-exponential (2), then expected demand is:

$$X_i = M\left(\frac{\tilde{y}}{p_i}\right)\left\{\frac{(p_i/a_i)^{-1/\mu}}{\sum_{j=1}^{N}(p_j/a_j)^{-1/\mu}}\right\} . \tag{8}$$

where $\tilde{y} = y/(1+\alpha)$ is the income not spent on the numeraire good. Thus, the demand functions have a constant elasticity of substitution (CES) form. These demand functions can be obtained by applying Roy's Identity to a representative consumer's indirect utility function:

$$V = Y^{1+\alpha}\left[\sum_{i=1}^{N}(p_i/a_i)^{-1/\mu}\right]^{\mu} . \tag{9}$$

Furthermore, it is shown that (9) is a monotonic transformation of the utilitarian social welfare function, that equals the sum over individuals of the expected value of maximized utility in (7).

2.3 General Utility Function

In this second case, the utility function in (6) take on the special form considered by Swan (1970), where the quantity of each good is multiplied by the quality a_i. Swan interprets a_i as a measure of durability, while Fisher and Shell (1972) call this a "repackaging" model of quality. In an intertemporal framework, Swan establishes the social optimality of the monopoly (or competitive) choice of durability. However, Spence (1975) showed that this result does not carry over to more general utility functions in which quantity and quality do not enter in a multiplicative form, or indirect utility functions where price and quality do not enter as a ratio. In this spirit, we will suppose in general that the utility obtained from the consumption of product i can be written in indirect form as:

$$V_i = \ln\phi_0(y) - \ln\phi_i(p_i, z_i) + \varepsilon_i, \qquad i=1,\dots,N, \tag{10}$$

where $z_i \in R_+^K$ denotes a vector of characteristics, and we assume that $\phi_0' > 0$ and $\partial\phi_i/\partial p_i > 0$, $\partial\phi_i/\partial z_{ik} \leq 0$. We can interpret $\phi_i(p_i, z_i)$ as a "quality-adjusted" price, by analogy from (1) and (7), and invert $q_i = \phi_i(p_i, z_i)$ to obtain $p_i = \pi_i(q_i, z_i)$. Then the *marginal value of characteristic k in good i* is defined by $\partial\pi_i(q_i, z_i)/\partial z_{ik}$, which is the increase in the price p_i that any consumer would be willing to pay for an increase in characteristic z_{ik}, keeping the quality-adjusted price q_i (and therefore utility V_i) constant.

For this specification of individual utility, the probability that product i is chosen is:

$$P_i = \text{Prob}\left[V_i = \max_{j=1,\dots,N} V_j \right], \tag{11}$$

where this probability is computed relative to the distribution $F(\varepsilon)$. Conditional on choosing product i, the quantity consumed by each consumer is computed from (10) using Roy's Identity as:

$$x_i = -\left(\frac{\partial V_i/\partial p_i}{\partial V_i/\partial y}\right) = \frac{\phi_0}{\phi_0'}\left(\frac{\partial \ln\phi_i}{\partial p_i}\right). \tag{12}$$

Then the expected demand for each product is given by $X_i = Mx_iP_i$.

The question we want to address is whether the demands X_i are consistent with the maximization of utility for some representative consumer, in which case X_i could be computed from the aggregate indirect utility function using Roy's Identity. If the utility function of the representative consumer can also be interpreted as a social welfare function, then it can be used to define an exact price index. In the next result, we draw on a theorem due to McFadden (1978, p. 80; 1983, pp. 227-228) to show that such an aggregate utility function exists for a broad family of distribution functions $F(\varepsilon)$ known as the *generalized extreme value:*

Proposition 1 (McFadden 1978, 1983)

Suppose that individual utility is specified by (10), and let G be a nonnegative function defined over R_{+}^{N} that satisfies the following properties:

(i) G is homogeneous of degree one;

(ii) $G \to \infty$ as any of its arguments approach infinity;

(iii) The partial derivatives of G with respect to n distinct variables exist and are continuous, nonnegative if n is odd, and positive if n is even, $n=1,\ldots,N$.

Assume that the distribution function $F(\varepsilon) \equiv \exp\left[-G\left(e^{-\varepsilon_1},\ldots,e^{-\varepsilon_N}\right)\right]$ has finite first moments, and define an aggregate indirect utility function by:

$$V\left[\phi_1(p_1,z_1),\ldots,\phi_N(p_N,z_N),Y\right] \equiv M\ln\phi_0(Y/M) + M\ln G\left[\phi_1(p_1,z_1)^{-1},\ldots,\phi_N(p_N,z_N)^{-1}\right]. \quad (13)$$

Then:

(a) Expected demand $X_i = Mx_iP_i$ computed from (11) and (12) equals $-\left(\frac{\partial V/\partial p_i}{\partial V/\partial Y}\right)$;

(b) V is a utilitarian social welfare function for the individual utilities in (10);

(c) V is convex in $(p_1,\ldots,p_N)$ provided that $\ln\phi_i(p_i,z_i)$ is concave in p_i, $i=1,\ldots,N$.

This result indicates there is a broad class of aggregate utility functions V that are consistent with individual utility maximization, in the sense that demand from the representative consumer equals total expected demands from individuals, as indicated in part (a). Applying the transformation $\tilde{V} \equiv \exp(V/M)$, the aggregate indirect utility function can be written as,

$$\tilde{V} = \phi_0(Y/M)\, G\left[\phi_1(p_1,z_1)^{-1},\ldots,\phi_N(p_N,z_N)^{-1}\right], \quad (13')$$

where G is homogeneous of degree one in its arguments, from property (i). Property (ii) means that as $\phi_i \to 0$ then $G \to \infty$. Noting that the utility of any individual in (10) is unbounded above as $\phi_i \to 0$, then property (ii) ensures that when all individuals have utility approaching infinity, so does the representative consumer. Property (iii) is a technical condition needed to ensure that the density function corresponding to the cumulative distribution F is nonnegative.

Examples of part (b) were provided by the MNL and CES cases discussed above, where the errors were i.i.d. according to the double-exponential (2). In that case the function G takes the form $G\left(e^{-\tilde{\varepsilon}_1},\ldots,e^{-\tilde{\varepsilon}_N}\right) = \sum_{i=1}^{N} e^{-\tilde{\varepsilon}_i}$, where $\tilde{\varepsilon}_i \equiv \varepsilon_i/\mu$ in (2) is the standardized double-exponential random variable. In contrast, Proposition 1 allows for a general pattern of correlation in the random terms across model. This eliminates the "red bus/blue bus" problem that plagues the conventional MNL model. Part (c) confirms that the aggregate utility function in (13) is well-behaved in prices.[5]

This key feature of the aggregate utility function (13) is the *weak separability* of (p_i, z_i) across product varieties. This follows directly from the specification of individual utility in (10), where the same function $\phi_i(p_i, z_i)$ applies to all individuals: the heterogeneity across consumers occurs only in the additive random term in utility. While this additive form has been used extensively, it should be noted that some of the most recent applications of discrete choice models have gone beyond it. For example, Berry, Levinsohn and Pakes (1995) allow for interactions between the random term in utility and product characteristics, while Goldberg (1995) allows for interactions between individual attributes (such as income) and characteristics. Both these recent applications therefore lie outside the scope of Proposition 1. However, McFadden (1983, pp. 206-216) demonstrates that even when individual preferences depend on individual attributes (such as historical income), then aggregate demands can still be obtained from the utilitarian social welfare function using Roy's Identity. Thus, it is possible that the analysis in this paper could be extended to include interactions between individual attributes and product characteristics.

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[5] Recall that we are measuring prices and income relative to the numeraire good. If we explicity introduce its price p_0, and replace p_i by p_i/p_0 and Y by Y/p_0 in (13), then it is immediate that V is homogeneous of degree zero in $(p_0, p_1, \ldots, p_N, Y)$. Furthermore, it is confirmed in the Appendix that V is quasi-convex in $(p_0, p_1, \ldots, p_N)$ provided that $\ln\phi_i(p_i/p_0, z_i)$ is concave in p_i/p_0, $i=1,\ldots,N$, and also $\phi_0' > 0$, as we have assumed.

3. FIRM CHOICE OF CHARACTERISTICS

We will suppose that firms choose the characteristics z_i of each variety and the prices p_i simultaneously, treating the characteristics and prices of other firms as fixed. Each firm may produce multiple product varieties, and indexing firms by $\ell=1,\ldots,L$, we let $I_\ell \subset \{1,\ldots,N\}$ denote the set of products produced by firm ℓ. Initially we suppose that each variety is produced by only one firm, but later allow for competition in the same variety. Let $c_{it}(z_{it})$ denote the cost of producing one unit of product i in year t, where these unit-costs depend on the level of characteristics and can also vary over time, but do not depend on output.[6] Adding a time subscript to all other variables, the profit-maximization problem for firm ℓ is stated as:

$$\max_{p_{it}, z_{it} \geq 0} \sum_{i \in I_\ell} [p_{it} - c_{it}(z_{it})] X_{it} \ , \qquad (14a)$$

where X_{it} is given by Proposition 1(a):

$$X_{it} = -\left(\frac{\partial V/\partial p_{it}}{\partial V/\partial Y_t}\right) = -\left(\frac{\partial \phi_i}{\partial p_{it}}\right)\left(\frac{\partial V/\partial \phi_i}{\partial V/\partial Y_t}\right) = -\left(\frac{\partial \pi_i}{\partial q_{it}}\right)^{-1}\left(\frac{\partial V/\partial \phi_i}{\partial V/\partial Y_t}\right) . \qquad (14b)$$

and the last equality follows because $p_{it} = \pi_i(q_{it}, z_{it})$ is the inverse function of $q_{it} = \phi_i(p_{it}, z_{it})$.

There may also be fixed costs involved in the production of each variety, which would influence the equilibrium number of varieties, but these are not made explicit. Thus, the Nash equilibrium we consider treats the number of product varieties as exogenous. Even keeping the number of varieties fixed, it is not guaranteed that a Nash equilibrium in prices and characteristics will exist. One condition that could be used to obtain existence is the quasi-concavity of profits in a firm's own prices and characteristics. Caplin and Nalebuff (1991) have recently established that for demand

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6 Note that marginal costs $c_{it}(z_{it})$ are independent of output i, but also the outputs of all other product varieties. This rules out economies of scale and scope, which is a significant limitation in monopolistic competition setting. See also note 9.

specifications that overlap with the discrete choice problem discussed in the last section, the profit function is quasi-concave in a firm's own price when it produces a single product, but their results do not apply to multi-product firms.[7] Furthermore, they treat the characteristics of products as exogenous. With the simultaneous choice of prices and characteristics, some special conditions will need to be satisfied to obtain quasi-concavity of profits, as we will discuss below.

3.1 Oligopoly Choice of Characteristics

Let $\{p^*_{it}, z^*_{it}\}$ denote a Nash equilibrium at which profits are maximized for the choice of prices and characteristics, where $q^*_{it} \equiv \phi_i(p^*_{it}, z^*_{it})$. Then consider alternative choices of (p_{it}, z_{it}) satisfying $p_{it} = \pi_i(q^*_{it}, z_{it})$. By construction, these choices hold constant the quality-adjusted price q^*_{it}, so that the demand for all products other than variety i are constant. In addition, the arguments of V are unaffected, so that demand X_{it} changes only due to the term $(\partial\pi_i/\partial q_{it})^{-1}$ on the right-side of (14b). It follows that the maximization of profits in (14) means that the firm will solve the sub-problem:

$$\max_{z_{it} \geq 0} [(\pi_i(q^*_{it}, z_{it}) - c_{it}(z_{it})] \left(\frac{\partial\pi_i(q^*_{it}, z_{it})}{\partial q_{it}}\right)^{-1} . \tag{15}$$

The quasi-concavity of profits in prices and characteristics implies that the objective function in (15) is quasi-concave in characteristics. Furthermore, if a maximum is obtained at an interior position z^*_{it}, then the objective function must be locally concave around this position (in the sense that the matrix of second-derivatives with respect to z_{it} must be negative semi-definite in a neighborhood of z^*_{it}). To see the implications of this condition, we can consider the specific models introduced in the last section.

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[7] Milgrom and Roberts (1990) establish existence of Nash equilibria under alternative "supermodularity" conditions, requiring that the reaction functions are monotonically increasing in the strategic variables of the other firms. They show (pp. 1271-1272) that this property holds for single-product firms with CES, MNL and other demand functions, and it appears that their analysis would extend naturally to multi-product firms.

In the LRUM model, quality-adjusted prices are $\ln q_{it} = p_{it} - g_i(z_{it})$, so that $p_{it} = \pi_i(q_{it}, z_{it}) = \ln q_{it} + g_i(z_{it})$. Using this in (15), it is immediate that for fixed values of q^*_{it} the firm will be maximizing $g_i(z_{it}) - c_{it}(z_{it})$, or just the difference between product quality and marginal cost. If costs are linear in characteristics, for example, then $g_i(z_{it})$ must be locally concave around an interior maximum z^*_{it}. In the CES model we had $\ln q_{it} = \ln p_{it} - \ln g_i(z_{it})$, so that $p_{it} = \pi_i(q_{it}, z_{it}) = q_{it} g_i(z_{it})$. Using this in (15), it follows that for fixed values of q^*_{it} the firm will be maximizing $g_i(z_{it})/c_{it}(z_{it})$, which is interpreted as product quality per unit of costs. The value $\ln[g_i(z_{it})/c_{it}(z_{it})]$ will also be maximized, so if costs are specified as log-linear in characteristics, then $\ln g_i(z_{it})$ must be locally concave around an interior maximum z^*_{it}. We will make use of these necessary second-order conditions for an interior maximum in later sections. For now, we shall focus on the first-order condition for the maximization of (15), which is summarized by:

<u>Proposition 2</u>

Assume there exists a Nash equilibrium $\{p^*_{it}, z^*_{it}\}$ in prices and characteristics, at which (15) is solved at an interior maximum z^*_{it}, and let $q^*_{it} \equiv \phi_i(p^*_{it}, z^*_{it})$. Then:

$$\frac{\partial c_{it}(z^*_{it})}{\partial z_{ikt}} = \frac{\partial \pi_i(q^*_{it}, z^*_{it})}{\partial z_{ikt}} \left[1 - \left(\frac{p^*_{it} - c_{it}(z^*_{it})}{p^*_{it}} \right) \left(\frac{1}{\sigma_{ikt}} \right) \right]. \tag{16}$$

where $\sigma_{ikt} = \left(\frac{\partial \pi_i}{\partial q_{it}}\right)\left(\frac{\partial \pi_i}{\partial z_{ikt}}\right) / p^*_{it} \left(\frac{\partial^2 \pi_i}{\partial q_{it} \partial z_{ikt}}\right)$, evaluated at $(q^*_{it}, z^*_{it}.)$

This result indicates that the marginal cost of each characteristic will differ from its marginal value by an amount that depends on the price-cost markups for each product, along with the term σ_{ikt}, which measures the elasticity of substitution between q_{it} and z_{ikt} in the function $p_{it} = \pi_i(q_{it}, z_{it})$. For example, in the LRUM model the elasticity of substitution is $\sigma_{ikt} = \infty$, so that the equilibrium condition (16) yields *equality* of the marginal cost and value of characteristics. In comparison, in the CES model we have $\sigma_{ikt}=1$, so that equality of marginal cost and value is obtained in *elasticity form:*

$$\left(\frac{1}{c_{it}}\right)\frac{\partial c_{it}(z^*_{it})}{\partial z_{ikt}} = \left(\frac{1}{p^*_{it}}\right)\frac{\partial \pi_i(q^*_{it}, z^*_{it})}{\partial z_{ikt}} \quad . \tag{17}$$

This equilibrium condition applies to the specification of quality in Swan (1970).

It should be noted that our model does little to narrow the range of σ_{ikt}, beyond requiring that (16) is nonnegative, and that $\sigma_{ikt} < 0$ is quite possible. The only restriction we have made on the functions ϕ_i is that $\ln\phi_i(p_{it}, z_{it})$ is concave in p_{it} [as in Proposition 1(c)], but this does not limit the elasticity between q_{it} and z_{it} in the inverse function $\pi_i(q_{it}, z_{it})$. For elasticities $\sigma_{ikt} > 0$, the marginal value of characteristics is greater than the marginal cost in Proposition 2, and conversely for $\sigma_{ikt} < 0$. When measuring the marginal cost and value of characteristics in elasticity form, as in (17), then $0 < \sigma_{ikt} < 1$ implies the marginal value is greater than the marginal cost, and conversely for $\sigma_{ikt} < 0$ or $\sigma_{ikt} > 1$.

3.2 Competition in the Same Product Variety

So far, we have assumed that firms do not compete in the same product varieties. Our analysis can be readily extended to deal with this case, and we will show that Proposition 2 continues to hold as stated. However, since firms producing the same product variety act as Bertrand competitors, their price will be driven to marginal cost, with the result that the marginal cost of each characteristic will equal its marginal value.

To analyze this case, we need to introduce some additional notation. If two firm labeled $\ell=1,2$ sell the same variety i, at the prices p^{ℓ}_{it} and characteristics z^{ℓ}_{it}, then we will write individual utility in (10) as:

$$V_i = \ln\phi_0(y) - \min\{\ln\phi_i(p^1_{it}, z^1_{it}), \ln\phi_i(p^2_{it}, z^2_{it})\} + \varepsilon_i \ . \tag{10'}$$

The probability that product i is chosen is given by P_i in (11), and then the probability that product i produced by firm 1 is chosen is given by:

$$P^1_{it} = \begin{cases} 0 \text{ if } \ln\phi_i(p^1_{it}, z^1_{it}) > \ln\phi_i(p^2_{it}, z^2_{it}) \\ \frac{1}{2} P_{it} \text{ if } \ln\phi_i(p^1_{it}, z^1_{it}) = \ln\phi_i(p^2_{it}, z^2_{it}) \\ P_{it} \text{ if } \ln\phi_i(p^1_{it}, z^1_{it}) < \ln\phi_i(p^2_{it}, z^2_{it}) \end{cases} \qquad (11')$$

and similarly for P^2_{it}. Note that when the products from the two firms have identical quality-adjusted prices, there is a equal probability that the consumer will purchase from either one; we would use the same condition when more than two firms sell a product at the same quality-adjusted price.

Conditional on choosing product i from firm ℓ, the quantity consumed by each consumer is computed as in (12) where we add the superscript ℓ, and the expected demand for product i from each firm is $X^\ell_{it} = M x^\ell_{it} P^\ell_{it}$. The marginal costs of producing variety i for firm ℓ are $c_{it}(z^\ell_{it})$, where we are supposing that all firms producing variety i have access to the same technology. With these changes in notation, the profit-maximization problem for firm ℓ is stated as:

$$\max_{p^\ell_{it}, z^\ell_{it} \geq 0} \sum_{i \varepsilon I_\ell} [p^\ell_{it} - c_{it}(z^\ell_{it})] X^\ell_{it} \; . \qquad (14')$$

In the Nash equilibrium $\{p^{\ell*}_{it}, z^{\ell*}_{it}\}$, all firms are maximizing profits for their choices of prices and characteristics over the (exogenous) sets of product I_ℓ, $\ell = 1, \ldots, L$.

With this additional notation, the proof of Proposition 2 continues to hold in a slightly modified form. Starting in a Nash equilibrium position where firms 1 and 2 have identical quality-adjusted prices for good i, we can consider alternative choices of $(p^\ell_{it}, z^\ell_{it})$ satisfying $p^\ell_{it} = \pi_i(q^{\ell*}_{it}, z^\ell_{it})$. This will not affect the demand for any products other than i, nor will it affect the probability P^ℓ_{it} that a consumer selects product i from each firm. It follows that the each firm still solves the sub-problem (15), except with the superscript ℓ added to the price and characteristics. The first-order condition for this problem yields Proposition 2, again with the superscript ℓ added to price and characteristics.

Thus, the equilibrium condition determining the choice of characteristics is identical whether multiple firms sell the same product variety or not. The difference, of course, is that with more than one firm competing in a given variety, Nash competition in prices will force price to equal marginal cost, by the usual Bertrand argument. Thus, if firms 1 and 2 were initially selling at the same quality-adjusted price, then a slight reduction in p_{it}^{1} would double the probability that consumers select the product from firm 1 in (11'), and double expected demand. This action would raise profits unless price equaled marginal cost initially, which must hold for all varieties produced by multiple firms in the Nash equilibrium. We shall refer to this as the "competitive case", and it implies that the marginal cost of producing each characteristic equals its marginal value, from Proposition 2.

3.3 Social Optimum

In order to further interpret the equilibrium condition (16), it is useful to solve for the socially optimal level of characteristics. As noted by Spence (1975), the optimal level of characteristics should be defined using fixed quantities for all goods produced. To this end, we can follow Diewert (1974, p. 123) and define the *direct* utility function of the representative consumer as:

$$U(X_{0t}, X_t, z_t) = \min_{p_{it} \geq 0} V\left[\phi_1(p_{1t}, z_{1t}), \ldots, \phi_N(p_{Nt}, z_{Nt}), X_{0t} + \sum_{i=1}^{N} p_{it} X_{it}\right], \qquad (18)$$

where X_{0t} is the aggregate consumption of the numeraire commodity, $X_t = (X_{1t}, \ldots, X_{tN})$, and z_t is the NxK vector of characteristics over all product varieties.[8] Applying the envelope theorem to (18) to compute $\partial U / \partial z_{ikt}$, and using Roy's Identity, we obtain:

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[8] As in (5), the direct utility function is defined by (18) only for values of $X_t > 0$ that are feasible outcomes from the individual choice problem described in (10)-(12). We would define U as $-\infty$ for any other values of X_t.

$$\frac{\partial U/\partial z_{ikt}}{\partial V/\partial Y_t} = -X_{it}\left(\frac{\partial\phi_i/\partial z_{ikt}}{\partial\phi_i/\partial p_{it}}\right) = X_{it}\,\frac{\partial\pi_i}{\partial z_{ikt}}\,, \tag{19}$$

where the last equality follows because $p_{it}=\pi_i(q_{it},z_{it})$ is the inverse function of $q_{it}=\phi_i(p_{it},z_{it})$. The expression on the left-side of (19) is the marginal value of characteristic k in good i, holding the *quantity* of every product constant, while the expression on the right is the marginal value of characteristic k in good i (multiplied by the demand for good i) holding the *quality-adjusted price* q_{it} constant. The equality of these two marginal values allows the social optimum to be characterized very easily.

The total costs of production are $\sum_{i=1}^{N} c_{it}(z_{it})X_{it}$, and holding constant the quantity of every good, the marginal cost of characteristic k in good i is simply $X_{it}\partial c_{it}/\partial z_{ikt}$. In the social optimum this should equal the marginal value in (19), so that denoting the social optimum by an over-bar:

$$\frac{\partial c_{it}(\overline{z}_{it})}{\partial z_{ikt}} = \frac{\partial \pi_i(\overline{q}_{it},\overline{z}_{it})}{\partial z_{ikt}}\,, \qquad i=1,\ldots,N. \tag{20a}$$

Note that a direct comparison of (20a) with the market equilibrium in (17) is complicated by quality-adjusted prices $q_{it}=\phi(p_{it},z_{it})$, that affect the marginal value of characteristics in both cases. In the social optimum, the (shadow) prices $\overline{p}_{it}$ for consumption of each good must equal the marginal cost $c_{it}(\overline{z}_{it})$, so that the quality-adjusted prices are determined by:

$$\overline{q}_{it} = \phi_i[c_{it}(\overline{z}_{it}),\overline{z}_{it}], \qquad i=1,\ldots,N, \tag{20b}$$

which is the second condition determining the choice of characteristics.

Comparing (20) with (16), we obtain the following results:

Proposition 3

The level of characteristics chosen by firms is socially optimal if:

(a) prices p_{it} equal the marginal cost of production; or,

(b) the quality-adjusted prices $q_{it}=\phi_i(p_{it},z_{it})$ are of the form:

$$\phi_i(p_{it},z_{it}) = \psi_i\left[\frac{p_{it} - g_i(z_{it})}{h_i(z_{it})}\right], \qquad (21a)$$

where $\psi_i>0$, $\psi_i'>0$, $h_i>0$, $\partial g_i/\partial z_{it}\geq 0$ and $\partial h_i/\partial z_{it}\geq 0$. If we also choose $\ln\phi_0(y)=y$ so the indirect utility function in (13) is linear in income, then the *direct* utility function corresponding to these quality-adjusted prices is of the form:

$$U(X_{0t},X_t,z_t) = X_{0t} + \sum_{i=1}^{N} g_i(z_{it})X_{it} + H[h_1(z_{1t})X_{1t},\ldots,h_N(z_{Nt})X_{Nt}], \qquad (21b)$$

where U is defined over those values of $X_t=Mx_{it}P_{it}>0$ that are feasible outcomes from the individual maximization (11)-(12), for each $z_t>0$.

Part (a) is an immediate consequence of setting $p^{*}_{it}=c_{it}(z^{*}_{it})$ in (16), as appropriate when firms compete in the same product variety, and comparing it to (20). While the social optimality of the "competitive case" is not too surprising perhaps, we note that it is sensitive to our assumption of constant returns to scale, whereby the marginal costs of production are independent of output.[9]

Part (b) shows that the two examples introduced in the last section - the LRUM model and the CES model - both yield the socially optimal level of characteristics in the Nash equilibrium.[10] The quality-adjusted prices introduced in (21a) combines these two models by allowing each characteristic to enter the function g_i, which equals the

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[9] Abel (1983) examines the social optimality of the competitive and monopolistic choice of durability in the Swan model, but without constant returns to scale. He identifies some cases where the competitive firm will not choose the socially optimal level of durability, while the monopolist does.

[10] Social optimality in the MNL case of the LRUM model has been noted by Anderson, de Palma and Thisse (1992, Prop. 7.2).

value individuals place on the observed characteristics (in terms of the numeraire), and the function h_i, which we can interpret as durability. For this specification of the quality-adjusted prices, the corresponding direct utility function in given by (21b). An example of direct utility for the MNL model was given in (5), where $h_i \equiv 1$. Alternatively, if we choose $g_i \equiv 0$ then the quality-adjusted prices in (21a) are just the ratio of price and durability (up to a monotonic transformation), and the direct utility function H in (21b) takes on the Swan (1970) form, where the durability of each product multiplies its quantity. More generally, the direct utility function in (21b) has the two quality measures g_i and h_i multiplying the quantity of each product. This specification is very close to that of Swan, which helps to explain why the level of characteristics obtained is socially optimal. We shall make use of the quality-adjusted prices and utility function in (21) in later sections.

4. EXACT HEDONIC PRICE INDEXES

In order to determine how a change in characteristics and prices affects aggregate utility, we will proceed on the assumption that the marginal values - denoted by γ_{it} - are known, where the asterisk indicating the Nash equilibrium is now omitted. In the competitive case γ_{it} equals the marginal cost of each characteristic, and can be measured by the coefficients of a hedonic regression. In the oligopoly case, the marginal values are not measured so easily, since the price-cost markups (which are an omitted variable in the hedonic regression) will bias the coefficients on the characteristics. We defer a discussion of this case to section 5, and note here that the assumption that γ_{it} is known is reasonable primarily in the competitive case.

We shall use the expenditure function for the representative consumer, which is obtained by inverting the indirect utility function in (13) or (13'). Diewert (1976) defines a price index - depending on observed prices and quantities in two periods - as *exact* if it equals the ratio of expenditure functions in two periods at a constant level of utility, but with changing prices. This means that the price index measures the change in expenditure needed to keep utility constant, or the change in the cost-of-living. Analogously, we shall define a *hedonic price index* - depending on observed prices, quantities, marginal values γ_{it} and characteristics - as *exact* if it equals the ratio of expenditure functions at constant utility, but allowing for changing prices and characteristics. While we will not generally obtain a unique measure for the exact hedonic index, we are able to establish upper and lower bounds. We derive these results first for the CES and MNL models introduced in section 2, and then for general utility and expenditure functions.

4.1 CES Expenditure Function

The CES expenditure function obtained by inverting (9) is:

$$E_c(p_{1t}/a_{1t},\dots,p_{Nt}/a_{Nt},U_t) = U_t^{1/(1+\alpha)}\left[\sum_{i=1}^{N}(p_{it}/a_{it})^{-1/\mu}\right]^{-\mu/(1+\alpha)}, \tag{22}$$

where $1/(1+\alpha_t)$ is the share of income devoted to the numeraire commodity, and $a_{it}=g_{it}(z_{it})$ is the (unobserved) measure of product quality. If these quality measures are held constant over time at a_i, then from Sato (1976) the following price index is exact:

$$\frac{E_c(p_{1t}/a_1,\dots,p_{Nt}/a_N,U)}{E_c(p_{1t-1}/a_1,\dots,p_{Nt-1}/a_N,U)} = P_c(p_{t-1},p_t,s_{t-1},s_t) \equiv \prod_{i-1}^{N}\left(\frac{p_{it}}{p_{it-1}}\right)^{\omega_{it}/(1+\alpha)} \tag{23a}$$

where,

$$\omega_{it} = \frac{\Delta s_{it}}{\Delta \ln s_{it}} \Big/ \sum_{j=1}^{N}\frac{\Delta s_{jt}}{\Delta \ln s_{jt}}, \tag{23b}$$

and $s_{i\tau}$ is the expenditure share on product i in year $\tau=t-1,t$, and $\Delta s_{it}=s_{it}-s_{it-1}$. The expression $\Delta s_{it}/\Delta \ln s_{it}$ in (23b) is a logarithmic mean of the expenditure shares s_{it-1} and s_{it}, and lies between these two share. Thus, the weights ω_i are an average of the expenditure shares across the periods, normalized to sum to unity. Then the index on the right-side of (23a) is a geometric mean of the ratios (p_{it}/p_{it-1}), using the weights ω_{it} adjusted for the share of expenditure on the numeraire good. The important feature of this index is that it does not depend on the unobserved qualities a_i, but its limitation is that the product qualities are held constant.

When the characteristics are changing over time, a formula like (23) still applies, except that we replace p_{it} by the quality-adjusted price $q_{it}=p_{it}/a_{it}$. While $a_{it}=g_i(z_{it})$ is not observed, the *ratio* of these quality measures can be bounded using the marginal values, along with a concavity assumption. In particular, let us assume that $\ln g_i(z_{it})$ is concave in characteristics. The motivation for this assumption was discussed in section 3.1, where we argued that if costs were log-linear in characteristics, then $\ln g_i(z_{it})$ must be locally concave around all equilibria. We are strengthening this condition to apply globally over a convex set containing all the observed values of characteristics. In that case, from the properties of concave functions we have:

$$\gamma_{it}'(z_{it}-z_{it-1}) \leq \ln g_i(z_{it}) - \ln g_i(z_{it-1}) \leq \gamma_{it-1}'(z_{it}-z_{it-1}), \tag{24}$$

where $\gamma_{it}\equiv\partial\ln g_i(q_{it},z_{it})/\partial z_{it}$, and all vectors are treated as columns unless transposed with $'$. Note that since the quality-adjusted prices are $q_{it}=p_{it}/g_i(z_{it})$, then $\pi_i(q_{it},z_{it})=q_{it}g_i(z_{it})$, so that γ_{it} is identical to the marginal values $\partial\ln\pi_i(q_{it},z_{it})/\partial z_{it}$.

Making use of the inequalities in (24), we can establish the following bounds on the exact hedonic index for the CES model:

<u>Proposition 4</u>

For the CES expenditure function (22), assume that $\ln g_i(z_{it})$ is concave in z_{it}.
Let P_c denote the exact index defined in (23), and let γ_{it} denote the marginal values $\partial\ln\pi_i(q_{it},z_{it})/\partial z_{it}$. Then:

$$P_c(\hat{p}_{t-1},p_t,s_{t-1},s_t) \leq \frac{E_c(p_{1t}/a_{1t},\ldots,p_{Nt}/a_{Nt},U)}{E_c(p_{1t-1}/a_{1t-1},\ldots,p_{Nt-1}/a_{Nt-1},U)} \leq P_c(p_{t-1},\hat{p}_t,s_{t-1},s_t), \tag{25}$$

where $\hat{p}_{it-1}\equiv p_{it-1}\exp[\gamma_{it-1}'(z_{it}-z_{it-1})]$, and $\hat{p}_{it}\equiv p_{it}\exp[-\gamma_{it}'(z_{it}-z_{it-1})]$.

This result provides bounds on the ratio the expenditure functions when product prices and characteristics are changing over time. To understand the nature of these bounds, notice that $\hat{p}_{it}$ and $\hat{p}_{it-1}$ correct the observed prices for changes in the characteristics between the two periods. This procedure corresponds closely to the "explicit quality adjustment" described by Triplett (1990, p. 39). The adjustment is made using the period t-1 marginal values γ_{it-1} in the lower-bound, and period t values γ_{it} in the upper-bound of Proposition 4. If these marginal values do not change much over time, then the bounds in (25) will be quite close, and if $\gamma_{it}=\gamma_{it-1}$ then the bounds are identical, so that a unique measure of the exact hedonic index is obtained.

4.2 Multinomial Logit Expenditure Function

A second example of a hedonic price index can be obtained for the multinomial logit (MNL) case of the LRUM model considered in section 2, for which the aggregate utility function is (4). Inverting this, the expenditure function is given by:

$$E_m(p_t-a_t,U_t) = U_t - M\mu \ln\left[\sum_{i=1}^{N} \exp\left(\frac{a_i - p_i}{\mu}\right)\right]. \tag{26}$$

where the number of individuals M is held constant over time, and $a_{it}=g_{it}(z_{it})>0$ is an (unobserved) measure of product quality. Since expenditure is linear in utility, the ratio of the expenditure functions will depend on the utility level chosen. Holding the product qualities constant at $a=(a_1,\dots,a_n)$, it is shown in the Appendix that the ratio of expenditure evaluated at the previous-period utility is:

$$\frac{E_m(p_t-a,U_{t-1})}{E_m(p_{t-1}-a,U_{t-1})} = P_m(p_{t-1},p_t,X_{0t-1},X_{t-1},X_t) \equiv 1 + \sum_{i=1}^{N}\left(\frac{\omega_{it}}{Y_{t-1}}\right)(p_{it}-p_{it-1}). \tag{27a}$$

where,

$$\omega_{it} = \frac{\Delta X_{it}}{\Delta \ln X_{it}} / \sum_{i=1}^{N}\frac{\Delta X_{it}}{\Delta \ln X_{it}} . \tag{27b}$$

and $Y_{t-1}=X_{0t-1}+\sum_{i=1}^{N}p_{it-1}X_{it-1}$ denotes total expenditure in period t-1, including the numeraire good. The weights in (27b) are interpreted as the logarithmic mean of the quantity purchased over the two periods, normalized to sum to unity.

When the characteristics are changing, then the index (27a) still applies, but with $(p_{it}-p_{it-1})$ replaced by $([p_{it}-a_{it})-(p_{it-1}-a_{it-1})]$. Then we can established bounds on the *difference* in the unobserved qualities $a_{it}=g_i(z_{it})$ by assuming that $g_i(z_{it})$ is concave in characteristics. This is the second-order necessary condition for (15) to be maximized if costs are linear in characteristics, and we strengthen this condition by applying it globally. This concavity condition implies:

$$\gamma_{it}'(z_{it}-z_{it-1}) \leq g_i(z_{it})-g_i(z_{it-1}) \leq \gamma_{it-1}'(z_{it}-z_{it-1}), \qquad (28)$$

where $\gamma_{it}\equiv\partial g_i(q_{it},z_{it})/\partial z_{it}$. Since the quality-adjusted prices are $q_{it}=p_{it}-g_i(z_{it})$, then $\pi_i(q_{it},z_{it})=q_{it}+g_i(z_{it})$, and γ_{it} is identical to the marginal values $\partial\pi_i(q_{it},z_{it})/\partial z_{it}$.

Using this concavity condition, we have:

<u>Proposition 5</u>

For MNL expenditure functions (26), assume that g_i is concave in z_{it}. Let P_m denote the exact index defined by (27), and γ_{it} denote the marginal values $\partial\pi_i(q_{it},z_{it})/\partial z_{it}$. Then:

$$P_m(\hat{p}_{t-1},p_t,X_{0t-1},X_{t-1},X_t) \leq \frac{E_m(p_t-a_t,U_{t-1})}{E_m(p_{t-1}-a_{t-1},U_{t-1})} \leq P_m(p_{t-1},\hat{p}_t,X_{0t-1},X_{t-1},X_t), \qquad (29)$$

where $\hat{p}_{it-1}\equiv p_{it-1}+\gamma_{it-1}'(z_{it}-z_{it-1})$ and $\hat{p}_{it}\equiv p_{it}-\gamma_{it}'(z_{it}-z_{it-1})$.

Once again, we obtain bounds on the exact hedonic index, that are constructed using the observed prices corrected for changes in the characteristics between the two periods. In this case, the correction to the prices is additive, and since prices enter the index P_m in a difference form, if $\gamma_{it-1}=\gamma_{it}$ then a unique measure of the exact hedonic index is again obtained.

4.3 General Expenditure Function

Both the expenditure functions considered in Propositions 4 and 5 arise when the distribution of the random term in the individual utility functions (1) or (6) are i.i.d. with the double-exponential distribution (2). This is a rather narrow case of the general family of distribution and utility functions dealt with in Proposition 1. Fortunately, analogous results to those above hold quite generally, and depend only on concavity properties of the individual utility functions.

To generate bounds on the exact hedonic index in the general case, we use the expenditure function corresponding to the indirect utility function in Proposition 1. Denoting utility by $U_t=\tilde{V}_t$ in (13'), and letting ϕ_0^{-1} denote the inverse function of ϕ_0, we solve for the level of expenditure $E(p_t,z_t,U_t)$ needed to obtain aggregate utility of U_t as:

$$E(p_t,z_t,U_t) = M\phi_0^{-1}\Big[U_t G\big[\phi_1(p_{1t},z_{1t})^{-1},\ldots,\phi_N(p_{Nt},z_{Nt})^{-1}\big]^{-1}\Big], \tag{30}$$

where z_t is the NxK vector of characteristics over all product varieties. $E(p_t,z_t,U_t)$ measures the expenditure on the varieties $i=1,\ldots,N$, plus the numeraire X_{0t}. As noted in Proposition 1(c), the indirect utility function is convex in prices p_t provided that $\ln\phi_i(p_{it},z_{it})$ is concave in p_{it}, $i=1,\ldots,N$, and under this same condition the expenditure function is concave in p_t.[11] In order to generate bounds on the exact hedonic index under this condition, we will need to assume that the prices $\pi_i(q_{it},z_{it})$ are concave in characteristics. This leads to results similar to the MNL case, in terms of the adjustment made for changing characteristics:

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[11] Recall from note 5 that the indirect utility function V is quasi-convex in the prices $(p_{0t},p_{1t},\ldots,p_{Nt})$, including the numeraire. If we explicity introduce its price and replace p_{it} by p_{it}/p_{0t} in (30), while also multiplying (30) by p_{0t}, then it is clear that the expenditure function is homogeneous of degree one in $(p_{0t},p_{1t},\ldots,p_{Nt})$. The quasi-convexity of V implies that the expenditure function is quasi-concave in $(p_{0t},p_{1t},\ldots,p_{Nt})$, but since it is also homogeneous of degree one in these variables, then it must be concave. Setting $p_{0t}=1$, it follows that the expenditure function in (30) is concave in p_t.

Proposition 6

For $E(p_t,z_t,U_t)$ in (30), assume that $\ln\phi_i(p_{it},z_{it})$ is concave in p_{it} and $\pi_i(q_{it},z_{it})$ is concave in z_{it}, $i=1,\ldots,N$. Denoting the marginal values by $\gamma_{it}\equiv\partial\pi_i(q_{it},z_{it})/\partial z_{it}$:

(a) $$\frac{E(p_t,z_t,U_{t-1})}{E(p_{t-1},z_{t-1},U_{t-1})} \le \left[\frac{x_{0t-1}+\sum_{i=1}^{N} x_{it-1}\hat{p}_{it}}{x_{0t-1}+\sum_{i=1}^{N} x_{it-1}p_{it-1}}\right], \text{ where } \hat{p}_{it}\equiv p_{it}-\gamma'_{it}(z_{it}-z_{it-1});$$

(b) $$\frac{E(p_t,z_t,U_t)}{E(p_{t-1},z_{t-1},U_t)} \ge \left[\frac{x_{0t}+\sum_{i=1}^{N} x_{it}p_{it}}{x_{0t}+\sum_{i=1}^{N} x_{it}\hat{p}_{it-1}}\right], \text{ where } \hat{p}_{it-1}\equiv p_{it-1}+\gamma'_{it-1}(z_{it}-z_{it-1}).$$

The price index on the right of (a) is a Laspeyres formula using the previous-period quantity as weights, and correcting the current-period prices for changes in the characteristics. Similarly, the index on the right of (b) is a Paasche formula using the current-period quantities as weights, and correcting the previous-period prices for changes in the characteristics. This result therefore shows how the conventional Paasche and Laspeyres bounds on the ratio of expenditure functions can be extended to deal with changing characteristics; related results are obtained by Pollak (1978, 1983).

A second case can be obtained by making the weaker assumption that $\ln\pi_i(q_{it},z_{it})$ is concave in z_{it}, along with the stronger assumption that $\ln\phi_i(p_{it},z_{it})$ is concave in $\ln p_{it}$, $i=1,\ldots,N$. This latter assumption was satisfied for the CES model, for example, where $\ln\phi_i(p_{it},z_{it})=\ln p_{it}-\ln g_i(z_{it})$. It is also satisfied for the LRUM model, and more generally for the quality-adjusted prices in (21a), provided that $g_i(z_{it})\ge 0$. The assumption that $\ln\phi_i(p_{it},z_{it})$ is concave in $\ln p_{it}$ will imply that $\ln[E(p_t,z_t,U_t)]$ in concave in $\ln p_{it}$, so that the expenditure *shares* are decreasing functions of their own price: thus, product demand is elastic. This is a quite natural case to consider if we believe that the product varieties $i=1,\ldots,N$ are close substitutes for each other. We also assume that the function ϕ_0 in (30) is chosen as $\phi_0(y)=y$, which implies that the $E(p_t,z_t,U_t)$ is homogeneous of degree one in utility, so the ratio of expenditure can be evaluated at any level of utility:

Proposition 7

Let $\phi_0(y)=y$ in (30), and assume that $\ln\phi_i(p_{it},z_{it})$ is concave in $\ln p_{it}$ and $\ln\pi_i(q_{it},z_{it})$ is concave in z_{it}, $i=1,\ldots,N$. Then for $\gamma_{it}\equiv\partial\ln\pi_i(q_{it},z_{it})/\partial z_{it}$:

$$\prod_{i=1}^{N}\left(\frac{p_{it}}{\hat{p}_{it-1}}\right)^{s_{it}} \leq \frac{E(p_t,z_t,U)}{E(p_{t-1},z_{t-1},U)} \leq \prod_{i=1}^{N}\left(\frac{\hat{p}_{it}}{p_{it-1}}\right)^{s_{it-1}} \tag{31}$$

where $\hat{p}_{it-1}\equiv p_{it-1}\exp[\gamma'_{it-1}(z_{it}-z_{it-1})]$, $\hat{p}_{it}\equiv p_{it}\exp[-\gamma'_{it}(z_{it}-z_{it-1})]$, and $s_{i\tau}\equiv p_{i\tau}X_{i\tau}/Y_\tau$ denotes the expenditure shares on products $i=1,\ldots,N$, for $\tau=t-1,t$.

In this result, the adjustment to the prices due to changing characteristics is the same as what we found for the CES expenditure function. If $\hat{p}_{i\tau}$ in Propositions 6 and 7 are the same, $\tau=t-1,t$, then the latter will give bounds that are at least as tight as the former. This is because the Laspeyres index in Proposition 6(a) can be written as an arithmetic mean of the price ratios $(\hat{p}_{it}/p_{it-1})$, $i=0,1,\ldots,N$, where $\hat{p}_{0\tau}=p_{0\tau}\equiv1$, $\tau=t-1,t$. This will be greater than or equal to the geometric mean on the right-side of (31). In addition, the Paasche index in Proposition 6(b) can be written as a harmonic mean of the price ratios $(p_{it}/\hat{p}_{it-1})$, $i=0,1,\ldots,N$, which is less than or equal to the geometric mean on the left-side of (31).[12] In practice, the bounds in Proposition 7 will give a narrow range for the exact hedonic price index provided that the marginal values γ_{it} and expenditure shares do not change much over time.

5. HEDONIC REGRESSIONS

5.1 Competitive Case

We turn now to the estimation of the marginal value of characteristics, which we assumed were available to construct bounds on the exact hedonic price indexes. We initially consider the "competitive case," where price equal marginal cost for each product. Suppose that the varieties {1,...,N} can be divided into groups (e.g. small cars

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12 Harmonic means are discussed in Hardy, Littlewood and Polya (1973), who demonstrate the ranking with the geometric and arithmetic means (p. 26).

and large cars), within which the marginal costs take on a log-linear form:

$$\ln p_{it} = \ln c_{it} = \alpha_t + \beta_t' z_{it} + \upsilon_{it} \ . \qquad (32)$$

where: α_t is a fixed-effect or "time dummy" reflecting the change in marginal costs over time; the coefficients β_t equal the marginal cost of increasing characteristics (expressed as an elasticity), that are common within a group; and the random term δ_{it} incorporates all other factors that influence marginal cost. For example, δ_{it} could reflect unmeasured characteristics, that may be correlated with the measured characteristics z_{it}. In this case, we can model the errors as:

$$\upsilon_{it} = \delta_t + \eta_t' z_{it} + \xi_{it} \ . \qquad (33)$$

where we assume that the errors ξ_{it} are uncorrelated with z_{it}. Substituting (33) into (32) we obtain:

$$\ln p_{it} = \ln c_{it} = (\alpha_t + \delta_t) + (\beta_t + \eta_t)' z_{it} + (\upsilon_t + \xi_{it}) \ . \qquad (32')$$

If characteristic k is correlated with unmeasured attributes according to (33) - and this relation is known by the firm - then it is rational for the firm to treat $\beta_k + \eta_k$ as the total expected marginal cost of increasing characteristic k, along with the correlated but unmeasured attributes. Note that the error in (32') is uncorrelated with the observed characteristics. Then with a suitable change in notation, we can treat the errors υ_{it} in (32) as uncorrelated with the characteristics z_{it}, where it is understood that the coefficients β_t reflect the marginal cost of the measured and unmeasured characteristics. We can then assume that the least-squares estimates $(\hat{\alpha}_t, \hat{\beta}_t)$ obtained from (32) are consistent.

From Proposition 3(a), the coefficients $\hat{\beta}_t$ can be used as estimates of the marginal value of each characteristic (expressed as an elasticity). These are precisely the marginal values referred to in Propositions 4 and 7. For Proposition 5 and 6, the marginal values would instead be measured by a linear regression of prices on characteristics. Thus, in the competitive case the bounds we have obtained for the exact hedonic price index can be measured quite easily.

It is also of interest to examine the estimate of the "time dummies" α_t, which Triplett (1986, p. 39) identifies as one method to obtain an hedonic price index. The value of $\hat{\alpha}_t$ is chosen so that measured residuals in (32) for each value of t sum to zero. We will consider applying weighted least-squares to (32), using the weights w_i on observation i=1,...,N, where $\Sigma_{i=1}^{N} w_i = 1$. Let us also assume that the value of β_t is treated as constant over time, $\beta_t \equiv \beta$, so that (32) is estimated by pooling over time. Then it follows that the estimate of $(\alpha_t - \alpha_{t-1})$ is:

$$\hat{\alpha}_t - \hat{\alpha}_{t-1} = \sum_{i=1}^{N} w_i[(\ln p_{it} - \ln p_{it-1}) - \beta'(z_{it} - z_{it-1})] \quad . \tag{34}$$

Thus, the difference in the "time dummies" can be interpreted as a *fixed-weight index* of the change in prices, adjusted for the change in characteristics. This can be directly compared to the bounds on the exact index in Propositions 4 and 7. Setting $\gamma_{it-1} = \gamma_{it} = \beta$ in Proposition 4, for example, the upper and lower bound for the exact CES index coincide, and differs from (34) only in the specification of the weights. It should be noted the weights ω_{it} used in the CES index would not be appropriate to use as the weights w_i for WLS, because the former reflect the average expenditure shares across periods t-1 and t, and are therefore endogenous variables. In contrast, the weights w_i used in the estimation should be exogenous to the error terms in (32). Thus, while the fixed weight index in (34) would not generally give identical results to the CES index, or the bounds in (31), we see that there is a close numerical relation between the fixed-weight and exact hedonic price index.

5.2 Pricing above Marginal Costs

Estimating the marginal value of each characteristic is more difficult when prices are above marginal cost. If marginal costs still take the form in (31), then the hedonic regression becomes:

$$\ln p_{it} = \alpha_t + \beta_t' z_{it} + (\ln p_{it} - \ln c_{it}) + \upsilon_{it} \quad . \tag{35}$$

where the price-cost markups ($\ln p_{it} - \ln c_{it}$) are an omitted variable. Any correlation between these markups and the characteristics z_{ikt} will bias the estimates of β_t. In his study of automobiles, for example, Bresnahan (1981) found that larger models have higher markups (even in percentage terms), which would bias upwards the estimates of β_t obtained from a hedonic regression.

There is, however, an intriguing possibility that should be explored. Given that the bias in estimating β_t depends on the correlation between price-cost margins and characteristics, but also that the price-cost margins determine the difference between the marginal cost and value of characteristics in Proposition 2, perhaps these two effects could just offset each other so that the hedonic regression (35) yields a consistent estimate of the marginal values, if not the marginal costs. Such a view is expressed by Triplett (1986, p. 38):

> If price differentials among models are set by sellers on the basis of their estimates of demand elasticities for characteristics, rather than on the basis of cost, then estimated implicit prices for characteristics will reflect user valuations, but not resource cost.

To investigate this possibility in our model, it turns out to be easier to first consider a linear relation between marginal costs and characteristics. Thus, let us respecify (35) as:

$$p_{it} = \alpha_t + \beta_t' z_{it} + (p_{it} - c_{it}) + \upsilon_{it} \ . \tag{35'}$$

To determine whether this pricing equation can be written in terms of marginal values, substitute the equilibrium condition (16) into (35') to obtain:

$$p_{it} = \alpha_t + \gamma_{it}' z_{it} + (p_{it} - c_{it}) \left[1 - \sum_{k=1}^{K} \frac{\gamma_{ikt} z_{ikt}}{p_{it} \sigma_{ikt}} \right] + \upsilon_{it} \ . \tag{36}$$

where $\gamma_{ikt} \equiv \partial \pi_i(q_{it}, z_{it}) / \partial \ln z_{it}$ denotes the marginal value of characteristics. This expression would be simplified if the third term on the right-side vanished for some specification of quality-adjusted prices. It is straightforward to calculate that this occurs for the quality-adjusted prices in (21a). with the added restriction that h_i is homogeneous of degree one.[13] In that case we obtain:

Proposition 8

Suppose that marginal costs are linear in characteristics, and the quality-adjusted prices and utility function take the form in (21), where h_i is homogeneous of degree one in z_{it}. Assume that $[g_i(z_{it})-c_{it}(z_{it})]/h_i(z_{it})]$ has an interior maximum in z_{it}. Then the equilibrium relation between prices and characteristics is:

$$p_{it} = \alpha_t + \gamma_{it}' z_{it} + \upsilon_{it}, \tag{37}$$

where: $p_{it}=g_i(z_{it})+H_i h_i(z_{it})$; $\gamma_{it}=\partial g_i/\partial z_{it}+H_i \partial h_i/\partial z_{it}$ are the marginal values; and H_i denotes the derivative of $H[h_1(z_{1t})X_{1t},\ldots,h_N(z_{Nt})X_{Nt}]$ with respect to its i^{th} argument.

Remarkably, for linear marginal costs and the quality-adjusted prices in (21a) (with h_i homogeneous of degree one), the coefficients of the hedonic regression indeed measure the marginal values γ_{it}. The price $p_{it}=g_i(z_{it})+H_i h_i(z_{it})$ is a nonlinear function of the characteristics of product i, and of all other products through the derivative $H_i[h_1(z_{1t})X_{1t},\ldots,h_N(z_{nt})X_{nt}]$. In practice, it would be convenient to treat H_i as a constant and estimate (37) by pooling across products, and possibly over time. Pooling across products means that we must assume the functions $g_i(z_{it})+H_i h_i(z_{it})$ are identical for some subset of varieties. Then choosing a specific functional form (e.g. a quadratic function of characteristics), we would regress prices on this function, and calculate the marginal values γ_{it} as the derivatives of the estimated function.[14] Of course, the simplest case is where we estimate $g_i(z_{it})+H_i h_i(z_{it})$ as a linear function

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[13] Actually, the third term in (36) vanishes for a somewhat more general specification of quality-adjusted prices, as described in the proof of Proposition 8 in the Appendix,

[14] Note that differing characteristics for the varieties can still be explained by differing marginal cost parameter β_{it} for products within the group. Proposition 8 remains true if (35') is respecified in this way, as $p_t = \alpha_t + \beta_{it}' z_{it} + \varepsilon_{it}$.

of characteristics, leading to a linear hedonic regression.

Proposition 8 can be used to illustrate a result of Jones (1988), who argues that under a certain specification of individual utility functions, prices must be a linearly homogeneous (and convex) function of characteristics. He assumes that the utility of each individual depends on the *total consumption of each characteristic.* This is obtained from (21b) is we also assume that g_i is homogeneous of degree one, in which case aggregate utility depends on the total characteristics $X_i z_{it}$ of each product. To see the implications of this, we can substitute the expressions for p_{it} and γ_{it} in Proposition 8 into (37), and use the linear homogeneity of h_i, to obtain the following equilibrium condition for characteristics:

$$g_i(z_{it}) = \alpha_t + (\partial g_i/\partial z_{it})' z_{it} + \upsilon_{it} \ . \qquad (38)$$

In the case where g_i is also homogeneous of degree one, then (38) implies that $\alpha_t + \upsilon_{it} = 0$. Substituting this into (37), we see that prices take the very simple form $p_{it} = g_i(z_{it}) + H_i h_i(z_{it}) = \gamma'_{it} z_{it}$, which is linearly homogeneous in characteristics (for fixed H_i). Jones obtains this result under arbitrary differences in the utility functions of individuals, provided that they depend only on the total consumption of characteristics, and any form of oligopoly pricing.

The restriction that $\alpha_t + \upsilon_{it} = 0$ in the above discussion is unusual, because it limits the form of marginal costs in (35'). In fact, this restriction is intimately related to the existence of a Nash equilibrium, when both g_i and h_i are homogeneous of degree one. To see this, note that for the quality-adjusted prices in (21a), the objective function for the firm in (15) can be rewritten as:

$$[\pi_i(q^*_{it}, z_{it}) - c_{it}(z_{it})] \left(\frac{\partial \pi_i(q^*_{it}, z_{it})}{\partial q_{it}} \right)^{-1} = \psi_i^{-1} \psi'_i + \psi'_i \left[\frac{g_i(z_{it}) - c_{it}(z_{it})}{h_i(z_{it})} \right] , \qquad (39)$$

where ψ_i^{-1} denotes the inverse function of ψ_i. The values of ψ_i^{-1} and ψ'_i depend on q^*_{it}, which is held constant in maximizing (39). Thus, the choice of characteristics is determined by the maximizing $[g_i(z_{it})-c_{it}(z_{it})]/h_i(z_{it})]$, and we have assumed it obtains an interior maximum in Proposition 8. However, when both g_i and h_i are homogenous of degree one, this condition will generally fail for the marginal costs in (35') unless $\alpha_t+\upsilon_{it}=0$.[15] Thus, the condition $\alpha_t+\upsilon_{it}=0$ is needed allow to for the existence of an interior Nash equilibrium, when both g_i and h_i are homogeneous of degree one.

We now consider the case of log-linear case marginal costs, as in (35). Making use of the approximation $(\ln p_{it}-\ln c_{it})\approx(p_{it}/c_{it})-1$ in (35), and substituting the equilibrium condition (16), the relation between prices and characteristics is:

$$\ln p_{it} \approx \alpha_t + \left(\frac{p_{it}}{c_{it}}\right)\gamma_{it}' z_{it} + \left(\frac{p_{it}}{c_{it}}-1\right)\left[1-\sum_{k=1}^{K}\frac{\gamma_{ikt}z_{ikt}}{\sigma_{ikt}}\right] + \upsilon_{it}, \tag{40}$$

where $\gamma_{it}\equiv\partial\ln\pi_i(q_{it},z_{it})/\partial\ln z_{it}$. The third term on the right-side of (40) is identical to the third term in (36), since the marginal values γ_{it} are now measured as an elasticity. Thus, this term again vanishes when the quality-adjusted prices take the form (21a) (with h_i homogeneous of degree one). In that case:

$$\ln p_{it} \approx \alpha_t + \left(\frac{p_{it}}{c_{it}}\right)\gamma_{it}' z_{it} + \upsilon_{it}. \tag{41}$$

The coefficients of the characteristics in (41) are $(p_{it}/c_{it})\partial\ln\pi_i(q_{it},z_{it})/\partial z_{it} = (1/c_{it})\partial\pi_i(q_{it},z_{it})/\partial z_{it}$, which measure the marginal value to the consumer *relative to the cost of the product.* This amount exceeds the marginal value relative to the product price - $\partial\ln\pi_i(q_{it},z_{it})/\partial z_{it}$ - by exactly the amount of the price-cost ratio. We have seen in the last section (Propositions 4 and 7) that the correct marginal value to use in the hedonic price index when prices are measured in logs is $\gamma_{it}\equiv\partial\ln\pi_i(q_{it},z_{it})/\partial z_{it}$. Thus, for the quality-adjusted prices in (21a) with h_i homogeneous of degree one, a log-linear specification for marginal cost leads to an

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15 When $\alpha_t+\upsilon_{it}=0$ in (35'), then it is immediate that $[g_i(z_{it})-c_{it}(z_{it})]/h_i(z_{it})]$ is homogeneous of degree zero in characteristics. In this case, the optimal characteristics are determined only up to a scalar multiple.

upward bias in the value of characteristics from the hedonic regression (41).[16]

While this results depends on a certain specification of the quality-adjusted prices, and therefore the individual utility functions, it is still of considerable interest given the common use of log-linear hedonic regressions in practice. The upward bias in the marginal values leads to a downward bias in the hedonic index when characteristics are growing. One method for checking for this bias would be to estimate the hedonic regression in both linear and log-linear form, and use the former to construct the bounds in Proposition 6, and the latter to construct the bounds in Proposition 7. If desired, a unique measure of the price index in each case could be obtained by taking the geometric mean of these bounds: for Proposition 6 this corresponds to a Fisher Ideal price index, and for Proposition 7 a Divisia index. Then if the bias we have identified is important, and characteristics are growing over time, we would expect the Fisher Ideal index to exceed the Divisia index. In this case, the former should be accepted as a preferred measure of the exact hedonic price index. If the Fisher Ideal and Divisia indexes are quite close, then the potential bias in the log-linear regression due to noncompetitive pricing would appear to be not too important.

6. CONCLUSIONS

We began this paper with the suggestion that, at best, hedonic regressions and associated hedonic price indexes should be viewed as an "approximation" to the true measure of consumer welfare. Since the paper has been long, it would be useful to look back and see what assumptions are needed to support this claim.

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[16] The reader might wonder about the case $g_i \equiv 0$, so that the quality-adjusted prices in (21a) take on the CES or Swan (1970) form, and the marginal cost and value of characteristics are equal in elasticity form. Then (41) continues to hold, but the choice of characteristics by firms satisfies $\gamma_{it}' z_{it} = 1$. Then (35) becomes $\ln c_{it} = \alpha_t + 1$ and (41) is $\ln p_{it} \approx \alpha_t + (p_{it}/c_{it})$, which leads us back to the approximation $\ln(p_{it}/c_{it}) \approx (p_{it}/c_{it}) - 1$. Since $\gamma_{it}' z_{it} = 1$ the characteristics in (41) are highly collinear, which would lead to large standard errors on estimated marginal values.

Proposition 1 provided us with an aggregate utility function that we could use to define an exact price index. The key condition imposed on the individual utility functions was they were identical up to an additive random term. This formulation allows for a general distribution of random terms across models - so that we are not restricted to the multinomial logit, for example - but rules out interactions between these random terms and individual characteristics. As noted, some of the most recent work in discrete choice models (e.g. Berry, Levinsohn and Pakes, 1995; Goldberg, 1995) have introduced such interaction terms, which is beyond the scope of Proposition 1.

Given this formulation of individual and aggregate utility, the link between the marginal cost and marginal value of characteristics was summarized in Proposition 2. In general, knowledge of the marginal costs of characteristics is insufficient to compute the marginal values: one also needs to know the price-cost markups and the elasticity σ_{ikt}. A simplification occurs in the competitive case, when firms price at the marginal cost of production, resulting in equality of the marginal cost and value of characteristics. If firms within some groups have the same marginal costs of characteristics, then these can be estimated from a hedonic regression, so that we also obtain the marginal values. Using these, we showed in section 4 that bounds on the exact hedonic price index could be constructed. Thus, in the competitive case, we have justified the use of hedonic methods with few assumptions beyond the additive random term and concavity properties of the individual utility functions, along with identical marginal costs of characteristics within groups of firms. While readers will want to judge the restrictiveness of these assumptions for themselves, they are weaker than what might have been expected.

With pricing above marginal costs, the situation is more complex, because the price-cost margins are an omitted variable in the hedonic regression. For a special class of quality-adjusted prices (or individual utility functions), we argued that a *linear* hedonic regression would still provide a measure of the marginal value of characteristics, but a *log-linear* regression would overstate these values. This result seems to be of some practical importance in view of the widespread use of log-linear regressions, such as in the measurement of the price index for computers (Cole *et al*, 1986, Cartwright, 1986). When characteristics are growing, the upward bias in the marginal values will lead to a downward bias in the hedonic price index. A practical method to check for this bias is to estimate the hedonic regression in both linear and log-linear form, construct the appropriate bounds on the exact hedonic index (as in Propositions 6 and 7), and compare these.

We conclude by commenting on the practice of imputing "missing" prices from the hedonic regression. Since many index number formulas require that a price is used for the period before the product first appears, the common practice has been to predict the "missing" price from the hedonic regression for that previous period, using the observed characteristics of that variety when it first appears. Indeed, Griliches (1990, p. 189) lists this "imputation method" as his preferred interpretation of the hedonic approach. The question is under what circumstances the resulting price index accurately reflects consumer preferences.

To answer this, let us first consider how the new product would affect the "time dummies", or fixed-weight index, discussed in section 5.1. If a new observation is added in period t, this will have only a minor effect on the "time dummies" estimated: $\hat{\alpha}_t$ will be lowered (raised) depending on whether the new product has a negative (positive) residual in the period t regression, and the impact on $\hat{\alpha}_t$ will be proportional to the weight the new product receives in the regression. Furthermore, if we added an artificial observation for the new product in period t-1, using the period t characteristics and the *predicted price* from the hedonic regression in period t-1, then this would have no further impact on the "time dummies" (since by construction, this artificial observation has a zero residual in period t-1).

We conclude that the fixed-weight index is affected only slightly by the addition of a new product, and in the same direction as the residual on the new product. Since we do not expect price indexes to be that sensitive to the weights that are used, the same will be true for a hedonic price index that uses endogenous rather than fixed weights: the imputation of "missing" prices will lower (raise) the price index when the new product has a negative (positive) residual in the hedonic regression. *If* the new product happens to fit the hedonic regression perfectly when it first appears, then it will have no impact on the price index. The economic interpretation of this result is that we are *implicitly treating the new product variety as a perfect substitute for existing varieties.* That is, the imputation method is guaranteed to have a small impact on the hedonic price index whenever the new product fits the regression closely when it first appears. This provides an approximate measure of consumer welfare if and only if the new product is a perfect substitute for an existing variety, in which case utility would unchanged by its introduction.

We can illustrate this argument for the case of desktop computers. New generations of these machines eliminate all purchases of the old generation, and for this reason, the generations could be treated as perfect substitutes after correcting for differences in their characteristics. Therefore, the imputation of "missing" prices by using the predicted price from the hedonic regression seems acceptable in this context. However, laptop and desktop computers should not be considered perfect substitutes, and the hedonic imputation method *would not* give an accurate measure of the welfare gain from introducing the first laptop, notebook, etc. In general, calculating the welfare gain or exact price index when new products *are not* perfect substitutes for existing ones, will require estimating at least some parameters of the underlying utility or expenditure function. Results along these lines for CAT scanners are contained in Trajtenberg (1989), for example. Estimating the expenditure function in the presence of new products is a topic we have not dealt with, and goes well beyond the conventional hedonic methods discussed in this paper.

APPENDIX

Proof of Proposition 1

(a) From the theorem of the generalized extreme value in McFadden (1978, p. 80; 1983, pp. 227-228), also discussed by Anderson, de Palma and Thisse (1992, p. 48), the probabilities P_i defined in (11) are given by:

$$P_i = \frac{\partial}{\partial u_i} \ln G\left(e^{u_1},\dots,e^{u_N}\right), \tag{A1}$$

where $u_i \equiv \ln\phi_0(y) - \ln\phi_i(p_i,z_i)$. Thus, $P_i = e^{u_i}G_i/G$ where G_i is the derivative of G with respect to its i^{th} argument. It follows using (12) that $X_i = Mx_iP_i$ equals:

$$X_i = M\left[\frac{\phi_0^2(\partial\phi_i/\partial p_i)G_i}{\phi_i^2\,\phi_0'G}\right]. \tag{A2}$$

Computing $-(\partial V/\partial p_i)/(\partial V/\partial Y)$ from (13), it is immediate that it equals (A2).

(b) The sum of expected maximized surplus over the M individuals is:

$$V^* = M\int_{R^N}\left(\max_{j=1,\dots,N} V_j\right)dF(\varepsilon) = M\ln\phi_0(Y/M) + M\int_{R^N}\left(\max_{j=1,\dots,N} \varepsilon_j - \ln q_j\right)dF(\varepsilon), \tag{A3}$$

where the second equality follows from the specification of V_j in (10). McFadden (1983, p. 227, eq. 5.36) demonstrates that the value of the integral on the right-side of (A3) differs from MlnG in (13) by only a constant. It follows that utility V in (13) differs from V^* in (A3) by only a constant.

(c) The assumption that $\ln\phi_i(p_i,z_i)$ in concave in p_i implies that V_i is convex in p_i, $i=1,\dots,N$. Then in (A3), the maximum of V_j will still be convex in $(p_1,\dots,p_N)$, and this property is also preserved by taking the integral. Then since V and V^* differ by only a constant, it follows that V in (13) is convex in $(p_1,\dots,p_N)$.

If we explicitly introduce the prices of the numeraire good, then the individual utility functions can be rewritten as:

$$V_i = \ln\phi_0(y/p_0) - \ln\phi_i(p_i/p_0,z_i) + \varepsilon_i, \qquad i=1,\ldots,N. \tag{A4}$$

Since $1/p_0$ is a convex function of p_0, if $-\ln\phi_i(p_i/p_0,z_i)$ is convex in its first argument then it is convex in both p_i and p_0. Again, this property is preserved by taking the maximum and the integral in the first line of (A3). Furthermore, when $\phi_0'>0$ as we have assumed, then $\ln\phi_0(y/p_0)$ will be quasi-convex in p_0. Since the sum of a convex and quasi-convex function is quasi-convex, it follows that V^* is quasi-convex in $(p_0,p_1,\ldots,p_N)$. The same property applies to V, which differs from V^* by only a constant. QED

Proof of Proposition 2

Follows directly from setting the first-derivative of (15) equal to zero.

Proof of Proposition 3

(a) Follows directly from setting $p^*_{it}=c_{it}(z^*_{it})$ in (16), and comparing it to (20).

(b) When quality-adjusted prices take the form in (21a), then the objective function in (15) can be rewritten as (39). The first-order condition for this problem is:

$$\frac{\partial g_i}{\partial z_{it}} - \frac{\partial c_{it}}{\partial z_{it}} = [g_i(z^*_{it})-c_{it}(z^*_{it})]\,\frac{\partial \ln h_i}{\partial z_{it}} \,. \tag{A5}$$

Since (A5) does not depend on the product price p_{it}, it will lead to an identical choice of characteristics for the competitive case, and for a firm with price exceeding marginal cost. Then it follows from part (a) that the characteristics satisfying (A5) are at the socially optimal level.

Setting $\phi_0(y)=y$ in (13), and using (18), the direct utility function corresponding to the quality-adjusted prices in (21a) is defined by:

$$U(X_{0t},X_t,a_t,b_t) \equiv \min_{p_{it}\geq 0}\left[M\ln G(\psi_{1t}^{-1},\ldots,\psi_{Nt}^{-1})+X_{0t}+\sum_{i=1}^{N} p_{it}X_{it}\right], \tag{A6}$$

where $a_{it}\equiv g_i(z_{it})$, $b_{it}\equiv h_i(z_{it})$, and $\psi_{it}^{-1}\equiv\psi_i[(p_{it}-a_{it})/b_{it}]^{-1}$. The first-order condition for (A6) is:

$$-M(G_i \psi_{it}' / G\psi_{it}^2 b_{it}) + X_{it} = 0, \tag{A7}$$

where G_i denotes the derivative of G with respect to its i^{th} argument. If p_i satisfies (A7) for the exogenously specified (a_{it}, b_{it}, X_{it}), then it is immediate that the value $\lambda_i p_i$ will satisfy (A7) for $(\lambda_i a_{it}, \lambda_i b_{it}, X_{it}/\lambda_i)$, $\lambda_i>0$, since the values of ψ_{it} and ψ_{it}' are unchanged. Furthermore, the values of the direct utility function in (A6) will unchanged for any $\lambda_i>0$. If we choose $\lambda_i=1/X_{it}$, $i=1,\ldots,N$, it follows that the direct utility function can be written as a function of $a_{it}X_{it}$ and $b_{it}X_{it}$, $i=1,\ldots,N$:

$$U(X_{0t}, X_t, a_t, b_t) = X_{0t} + \tilde{H}(a_{1t}X_{1t},\ldots,a_{Nt}X_{Nt}, b_{1t}X_{1t},\ldots,b_{Nt}X_{Nt}) \ . \tag{A8}$$

By applying the envelope theorem to (A6) and using (A8) we can compute:

$$\partial U/\partial a_{it} = M(G_i \psi_{it}' / G\psi_{it}^2 b_{it}) = \tilde{H}_i X_{it} \ , \tag{A9}$$

where $\tilde{H}_i$ denotes the derivative of $\tilde{H}$ with respect to its i^{th} argument. Comparing (A9) and (A7) it follows that $\tilde{H}_i \equiv 1$ for $X_{it}>0$, $i=1,\ldots,N$. Thus, the direct utility function in (A8) can be written as in (21b). QED

Proof of Proposition 4

We need to extend the price index of Sato (1976) in (23) to the case where $a_{it-1} \neq a_{it}$. Taking the difference in the log of the expenditure functions (22) and using the product demands in (8) we obtain:

$$\ln E_t - \ln E_{t-1} = \frac{1}{1+\alpha}\Big[(\ln U_t - \ln U_{t-1}) + \mu \ln(\tilde{s}_{it}/\tilde{s}_{it-1}) + \ln(q_{it}/q_{it-1})\Big], \tag{A10}$$

where $E_\tau \equiv E_c(p_{1\tau}/a_{1\tau},\ldots,p_{N\tau}/a_{N\tau}, U_\tau)$, $q_{i\tau} \equiv (p_{i\tau}/a_{i\tau})$, and $\tilde{s}_{i\tau} \equiv p_{i\tau}X_{i\tau}/M\tilde{y}$ is the expenditure share on product i relative to total expenditure on products $i=1,\ldots,N$, $\tau=t-1,t$. Since the numeraire good receives a constant fraction $1/(1+\alpha)$ of expenditure, then $\tilde{s}_{i\tau}/(1+\alpha) = s_{i\tau}$ is the expenditure share on product i relative to total expenditure on all products, including the numeraire. Thus, $\ln(\tilde{s}_{it}/\tilde{s}_{it-1}) = \ln(s_{it}/s_{it-1})$ so we can use the latter term in (A10). Multiplying (A10) by the weights ω_{it} defined in (23b), and summing over $i=1,\ldots,N$, we obtain:

$$\ln E_t - \ln E_{t-1} = \frac{(\ln U_t - \ln U_{t-1})}{1+\alpha} + \frac{1}{1+\alpha}\sum_{i-1}^{N}\omega_{it}\Big[\mu\ln(s_{it}/s_{it-1}) + \ln(q_{it}/q_{it-1})\Big]. \qquad \text{(A11)}$$

The first term in the above summation is zero, because $\omega_{it}\ln(s_{it}/s_{it-1})$ is proportional to Δs_{it}, which sums to zero since $\sum_{i=1}^{N} s_{i\tau}=1/(1+\alpha)$, $\tau=t-1,t$. Furthermore, evaluating $E_\tau \equiv E_c(p_{1\tau}/a_{1\tau},\ldots,p_{N\tau}/a_{N\tau},U_\tau)$ at any other utility level U will have no impact on the product shares $s_{i\tau}$, $i=1,\ldots,N$, because the utility function is homothetic. Thus, taking the exponent of (A11) we obtain:

$$\frac{E_c(p_{1t}/a_{1t},\ldots,p_{Nt}/a_{Nt},U)}{E_c(p_{1t-1}/a_{1t-1},\ldots,p_{Nt-1}/a_{Nt-1},U)} = \prod_{i-1}^{N}\left(\frac{p_{it}/a_{it}}{p_{it-1}/a_{it-1}}\right)^{\omega_{it}/(1+\alpha)}.$$

Setting $a_t=a_{t-1}$ we obtain (23). With $a_t \neq a_{t-1}$, we can use the inequalities in (24) to establish Proposition 4. QED

<u>Proof of Proposition 5</u>

We need to establish (27), and also extend this price index to the case where $a_{it-1} \neq a_{it}$. Taking the difference in the expenditure functions (26) and using the product demands in (3) we obtain:

$$E_t - E_{t-1} = (U_t - U_{t-1}) + M\mu\ln(X_{it}/X_{it-1}) + M(q_{it}-q_{it-1}), \qquad \text{(A12)}$$

where $E_\tau \equiv E_m(p_\tau - a_\tau, U_\tau)$ and $q_{i\tau} \equiv (p_{i\tau}-a_{i\tau})$, $\tau=t-1,t$. Multiplying (A12) by the weights ω_{it} defined in (27b), and summing over $i=1,\ldots,N$, we obtain:

$$E_t - E_{t-1} = (U_t - U_{t-1}) + \sum_{i-1}^{N}\omega_{it}[M\mu\ln(X_{it}/X_{it-1}) + M(q_{it}-q_{it-1})]. \qquad \text{(A13)}$$

The first term in the above summation is zero, because $\omega_{it}\ln(X_{it}/X_{it-1})$ is proportional to ΔX_{it}, which sums to zero since $\sum_{i=1}^{N} X_{i\tau}=M$, $\tau=t-1,t$. Furthermore, evaluating $E_t \equiv E_m(p_t - a_t, U_t)$ at the utility level U_{t-1} will have no impact on the product demands X_{it}, i=1,...,N, because of the additively separable numeraire good. Thus, we can divide (A13) by $Y_{t-1} \equiv E_{t-1}$ and obtain:

$$\frac{E_m(p_t - a_t, U_{t-1})}{E_m(p_{t-1} - a_{t-1}, U_{t-1})} = 1 + \sum_{i=1}^{N} \left(\frac{\omega_{it}}{Y_{t-1}} \right) [(p_{it} - a_{it}) - (p_{it-1} - a_{t-1})].$$

Setting $a_t = a_{t-1}$ we obtain (27). With $a_t \neq a_{t-1}$, we can use the inequalities in (28) to establish Proposition 5. QED

<u>Proof of Proposition 6</u>

Using the assumption that $\ln\phi_i(p_{it}, z_{it})$ is concave in p_{it}, i=1,...,N, the concavity of the expenditure function $E(p_t, z_t, U_t)$ in p_t is established as in footnote 11.

Define ${}_{t-1}\tilde{p}_{it} \equiv \pi_i(q_{it}, z_{it-1})$ as the hypothetical price in period t that would yield the same quality-adjusted prices as actually obtained, but with the characteristics z_{it-1}. Similarly, we define ${}_t\tilde{p}_{it-1} \equiv \pi_i(q_{it-1}, z_{it})$. Then letting $\gamma_{it} \equiv \partial\pi_i(q_{it}, z_{it})/\partial z_{it}$ denote the vector of marginal values, the concavity of $\pi_i(q_{it}, z_{it})$ in z_{it} implies:

$$\begin{aligned} {}_{t-1}\tilde{p}_t &\equiv \pi_i(q_{it}, z_{it-1}) \leq \pi_i(q_{it}, z_{it}) - \gamma'_{it}(z_{it} - z_{it-1}), \text{ and} \\ {}_t\tilde{p}_{t-1} &\equiv \pi_i(q_{it-1}, z_{it}) \leq \pi_i(q_{it-1}, z_{it-1}) + \gamma'_{it-1}(z_{it} - z_{it-1}). \end{aligned} \tag{A14}$$

Notice that keeping the quality-adjusted prices q_{it} constant in the definition of ${}_{t-1}\tilde{p}_t$ ensures that $E(p_t, z_t, U) = E({}_{t-1}\tilde{p}_t, z_{t-1}, U)$ for any utility level U, since the expenditure function in (30) depends on $q_{it} = \phi_i(p_{it}, z_{it}) = \phi_i({}_{t-1}\tilde{p}_{it}, z_{it-1})$. It follows that:

$$\begin{aligned} E(p_t, z_t, U_{t-1}) - E(p_{t-1}, z_{t-1}, U_{t-1}) &= E({}_{t-1}\tilde{p}_t, z_{t-1}, U_{t-1}) - E(p_{t-1}, z_{t-1}, U_{t-1}) \\ &\leq \sum_{i=1}^{N} X_{it-1}\left({}_{t-1}\tilde{p}_t - p_{t-1}\right) \\ &\leq \sum_{i=1}^{N} X_{it-1}[(p_t - p_{t-1}) - \gamma'_{it}(z_{it} - z_{it-1})] . \end{aligned} \tag{A15}$$

The second line in (A15) follows from the concavity of the expenditure function, where $X_{it-1} = \partial E(p_{t-1}, z_{t-1}, U_{t-1})/\partial p_{it-1}$, and the third line follows from the first inequality in (A14). Dividing (A15) by $E(p_{t-1}, z_{t-1}, U_{t-1}) = X_{0t-1} + \sum_{i=1}^{N} p_{it-1} X_{it-1}$, we obtain Proposition 6(a). Part (b) can be proved in a similar fashion. QED

Proof of Proposition 7

The assumption that $\ln\phi_i(p_{it}, z_{it})$ is concave in $\ln p_{it}$ means that the individual function V_i in (10) are convex in $\ln p_{it}$. It follows that V^* defined in (A3) of the proof of Proposition 1 in convex in $\ln p_{it}$, and since V^* and V differ by only a constant, V in also convex in $\ln p_{it}$, $i=1,\dots,N$. Setting $\phi_0(y)=\phi_0^{-1}(y)=y$ in (30), and taking the log of the expenditure function, we obtain:

$$\ln E(p_t, z_t, U_t) = \ln(MU_t) - \ln G\left[\phi_1(p_{1t}, z_{1t})^{-1}, \dots, \phi_N(p_{Nt}, z_{Nt})^{-1}\right]. \qquad (A16)$$

Comparing this with the indirect utility function in (13), it is immediate that the convexity of V in $\ln p_{it}$ implies that $\ln E(p_t, z_t, U_t)$ in concave in $\ln p_{it}$, $i=1,\dots,N$.

We define ${}_{t-1}\tilde{p}_{it} \equiv \pi_i(q_{it}, z_{it-1})$ and ${}_{t}\tilde{p}_{it-1} \equiv \pi_i(q_{it-1}, z_{it})$ as in the proof of Proposition 6. Then letting $\gamma_{it} \equiv \partial \ln\pi_i(q_{it}, z_{it})/\partial z_{it}$ denote the vector of marginal values, the concavity of $\ln\pi_i(q_{it}, z_{it})$ in z_{it} implies:

$$\begin{aligned} \ln({}_{t-1}\tilde{p}_t) &\equiv \ln\pi_i(q_{it}, z_{it-1}) \leq \ln\pi_i(q_{it}, z_{it}) - \gamma'_{it}(z_{it} - z_{it-1}), \text{ and} \\ \ln({}_{t}\tilde{p}_{t-1}) &\equiv \ln\pi_i(q_{it-1}, z_{it}) \leq \ln\pi_i(q_{it-1}, z_{it-1}) + \gamma'_{it-1}(z_{it} - z_{it-1}). \end{aligned} \qquad (A17)$$

Keeping the quality-adjusted prices q_{it} constant in the definition of ${}_{t-1}\tilde{p}_t$ ensures that $\ln E(p_t, z_t, U) = \ln E({}_{t-1}\tilde{p}_t, z_{t-1}, U)$ for any utility level U, since the expenditure function in (A16) depends on $q_{it} = \phi_i(p_{it}, z_{it}) = \phi_i({}_{t-1}\tilde{p}_{it}, z_{it-1})$. It follows that:

$$\begin{aligned} \ln E(p_t, z_t, U) - \ln E(p_{t-1}, z_{t-1}, U) &= \ln E({}_{t-1}\tilde{p}_t, z_{t-1}, U) - \ln E(p_{t-1}, z_{t-1}, U) \\ &\leq \sum_{i=1}^{N} s_{it-1}[\ln({}_{t-1}\tilde{p}_t) - \ln(p_{t-1})] \qquad (A18) \\ &\leq \sum_{i=1}^{N} s_{it-1}[\ln(p_t/p_{t-1}) - \gamma'_{it}(z_{it} - z_{it-1})] \, . \end{aligned}$$

The second line in (A18) follows from the concavity of the expenditure function in $\ln p_{it}$, where $s_{it-1}=\partial \ln E(p_{t-1},z_{t-1},U)/\partial \ln p_{it-1}$, which is independent of the utility level U with homothetic preferences. The third line follows from the first inequality in (A17). Taking the exponent of (A18) we obtain the right-side inequality of Proposition 7, and the left-side is proved in a similar fashion. QED

Proof of Proposition 8

We need to show that the third term on the right-side of (36) vanishes for the quality-adjusted prices in (21a), with h_i homogeneous of degree zero. Using the definition of σ_{ikt} in Proposition 2, the third term in (36) vanishes if and only if:

$$\frac{\partial \pi_i}{\partial q_{it}} = \sum_{k=1}^{K} \frac{\partial^2 \pi_i}{\partial q_{it} \partial z_{ikt}} z_{ikt} \ . \tag{A19}$$

This equality holds if and only if $\partial \pi_i/\partial q_{it}$ is homogeneous of degree one in z_{it}. This implies that $\pi_i(q_{it},z_{it})$ must be of the form:

$$\pi_i(q_{it},z_{it}) = \tilde{\pi}_i(q_{it},z_{it}) + g_i(z_{it}) \ , \tag{A20}$$

where $\tilde{\pi}_i(q_{it},z_{it})$ is homogeneous of degree one in z_{it}. Inverting (A20), we see that the quality-adjusted prices must be of the general form:

$$\phi_i(q_{it},z_{it}) = \tilde{\phi}_i[p_{it} - g_i(z_{it}),z_{it}] \ , \tag{A21}$$

where $\tilde{\phi}_i$ is homogeneous of degree zero in its N+1 arguments. A special case of the quality-adjusted prices in (A21) are those in (21a), with the restriction that h_i is homogeneous of degree one.

Equation (37) now follows directly from (36). The prices $p_{it}=g_i(z_{it})+H_i h_i(z_{it})$ are obtained by differentiating the direct utility function in (21b) with respect to X_{it}, i=1,...,N. Alternatively, we can invert the quality-adjusted prices in (21a) to obtain $p_{it} = g_i(z_{it})+\psi_i^{-1}(q_{it})h_i(z_{it})$. It follows that $H_i=\psi_i^{-1}(q_{it})$. Thus, in computing the marginal values $\gamma_{it}=\partial g_i/\partial z_{it}+H_i \partial h_i/\partial z_{it}$ we keep $H_i=\psi_i^{-1}(q_{it})$ constant. QED

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