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EXACT SOLUTION OF MHD MIXED CONVECTION PERIODIC FLOW IN A ROTATING VERTICAL CHANNEL WITH HEAT RADIATION

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Magnetohydrodynamic (MHD) mixed convection flow of a viscous, incompressible and electrically conducting fluid in a vertical channel is analyzed analytically. A magnetic field of uniform strength is applied perpendicular to the planes of the channel walls. The fluid is acted upon by a periodic variation of the pressure gradient in the vertically upward direction. The temperature of one of the plates is non-uniform and the temperature difference of the walls of the channel is high enough to induce heat transfer due to radiation. The fluid and the channel rotate in unison with an angular velocity about the axis normal to the plates of the channel. An exact analytical solution of the problem is obtained. Two cases of small and large rotation have been considered to assess the effects of different parameters involved in the flow problem. The velocity field, the amplitude and the phase angle of the shear stress are shown graphically and discussed in detail. During analysis it is found that the flow problem studied by Makinde and Mhone (2005) is incorrect physically and mathematically.

Key words: periodic flow, mixed convection, magnetohydrodynamic (MHD), rotating system, heat radiation.

1. Introduction

The hydrodynamic rotating flow of electrically conducting viscous incompressible fluids has gained considerable attention because of its numerous applications in physics and engineering. In geophysics it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth which rotates with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics nowadays has become an important branch of fluid dynamics due to the increasing interest to study the environment. In astrophysics it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms and flares, etc. In engineering it finds its application in MHD generators, ion propulsion, MHD bearings, MHD pumps, MHD boundary layer control of reentry vehicles etc. Several scholars viz. Crammer and Pai (1973), Ferraro and Plumpton (1966), Shercliff (1965) have studied such flows on account of their varied importance. MHD channel or duct flows are important from the practical point of view. Chang and Lundgren (1961) studied a hydromagnetic flow in a duct. Yen and Chang (1964) analyzed the effect of wall electrical conductance on the magnetohydrodynamic Couette flow. Attia and Kotb (1996) investigated on MHD flow between two parallel porous plates.

In recent years a number of studies have also appeared in the literature on the fluid phenomena on earth involving rotation to a greater or lesser extent, viz. Vidyanidhu and Nigam (1967) Gupta (1972) Jana and Datta (1977). An unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid in a rotating system is analyzed by Seth *et al.* (1982). Chandran *et al.* (1993) studied the rotational effect on an unsteady hydromagnetic Couette flow. The transient effect on a magnetohydrodynamic Couette flow with rotation is investigated by Singh *et al.* (1994). An exact solution of an oscillatory Couette flow in rotating system in the presence of transverse magnetic field has been investigated by Singh (2000). Prasad Rao *et al.* (1982) studied the combined effect of free and forced convection on an MHD flow in a rotating porous channel. Soundalgekar and Pop (1973) analyzed a hydromagnetic flow in a rotating fluid past an infinite porous plate. Singh *et al.* (2005) studied a periodic solution of an oscillatory Couette flow through a porous medium in a rotating system. Transient effects on a magnetohydrodynamic Couette flow with rotation such as: accelerated motion is analyzed by Singh *et al.* (1994). Singh and Kumar (2009) studied theoretically the combined effects of rotation and Hall current on free convection of an MHD flow in a vertical porous channel. Singh and Garg (2010) also obtained an exact solution of an oscillatory free convection MHD flow in a rotating channel in the presence of heat transfer due to radiation. Makinde and Mhone (2005) investigated the combined effects of the transverse magnetic field and radiative heat transfer in an unsteady flow of a conducting optically thin fluid through a channel filled with a porous medium. Singh (2011) obtained an exact solution of an oscillatory MHD flow in a channel filled with a porous medium. Singh and Reena (2010) conducted an analysis of an oscillatory rotating MHD Poiseuille flow with injection/suction and Hall currents.

The purpose of the present analysis is to study an oscillatory mixed convection of an electrically conducting viscous incompressible flow in a vertical channel. The entire system rotates about an axis perpendicular to the planes of the plates of the channel and a uniform magnetic field is also applied along this axis of rotation. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is neglected. During mathematical analysis it is found that the study presented by Makinde and Mhone (2005) is incorrect.

2. Formulation of the problem

Consider the flow of a viscous, incompressible and electrically conducting fluid in a rotating vertical channel as shown in Fig.1.

$$z^* = -\frac{d}{2}, \quad u^* = v^* = 0, \quad T^* = 0, \quad z^* = \frac{d}{2}, \quad u^* = v^* = 0, \quad T^* = T_0 \cos\omega^* t^* .$$

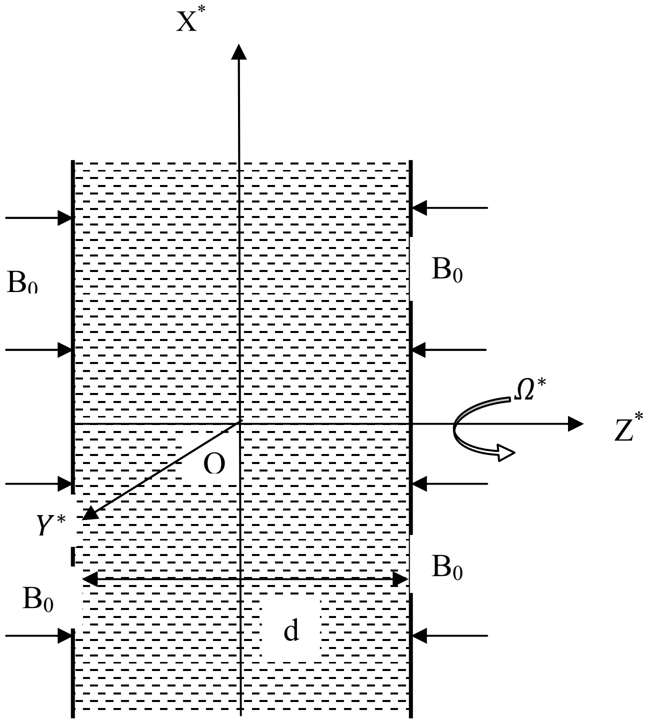

Fig.1. Physical configuration of the physical problem.

In order to derive basic equations for the problem under consideration the following assumptions are made:

(I) The flow considered is unsteady and laminar.
(II) The fluid is finitely conducting and with constant physical properties.
(III) A magnetic field of uniform strength is applied normal to the flow.
(IV) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(V) Hall effect, electrical and polarization effects are neglected.
(VI) It is assumed that the fluid is optically thin with relatively low density.
(VII) The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.
(VIII) Since the plates are infinite so all physical quantities except pressure depend only on z^* and t^*.

Under these assumptions, we write hydromagnetic equations of continuity, motion and energy in a rotating frame of reference as

$$\nabla \cdot \boldsymbol{V} = 0, \tag{2.1}$$

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V} \cdot \nabla V) + 2\boldsymbol{\Omega} \cdot \boldsymbol{V} = -\frac{1}{\rho}\nabla p^* + \vartheta \nabla^2 \boldsymbol{V} + \frac{1}{\rho}(\boldsymbol{J} \cdot \boldsymbol{B}) + \boldsymbol{F}, \tag{2.2}$$

$$\rho c_p \frac{\partial T^*}{\partial t^*} = k \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\partial q}{\partial z^*}. \tag{2.3}$$

In Eq.(2.2) the last term on the left hand side is the Coriolis force. On the right hand side of Eq.(2.2) the last term $\boldsymbol{F}\left(= g\beta T^*\right)$ accounts for the force due to buoyancy and the second last term is the Lorentz force due to magnetic field $\boldsymbol{B}$ and is given by

$$\boldsymbol{J} \times \boldsymbol{B} = \sigma\left(\boldsymbol{V} \times \boldsymbol{B}\right) \times \boldsymbol{B}, \tag{2.4}$$

and the modified pressure, $p^* = p' - \frac{\rho}{2}\left(\boldsymbol{\Omega} \cdot \boldsymbol{R}\right)^2$, where $\boldsymbol{R}$ denotes the position vector from the axis of rotation, p' denotes the fluid pressure, $\boldsymbol{J}$ is the current density and all other quantities have their usual meanings and have been defined in the text. Following Cogley *et al.* (1968) the last term in the energy Eq.(2.3) stands for the radiative heat flux, which is given by

$$\frac{\partial q}{\partial z^*} = 4\alpha^2 T^*. \tag{2.5}$$

In the present analysis we consider an unsteady flow of a viscous incompressible and electrically conducting fluid bounded by two infinite insulated vertical plates distance 'd' apart. A coordinate system is chosen such that the X^* -axis is oriented upward along the centerline of the channel and the Z^*-axis is taken perpendicular to the planes of the plates lying in $z^* = \pm\frac{d}{2}$ planes. The non-uniform temperature of the plate at $z^* = +\frac{d}{2}$ is assumed to be varying periodically with time. The Z^*- axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity Ω^*. A transverse magnetic field of uniform strength $\boldsymbol{B}$ $(0, 0, B_0)$ is also applied along the axis of rotation. The velocity may reasonably be assumed with its components along x^*, y^*, z^* directions as $\boldsymbol{V}(u^*, v^*, 0)$. The equation of continuity is then satisfied identically. Using the velocity and the magnetic field distribution as stated above the magnetohydrodynamic (MHD) flow in the rotating channel is governed by the following Cartesian equations

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\frac{\partial^* u}{\partial z^{*2}} + 2\Omega^* v^* - \frac{\sigma B_0{}^2}{\rho}u^* + g\beta T^*, \tag{2.6}$$

$$\frac{\partial v^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta\frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho}v^*, \tag{2.7}$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}, \tag{2.8}$$

$$\rho c_p \frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial z^{*2}} - 4\alpha^2 T^* \tag{2.9}$$

where ρ is the density ϑ is the kinematic viscosity, p^* is the modified pressure, t^* is the time, σ is the electric conductivity, g is the acceleration due to gravity, k is the thermal conductivity, c_p is the specific heat at constant pressure and α is the mean radiation absorption coefficient. Equation (2.8) shows the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flows under the influence of the pressure gradient varying periodically with time in the X^*-axis which is of the form

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = A\cos\omega^* t^* \tag{2.10}$$

where A is a constant.

The boundary conditions for the problem are

$$z^* = \frac{d}{2}: \qquad u^* = v^* = 0, \qquad T^* = T_0\cos\omega^* t^*, \tag{2.11}$$

$$z^* = -\frac{d}{2}: \qquad u^* = v^* = 0, \qquad T^* = 0 \tag{2.12}$$

where T_0 is the mean temperature and ω^* is the frequency of oscillations.

Introducing the following non-dimensional quantities into Eqs (2.4) and (2.5)

$$\eta = \frac{z^*}{d}, \qquad x = \frac{x^*}{d}, \qquad y = \frac{y^*}{d}, \qquad u = \frac{u^*}{U}, \qquad v = \frac{v^*}{U},$$
$$T = \frac{T^*}{T_0}, \qquad t = \frac{t^* U}{d}, \qquad \omega = \frac{\omega^* d}{U}, \qquad p = \frac{p^*}{\rho U^2}, \tag{2.13}$$

we get

$$\mathrm{Re}\frac{\partial u}{\partial t}=-\mathrm{Re}\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial \eta^2}+2\Omega v-M^2u+\mathrm{Gr}T, \tag{2.14}$$

$$\mathrm{Re}\frac{\partial v}{\partial t}=-\mathrm{Re}\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial \eta^2}-2\Omega u-M^2v, \tag{2.15}$$

$$\mathrm{Pe}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial \eta^2}-N^2T \tag{2.16}$$

where U is the mean axial velocity, '*' represents the dimensional physical quantities,

$\mathrm{Re}=\dfrac{Ud}{\vartheta}$ is the Reynolds number,

$\Omega=\dfrac{\Omega^* d^2}{\vartheta}$ is the rotation parameter,

$\mathrm{M}=B_0 d\sqrt{\dfrac{\sigma}{\mu}}$ is the Hartmann number,

$\mathrm{Gr}=\dfrac{g\beta d^2 T_0}{\vartheta U}$ is the Grashof number,

$\mathrm{Pe}=\dfrac{k}{\rho c_p dU}$ is the Peclet number,

$N=\dfrac{2\alpha d}{\sqrt{k}}$ is the radiation parameter.

The boundary conditions in the dimensionless form become

$$\eta=\frac{1}{2}: \qquad u=v=0, \qquad T=\cos\omega t, \tag{2.17}$$

$$\eta=-\frac{1}{2}: \qquad u=v=0, \qquad T=0. \tag{2.18}$$

For the oscillatory internal flow we shall assume that the fluid flows only under the influence of a non-dimension pressure gradient oscillating in the direction of the x-axis only which is of the form

$$-\frac{\partial p}{\partial x}=A\cos\omega t. \tag{2.19}$$

3. Solution of the problem

Now combining Eqs (2.14) and (2.15) into a single equation by introducing a complex function of the form $F = u + iv$, we get

$$\mathrm{Re}\frac{\partial F}{\partial t} = \mathrm{Re}A\cos\omega t + \frac{\partial^2 F}{\partial\eta^2} - \left(\mathrm{M}^2 + 2i\Omega\right)F + \mathrm{Gr}T, \tag{3.1}$$

with corresponding boundary conditions as

$$\eta = \frac{1}{2}: \qquad F = 0, \qquad T = \cos\omega t, \tag{3.2}$$

$$\eta = -\frac{1}{2}: \qquad F = 0, \qquad T = 0. \tag{3.3}$$

In order to solve Eqs (3.1) and (2.16) under boundary conditions Eqs (3.2) and (3.3) we assume in complex notations the solution of the problem as

$$F(\eta,t) = F_0(\eta)e^{i\omega t}, \qquad T = \theta_0(\eta)e^{i\omega t} - \frac{\partial y}{\partial x} = Ae^{i\omega t}. \tag{3.4}$$

The boundary conditions Eqs (3.2) and (3.3) in complex notations can also be written as

$$\eta = \frac{1}{2}: \qquad F = 0, \qquad T = e^{i\omega t}, \tag{3.5}$$

$$\eta = -\frac{1}{2}: \qquad F = 0, \qquad T = 0. \tag{3.6}$$

Substituting these expressions in Eqs (2.16) and (3.1), we get

$$\frac{d^2F_0}{d\eta^2} - m^2F_0 = -\mathrm{Re}\,A - \mathrm{Gr}\,\theta_0, \tag{3.7}$$

$$\frac{d^2\theta_0}{d\eta^2} - n^2\theta_0 = 0 \tag{3.8}$$

where

$$m = \sqrt{\mathrm{M}^2 + 2i\Omega + i\omega\mathrm{Re}}, \qquad n = \sqrt{N^2 + i\omega\mathrm{Pe}}.$$

The transformed boundary conditions reduce to

$$\eta = \frac{1}{2}: \qquad F_0 = 0, \qquad \theta_0 = 1, \tag{3.9}$$

$$\eta = -\frac{1}{2}: \qquad F_0 = 0, \qquad \theta_0 = 1. \tag{3.10}$$

The ordinary differential Eqs (3.7) and (3.8) are solved under the boundary conditions Eqs (3.9) and (3.10) for the velocity and temperature fields. The solution of the problem is obtained as

$$u(\eta,t) = \left[\frac{\operatorname{Re} A}{m^2}\left(1 - \frac{\cosh m\eta}{\cosh\frac{1}{2}}\right) + \frac{\operatorname{Gr}}{\left(n^2 - m^2\right)}\left\{\frac{\sin m\left(\eta + \frac{1}{2}\right)}{\sinh m} - \frac{\sinh n\left(\eta + \frac{1}{2}\right)}{\sinh n}\right\}\right] e^{i\omega t}, \tag{3.11}$$

$$T(\eta,t) = \frac{\sin n\left(\eta + \frac{1}{2}\right)}{\sinh n} e^{i\omega t}. \tag{3.12}$$

The validity and correctness of the present solution is verified by taking Gr = Ω = M =*0* i.e. for the horizontal channel in the absence of rotation and the magnetic field. In this case the solution reduces to

$$u(\eta,t) = \frac{A}{i\omega}\left(1 - \frac{\cosh\sqrt{i\omega\operatorname{Re}}\,\eta}{\cosh\frac{\sqrt{i\omega\operatorname{Re}}}{2}}\right) e^{i\omega t}, \tag{3.13}$$

which is the well known solution reported by Schlichting and Gersten (2001) for a periodic variation of the pressure gradient along the axis of the channel.

From the velocity field we can now obtain the skin-friction τ_L at the left plate in terms of its amplitude and phase angle as

$$\tau_L = \left(\frac{\partial u}{\partial \eta}\right)_{\eta = -\frac{1}{2}} = |F|\cos(t + \varphi), \tag{3.14}$$

with
$$F_r + iF_i = \frac{\operatorname{Re} A}{m}\tanh\frac{m}{2} + \frac{\operatorname{Gr}}{\left(n^2 - m^2\right)}\left\{\frac{m}{\sinh m} - \frac{n}{\sinh n}\right\}. \tag{3.15}$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle $\varphi = \tan^{-1}\frac{F_i}{F_r}$. (3.16)

From the temperature field given in Eq.(3.12) the heat transfer coefficient Nu (Nusselt number) in terms of its amplitude and the phase angle can be obtained as

$$\operatorname{Nu} = \left(\frac{\partial T}{\partial \eta}\right)_{\eta = -\frac{1}{2}} = |H|\cos\left(\omega t + \psi\right) \tag{3.17}$$

where $$H_r + iH_i = \frac{n}{\sinh(n)}. \tag{3.18}$$

The amplitude $|H|$ and the phase angle ψ of the heat transfer coefficient Nu (Nusselt number) are given by

$$|H| = \sqrt{H_r^2 + H_i^2} \qquad \text{and} \qquad \psi = \tan^{-1}\left(\frac{H_i}{H_r}\right) \qquad \text{respectively.} \tag{3.19}$$

During mathematical analysis it is found that the mathematical formulation of the problem by Makinde and Mohne (2005) is not in consistency with the geometry of the physical problem shown in their Fig.1. Geometrically the channel is horizontal whereas the mathematical formulation is for the vertical channel where the Boussinesq incompressible fluid model is assumed to include buoyancy force in momentum Eq.(2.1). Boundary conditions Eqs (2.3) and (2.4) are not according to the choice of the Cartesian coordinate system with *Ox*-axis lying along the centerline of the channel. The temperatures of the walls are also not non-uniform as mentioned in the abstract. For this purely oscillatory flow the boundary conditions Eqs (2.13) and (2.14) cannot be obtained after the substitution of Eq.(2.11) into the boundary conditions Eqs (2.9) and (2.10). The energy Eq.(2.8) is incorrect and its solution Eq.(2.15) obtained under wrong boundary conditions Eqs (2.13) and (2.14) is obviously incorrect. This solution is further used in the equation of motion Eq.(2.12) which consequently yields a wrong solution again. For $\Omega = 0$ in Eq.(3.11) of the present analysis gives the correct form of the velocity distribution and Eq.(3.12) gives the correct form of temperature distribution of the problem by Makinde and Mhone (2005) for the case of an ordinary medium.

4. Discussion of the results

The hydrodynamic mixed convection flow in an infinite vertical channel is analyzed when the entire system rotates about an axis perpendicular to the planes of the plates. In the presence of a transverse magnetic field an exact solution of the problem is obtained. The velocity field and Ω, the shear stress in terms of its amplitude and phase angle are evaluated numerically for different sets of the values of rotation parameter Ω, Reynolds number Re, Hartmann number M, pressure gradient A, Grashof number Gr, Peclet number Pe, radiation parameter N and the frequency of oscillations ω. These numerical values are then shown graphically to assess the effect of each parameter for the two cases of small ($\Omega = 5$) and large ($\Omega = 20$) rotations.

Figure 2 illustrates the variation of the velocity with the increasing rotation of the system. It is quite obvious from this figure that velocity goes on decreasing with increasing rotation Ω of the entire system. The velocity profiles initially remain parabolic with maximum at the centre of the channel for small values of the rotation parameter Ω and then as rotation increases the velocity profiles flatten. For a further increase in Ω (= *20*) the maximum of velocity profiles no longer occurs at the centre but shifts towards the walls of the channel. It means that for large rotation there arise boundary layers on the walls of the channel. The variation of the velocity profiles with the Reynolds number Re is presented in Fig.3. Two cases of small rotation (Ω=*5*) and (Ω=*20*) are considered to ascertain the effect of the Reynolds number. For small Ω (=*5*) velocity goes on increasing with increasing Re and remains parabolic with the maximum at the centerline. However, for large Ω (=*20*) although velocity increases with increasing Re but the maximum of velocity shifts towards the walls of the channel. From Fig.4 it is evident that velocity goes on increasing with the increasing favorable pressure gradient A (>0). The velocity profiles for small rotation (Ω=*5*) remain parabolic with the maximum at the centerline. But for large rotation (Ω=*20*) the maximum of velocity shift towards the walls of the channel.

The effects of the magnetic field on the velocity field are depicted in Fig.5. It is observed that for both cases of small (Ω=*5*) and large (Ω=*20*) rotations velocity increases with the increasing Hartmann number M.

This means that the increasing Lorentz force due to increasing magnetic field strength resists the backward flow caused by the rotation of the system. The variations of the velocity profiles with the Grashof number Gr are presented in Fig.6. For small rotations (Ω=*5*) velocity increases with the increasing Grashof number.

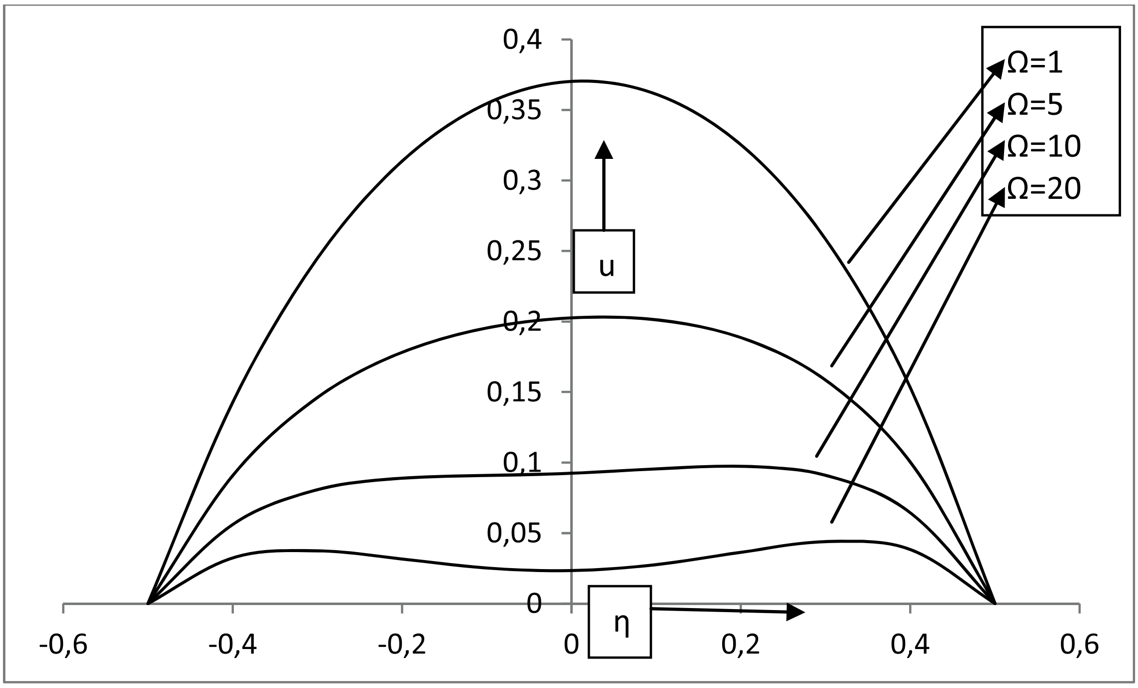

Fig.2. Velocity profiles for Re=*1*, *A*=*5*, M=*2*, Gr=*1*, Pe=*0.7*, *N*=*1*, ω =5 and *t*=*0*.

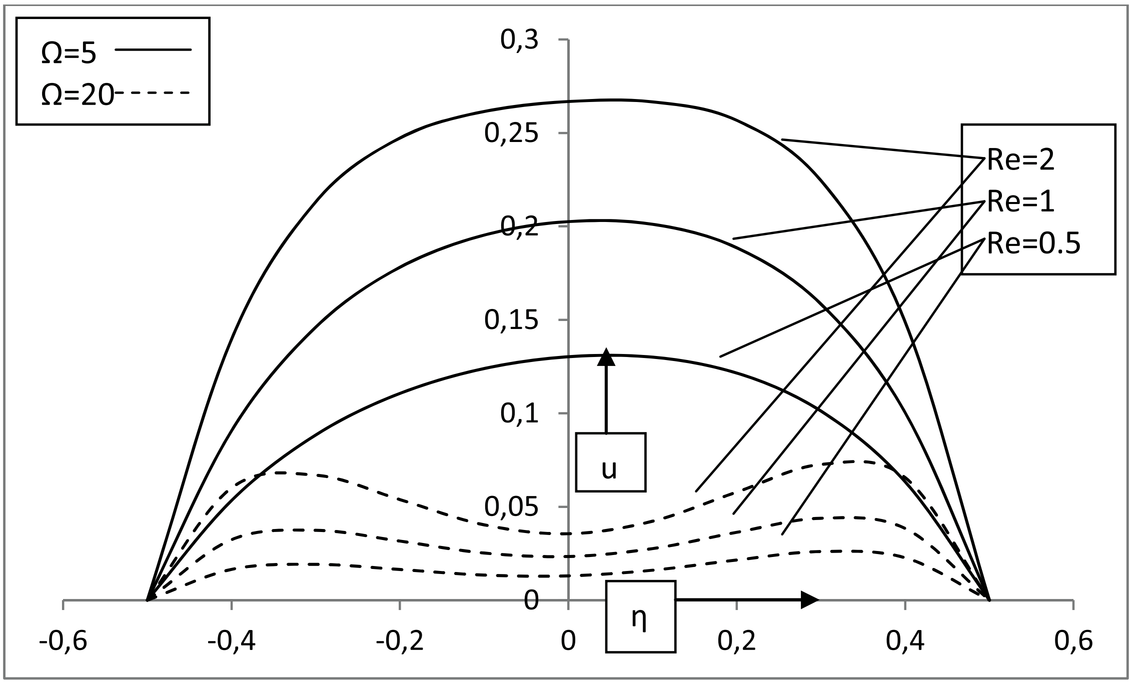

Fig.3. Velocity profiles for *A*=*5*, M=*2*, Gr=*1*, Pe=*0.7*, *N*=*1*, ω =5 and *t*=*0*.

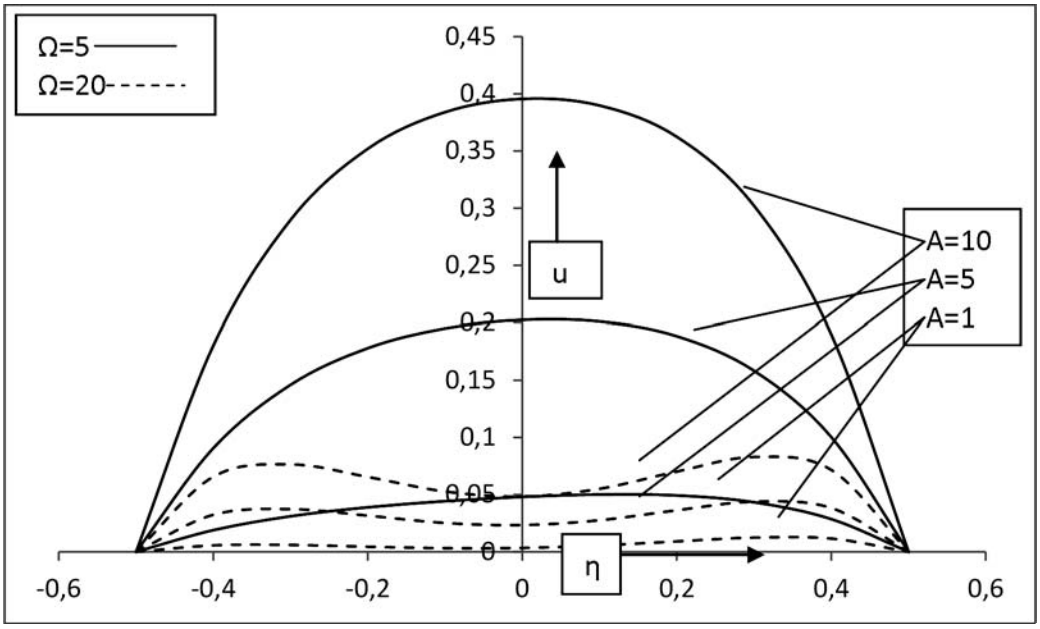

Fig.4. Velocity profiles for Re=*1*, M=*2*, Gr=*1*, Pe=*0.7*, *N=1*, ω =*5* and *t=0*.

The maximum of the velocity profiles shifts toward the right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of the hotter plate. For large rotation (**Ω**=*20*) the Grashof number has an opposite effect on the velocity profiles in the right half and the left half of the channel. In the right half there lies the hot plate at η = *1/2* and heat is transferred from the hot plate to the fluid and consequently the buoyancy force enhances the flow velocity further. In the left half of the channel the transfer of heat takes place from the fluid to the cooler plate at η = -*1/2*. Thus, the effect of the Grashof number on velocity is reversed, i.e., velocity decreases with increasing Gr.

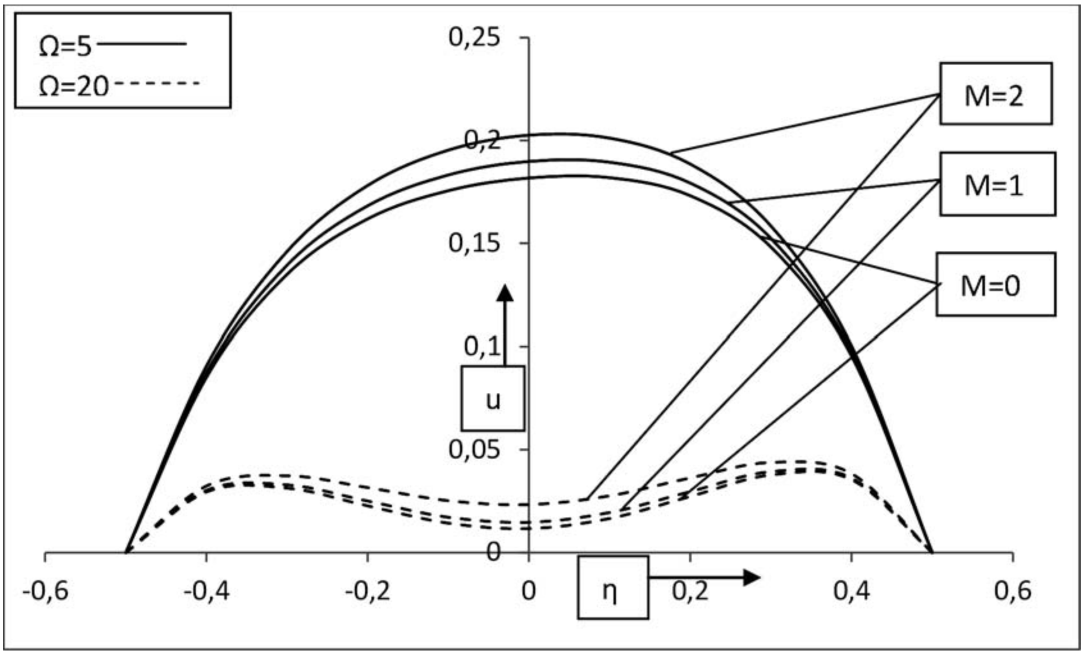

Fig.5. Velocity profiles for Re=*1*, *A=5*, Gr=*1*, Pe=*0.7*, *N=1*, ω =*5* and *t=0*.

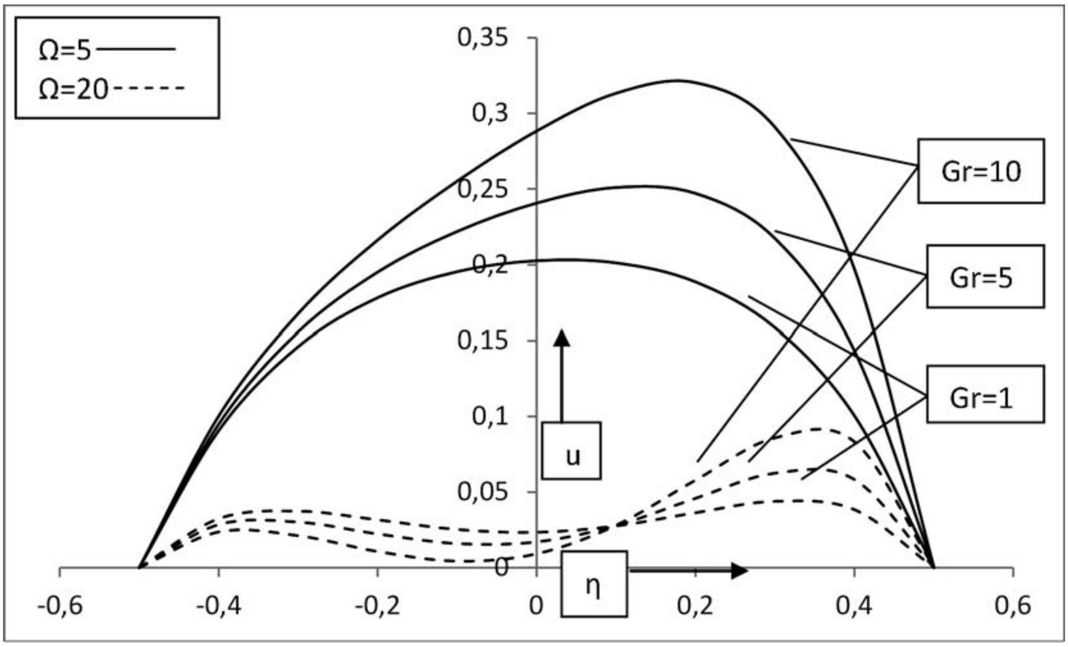

Fig.6. Velocity profiles for Re=*1*, *A=5*, M=*2*, Pe=*0.7*, *N=1*, ω =5 and *t=0*.

We find from Fig.7 that with the increase of the Peclet number Pe velocity decreases for the case of small rotation (**Ω**=*5*) while for large rotation (**Ω**=*20*) velocity increases slightly in the left half but decreases in the right half of the channel. For Re=*1* the two values of Pe (= *0.7* and *7*) have been chosen to represent real fluids, i.e., air and water, respectively. Figure 8 shows that velocity decreases with the increasing radiation parameter for small rotations. For large rotation, velocity slightly increases in the left half then decreases in the right half of the channel. The effect of the frequency of oscillations ω on velocity is exhibited in Fig.9. It is noticed that velocity decreases with increasing frequency ω for either case of channel rotation, large or small.

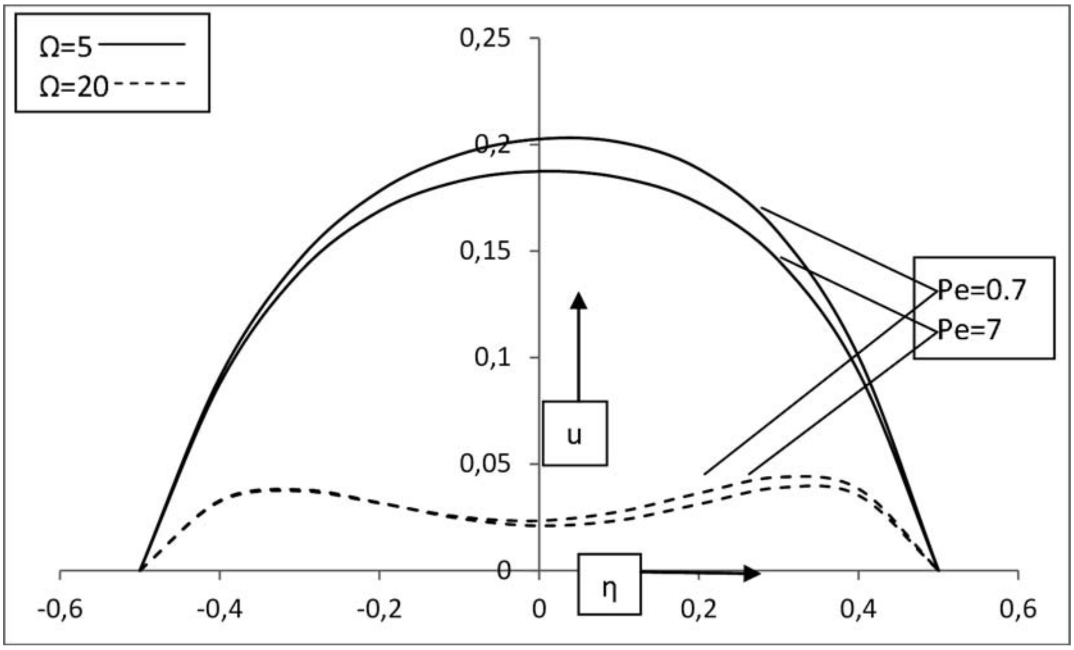

Fig.7. Velocity profiles for Re=*1*, *A=5*, M=*2*, Gr=*1*, *N=1*, ω =*5* and *t=0*.

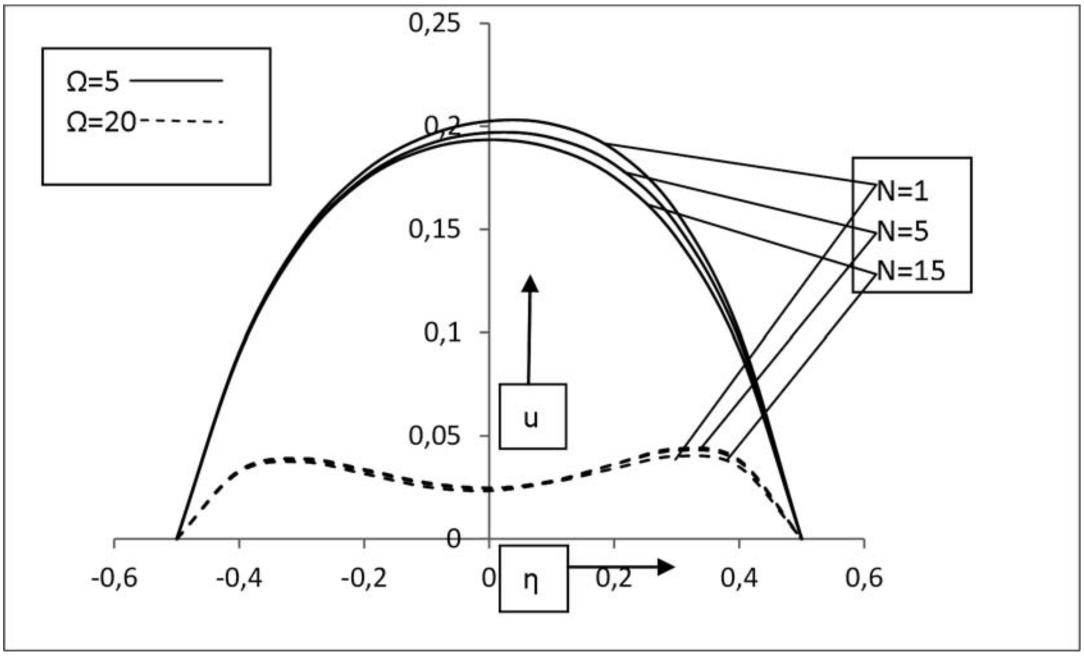

Fig.8. Velocity profiles for Re=*1*, *A=5*, M=*2*, Gr=*1*, Pe=*0.7*, ω =*5* and *t=0*.

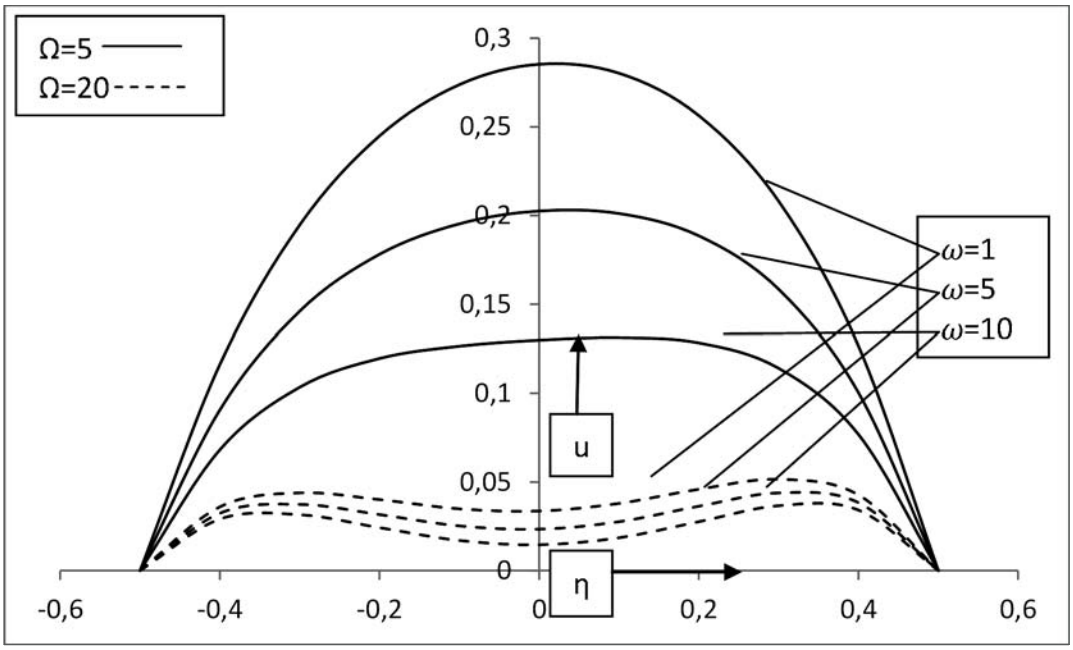

Fig.9. Velocity profiles for Re=*1*, A=*5*, M=*2*, Gr=*1*, Pe=*0.7*, *N*=*1* and *t=0*.

The skin-friction τ_L in terms of its amplitude $|F|$ and phase angle φ has been shown in Figs 10 and 11, respectively. The effect of each of the parameters on $|F|$ and φ is assessed by comparing each curve with dotted curves I in these figures. In Fig.10 the comparison of the curves III, IV and VIII with the dotted curve I indicate that the amplitude increases with the increase of the Grashof number Gr, Reynolds number Re and the pressure gradient parameter *A*. Physically, it is also expected because due to the increase of these parameters for small rotation (**Ω**=*5*) velocity increases and consequently the faster flows gives rise to more skin-friction. Similarly the comparison of curves II, V, VI and VII with the dotted curve I shows that the skin-friction amplitude decreases with the increase of the rotation parameter **Ω**, Hartmann number M, radiation parameter *N* and the Peclet number Pe because velocity due to these parameters decreases and for slow flows skin-friction is less. It is obvious that $|F|$ goes on decreasing with increasing frequency of oscillations ω. From Fig.11 showing the variations of the phase angle of the skin-friction it is clear that there is always a phase lag because the values of φ remain negative throughout. Comparing curves II, III, IV and VII with the dotted curve I it is observed that the phase lag increases with the increase of the rotation parameter **Ω**, Grashof number Gr, Reynolds number Re and the pressure gradient parameter *A*. Also, the comparison of curves V, VI and VII with the dotted curve I indicates that the phase lag decreases with the increase of the Hartmann number M, radiation parameter *N* and the Peclet number Pe although the decrease due to *N* and Pe is not significant. The phase angle goes on increasing with increasing frequency of oscillations ω but the trend reverses slightly for large frequency.

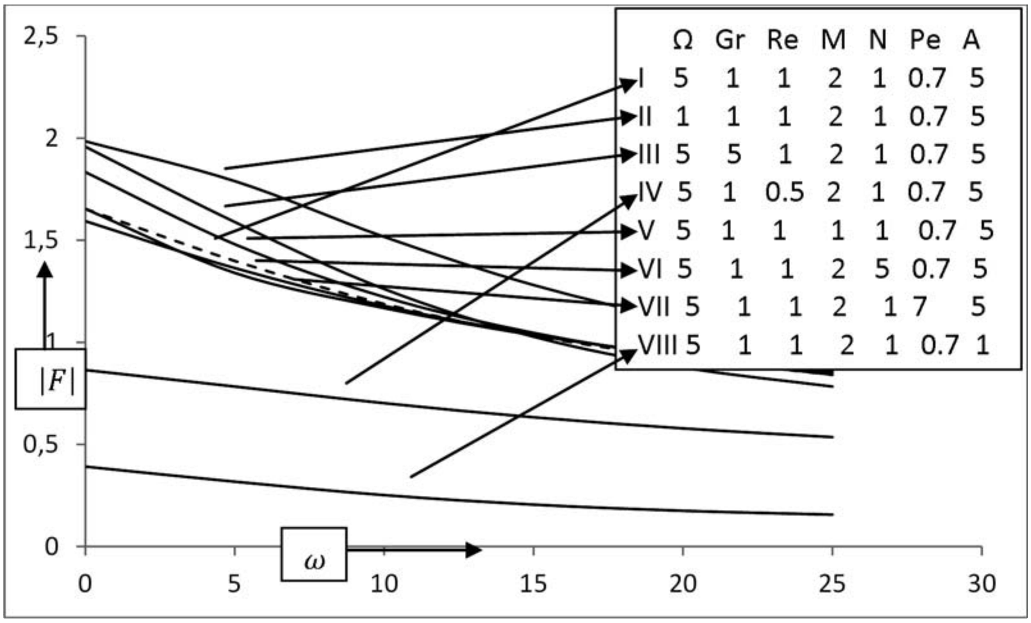

Fig.10. Amplitude of the skin-friction for *t=0*.

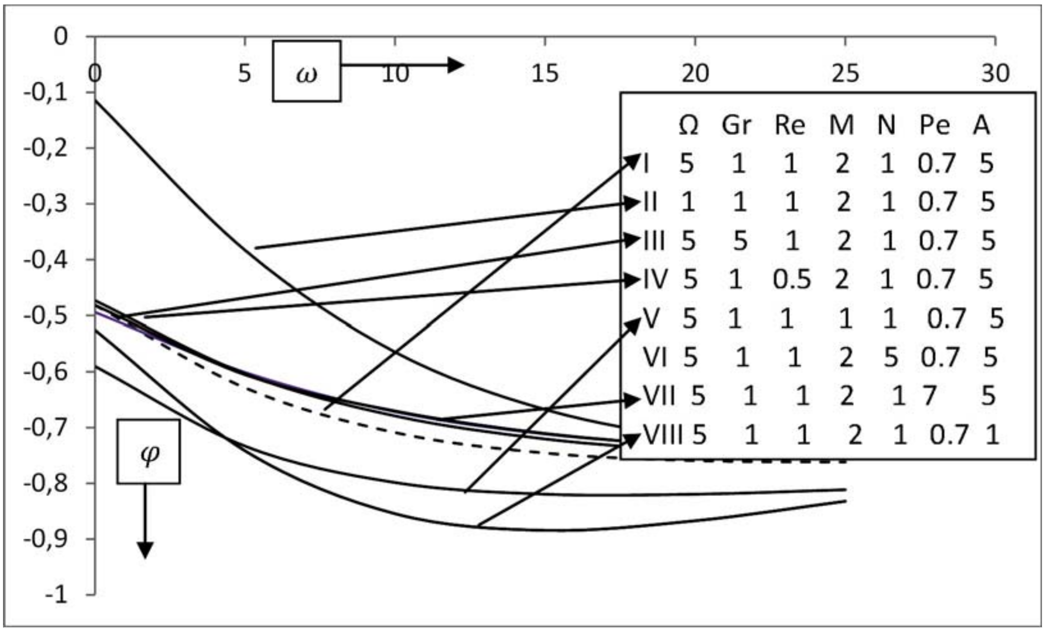

Fig.11. Phase angle of the skin-friction for *t=0*.

The temperature profiles are shown in Fig.12 for various values of the parameters involved. This figure reveals that the temperature decreases with the increase of either of the parameters, may it be the Peclet number (Pe), or radiation parameter (N) or the frequency of oscillations (ω). The decrease in temperature is noticed more in the left half of the channel. The temperature field is influenced more by the conduction heat than the radiation heat and becomes negative near the left plate due to the increase of Pe.

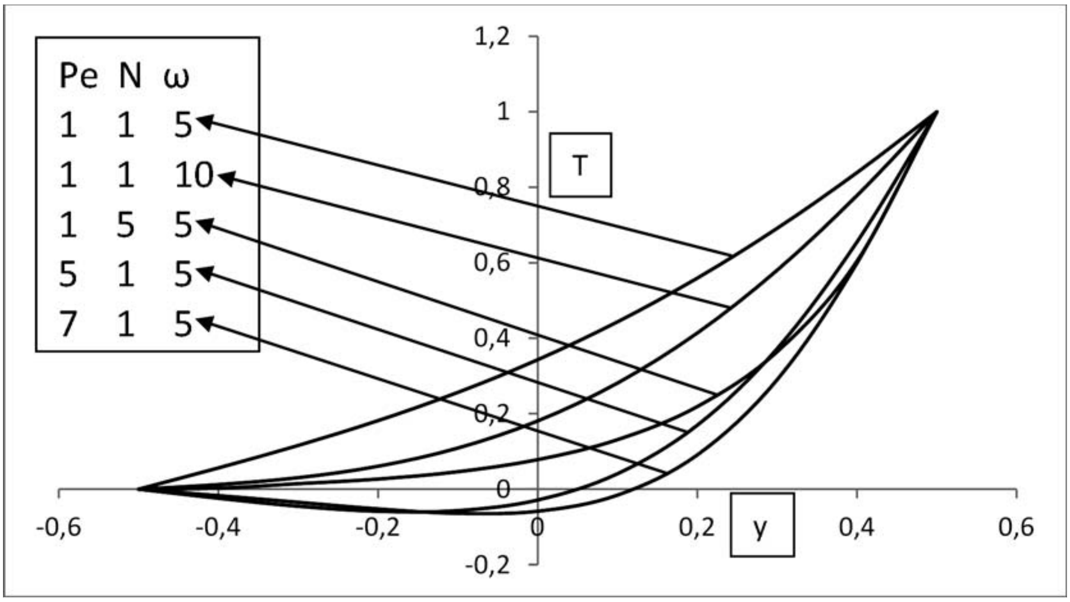

Fig.12. Temperature profiles for $t=0$.

The amplitude of the Nusselt number is presented in Fig.13. This figure shows that decreases with the increase of the radiation parameter N and the Peclet number Pe. The phase angle ψ of the rate of heat transfer is depicted in Fig.14. It is noticed that there is a phase lag initially for small values of the frequency of oscillation ω and thereafter the phase oscillates between the phase lag and the phase lead.

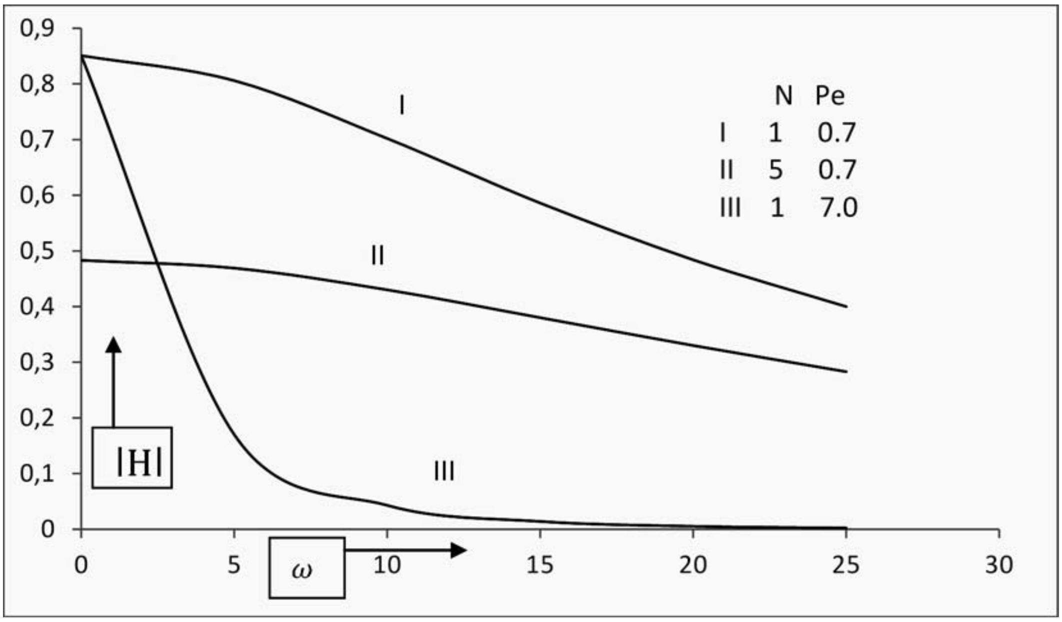

Fig.13. Amplitude $|H|$ of the Nusselt number.

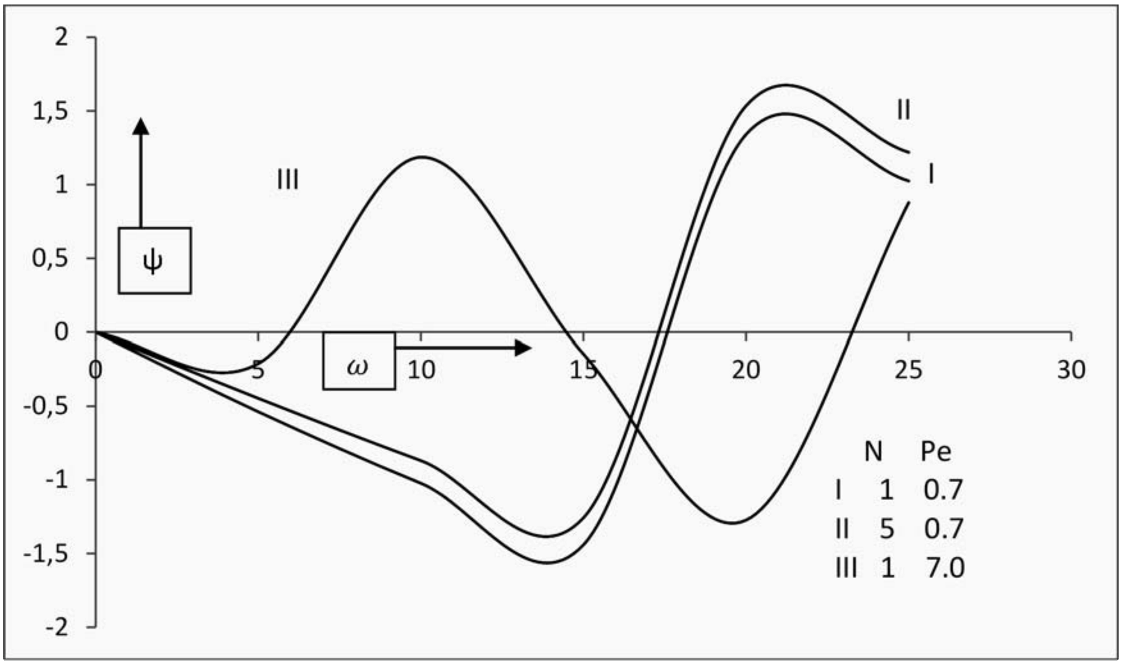

Fig.14. Phase angle ψ of the Nusselt number.

Nomenclature

A	– a constant
B_0	– magnetic field applied
c_p	– specific heat at constant pressure
e	– electric charge
$\|F\|$	– amplitude of skin friction
Gr	– Grashof number
g	– gravitational force
$\|H\|$	– amplitude of Nusselt number
$\boldsymbol{J}$	– current density
k	– thermal conductivity
M	– Hartmann number
N	– heat radiation parameter
Nu	– Nusselt number
Pe	– Peclet number
p	– pressure
Re	– Reynolds number
T	– fluid temperature
T_0	– constant temperature
t	– time variable
U	– mean flow velocity
u, v, w	– velocity components
x, y, z	– axial variables
α	– mean radiation absorption coefficient
β	– coefficient of volume expansion
θ_0	– mean non-dimensional temperature
μ	– viscosity
ρ	– fluid density

σ – electric conductivity
τ_L – skin-friction at the left wall
φ – phase angle of the skin-friction
ψ – phase angle of Nusselt number
Ω – angular velocity
ω – frequency of oscillations
* – superscript representing dimensional quantities

References

Attia H.A. and Kotb N.A. (1996): *MHD flow between two parallel plates with heat transfer*. – Acta Mech., vol.117, pp.215-220.

Chandran P., Sacheti N.C. and Singh A.K. (1993): *Effect of rotation on unsteady hydromagnetic Couette flow*. – Astrophysics and Space Sci., vol.202, pp.1-10.

Chang C.C. and Lundgren T.S. (1961): *Duct flow in magnetohydrodynamics*. – Z. Angew. Math. Phys. (ZAMP), vol.12, pp.100-114.

Cogley A.C.L., Vinvent W.G. and Giles E.S. (1968): *Differential approximation for radiative transfer in a non-gray near equilibrium*. – American Institute of Aeronautics and Astronautics, vol.6, pp.551-553.

Crammer K.R. and Pai S.I. (1973): *Magneto Fluid Dynamics for Engineers and Applied Physicists*. – New York: McGraw-Hill Book Co.

Ferraro V.C.A. and Plumpton C. (1966): *An Introduction to Magneto Fluid Mechanics*. – Oxford: Clarandon Press.

Gupta A.S. (1972): M*agnetohydrodynamic Ekmann layer* – Acta Mech., vol.13, pp.155-160.

Jana R.N. and Datta N. (1977): *Couette flow and heat transfer in a rotating system*. – Acta Mech., vol.26, pp.301-306.

Makinde O.D. and Mhone P.Y. (2005): *Heat transfer to MHD oscillatory flow in a channel filled with oporous medium*. – Rom. Journ. Phys., vol.50, pp.931-938.

Prasad R.D.R.V., Krishna D.V. and Devnath L. (1982): *Combined effect of free and forced convection on MHD flow in a rotating porous channel*. – Int. J. Math. And Math. Sci., vol.5, pp.165-182.

Schlichting H. and Gersten K. (2001): *Boundary layer theory*. – McGraw-Hill, pp.137.

Seth G.S., Jana R.N. and Maiti M.K. (1982):*Unsteady hydromagnetic Couette flow in a rotating system*. – Int. J. Engng. Sci., vol.20, pp.989-999.

Shercliff J.A. (1965): *A Text Book of Magneto Hydrodynamics*. – New York: Pergamon Press Ltd.

Singh A.K., Sacheti N.C. and Chandran P. (1994): *Transient effects on magnetohydrodynamic Couette flow with rotation: Accelerated motion*. – Int. J. Engng. Sci., vol. 32, pp.133-139.

Singh K.D. (2000): *An oscillatory hydromagnetic Couette flow with transpiration cooling*. – Z. Angew. Math. Mach. (ZAMM), vol.80, pp.429-432.

Singh K.D. (2011): *Exact solution of an oscillatory MHD flow in a channel filled with porous medium*. – Int. J. Appl. Mech. And Engng. (IJAME), vol.16, No.1, pp.277-283.

Singh K.D. and Garg B.P. (2010): *Exact solution of an oscillatory free convective MHD flow in a rotating porous channel with radiative heat*. – Proc. Nat. Acad. Sci. India, vol.80, pp.81-89.

Singh K.D. and Reena P. (2010): *An analysis of an oscillatory rotating MHD Poiseuille flow with injection/suction and Hall currents*. – Proc. Indian Natn. Sci. Acad., vol.76, pp.201-207.

Singh K.D., Gorla M.G. and Hans R. (2005): *A periodic solution of an oscillatory Couette flow through porous medium in a rotating system*. – Indian J. Pure and Appl. Math., vol.36, pp.151-159.

Singh K.D. and Rakesh K. (2009): *Combined effects of Hall current and rotation on free convection MHD flow in a porous channel*. – Indian J. Pure and Appl. Phys., vol.47, pp.617-623.

Soundalgekar V.M. and Pop I. (1973): *On hydromagnetic flow in a rotating fluid past an infinite porous plate*. – Z. Angew. Math. Mach. (ZAMM), vol.53, pp.718-719.

Vidyanidhu V. and Nigam S.D. (1967): *Secondary flow in a rotating channel*. – J. Math. And Phys. Sci., vol.1, pp.85-100.

Yen J.T. and Chang C.C. (1964): *Magnetohydrodynamic Couette flow as affected by wall conductance*. – Z. Angew. Math. Phys. (ZAMP), vol.15, pp.400-407.

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