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Exact solutions and conservation laws of a coupled integrable dispersionless system

Chaudry Masood Khalique

*International Institute for Symmetry Analysis and Mathematical Modelling,
Department of Mathematical Sciences, North-West University, Mafikeng Campus,
Private Bag X 2046, Mmabatho 2735,
Republic of South Africa*

Abstract. In this paper we study the coupled integrable dispersionless system (CIDS), which arises in the analysis of several problems in applied mathematics and physics. Lie symmetry analysis is performed on CIDS and symmetry reductions and exact solutions with the aid of simplest equation method are obtained. In addition, the conservation laws of the CIDS are also derived using the multiplier (and homotopy) approach.

1. Introduction

Recently, large amount of interest has been shown in the study of the dispersionless or quasiclassical limits of integrable equations and hierarchies. Since the dispersionless hierarchies arise in the analysis of several problems in applied mathematics and physics from the theory of quantum fields and strings to the theory of conformal maps on the complex plane [1, 5–7, 9–12, 21, 22], their study is of great significance. Various methods, including the inverse scattering transformation, Painlevè analysis, Bäcklund transformation and Darboux transformation, have been used in the literature to study dispersionless equations and hierarchies. In particular, several (1+1)-dimensional equations and systems have been analyzed by the quasiclassical version of the inverse scattering transform, including the local Riemann-Hilbert problem approach.

In nineteen nineteens, Konno et al. [10, 11] introduced a more general set of coupled integrable dispersionless system (CIDS), viz.,

$$q_{xt} - 2\alpha q r_x - 2\beta q s_x + \gamma (rs)_x = 0, \tag{1a}$$

$$r_{xt} - 2\alpha r r_x + 2\beta \left(2 q q_x + r_x s\right) - 2\gamma q_x r = 0, \tag{1b}$$

$$s_{xt} - 2\beta s s_x + 2\alpha \left(2 q q_x + r s_x\right) - 2\gamma s q_x = 0, \tag{1c}$$

where α, β and γ are constants. The coupled integrable dispersionless system (1a)–(1c) physically describes a current-fed string interacting with an external magnetic field in three-dimensional Euclidean space [9–11, 17]. It also appears geometrically as the parallel transport of each point of the curve along the direction

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Email address: Masood.Khalique@nwu.ac.za (Chaudry Masood Khalique)

of time where the connection is magnetic-valued [9–11, 17]. Although these CID equations have been shown to possess the Painlevé property, their complete integrability have been investigated by Zhao and Lu [20] while performing the prolongation structure analysis of the system. In view of the above interests, Zhaqilao, Zhao and Li [9] have derived new CID equations from the viewpoint of spectral transform. Many special cases of the CIDS (1) have been studied by various authors. See, for example, [5–7, 12]. In Ref. [21], Zhaqilao studied CIDS (1) and derived the Darboux transformation and Lax pair for the system (1) and obtained N-soliton solutions.

It is normal to point out that solitons arise from the balance between nonlinearity and dispersion. It is not necessary for the equation to possess dispersion in order for solitons to exist. The dispersionless equations, such as Burgers equation, sine-Gordon equation and the CIDs all give rise to kinks, a well known type of solitons although these equations do not posses any dispersion. This fact is examined thoroughly in [5–7, 9–12, 17, 20–22]. In addition most of these equations are integrable and generate multiple soliton, or kink, solutions. Moreover, these equations include the nonlinearty term uu_x and the dissipation term u_{xx}. Kink waves are travelling waves which rise or descend from one asymptotic state to another. The kink solution approaches a constant at infinity.

In this paper we use Lie symmetry method along with the simplest equation method to obtain exact solutions of the CIDS (1). Subsequently, the conservation laws are derived for the underlying system.

A Lie point symmetry of a differential equation is an invertible transformation of the dependent and independent variables that leaves the equation unchanged. Finding all the symmetries of a differential equation is an alarming task. However, in the middle of the nineteenth, Sophus Lie (1842-1899) realized that if we restrict ourself to symmetries that depend continuously on a small parameter and that form a group (continuous one-parameter group of transformations), one can linearize the symmetry conditions and end up with an algorithm for calculating continuous symmetries. Lie's continuous symmetry groups have applications in such diverse fields as invariant theory, control theory, classical mechanics, relativity etc. For the theory and application of the Lie group analysis methods, see for example, the Refs. [4, 8, 15, 16].

The notion of conservation laws plays an important role in the solution process of differential equations. Finding the conservation laws of system of differential equations (DEs) is often the first step towards finding the solution. In fact, the existence of a large number of conservation laws of a system of partial differential equations (PDEs) is a strong indication of its integrability [4]. In [13], the invariance of a conservation law was used to obtain solutions for a problem in thin films. In jet problems, the conserved quantity plays an essential role in the derivation of the solution. Recently, in [14] the conserved quantity was used to determine the unknown exponent in the similarity solution which cannot be obtained from the homogeneous boundary conditions.

The outline of the paper is as follows. In Section 2, we obtain some symmetry reductions of the coupled integrable dispersionless system (1) using Lie group analysis. Exact solutions are obtained in Section 3 using the simplest equation method by taking the Bernuolli and Ricatti equations as simple equations. Then in Section 4, we construct conservation laws for (1) using the multiplier method. Finally, in Section 5 concluding remarks are presented.

2. Some symmetry reductions of (1)

The symmetry group of the coupled integrable dispersionless system (1) will be generated by the vector field of the form

$$X = \xi^1(x,t,q,r,s)\frac{\partial}{\partial x} + \xi^2(x,t,q,r,s)\frac{\partial}{\partial t} + \eta^1(x,t,q,r,s)\frac{\partial}{\partial q} + \eta^2(x,t,q,r,s)\frac{\partial}{\partial r} + \eta^3(x,t,q,r,s)\frac{\partial}{\partial s}.$$

Applying the second prolongation $\mathrm{pr}^{(2)}X$ [15] to (1) and solving the resultant overdetermined system of linear PDEs one obtains the following infinitesimals coefficients:

$$
\begin{aligned}
&\xi^1(x,t,q,r,s) = F_1(x),\\
&\xi^2(x,t,q,r,s) = C_2 - \gamma C_5 t,\\
&\eta^1(x,t,q,r,s) = C_4(\beta s - \alpha r) + C_5(\gamma q + \alpha r - \beta s) + \gamma F_2(t),\\
&\eta^2(x,t,q,r,s) = C_4(\gamma r - 2\beta q) + 2\beta C_5 q + 2\beta F_2(t),\\
&\eta^3(x,t,q,r,s) = C_4(2\alpha q - \gamma s) + C_5(2\gamma s - 2\alpha q) + 2\alpha F_2(t).
\end{aligned}
$$

Here C_2, C_4, C_5 are constants, and $F_1(x)$ and $F_2(t)$ are arbitrary functions of x and t, respectively. By taking $F_1(x)$ and $F_2(t)$ to be constants, we obtain the following five Lie point symmetries of CIDS (1):

$$
\begin{aligned}
&X_1 = \frac{\partial}{\partial x},\\
&X_2 = \frac{\partial}{\partial t},\\
&X_3 = \gamma\frac{\partial}{\partial q} + 2\beta\frac{\partial}{\partial r} + 2\alpha\frac{\partial}{\partial s},\\
&X_4 = (\beta s - \alpha r)\frac{\partial}{\partial q} + (\gamma r - 2\beta q)\frac{\partial}{\partial r} + (2\alpha q - \gamma s)\frac{\partial}{\partial s},\\
&X_5 = -\gamma t\frac{\partial}{\partial t} + (\gamma q + \alpha r - \beta s)\frac{\partial}{\partial q} + 2\beta q\frac{\partial}{\partial r} + 2(\gamma s - \alpha q)\frac{\partial}{\partial s}.
\end{aligned}
$$

We now present some symmetry reductions for the coupled integrable dispersionless system (1).

Case 1. $\nu X_1 + X_2$
The symmetry generator $\nu X_1 + X_2$ gives rise to the group-invariant solution

$$q = E(z), \quad r = F(z), \quad s = G(z), \tag{2}$$

where $z = x - \nu t$ is an invariant of the symmetry $\nu X_1 + X_2$. Substituting (2) into (1) yields the system of ODEs

$$-\nu E'' - 2\beta G' E - 2\alpha F' E + \gamma F' G + \gamma G' F = 0, \tag{3a}$$
$$-\nu F'' - 2\gamma E' F - 2\alpha F' F + 4\beta E' E + 2\beta F' G = 0, \tag{3b}$$
$$-\nu G'' - 2\gamma E' G - 2\beta G' G + 4\alpha E' E + 2\alpha G' F = 0. \tag{3c}$$

Case 2. $\mu_1 X_1 + X_2 + \mu_3 X_3$
The symmetry operator $\mu_1 X_1 + X_2 + \mu_3 X_3$ provides us with the group-invariant solution of the form

$$q = E(z) + \mu_3\gamma t, \quad r = F(z) + 2\mu_3\beta t, \quad s = G(z) + 2\mu_3\alpha t, \tag{4}$$

where $z = x - \mu_1 t$ is an invariant of $\mu_1 X_1 + X_2 + \mu_3 X_3$ and the functions E, F and G satisfy the following system of ODEs:

$$
\begin{aligned}
&-\mu_1 E'' - 2\alpha F' E - 2\beta G' E + \gamma F' G + \gamma G' F = 0,\\
&-\mu_1 F'' - 2\alpha F' F + 4\beta E' E + 2\beta F' G - 2\gamma E' F = 0,\\
&-\mu_1 G'' - 2\beta G' G + 4\alpha E' E + 2\alpha G' F - 2\gamma E' G = 0.
\end{aligned}
$$

Case 3. X_2
The symmetry X_2 gives rise to the steady state group-invariant solution

$$q = E(z), \quad r = F(z), \quad s = G(z), \tag{5}$$

where $z = x$ is an invariant of the symmetry X_2. Substitution of (5) into (1) results in the system of ODEs

$$-2\beta G' E - 2\alpha F' E + \gamma F' G + \gamma G' F = 0,$$
$$-2\gamma E' F - 2\alpha F' F + 4\beta E' E + 2\beta F' G = 0,$$
$$-2\gamma E' G - 2\beta G' G + 4\alpha E' E + 2\alpha G' F = 0.$$

3. Exact solutions using simplest equation method

In this section we employ the simplest equation method, which was introduced by Kudryashov [**?**] and modified by Vitanov [19], to solve the ODE system (3) and as a result we obtain the exact solutions of our coupled integrable dispersionless system (1). The simplest equations that we use are the Bernoulli and Riccati equations.

We first briefly recall the simplest equation method here. Let us consider the solutions of the ODE system (3) in the form

$$E(z) = \sum_{i=0}^{M} \mathcal{A}_i (H(z))^i, \quad F(z) = \sum_{i=0}^{M} \mathcal{B}_i (H(z))^i, \quad G(z) = \sum_{i=0}^{M} C_i (H(z))^i, \tag{6}$$

where $H(z)$ satisfies the Bernoulli and Riccati equations, M is a positive integer that can be determined by balancing procedure as in [19] and $\mathcal{A}_i$, $\mathcal{B}_i$ and C_i $(i = 0, 1, \cdots, M)$, are parameters to be determined. The Bernoulli and Riccati equations are well-known nonlinear ODEs whose solutions can be expressed in terms of elementary functions.

We consider the Bernoulli equation

$$H'(z) = aH(z) + bH^2(z), \tag{7}$$

where a and b are constants. The solution of the Bernoulli equation (7) can be written in the form

$$H(z) = a\left\{\frac{\cosh[a(z+C)] + \sinh[a(z+C)]}{1 - b\cosh[a(z+C)] - b\sinh[a(z+C)]}\right\},$$

with C is a constant of integration.

For the Riccati equation

$$H'(z) = aH^2(z) + bH(z) + c, \tag{8}$$

where a, b and c are constants, we use the solutions

$$H(z) = -\frac{b}{2a} - \frac{\theta}{2a}\tanh\left[\frac{1}{2}\theta(z+C)\right]$$

and

$$H(z) = -\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{1}{2}\theta z\right) + \frac{\operatorname{sech}\left(\frac{\theta z}{2}\right)}{C\cosh\left(\frac{\theta z}{2}\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta z}{2}\right)},$$

with $\theta^2 = b^2 - 4ac > 0$ and C is a constant of integration.

3.1. Solutions of (1) using the Bernoulli equation as the simplest equation

The balancing procedure yields $M = 1$ so the solutions of the ODE system (3) are of the form

$$E(z) = \mathcal{A}_0 + \mathcal{A}_1 H, \quad F(z) = \mathcal{B}_0 + \mathcal{B}_1 H, \quad G(z) = C_0 + C_1 H. \tag{9}$$

Substituting (9) into (3) and making use of the Bernoulli equation (7) and then equating the coefficients of the functions H^i to zero, we obtain an algebraic system of equations in terms of $\mathcal{A}_i$, $\mathcal{B}_i$ and C_i $(i = 0, 1)$.

Solving the resultant system of algebraic equations with the aid of Mathematica, one possible set of values of $\mathcal{A}_i$, $\mathcal{B}_i$ and C_i $(i = 0, 1, 2)$ are

$$\begin{aligned}
\alpha &= -\frac{\gamma^2}{4\beta},\\
\mathcal{A}_1 &= \frac{4b\nu}{\gamma},\\
\mathcal{B}_0 &= \frac{4\mathcal{A}_0 b\beta - a\mathcal{A}_1\beta}{2b\gamma},\\
\mathcal{B}_1 &= -\frac{b\nu}{\alpha},\\
C_0 &= \frac{\alpha\,(a\mathcal{A}_1 + 2\mathcal{A}_0 b)}{b\gamma},\\
C_1 &= -\frac{4b\nu}{\beta}.
\end{aligned}$$

As a result, a solution of (1) is

$$q(x,t) = \mathcal{A}_0 + \mathcal{A}_1 a\left\{\frac{\cosh[a(z+C)] + \sinh[a(z+C)]}{1 - b\cosh[a(z+C)] - b\sinh[a(z+C)]}\right\}, \tag{10a}$$

$$r(x,t) = \mathcal{B}_0 + \mathcal{B}_1 a\left\{\frac{\cosh[a(z+C)] + \sinh[a(z+C)]}{1 - b\cosh[a(z+C)] - b\sinh[a(z+C)]}\right\}, \tag{10b}$$

$$s(x,t) = C_0 + C_1 a\left\{\frac{\cosh[a(z+C)] + \sinh[a(z+C)]}{1 - b\cosh[a(z+C)] - b\sinh[a(z+C)]}\right\}, \tag{10c}$$

where $z = x - \nu t$ and C is a constant of integration.

3.2. *Solutions of (1) using the Riccati equation as the simplest equation*

The balancing procedure yields $M = 1$ so the solutions of the ODE system (3) are of the form

$$E(z) = \mathcal{A}_0 + \mathcal{A}_1 H, \quad F(z) = \mathcal{B}_0 + \mathcal{B}_1 H, \quad G(z) = C_0 + C_1 H. \tag{11}$$

Substituting (11) into (3) and using the Ricatti equation (8), we obtain an algebraic system of equations in terms of $\mathcal{A}_i$, $\mathcal{B}_i$ and C_i $(i = 0, 1)$ by equating all coefficients of the functions H^i to zero. Solving the resultant system, one possible set of values are

$$\begin{aligned}
\alpha &= -\frac{\gamma^2}{4\beta},\\
\mathcal{A}_1 &= \frac{4a\nu}{\gamma},\\
\mathcal{B}_0 &= \frac{\beta\,(4a\mathcal{A}_0 - \mathcal{A}_1 b)}{2a\gamma},\\
\mathcal{B}_1 &= -\frac{a\nu}{\alpha},\\
C_0 &= \frac{\alpha\,(2a\mathcal{A}_0 + \mathcal{A}_1 b)}{a\gamma},\\
C_1 &= -\frac{4a\nu}{\beta}.
\end{aligned}$$

Hence solutions of (1) are

$$q(x,t) = \mathcal{A}_0 + \mathcal{A}_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left[\frac{1}{2}\theta(z+C)\right]\right\}, \tag{12a}$$

$$r(x,t) = \mathcal{B}_0 + \mathcal{B}_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left[\frac{1}{2}\theta(z+C)\right]\right\}, \tag{12b}$$

$$s(x,t) = \mathcal{C}_0 + \mathcal{C}_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left[\frac{1}{2}\theta(z+C)\right]\right\} \tag{12c}$$

and

$$q(x,t) = \mathcal{A}_0 + \mathcal{A}_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{1}{2}\theta z\right) + \frac{\operatorname{sech}\left(\frac{\theta z}{2}\right)}{C\cosh\left(\frac{\theta z}{2}\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta z}{2}\right)}\right\}, \tag{13a}$$

$$r(x,t) = \mathcal{B}_0 + \mathcal{B}_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{1}{2}\theta z\right) + \frac{\operatorname{sech}\left(\frac{\theta z}{2}\right)}{C\cosh\left(\frac{\theta z}{2}\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta z}{2}\right)}\right\}, \tag{13b}$$

$$s(x,t) = \mathcal{C}_0 + \mathcal{C}_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{1}{2}\theta z\right) + \frac{\operatorname{sech}\left(\frac{\theta z}{2}\right)}{C\cosh\left(\frac{\theta z}{2}\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta z}{2}\right)}\right\}, \tag{13c}$$

where $z = x - vt$ and C is a constant of integration.

A profile of the solution (12) is given in Figure 1.

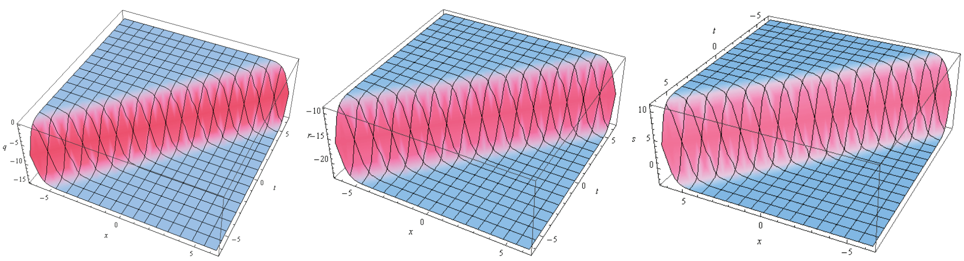

Figure 1: Profile of kink solution (12)

4. Conservation laws

In this section, using the multiplier method [2, 3, 15, 18], we construct conservation laws for our coupled integrable dispersionless system (1).

Consider a kth-order system of PDEs of n independent variables $x = (x^1, x^2, \ldots, x^n)$ and m dependent variables $u = (u^1, u^2, \ldots, u^m)$, viz.,

$$E_\alpha(x, u, u_{(1)}, \ldots, u_{(k)}) = 0, \quad \alpha = 1, \ldots, m, \tag{14}$$

where $u_{(1)}, u_{(2)}, \ldots, u_{(k)}$ denote the collections of all first, second, ..., kth-order partial derivatives, that is, $u_i^\alpha = D_i(u^\alpha), u_{ij}^\alpha = D_jD_i(u^\alpha), \ldots$ respectively, with the *total derivative operator* with respect to x^i given by

$$D_i = \frac{\partial}{\partial x^i} + u_i^\alpha\frac{\partial}{\partial u^\alpha} + u_{ij}^\alpha\frac{\partial}{\partial u_j^\alpha} + \ldots, \quad i = 1, \ldots, n, \tag{15}$$

and where the summation convention is used whenever appropriate [8].

The *Euler-Lagrange operator*, for each α, is given by

$$\frac{\delta}{\delta u^\alpha} = \frac{\partial}{\partial u^\alpha} + \sum_{s\geq 1}(-1)^s D_{i_1}\dots D_{i_s}\frac{\partial}{\partial u^\alpha_{i_1 i_2\dots i_s}}, \qquad \alpha = 1,\dots,m. \tag{16}$$

The n-tuple vector $T = (T^1, T^2, \dots, T^n)$, $T^j \in \mathcal{A}$, $j = 1,\dots,n$, ($\mathcal{A}$ is the space of differential functions) is a *conserved vector* of (14) if T^i satisfies

$$D_i T^i|_{(14)} = 0. \tag{17}$$

The equation (17) defines a *local conservation law* of system (14). A multiplier $\Lambda_\alpha(x, u, u_{(1)}, \dots)$ has the property that

$$\Lambda_\alpha E_\alpha = D_i T^i \tag{18}$$

holds identically. Here we consider multipliers of the zeroth order, i.e., $\Lambda_\alpha = \Lambda_\alpha(t, x, q, r, s)$. We note that the right hand side of (18) is a divergence expression. The determining equation for the multiplier Λ_α is

$$\frac{\delta(\Lambda_\alpha E_\alpha)}{\delta u^\alpha} = 0. \tag{19}$$

Once the multipliers are obtained the conserved vectors are calculated via a homotopy formula [2].

Conservation laws of (1)

For the coupled integrable dispersionless system (1), we consider the following multipliers, $\Lambda_1(t, x, q, r, s)$, $\Lambda_2(t, x, q, r, s)$ and $\Lambda_3(t, x, q, r, s)$ that are given by

$$\Lambda_1 = -\frac{2\alpha C_1 r}{\gamma} + \frac{2\beta C_1 s}{\gamma} + \frac{\gamma f(t)}{\beta}, \tag{20}$$

$$\Lambda_2 = \frac{2\alpha C_1 q}{\gamma} - C_1 s + \frac{\alpha f(t)}{\beta}, \tag{21}$$

$$\Lambda_3 = -\frac{2\beta C_1 q}{\gamma} + C_1 r + f(t), \tag{22}$$

where C_1 is a constant and $f(t)$ is an arbitrary function of t. Corresponding to the above multipliers we obtain the following two local conserved vectors of (1):

$$\begin{aligned} T_1^t &= \frac{1}{2\gamma}\Big\{ -2\alpha q_x r + 2\alpha r_x q + 2\beta q_x s - 2\beta s_x q - \gamma r_x s + \gamma s_x r\Big\}, \\ T_1^x &= \frac{1}{2\gamma}\Big\{ -2\alpha q_t r + 2\alpha r_t q + 2\beta q_t s - 2\beta s_t q - \gamma r_t s + \gamma s_t r\Big\} \end{aligned}$$

and

$$\begin{aligned} T_2^t &= \frac{1}{2\beta}\Big\{\gamma f(t) q_x + \alpha f(t) r_x + \beta f(t) s_x\Big\}, \\ T_2^x &= \frac{1}{2\beta}\Big\{ -\gamma f'(t) q - \alpha f'(t) r - \beta f'(t) s - 4\alpha\gamma f(t) qr - 4\beta\gamma f(t) qs + 8\alpha\beta f(t) q^2 \\ &\quad +4\alpha\beta f(t) rs + 2\gamma^2 f(t) rs - 2\alpha^2 f(t) r^2 - 2\beta^2 f(t) s^2 + \gamma f(t) q_t + \alpha f(t) r_t + \beta f(t) s_t\Big\}. \end{aligned}$$

Remark. The components of the second conserved vector contains an arbitrary function $f(t)$ and hence one can obtain an infinite number of conservation laws of (1).

5. Concluding remarks

In this paper we studied the coupled integrable dispersionless system (1) from the Lie symmetry analysis standpoint. Similarity reductions and exact solutions with the aid of simplest equation method were obtained. The exact solutions obtained were kink solutions. Kink waves are travelling waves which rise or descend from one asymptotic state to another. The kink solution approaches a constant at infinity. Normally solitons arise from the balance between nonlinearity and dispersion. However, it is not necessary for the equation to possess dispersion in order for solitons to exist. The dispersionless equations, such as Burgers equation, sine-Gordon equation and the CIDs all give rise to kinks, a well known type of solitons although these equations do not posses any dispersion. This fact is examined thoroughly in [5–7, 9–12, 17, 20–22]. In addition most of these equations are integrable and generate multiple soliton, or kink, solutions. Moreover, these equations include the nonlinearty term uu_x and the dissipation term u_{xx}. We have checked the correctness of the solutions obtained here by substituting them back into the coupled integrable dispersionless (1). Finally, conservation laws for the underlying system (1) were derived by employing the multiplier method.

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