EXACT SOLUTIONS FOR A REACTION DIFFUSION EQUATION BY USING THE GENERALIZED TANH METHOD

Soluciones exactas para una ecuación de reacción difusión, usando el método generalizado de la tanh

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ABSTRACT

In this paper we present the generalized tanh method to obtain exact solutions of nonlinear partial differential equations. As a particular case, we obtain exact solutions for a reaction diffusion equation.

KEYWORDS: reaction diffusion equation; tanh method, Mathematica.

RESUMEN

En este artículo presentamos el método de la tanh hiperbólica generalizada para obtener soluciones exactas de de ecuaciones diferenciales parciales no lineales. Como un caso particular, obtenemos soluciones exactas para una ecuación de reacción difusión.

PALABRAS CLAVES: *Ecuación de reacción difusión, método generalizado de la tanh, Mathematica.*

1. INTRODUCTION

The search of exact solutions to nonlinear partial differential equations is of great importance, because these equations appear in complex physics phenomena, mechanics, chemistry, biology and engineering. A variety of powerful and direct methods have been developed in this direction. The principal objective of this paper, is to present the generalized tanh method [1][5], and to apply it to obtain exact solutions for a reaction diffusion equation. There are other methods for solving nonlinear differential equations, One of them is the projective Riccati equation method [4].

2. THE GENERALIZED TANH METHOD

Consider a given PDE, say in two variables

$$P(u, u_x, u_y, u_{xx}, u_{xt}, \ldots) = 0. \qquad (1)$$

Using the wave transformation

$$u(x,t) = v(\xi), \quad \xi = x + \lambda\, t, \qquad (2)$$

where λ is a constant, the equation (1) reduces to an ordinary differential equation

$$p(v, v', v'', \ldots) = 0. \qquad (3)$$

The generalized tanh method, which has been introduced by Fan [1], is based on the idea of looking for solutions to equation (3) in the form

$$v(\xi) = \sum_{i=0}^{m} a_i \phi^i, \qquad (4)$$

where the new variable $\phi = \phi(\xi)$ satisfies the Riccati equation

$$\phi' = \phi^2 + k, \qquad (5)$$

whose solutions are given by

$$\phi(\xi) = \begin{cases} -\dfrac{1}{\xi} & k = 0 \\ \sqrt{k}\tan(\sqrt{k}\xi - c) & k > 0 \\ -\sqrt{k}\cot(\sqrt{k}\xi - c) & k > 0 \\ -\sqrt{-k}\tanh(\sqrt{-k}\xi - c) & k < 0 \\ -\sqrt{-k}\coth(\sqrt{-k}\xi - c) & k < 0. \end{cases} \qquad (6)$$

The integer m can be determined balancing the highest derivative term with nonlinear term in (3), before the a_i can be computed. Substituting (4) along with (5) into (3) and collecting all terms with the same power ϕ^i, we get a polynomial in the variable ϕ. Equaling the coefficients of this polynomial to zero, we obtain a system of algebraic equations, from which the constants a_i, λ, k are obtained explicitly. Lastly, we find solutions to (1) in the original variables.

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3. EXACT SOLUTIONS FOR REACTION DIFFUSION EQUATION

In this section, we use the generalized tanh method, to obtain exact solutions to the reaction diffusion equation [2]

$$u_{tt} + \alpha u_{xx} + \beta u + \gamma u^3 = 0. \qquad (7)$$

The transformation $u(x,t) = v(\xi)$, $\xi = x + \lambda t$, reduces (7) to well known elliptic equation

$$v'' + k_1 v + k_2 v^3 = 0, \qquad (8)$$

where

$$k_1 = \frac{\beta}{\alpha + \lambda^2}, \; k_2 = \frac{\gamma}{\alpha + \lambda^2}.$$

After balancing we obtain $m = 1$, therefore we seek solutions to (8) in the form

$$v(\xi) = a_0 + a_1 \phi(\xi). \qquad (9)$$

Substituting (9) into (8), and using (5), we obtain the system

$$\begin{cases} \beta a_0 + \gamma a_0{}^3 = 0 \\ 3\gamma a_0 a_1{}^2 = 0 \\ \beta a_1 + 2\alpha k a_1 + 2k\lambda^2 a_1 + 3\gamma a_0{}^2 a_1 = 0 \\ 2\alpha a_1 + 2\lambda^2 a_1 + \gamma a_1{}^3 = 0. \end{cases} \qquad (10)$$

With the aid of Mathematica [3], we obtain the following set of solutions to the previous system:

$$a_0 = 0, \quad a_1 = \pm \frac{\sqrt{\beta}}{\sqrt{k}\sqrt{\gamma}}, \quad \lambda = \pm \sqrt{\frac{-\beta - 2\alpha k}{2k}}$$

.

Therefore, using (1.6), (1.9) and $u(x,t) = v(\xi)$, the solutions to (1.7) are given by

$$u_1 = \mp \frac{\sqrt{\beta}\cot(\sqrt{k}\ \xi)}{\sqrt{\gamma}} \qquad (k > 0, \quad \beta > 0).$$

$$u_2 = \mp \frac{\sqrt{\beta}\tan(\sqrt{k}\ \xi)}{\sqrt{\gamma}} \qquad (k > 0, \quad \beta > 0).$$

$$u_3 = \mp \frac{i\sqrt{\beta}\coth(\sqrt{k}\ i\,\xi)}{\sqrt{\gamma}} \qquad (k < 0, \quad \beta < 0)$$

$$u_4 = \mp \frac{i\sqrt{\beta}\tanh(\sqrt{k}\ i\xi)}{\sqrt{\gamma}} \qquad (k < 0, \quad \beta < 0)$$

.

In all cases, $\xi = x + \lambda t = x \pm \sqrt{\dfrac{-2k\alpha - \beta}{2k}}\, t$.

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