EXACT SOLUTIONS OF A FAMILY OF HIGHER-DIMENSIONAL SPACE-TIME FRACTIONAL KDV-TYPE EQUATIONS

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ABSTRACT

In this paper, based on the definition of conformable fractional derivative, the functional variable method (FVM) is proposed to seek the exact traveling wave solutions of two higher-dimensional space-time fractional KdV-type equations in mathematical physics, namely the (3+1)-dimensional space–time fractional Zakharov-Kuznetsov (ZK) equation and the (2+1)-dimensional space–time fractional Generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony (GZK-BBM) equation. Some new solutions are procured and depicted. These solutions, which contain kink-shaped, singular kink, bell-shaped soliton, singular soliton and periodic wave solutions, have many potential applications in mathematical physics and engineering. The simplicity and reliability of the proposed method is verified.

KEYWORDS

Functional Variable Method, Fractional Partial Differential Equations, Exact Solutions, Conformable Fractional Derivative

1. INTRODUCTION

In recent years, Fractional partial differential equations (FPDEs) have been extensively utilized to model complex physical phenomena that arise in various aspects of science and engineering, such as applied mathematics, physics, chemistry, biology, signal processing, control theory, finance and fractional dynamics [1,2]. The analytical solutions of FPDEs play a significant role in the study of nonlinear physical phenomena. Therefore, the efficient approaches to construct the solutions of FPDEs have attracted great interest by several groups of researchers. A large collection of analytical and computational methods has been introduced for this reason, for example the exp-function method [3,4], Adomian decomposition method [5], the (G'/G)-expansion method [6], the first integral method [7,8], the variational iteration method [9], the sub-equation method [10,11], the modified simple equation method [12], Jacobi elliptic function expansion method [13], the generalized Kudryashov method [14,15] and so on. One of the most powerful methods for seeking analytical solutions of nonlinear differential equations is the functional variable method, which was first proposed by Zerarka et al. [16,17] in 2010. It has received much interest since it has been employed to solve a wide class of problems by many authors [18-22]. The main advantage of this method over other existing methods is its capability

to reduce the size of computations during the solution procedure. Therefore, it can be applied without using any symbolic computation software.

As one of the most well-known nonlinear dispersive equations, the Korteweg-de Vries (KdV) equation has attracted much attention by many researchers in the scientific community duo to its significant role in various scientific disciplines. It describes a variety of important nonlinear phenomena, including finite amplitude dispersive wave phenomena, acoustic waves in a harmonic crystal and ion-acoustic waves in plasmas [23]. Several variations of this equation have been introduced in the literature. The (3+1)-dimensional Zakharov-Kuznetsov (ZK) equation was derived as a three-dimensional generalization of the KdV equation, which arises as a model for the propagation of nonlinear plasma-acoustic waves in the isothermal multi-component magnetized plasma [24,25]. If the nonlinear dispersion in KdV equation is incorporated, the Benjamin-Bona-Mahony (BBM) equation arises to describe a propagation of long waves. The (2+1)-dimensional Generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony (GZK-BBM) equation was developed by Wazwaz [26] as a combination of the well-known Benjamin-Bona-Mahony (BBM) equation with the ZK equation. It arises as a description of gravity water waves in the long-wave regime. Therefore, it is very interesting to examine the traveling wave solutions of KdV-type equations. It is worthwhile to mention that the (3+1)-dimensional space–time fractional ZK equation and the (2+1)-dimensional space–time fractional GZK-BBM equation have not been solved yet by using any existing analytical method.

There are many different definitions for fractional differential equations in fractional calculus; among these definitions are Riemann–Liouville, Grünwald–Letnikov, Caputo, Weyl, Marchaud, Hadamard, Canavati, Davidson-Essex, Riesz–Fischer, Jumarie fractional derivatives and so on [2,27]. However, these definitions have some shortcomings. For instance, they do not satisfy the product rule, the quotient rule and the chain rule for derivative operations. To overcome these drawbacks, Khalil et al. [28] introduced a completely new definition of the fractional derivative, which is more natural and fruitful than previous ones, called conformable fractional derivative.

The present paper is devoted to suggest the functional variable method for constructing new exact solutions of two higher-dimensional space-time fractional KdV-related equations, namely the (3+1)-dimensional space–time fractional ZK equation and the (2+1)-dimensional space–time fractional GZK-BBM equation. The fractional derivatives are presented in terms of the conformable sense. To the best of our knowledge, these equations have not been investigated previously by using the functional variable method in the sense of conformable derivative.

The rest of the paper is organized as follows: In Section 2, we describe some relevant materials and methods. In Section 3, the proposed approach is applied to establish the exact solutions of the underlying equations. The graphical representations of the obtained solutions are provided in Section 4, Results and discussion are presented in Section 5. Finally, conclusions are given in Section 6.

2. MATERIALS AND METHODS

2.1. Conformable fractional derivative and its properties

In this subsection, we present some basic definitions and properties of the conformable fractional calculus. Suppose a function $f : [0, \infty) \to \square$, then, the conformable fractional derivative of order α is defined as follows [28,29]:

$$T_\alpha(f)(t) = \lim_{\varepsilon \to 0} \frac{f\left(t + \varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon}, \tag{1}$$

in which $t > 0$ and $0 < \alpha \le 1$. If f is $\alpha-$differentiable in some $(0,a)$, $a > 0$, and $\lim_{t \to 0^+} f^{(\alpha)}(t)$ exists, then $f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$.

Now, we summarize some useful properties of the conformable derivative as follows [28-30]:

(i) $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$, for all $a,b \in \square$.

(ii) $T_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in \square$.

(iii) $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.

(iv) $T_\alpha\left(\dfrac{f}{g}\right) = \dfrac{gT_\alpha(f) - fT_\alpha(g)}{g^2}$.

(v) $T_\alpha(\lambda) = 0$, where λ is a constant.

(vi) If f is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha}\dfrac{df}{dt}$.

(vii) If f, g are differential functions, then $T_\alpha(f \circ g)(t) = t^{1-\alpha}g'(t)f'(g(t))$.

Moreover, some conformable fractional derivatives of certain functions can be found in [28]. The abovementioned properties will be utilized further in the forthcoming sections.

2.2. Description of the functional variable method

Consider the following general FPDE with four independent variables:

$$P\left(u, \frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial^\alpha u}{\partial x^\alpha}, \frac{\partial^\alpha u}{\partial y^\alpha}, \frac{\partial^\alpha u}{\partial z^\alpha}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^{2\alpha} u}{\partial x^{2\alpha}}, \ldots\right) = 0, \ 0 < \alpha \le 1 \tag{2}$$

where P is a polynomial of $u(x,y,z,t)$ and its fractional partial derivatives, in which the highest order derivatives and the nonlinear terms are involved.

The foremost steps of the FVM can be outlined as follows [18,19]:

Step 1: To find the exact solution of Eq. (2), we use the fractional complex transformation

$$u(x,y,z,t) = u(\xi), \quad \xi = \frac{k_1 x^\alpha}{\alpha} + \frac{k_2 y^\alpha}{\alpha} + \frac{k_3 z^\alpha}{\alpha} + \frac{ct^\alpha}{\alpha}, \tag{3}$$

where k_1, k_2, k_3 and c are nonzero arbitrary constants, to convert Eq. (2) into the following ordinary differential equation (ODE) of integer order:

$$\tilde{P}(u, cu', k_1 u', k_2 u', k_3 u', c^2 u'', k_1^2 u'', \ldots) = 0, \tag{4}$$

where $\tilde{P}$ is a polynomial in $u(\xi)$ and its total derivatives with respect to ξ.

Step 2: Let us make a transformation in which the unknown function $u(\xi)$ is considered as a functional variable in the form

$$u_\xi = F(u), \tag{5}$$

It is easy to find some higher order derivatives of $u(\xi)$ as follows:

$$u_{\xi\xi} = FF' = \frac{1}{2}\left(F^2\right)',$$

$$u_{\xi\xi\xi} = \frac{1}{2}\left(F^2\right)'' F = \frac{1}{2}\left(F^2\right)' \sqrt{F^2}, \tag{6}$$

$$u_{\xi\xi\xi\xi} = \frac{1}{2}\left(\left(F^2\right)''' F^2 + \frac{1}{2}\left(F^2\right)''\left(F^2\right)'\right),$$

and so on, where the prime denotes the derivative with respect to u.

Step 3: We substitute Eqs. (5) and (6) into Eq. (4) to reduce it to the following ODE:

$$R(u, F, F', F'', \ldots) = 0. \tag{7}$$

Step 4: After integration, Eq. (7) provides the expression of F, and this in turn together with Eq. (5) gives the appropriate solutions to the original equation.

3. APPLICATIONS

In this section, we apply the functional variable method, which described in the previous section, to look for the exact solutions of two higher-dimensional space-time fractional equations of KdV-type.

3.1 The (3+1)-dimensional space–time fractional ZK equation

Consider the (3+1)-dimensional space–time fractional ZK equation [24,25]

$$D_t^\alpha u + au D_x^\alpha u + D_x^{2\alpha} u + D_y^{2\alpha} u + D_z^{2\alpha} u = 0, \tag{8}$$

where $0 < \alpha \le 1$ and a is a nonzero constant.

To investigate Eq. (8) using the FVM, we use the fractional complex transformation given by Eq. (3) to reduce Eq. (8) into the following ODE:

$$cu_\xi + ak_1 u u_\xi + \left(k_1^2 + k_2^2 + k_3^2\right) u_{\xi\xi} = 0, \tag{9}$$

Integrating once w.r.t. ξ and setting the constant of integration to zero, yields

$$cu + \frac{ak_1}{2} u^2 + \left(k_1^2 + k_2^2 + k_3^2\right) u_\xi = 0, \tag{10}$$

Substituting Eq. (5) into Eq. (10), the function $F(u)$ reads

$$F(u) = -\frac{c}{k_1^2 + k_2^2 + k_3^2} u \left(1 + \frac{ak_1}{2c} u\right), \tag{11}$$

Separating the variables in Eq. (11) and then integrating, we obtain

$$\int \frac{-du}{u\left(1 + \frac{ak_1}{2c} u\right)} = \frac{c}{k_1^2 + k_2^2 + k_3^2}(\xi + \xi_0), \tag{12}$$

where ξ_0 is a constant of integration. After completing the integration of Eq. (12), we get the following exact solutions:

$$u_1(\xi) = -\frac{c}{ak_1}\left(1 - \tanh\left(\frac{c}{2\left(k_1^2 + k_2^2 + k_3^2\right)}(\xi + \xi_0)\right)\right), \tag{13}$$

$$u_2(\xi) = -\frac{c}{ak_1}\left(1 - \coth\left(\frac{c}{2\left(k_1^2 + k_2^2 + k_3^2\right)}(\xi + \xi_0)\right)\right). \tag{14}$$

where $\xi = \frac{k_1 x^\alpha}{\alpha} + \frac{k_2 y^\alpha}{\alpha} + \frac{k_3 z^\alpha}{\alpha} + \frac{ct^\alpha}{\alpha}$.

3.2 The (2+1)-dimensional space–time fractional GZK–BBM equation

Consider the (2+1)-dimensional space–time fractional GZK-BBM equation in the form [26]

$$D_t^\alpha u + D_x^\alpha u + aD_x^\alpha u^n + bD_x^\alpha\left(D_x^\alpha D_t^\alpha u + D_y^{2\alpha} u\right) = 0, \quad n > 1 \tag{15}$$

where $0 < \alpha \leq 1$ and a, b are nonzero constants.

To apply the FVM for Eq. (15). We exploit the fractional complex transformation

$$u(x, y, t) = u(\xi), \quad \xi = \frac{k_1 x^\alpha}{\alpha} + \frac{k_2 y^\alpha}{\alpha} + \frac{ct^\alpha}{\alpha}, \tag{16}$$

to convert Eq. (15) into the following ODE:

$$(c + k_1)u_\xi + ak_1\left(u^n\right)_\xi + bk_1\left(k_1 c u_{\xi\xi} + k_2^2 u_{\xi\xi}\right)_\xi = 0, \tag{17}$$

Integrating once w.r.t. ξ with zero constant of integration, we obtain

$$(c + k_1)u + ak_1 u^n + bk_1\left(k_1 c + k_2^2\right)u_{\xi\xi} = 0, \tag{18}$$

Substituting Eqs. (6) into Eq. (18), yields

$$\left(F^2\right)' = -\frac{2}{bk_1\left(k_1c + k_2{}^2\right)}\left[(c + k_1)u + ak_1u^n\right], \tag{19}$$

Integrating Eq. (19) w.r.t. u, we deduce the expression of the function $F(u)$ as follows

$$F(u) = \sqrt{\frac{-(c + k_1)}{bk_1\left(k_1c + k_2{}^2\right)}}\, u\sqrt{1 + \frac{2ak_1}{(c + k_1)(n + 1)}u^{n-1}}, \tag{20}$$

Separating the variables in Eq. (20) and then integrating, we obtain

$$\int \frac{du}{u\sqrt{1 + \dfrac{2ak_1}{(c + k_1)(n + 1)}u^{n-1}}} = \sqrt{\frac{-(c + k_1)}{bk_1\left(k_1c + k_2{}^2\right)}}(\xi + \xi_0), \tag{21}$$

where ξ_0 is a constant of integration. After completing the integration of Eq. (21), we can simply attain the following exact solutions:

(i) If $\dfrac{c + k_1}{bk_1\left(k_1c + k_2{}^2\right)} < 0$, we have the following hyperbolic solutions:

$$u_1(\xi) = \left\{-\frac{(c + k_1)(n + 1)}{2ak_1}\mathrm{sech}^2\left(\frac{n - 1}{2}\sqrt{\frac{-(c + k_1)}{bk_1\left(k_1c + k_2{}^2\right)}}(\xi + \xi_0)\right)\right\}^{\frac{1}{n-1}}, \tag{22}$$

$$u_2(\xi) = \left\{\frac{(c + k_1)(n + 1)}{2ak_1}\mathrm{csch}^2\left(\frac{n - 1}{2}\sqrt{\frac{-(c + k_1)}{bk_1\left(k_1c + k_2{}^2\right)}}(\xi + \xi_0)\right)\right\}^{\frac{1}{n-1}}, \tag{23}$$

where $\xi = \dfrac{k_1 x^\alpha}{\alpha} + \dfrac{k_2 y^\alpha}{\alpha} + \dfrac{ct^\alpha}{\alpha}$.

(ii) If $\dfrac{c + k_1}{bk_1\left(k_1c + k_2{}^2\right)} > 0$, we have the following trigonometric solutions:

$$u_3(\xi) = \left\{-\frac{(c + k_1)(n + 1)}{2ak_1}\sec^2\left(\frac{n - 1}{2}\sqrt{\frac{(c + k_1)}{bk_1\left(k_1c + k_2{}^2\right)}}(\xi + \xi_0)\right)\right\}^{\frac{1}{n-1}}, \tag{24}$$

$$u_4(\xi) = \left\{-\frac{(c + k_1)(n + 1)}{2ak_1}\csc^2\left(\frac{n - 1}{2}\sqrt{\frac{(c + k_1)}{bk_1\left(k_1c + k_2{}^2\right)}}(\xi + \xi_0)\right)\right\}^{\frac{1}{n-1}}. \tag{25}$$

where $\xi = \dfrac{k_1 x^\alpha}{\alpha} + \dfrac{k_2 y^\alpha}{\alpha} + \dfrac{ct^\alpha}{\alpha}$.

4. GRAPHICAL ILLUSTRATIONS

In this section, with the aid of Maple software, we show the graphical representation of some results in Figs. 1-3 by assigning appropriate values to the unknown parameters in order to visualize the mechanism of Eqs. (8) and (15). Some physical interpretations are also presented.

4.1 The (3+1)-dimensional space–time fractional ZK equation

The profiles of the kink-shaped solution $u_1(\xi)$ given by Eq. (13) is shown in Fig. 1 when $y = 0, z = 1, a = 1, \; k_1 = 1.5, k_2 = 0.25, k_3 = 1, c = -2, \xi_0 = 0$ for various values of α. We can observe that when the fractional derivative order α increased, the shape is closer to the known kink wave as the velocity of the propagation wave decreases. The kink wave keeps its height for various values of α. It should also be pointed out that the solution $u_2(\xi)$ given by Eq. (14) is a singular kink solution.
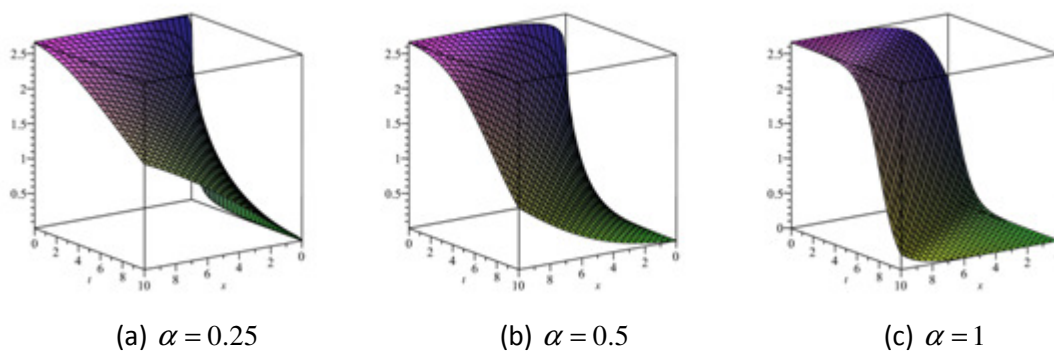

(a) $\alpha = 0.25$ (b) $\alpha = 0.5$ (c) $\alpha = 1$

Figure 1. The kink solution corresponding to Eq. (13) for various values of α

4.2 The (2+1)-dimensional space–time fractional GZK-BBM equation

The dynamics of the singular soliton solution $u_2(\xi)$ given by Eq. (23) is shown in Fig. 2 when $y = 1, \; a = 1, b = -2, k_1 = 1.25, k_2 = -4, c = 2, n = 4, \xi_0 = 0$ for various values of α. When α increased, the height of the wave changes as the velocity of the wave propagation decreases. Fig. 3 shows the motions of the periodic wave solution $u_3(\xi)$ given by Eq. (24) when $y = 0, a = 1,$ $b = 0.5, \; k_1 = 0.5, \; k_2 = 0.25, \; c = -2, n = 4, \xi_0 = 0.$ for various values of α. When α increased, the height of the wave becomes lower as the velocity of the wave propagation decreases. It should also be mentioned that the solution $u_1(\xi)$ given by Eq. (22) is a bell-shaped soliton solution.
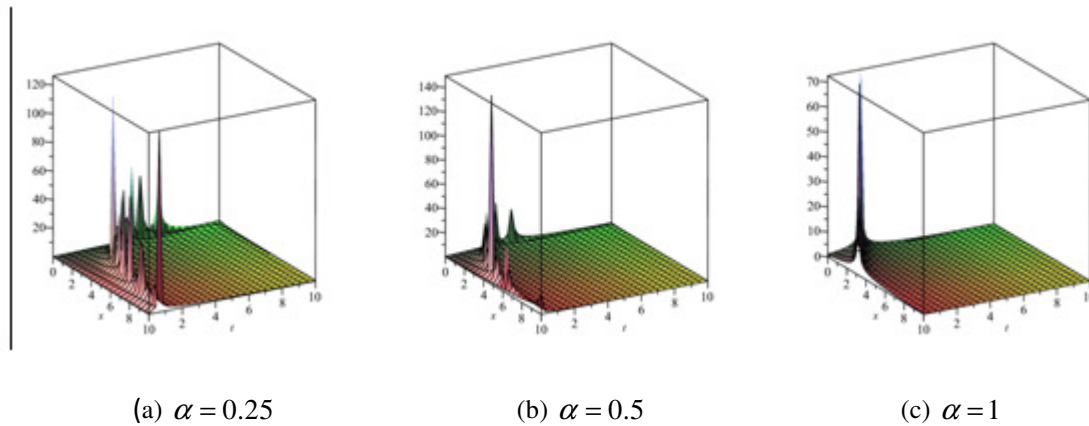
(a) $\alpha = 0.25$ (b) $\alpha = 0.5$ (c) $\alpha = 1$

Figure 2. The singular soliton solution corresponding to Eq. (23) for various values of α

(a) $\alpha = 0.25$ (b) $\alpha = 0.5$ (c) $\alpha = 1$
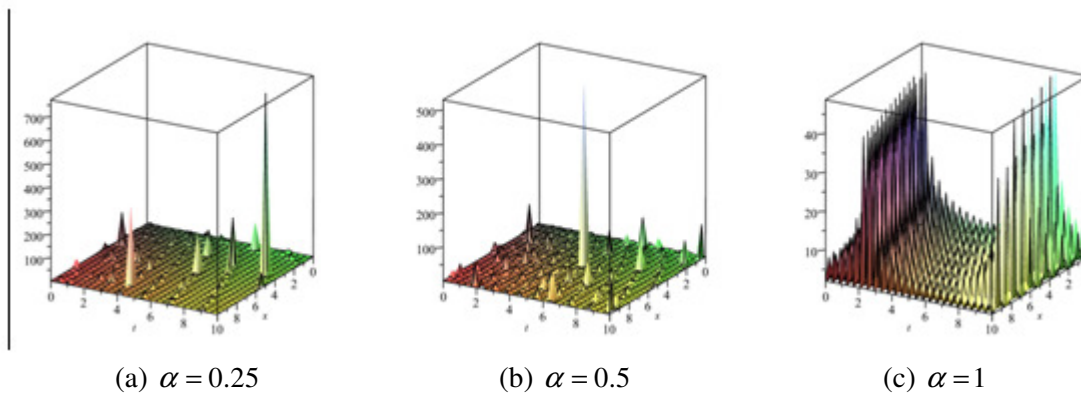
Figure 3. The periodic solution corresponding to Eq. (24) for various values of α

5. RESULTS AND DISCUSSION

For the first time, the analytical solutions of the (3+1)-dimensional space–time fractional ZK equation and the (2+1)-dimensional space–time fractional GZK-BBM equation have been attained via the functional variable method, the fractional derivative has been described in the conformable sense. Consequently, we deduce that our solutions (13), (14), (22)-(25) are new and not discussed heretofore. It is remarkable that the obtained solutions in this article have potential physical meaning for the underlying equations. In addition to the physical meaning, these solutions can be used to measure the accuracy of numerical results and to help in the study of stability analysis.

6. CONCLUSIONS

In this paper, we have successfully executed the functional variable method to attain new exact traveling wave solutions of a family of higher-dimensional space-time fractional KdV-type

equations arising in mathematical physics, namely the (3+1)-dimensional space–time fractional ZK equation and the (2+1)-dimensional space–time fractional GZK-BBM equation. Two types of solutions including hyperbolic function solutions and trigonometric function solutions are obtained and numerically simulated in Figs. 1-3. The obtained solutions are significant to reveal the inner mechanism of the nonlinear physical phenomena that described by the aforementioned equations. It is shown that the FVM is straightforward, powerful and can be extended to handle many other higher-dimensional fractional partial differential equations as it maintains the reduced volume of computational work. With the aid of the Maple, we have verified our results.

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