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Research Article

Exact Solutions of the Mass-Dependent Klein-Gordon Equation with the Vector Quark-Antiquark Interaction and Harmonic Oscillator Potential

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Using the asymptotic iteration and wave function ansatz method, we present exact solutions of the Klein-Gordon equation for the quark-antiquark interaction and harmonic oscillator potential in the case of the position-dependent mass.

1. Introduction

The movement of a particle with harmonic oscillations around an equilibrium position creates one of the most fundamental problems of physics. The basic structure of different systems such as vibration of the diatomic molecules, oscillations of atoms in crystal lattices, or nucleons in core is a harmonic oscillator problem [1]. Further, the quantum theory of electromagnetic fields is closely related to the examples of the harmonic oscillator [2, 3]. When a particle is in a strong potential field, the relativistic effect must be considered. However, in relativistic and nonrelativistic quantum mechanics, many authors have adequately the harmonic oscillator problem [4, 5]. But, it has never been investigated for relativistic particles with position-dependent mass (pdm). Quantum mechanical review of relativistic spin-0 particles with pdm in the harmonic oscillator potential is very important in terms of understanding physical behavior of systems as the above-mentioned systems. The studies of quantum and relativistic quantum systems with pdm have received increasing attention in the literature. Systems with pdm have been found to be very useful in studying the physical and electronic properties of semiconductors, quantum wells and quantum dots, quantum liquids, ^{3}He clusters, graded alloys, and semiconductor heterostructures [6]. To study the quasi-exactly solvable and exactly solvable nonrelativistic Schrödinger, relativistic Klein-Gordon and Dirac equations in the presence of pdm having a suitable mass distribution functions in one, three, and/or any arbitrary D-dimensional cases for different potentials have been used in different methods by many authors [7–21].

In this study, in addition to the examination of the harmonic oscillator potential, relativistic spin-0 particles with pdm have also been investigated in the quark-antiquark interaction potential. This type of potential and some of central potentials have recently been studied using different techniques [22–25]. The quark-antiquark interaction potential consists of harmonic, linear, and Coulomb potential terms. As we know, the quark-antiquark interaction potentials are a spherically symmetrical potential. The spherically symmetrical potential model also presents a good description of heavy quarkonium mass spectra such as charmonium and bottomonium. The interaction potentials for such systems are of a confining type called the Cornell potential. The Cornell potential consists of two terms, namely, the Coulomb and linear terms. The Coulomb term is responsible for the interaction at small distances and the linear term leads to the confinement. This type of interaction potential is also

supported by lattice quantum chromodynamics calculations [26]. The quark-antiquark interaction has also been studied using the Coulomb term plus the power potential [27].

The organization of this paper is as follows. In the second section, the AIM is given shortly. In the third section, the general formalism of the Klein-Gordon equation for spin-0 particles with pdm has been considered. The relativistic energy eigenvalues and corresponding eigenfunctions have been presented for the harmonic oscillator and quark-antiquark interaction potentials preserved in the fourth and fifth section, respectively. Finally, conclusions are given in the last section.

2. Basic Equations of the AIM

We briefly outline the AIM here; the details can be found in [28–30]. The AIM was proposed to solve second-order differential equations of the form:

$$y'' = \lambda_0(x)\, y' + s_0(x)\, y, \tag{1}$$

where $\lambda_0(x) \neq 0$ and $s_0(x)$ are in $C_\infty(a,b)$, and these variables are sufficiently differentiable. The differential equation (1) has a general solution as follows:

$$y(x) = \exp\left(-\int^x \alpha dx'\right) \times \left[C_2 + C_1 \int^x \exp\left(\int^{x'} \left[\lambda_0\left(x''\right) + 2\alpha\left(x''\right)\right] dx''\right) dx'\right] \tag{2}$$

if $n > 0$, for sufficiently large n,

$$\frac{s_n}{\lambda_n} = \frac{s_{n-1}}{\lambda_{n-1}} = \alpha_k, \tag{3}$$

where

$$\lambda_n(x) = \lambda'_{n-1}(x) + s_{n-1}(x) + \lambda_0(x)\,\lambda_{n-1}(x), \qquad s_n(x) = s'_{n-1}(x) + s_0(x)\,\lambda_{n-1}(x), \quad n = 1, 2, 3, \ldots \tag{4}$$

The termination condition of the method together with (4) can be also written as follows:

$$\delta(x) = \lambda_{n+1}(x)\, s_n(x) - \lambda_n(x)\, s_{n+1}(x) = 0. \tag{5}$$

For a given potential, the idea is to convert the relativistic wave equation to the form of (1). Then, s_0 and λ_0 are determined and s_n and λ_n parameters are calculated. The energy eigenvalues are obtained by the termination condition given by (5). However, the exact eigenfunctions can be derived from the following wave function generator:

$$y_n(x) = C_2 \exp\left(-\int^x \alpha_k dx'\right), \tag{6}$$

where $n = 0, 1, 2, \ldots$ and k is the iteration step number, and it is greater than n.

3. Formalism of the Klein-Gordon Equation with Pdm

In the relativistic quantum mechanics, for spin-0 particles with pdm, the Klein-Gordon equation is defined as follows:

$$\nabla^2\psi(r) + \frac{1}{\hbar^2 c^2}\left[\left(E_{nl} - V(r)\right)^2 - \left(m(r)\,c^2 + S(r)\right)^2\right]\psi(r) = 0, \qquad \nabla^2 = \sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2}, \tag{7}$$

where $V(r)$ and $S(r)$ are Lorentz vector and scalar potential, respectively, $m(r)$ is mass function, and E_{nl} is the energy of particle. Let us decompose the radial wave function $\psi(r)$ as follows:

$$\psi(r) = \frac{u(r)}{r} Y_m^l(\hat{r}), \tag{8}$$

where $u(r)$ is the radial wave function and $Y_m^l(\hat{r})$ is the angular dependent spherical harmonics, and this reduces (8) into the following Schrödinger-like equation with position-dependent mass:

$$\frac{d^2u(r)}{dr^2} + \frac{1}{\hbar^2 c^2} \times \left(\left[E_{nl} - V(r)\right]^2 - \left[m(r)\,c^2 + S(r)\right]^2 - \frac{l(l+1)}{r^2}\right) \times u(r) = 0. \tag{9}$$

4. In Case of Harmonic Oscillator Potential

4.1. The Eigenvalues. In case of harmonic oscillator to investigate spin-0 particles with pdm, we should solve (9). In this solution, we use atomic units $\hbar = c = 1$. However, in (9), we prefer to use mass function similar to type of harmonic oscillator potential as follows:

$$m(r) = m_0 + \frac{1}{2}kr^2, \tag{10}$$

where m_0 and k are positive constants. The selection as in (10) of position dependent mass function is more suitable both physically and mathematical. Already, in physical applications, the position-dependent mass creates a new effective potential by shifting potential profile of the system.

In this study, in the absence of scalar potential, vector harmonic oscillator potential is defined as

$$V(r) = \frac{1}{2}m(r)\,\omega^2 r^2, \tag{11}$$

where $\omega = \sqrt{k/m(r)}$ is the angular frequency and k is elastic coefficient.

In the presence of vector potential and by taking $\hbar = c = 1$, if (10) and (11) are inserted into (9), it is obtained that

$$\left[\frac{d^2}{dr^2} + \xi_0 - \xi_1 r^2 - \frac{\xi_2}{r^2}\right] u(r) = 0, \tag{12}$$

where $\xi_0 = E_{nl}^2 - m_0^2$, $\xi_1 = k(E_{nl} - m_0)$, $\xi_2 = l(l+1)$.

In (12), while r approaches zero and infinite, solving of (12) are $r^{(1/2)(1+\sqrt{1+4\xi_2})}$, $e^{-r^2\sqrt{\xi_1}/2}$, respectively. Therefore, the reasonable physical wave function is proposed as follows:

$$u_{nl}(r) = r^{(1/2)(1+\sqrt{1+4\xi_2})} e^{-r^2\sqrt{\xi_1}/2} f_{nl}(r). \tag{13}$$

Substituting (13) into (12), we have second-order homogeneous linear differential equation:

$$\begin{aligned} f''(r) = &\left(-\frac{1+\sqrt{1+4\xi_2}}{r} + 2r\right) f'(r) \\ &- \left(\xi_0 - \sqrt{\xi_1}\left(2 + \sqrt{1+4\xi_2}\right)\right) f(r). \end{aligned} \tag{14}$$

Defining a new variable $z = \sqrt{\xi_1} r^2$, so doing, we have solvable differential equation by AIM as follows:

$$\begin{aligned} f''(z) = &\left(1 - \frac{2+\sqrt{1+4\xi_2}}{2z}\right) f'(z) \\ &+ \frac{\sqrt{\xi_1}\left(2+\sqrt{1+4\xi_2}\right) - \xi_0}{4\sqrt{\xi_1} z} f(z). \end{aligned} \tag{15}$$

By comparing (15) with (1), $\lambda_0(z)$ and $s_0(z)$ values are obtained, and using (4) we calculate $\lambda_n(z)$ and $s_n(z)$. In this way,

$$\begin{aligned} \lambda_0(z) &= \left(1 - \frac{2+\sqrt{1+4\xi_2}}{2z}\right), \\ s_0(z) &= \frac{\sqrt{\xi_1}\left(2+\sqrt{1+4\xi_2}\right) - \xi_0}{4\xi_1 z}, \\ \lambda_1(z) &= \left(4\xi_1\left(2 - 2z + \sqrt{1+4\xi_2}\right)^2\right. \\ &\quad + 4\sqrt{\xi_1}\left(-2z\xi_0 + 2\sqrt{\xi_1}\left(2+\sqrt{1+4\xi_2}\right)\right) \\ &\quad \left. + z\sqrt{\xi_1}\left(2+\sqrt{1+4\xi_2}\right)\right) \times \left(16z^2\xi_1\right)^{-1}, \\ s_1(z) &= \left(\left(2 - \sqrt{\xi_1}\left(2+\sqrt{1+4\xi_2}\right)\right)\right. \\ &\quad \left.\times \left(4\sqrt{\xi_1} + 2\sqrt{\xi_1}\left(2 - 2z + \sqrt{1+4\xi_2}\right)\right)\right) \\ &\quad \times \left(16z^2\xi_1\right)^{-1} \\ &\vdots \end{aligned} \tag{16}$$

Combining these results obtained by the AIM with quantization condition given by (5) yields

$$\begin{aligned} s_0\lambda_1 - s_1\lambda_0 = 0 &\Longrightarrow \xi_{00} \\ &= \frac{1}{2}\sqrt{\xi_1}\left(2 + \sqrt{1+4\xi_2}\right), \quad \text{for } n = 0 \end{aligned} \tag{17a}$$

$$\begin{aligned} s_1\lambda_2 - s_2\lambda_1 = 0 &\Longrightarrow \xi_{01} \\ &= \frac{1}{2}\sqrt{\xi_1}\left(6 + \sqrt{1+4\xi_2}\right), \quad \text{for } n = 1 \end{aligned} \tag{17b}$$

$$\begin{aligned} s_2\lambda_3 - s_3\lambda_2 = 0 &\Longrightarrow \xi_{02} \\ &= \frac{1}{2}\sqrt{\xi_1}\left(10 + \sqrt{1+4\xi_2}\right), \quad \text{for } n = 2 \end{aligned} \tag{17c}$$

$$\vdots$$

If the set of equations (17a), (17b), and (17c) are generalized, the indirect energy eigenvalues statement turns out to be

$$\xi_{0n} = \frac{1}{2}\sqrt{\xi_1}\left(4n + 2 + \sqrt{1+4\xi_2}\right). \tag{18}$$

When (18) and $\xi_0 = E_{nl}^2 - m_0^2$ are compared, it is found for energy eigenvalues that

$$E_{nl}^2 = m_0^2 + \frac{\sqrt{k\left(E_{nl} - m_0\right)}}{2}\left(4n + 2 + \sqrt{1 + 4l(l+1)}\right). \tag{19}$$

4.2. The Eigenfunctions. The exact eigenfunctions can be derived from the following generator:

$$f_n(z) = C_2 \exp\left(-\int^z \alpha_k dz'\right). \tag{20}$$

Using (3) and (20), the eigenfunctions are obtained as follows:

$$\begin{aligned} f_0(z) &= 1, \\ f_1(z) &= \left(z - \frac{\left(2+\sqrt{1+4\xi_2}\right)}{2}\right), \\ f_2(z) &= \left(\frac{2+\sqrt{1+4\xi_2}}{2}\right)\left(1 + \frac{2+\sqrt{1+4\xi_2}}{2}\right) \\ &\quad - 2\left(1 + \frac{2+\sqrt{1+4\xi_2}}{2}\right) z + z^2, \\ &\vdots \end{aligned} \tag{21}$$

Finally, the following general formula for the exact solutions $f_n(z)$ is acquired as

$$\begin{aligned} f_n(z) = &(-1)^n \left(\frac{2+\sqrt{1+4\xi_2}}{2}\right)_n \\ &\times {}_1F_1\left(-n, \frac{2+\sqrt{1+4\xi_2}}{2}; z\right). \end{aligned} \tag{22}$$

Hence, we write the total radial wavefunction as follows:

$$u_{nl}(r) = N r^{(1/2)(1+\sqrt{1+4\xi_2})} e^{-r^2\sqrt{\xi_1}/2} \left(\frac{2+\sqrt{1+4\xi_2}}{2}\right)_n \times {}_1F_1\left(-n, \frac{2+\sqrt{1+4\xi_2}}{2}; \xi_1 r^2\right), \tag{23}$$

where N is normalization constant.

5. In Case of Quark-Antiquark Interaction Potential

5.1. The Eigenvalues and Corresponding Eigenfunctions. In this section, we present the solution of the Klein-Gordon equation for the quark-antiquark interaction potential. This potential is defined as

$$V(r) = ar^2 + br - \frac{c}{r}, \quad a > 0, \tag{24}$$

where a, b, and c are constants. In atomic units $\hbar = c = 1$, we solve (9) in the absence of scalar potential. Consider the position-dependent mass function:

$$m(r) = m_0 + ar^2 + br - \frac{c}{r}. \tag{25}$$

On substituting (24) and (25) into (9), we find that

$$\left[\frac{d^2}{dr^2} + \varepsilon - \varepsilon_0 r^2 - \varepsilon_1 r + \frac{\varepsilon_2}{r} - \frac{l(l+1)}{r^2}\right] u_l(r) = 0, \tag{26}$$

where $\varepsilon = E_{nl}^2 - m_0^2$, $\varepsilon_0 = 2a(E_{nl} + m_0)$, $\varepsilon_1 = 2b(E_{nl} + m_0)$, and $\varepsilon_2 = 2c(E_{nl} + m_0)$.

To solve (26), applying an ansatz to the radial wavefunction $u_l(r)$,

$$u_l(r) = e^{\alpha r + (\gamma/2) r^2} \sum_{n=0} a_n r^{n+\delta}. \tag{27}$$

If (27) is inserted into (26), it is obtained that

$$\sum_{n=0} a_n \underbrace{\left[\gamma + 2\gamma(n+\delta) + \varepsilon\right]}_{A_n} r^{n+\delta} + \sum_{n=0} a_n \underbrace{\left[2\alpha\gamma - \varepsilon_1\right]}_{2\alpha\gamma=\varepsilon_1} r^{n+\delta+1} + \sum_{n=0} a_n \underbrace{\left[\alpha^2 - \varepsilon_0 + \gamma^2\right]}_{\varepsilon_0=\alpha^2+\gamma^2} r^{n+\delta+2} + \sum_{n=0} a_n \underbrace{\left[2\alpha(n+\delta) + \varepsilon_2\right]}_{B_n} r^{n+\delta-1} + \sum_{n=0} a_n \underbrace{\left[(n+\delta)(n+\delta-1) - l(l+1)\right]}_{C_n} r^{n+\delta-2} = 0, \tag{28}$$

$$\varepsilon_1 = 2\alpha\gamma, \qquad \varepsilon_0 = \alpha^2 + \gamma^2 \tag{29}$$

α and γ can be obtained with the help of (29). Then, $\alpha = (-\sqrt{\varepsilon_0 + \varepsilon_1} - \sqrt{\varepsilon_0 - \varepsilon_1})/2$ and $\gamma = (-\sqrt{\varepsilon_0 + \varepsilon_1} + \sqrt{\varepsilon_0 - \varepsilon_1})/2$. Editing (28),

$$\sum_{n=0} a_n A_n r^{n+\delta} + \underbrace{\sum_{n=0} a_n B_n r^{n+\delta-1}}_{n\to n+1} + \underbrace{\sum_{n=0} a_n C_n r^{n+\delta-2}}_{n\to n+2} = 0, \tag{30}$$

$$\underbrace{(a_0 B_0 + a_1 C_1)}_{0} r^{\delta-1} + a_0 \underbrace{C_0}_{0} r^{\delta-2} + \sum_{n=0} \underbrace{(a_n A_n + a_{n+1} B_{n+1} + a_{n+2} C_{n+2})}_{0} r^{n+\delta} = 0. \tag{31}$$

In (31), if the first nonvanishing coefficient is $a_0 \neq 0$, C_0 should be equal to zero:

$$C_0 = \delta(\delta - 1) - l(l+1) = 0. \tag{32}$$

We choose $\delta = l + 1$ as a physically acceptable solution from (32). Moreover, if the pth nonvanishing coefficient is $a_p \neq 0$, but $a_{p+1} = a_{p+2} = \cdots = 0$, then, from (31), it has to be $A_p = 0$. At that rate,

$$\gamma + 2\gamma(p + \delta) + \varepsilon = 0. \tag{33}$$

Using together (29) and (33), we obtain the energy eigenvalues. Namely,

$$\frac{b(E_{nl} + m_0)}{\alpha} + \frac{2b(E_{nl} + m_0)}{\alpha}(p + l + 1) + E_{nl}^2 - m_0^2 = 0. \tag{34}$$

A_n, B_n, and C_n must satisfy the determinant relation for a nontrivial solution:

$$\det \begin{vmatrix} B_0 & C_1 & \cdots & \cdots & \cdots & 0 \\ A_0 & B_1 & C_2 & \cdots & \cdots & . \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & A_{p-1} & B_p \end{vmatrix} = 0. \tag{35}$$

In order to appreciate this method, we present the exact solutions for the cases $p = 0, 1$ as follows.

If $p = 0$, $\det|B_0| = 0$ and $B_0 = 0$. So,

$$B_0 = 0 \Longrightarrow 2(E_0 + m_0)c = -2(l+1)\alpha. \tag{36}$$

We will obtain energy eigenvalues by using (34). But, we cannot ignore (36) Because it is a restriction on the parameters of the potential and the l quantum number.

The corresponding eigenfunction for $p = 0$ is given as

$$u_l^0(r) = a_0 \exp\left[-\frac{\sqrt{\varepsilon_0 + \varepsilon_1} + \sqrt{\varepsilon_0 - \varepsilon_1}}{2} r - \frac{\sqrt{\varepsilon_0 + \varepsilon_1} - \sqrt{\varepsilon_0 - \varepsilon_1}}{4} r^2\right] r^{\delta}, \tag{37}$$

where a_0 is the normalization constant.

If $p = 1$, $\det \begin{vmatrix} B_0 & C_1 \\ A_0 & B_1 \end{vmatrix} = 0$. In this case, it is obtained that

$$\underbrace{\left(\frac{\varepsilon_1}{2\alpha}+\frac{\varepsilon_1}{\alpha}(l+1)+\varepsilon\right)2(l+1)-\left(\varepsilon_2+2\alpha(l+1)\right)\left(\varepsilon_2+2\alpha(l+2)\right)}_{\text{for } E_1} = 0, \tag{38}$$

which is a restriction on the parameters of the potential and the angular momentum quantum number.

The corresponding eigenfunction for $p = 1$ is given as

$$u_l^1(r) = (a_0 + a_1 r)\exp\left[-\frac{\sqrt{\varepsilon_0+\varepsilon_1}+\sqrt{\varepsilon_0-\varepsilon_1}}{2}r - \frac{\sqrt{\varepsilon_0+\varepsilon_1}-\sqrt{\varepsilon_0-\varepsilon_1}}{4}r^2\right] r^{\delta}, \tag{39}$$

where a_0 is the normalization constant.

Following in this way, we can generate a class of exact solutions by setting $p = 1, 2, \ldots$. Generally, if $a_p \neq 0$, but $a_{p+1} = a_{p+2} = \cdots = 0$. So, the energy eigenvalues E_p is obtained by using (34). The corresponding eigenfunction is

$$u_l^p(r) = \left(a_0 + a_1 r + \cdots + a_p r^p\right) \times \exp\left[-\frac{\sqrt{\varepsilon_0+\varepsilon_1}+\sqrt{\varepsilon_0-\varepsilon_1}}{2}r - \frac{\sqrt{\varepsilon_0+\varepsilon_1}-\sqrt{\varepsilon_0-\varepsilon_1}}{4}r^2\right] r^{\delta}, \tag{40}$$

where $a_0, a_1, \ldots, a_p$ are normalization constants.

6. Conclusions

This paper has presented a different approach, the AIM, to calculate the bound state solutions of the relativistic Klein-Gordon with the harmonic oscillator potential in the case of the pdm. For arbitrary quantum number l state, we have exactly obtained the energy eigenvalues and corresponding eigenfunctions for the case of mass function by AIM. The advantage of the AIM is that it gives the eigenvalues directly by transforming the second-order differential equation into a form of $y'' = \lambda_0(r)y' + s_0(r)y$. The exact wavefunctions are easily constructed by iterating the values of s_0 and λ_0. The method presented in this study is general and worth extending to the solution of other interactions. For the quark-antiquark interaction potential, to solve Klein-Gordon equation with pdm, we have used wave function ansatz method. While using this method, the most important factor to be considered is a corresponding restriction on the parameters of the quark-antiquark potential and the l quantum number.

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