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AUTHOR(S): M. J. Lahart

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los alamos scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87545

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EXAMPLES OF MAXIMUM LIKELIHOOD SPATIAL FILTERING*

M. J. Lahart

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87545

SUMMARY

We describe a maximum likelihood method for estimating spatial frequency components of truncated sections of data. The method is used to estimate the one-dimensional Fourier transform of short scans of images of an edge and of a slit. With the aid of a constrained least squares noise control, the frequency response of the imaging system is computed from the estimated Fourier transform.

The application of Fourier transform techniques to image processing usually requires large amounts of data. Much of spatial filtering theory is based on the Fourier integral with its infinite limits, which we must approximate in practice with a finite integration region. Images must be truncated, and the truncation leads to artifacts in the processed image. These tend to be most prominent near the edges, and, in practice, we often discard all but the center region of the image, where errors are smaller.

The effects of truncation must be considered more carefully when the data that is available is confined to a limited region. For example, the evaluation of the spatial frequency response of an optical system makes use of a target such as an edge, whose image is relatively small in extent. A larger image than necessary has more noise, but no more information than a smaller image, and inaccuracies arise because of this. If the image is too small, edge effects may lead to severe distortions of its Fourier transform. Distortions occur because the Fourier transform of the image of an edge, computed in the usual manner, is the same as that of a periodic repetition of the truncated edge image, with a discontinuity occurring once in each period that leads to large errors in the high-frequency part of the transform.

Truncation leads to errors in the determination of an infinite Fourier transform because this transform cannot be determined uniquely from finite sets of data. In order to compute the Fourier transform, we often assume that the data is zero outside the truncation region or that it repeats itself periodically. Both of these assumptions are usually incorrect. We can avoid them, however, by computing the least squares estimate or the most likely estimate of the Fourier transform that is compatible with the data. To compute these statistical estimates, we make assumptions only about the statistics of the data, but not about the data itself, outside the truncation region.

We will assume that the untruncated image is statistically stationary and compute the least squares estimate of the Fourier transform. This coincides with the maximum likelihood estimate when the data is normally distributed.[1] The Gaussian distribution is the maximum entropy distribution when, as in our case, information only about the first and second order moments is used,[2] and it is reasonable if the average value of the data is not too close to zero. When the data may be assumed to be normally distributed, our estimate of its Fourier transform is its most likely value; in any event, it is the estimate that minimizes the sum of the squares of the error.

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ESTIMATION OF FOURIER TRANSFORM

The estimate $\beta_F(f)$ of the Fourier transform of the image data may be expressed in terms of the Fourier transform $\beta(f)$ that is computed by assuming that the truncated section of the data is repeated periodically. If M is the matrix of correlation coefficients between different spectral components of $\beta(f)$ and M_F is the array of correlation coefficients between $\beta(f)$ and $\beta_F(f)$, the expression for the least squares estimate of $\beta_F(f)$ is[1]

$$\vec{\beta}_F(f) = (M_F M^{-1})\vec{\beta}(f) \ . \qquad (1)$$

The vector notation denotes a row of $\beta(f)$ whose components represent different values of the spatial frequency f.

We have shown previously[3] that an element of the correlation coefficient matrix M, of a section of length L of a stationary process is

$$M_{ij} = \int t(f)t(f + f_j - f_i)A(f_i - f)df \ , \qquad (2)$$

where A(f) is the Wiener spectrum of the stationary process and t(f) is

$$t(f) = \frac{\sin \pi f L/2}{\pi f L/2} \ . \qquad (3)$$

Equation (2) was derived by expressing the section of the stationary process as a product of the process and a rectangle function whose Fourier transform is given in Eq. (3). In the Fourier domain this product corresponds to a convolution, and the coefficient M_{ij} is the ensemble average of two of these convolutons. By a similar derivation, an element of M_F is

$$M_{Fij} = A(f_i)t(f_i - f_j) \ . \qquad (4)$$

EXAMPLES

We have used this maximum likelihood technique to estimate the response of an imaging system from the images it yields of edges and slits. We used Eq. (1) to compute the most likely values of the Fourier transform, performed a constrained least squares restoration on the result to reduce the effects of noise and divided the resulting spectral estimate by the Fourier transform of the original object. The constrained least squares noise reduction algorithm adjusts the difference between the image and the convolution of the object with the signal by minimizing the squares of the second derivatives, subject to the constraint that the noise equal some estimated value.[4] When the object is an edge, the constrained least squares filter has the form

$$\alpha(f) = \frac{(2\pi i f)^{-1}}{[(2\pi i f)^{-2} + \gamma f^4]} \beta_F(f) \quad , \tag{5}$$

where $\alpha(f)$ is the Fourier transform of the point spread function, and γ is a parameter that is related to the estimated value of the noise. In practice, point spread functions are computed for several values of γ, and the lowest value of γ which yields a reasonably smooth result is assumed to be optimal.

Equations (2) and (4) require an estimate of the Wiener spectrum A(f) of the untruncated stationary process. For both slits and edges, we have used f^{-2} since the square of the absolute values of the Fourier transform of both slits and edges is proportional to this quantity. (The Wiener spectrum estimate was arbitrarily set to unity at zero spatial frequency.) Although the A(f) that is used must be reasonable, we have found empirically that moderate errors in it have little influence on the estimate $\beta_F(f)$.

Figure 1 is a photograph of an edge that was used to evaluate our imaging system, and Fig. 2 is a one-dimensional trace of a scan across it by a slit. The section between points at 10 mm and 120 mm was processed, using the maximum likelihood Fourier technique and Eq. (5), to find the system line spread function. This is shown in Fig. 3, and its Fourier transform, the system frequency response, is in Fig. 4. Figures 5 and 6 show the trace of a slit and the system line spread function, computed in the same way. We have little information about the higher spatial frequency portion of $\beta_F(f)$ because of the large amount of noise in the scan and the relatively small magnitudes of the Fourier transforms of the edge and slit targets in these regions. We do have reliable estimates of the low frequency portion, however, and the point spread functions in Figs. 3 and 6 indicate that edge effects are relatively small. We were not required, in finding Fourier transforms, to use larger parts of the scan than necessary to include all of the available signal information.

The maximum likelihood technique outlined here is based entirely on second order moments. As a consequence of this, the equations that must be solved are linear. Inclusion of more information may lead to a knowledge of higher order moments and would possibly improve the accuracy of the estimate of $\beta_F(f)$, but this would lead to nonlinear equations. In our case, much information has been extracted from the data without taking this step.

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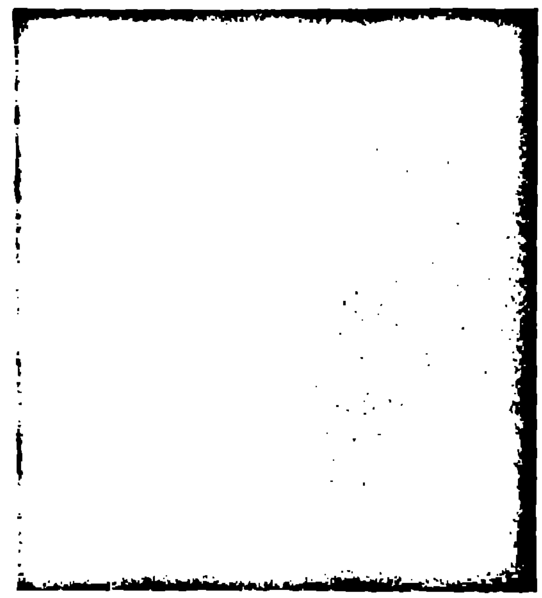

Fig. 1. Image of an edge target used to evaluate an imaging system.

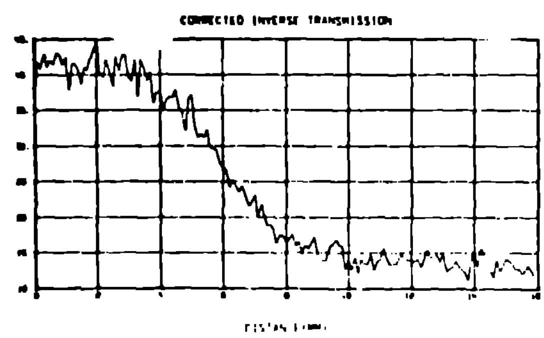

Fig 2. One-dimensional scan of the edge of target of Fig. 1.

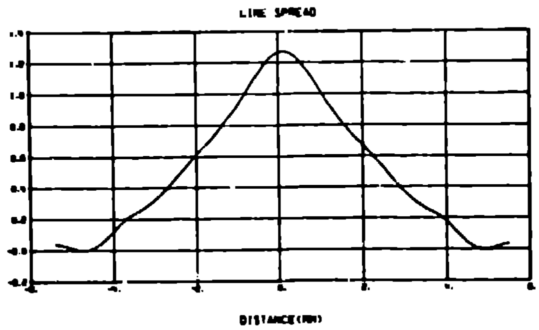

Fig. 3. Line spread function of system computed from 10 mm to 120 mm section of Fig. 2 using maximum likelihood Fourier estimation with constrained least squares noise control.

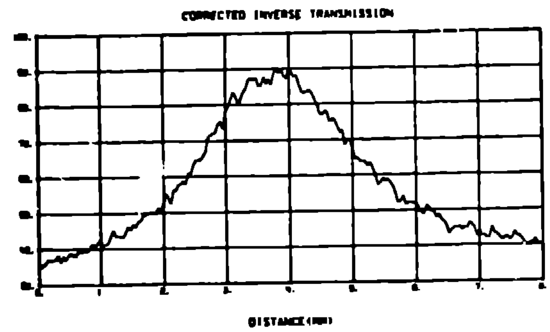

Fig. 5. One-dimensional scan of a slit target.

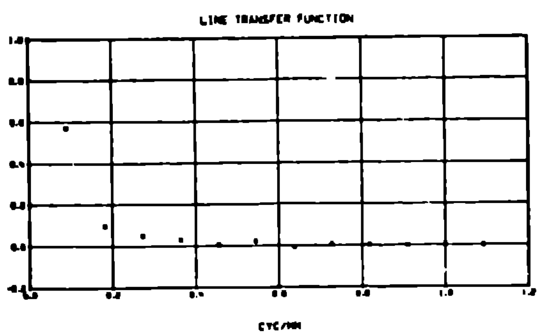

Fig. 4. System spatial frequency response, computed as Fourier transform of Fig. 3.

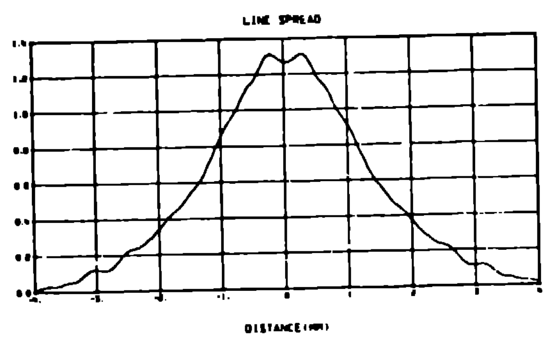

Fig. 6. System line spread function, computed from Fig. 5 using maximum likelihood Fourier estimation and constrained least squares noise control.