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EXCHANGE EFFECTS IN THE DIFFRACTIVE BEHAVIOUR
OF NUCLEAR COMPTON SCATTERING

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ABSTRACT

We suggest that nuclear Compton scattering below meson production threshold can provide a fundamental tool in the analysis of the exchange effects in nuclei. We evaluate exchange terms in a model-independent way and show that they strongly affect the diffractive behaviour even in the-low momentum transfer region.

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It is often said that electron scattering provides a better probe of nuclear structure with respect to photoreactions since it is possible to change energy and momentum transfer independently. The aim of the present letter is to show that, in fact, also Compton scattering can be exploited for the above purpose and that, in addition, it is also sensitive to exchange effects. We find overwhelming evidence of their presence even at small values of the momentum transfer (e.g., in the region around 100 MeV of incident photon energy where others like hard-core and short-range correlations do not yet come into play, different from electron scattering).

In earlier papers¹⁻³⁾ we have discussed the behaviour of the nuclear Compton scattering amplitude below pion production threshold, paying particular attention to the evaluation of exchange effects. For completeness, we begin by recalling the essential steps used to determine the amplitude M for the elastic process:

$$\gamma(\omega, \vec{k}) + N \rightarrow \gamma(\omega, \vec{k}') + N \quad (1)$$

(N being the nuclear ground state).

In the non-relativistic approximation it is necessary to introduce seagull terms in the evaluation of the Compton scattering amplitude in order to keep gauge invariance, since only positive frequency intermediate states survive in this limit. We obtain^{1,2)}

$$\begin{aligned} M \approx & \epsilon_\ell \epsilon'_m \left\{ \sum_n \int d^3x \int d^3y e^{i\vec{k}\cdot\vec{x}} e^{-i\vec{k}'\cdot\vec{y}} \right. \\ & \left. \frac{[\langle N | J_e(0, \vec{y}) | n \rangle \langle n | J_m(0, \vec{x}) | N \rangle + \text{crossed}] }{(\bar{E}_N - E_n + \omega - i\Gamma_n/2)} \right\} \quad (2) \\ & - (\bar{\epsilon} \bar{\epsilon}') \frac{e^2}{M} \langle N | \sum_i (1 + \tau_i^3) e^{i\vec{q}\cdot\vec{r}_i} | N \rangle \\ & + e^2 \langle N | \sum_{i < j} (\tau_i^3 - \tau_j^3)^2 (\bar{\epsilon} \bar{\epsilon}_{ij}) (\bar{\epsilon}' \bar{\epsilon}'_{ij}) V(\tau_{ij}) e^{i\vec{q}(\frac{\vec{r}_i + \vec{r}_j}{2})} | N \rangle \end{aligned}$$

where $\bar{\epsilon}, \bar{\epsilon}'$ are the photon polarizations, $\vec{q} = \vec{k} - \vec{k}'$, $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$, and V^{ex} is the exchange potential of the nuclear Hamiltonian. Equation (2) is a rearrangement of the results of Ref. 2) in which we have neglected the factor

$$\frac{\Delta \text{im}^2 \left(\frac{(\vec{k} + \vec{k}')}{4}, \vec{z}_{ij} \right)}{\left(\frac{(\vec{k} + \vec{k}')}{4}, \vec{z}_{ij} \right)^2} = \frac{\Delta \text{im}^2 \left(\frac{\vec{q}}{4}, \vec{z}_{ij} \right)}{\left(\frac{\vec{q}}{4}, \vec{z}_{ij} \right)^2}$$

which multiplies the last term and differs from one only by a few per cent in the energy region we are interested in. We also stress here that $J_1(0, \vec{y})$ is the fully conserved, non-relativistic electromagnetic current with explicit presence of exchange effects and that the two seagull terms are the static limit of the negative frequency part of the scattering off nucleons and off virtual exchanged particles (Z graphs).

It has been shown²⁾ that the different pieces in the amplitude [Eq. (2)] combine to reproduce the Thomson limit at zero energy and the usual resonant contribution in the giant resonance region. In the asymptotic region ($\omega \approx 100$ MeV) only the seagull terms survive. In the absence of exchange effects we would, therefore, obtain the expected $-Ze^2/M$ high-energy scattering off incoherent protons⁴⁾.

Corrections to Eq. (2) come from the tail of the T ordered resonant behaviour and from the composite structure of the nucleon⁵⁾; however, we have estimated their effect to be less than 20% and we will neglect them in the present discussion. Both these corrections are small because nuclear collective excitations of energy of the order of 20-40 MeV and the nucleon excitation region are very well separated. This can be seen, for example, from photo-absorption data⁶⁾ which exhibit a smooth behaviour above 50 MeV up to the pion production threshold.

For the same reason it is possible to neglect the contribution of the imaginary part automatically zero in our static approximation⁷⁾. With these approximations we can write in the region of energy ranging approximately between 60 and 120 MeV⁸⁾

$$\left(\frac{d\sigma}{d\Omega} \right)_{el.} \approx \left(\frac{1 + \cos^2 \theta}{2} \right) \left\{ -Z \frac{e^2}{M} F_p(q^2) - \frac{ZN e^2}{AM} K F_{ex}(q^2) \right\}^2 \quad (3)$$

where $q = 2\omega \sin \theta/2$ and

$$Z F_p(q^2) = \langle N | \sum_i \frac{(1 + \tau_i^3)}{2} e^{i\vec{q} \cdot \vec{z}_i} | N \rangle \quad (4)$$

$$\frac{ZN}{MA} K F_{ex}(q^2) = \langle N | \sum_{i < j} (\tau_i^3 - \tau_j^3)^2 V(\tau_{ij}) e^{i\vec{q} \cdot \left(\frac{\vec{z}_i + \vec{z}_j}{2} \right)} | N \rangle \quad (5)$$

where K is chosen in order to normalize F_{ex} to unity. $F_p(q^2)$ is the usual charge form factor and can be expressed in terms of the proton density $\rho_p(r)$:

$$F_p(q^2) = \frac{1}{Z} \int d^3z \rho_p(z) e^{i\vec{q}\cdot\vec{z}} \quad (6)$$

$F_{ex}(q^2)$ is given in terms of the proton density by:

$$K \frac{ZN}{MA} F_{ex}(q^2) = \int d^3z_1 \int d^3z_2 e^{i\vec{q}\cdot\frac{(\vec{z}_1+\vec{z}_2)}{2}} \cdot \rho_p(z_1) \rho_n(z_2) V^{ex}(z_{12}) F(z_{12})$$

where

$$F(z_{12}) = \left[\frac{3}{(k_F z_{12})^2} \left(\frac{\sin(k_F z_{12})}{(k_F z_{12})} - \cos(k_F z_{12}) \right) \right]^2 \quad (7)$$

with $k_f = 1.35 \text{ fermi}^{-1}$ is a correlation function which takes into account the Pauli principle. In a more transparent way, introducing $\bar{R} = (\bar{r}_1 + \bar{r}_2)/2$ and $\bar{r} = (\bar{r}_1 - \bar{r}_2)/2$, we obtain

$$K \frac{ZN}{AM} F_{ex}(q^2) = \int d^3R \rho_{ex}(R) e^{i\vec{q}\cdot\vec{R}} \quad (8)$$

where

$$\rho_{ex}(R) = \int d^3z V^{ex}(z) F(z) \rho_p(\vec{z}+\vec{R}) \rho_n(\vec{z}-\vec{R}) \quad (9)$$

and, of course,

$$\int d^3R \rho_{ex}(R) = K \left(\frac{ZN}{AM} \right) \quad (10)$$

Equations (9) and (10) define K as the mean value of the same two-body operator appearing in the Bethe-levinger dipole sum rule as a correction due to the exchange effects.

This result has an immediate physical interpretation: the first term of Eq. (3) measures the contribution coming from the scattering of the protons:

$$-e^2 \frac{Z}{M} F_p(q^2) = \langle N | \sum_{i=\text{protons}} -\frac{e_i^2}{M} e^{i\vec{q}\vec{r}_i} | N \rangle \quad (11)$$

and the second term arises from the scattering on charged-exchanged particles. In the hypothesis that pions dominate the exchange process we have:

$$-e^2 \frac{ZN}{AM} K F_{ex}(q^2) = \langle N | \sum_{i=\text{pions}} -\frac{e_i^2}{m_\pi} e^{i\vec{q}\vec{r}_i} | N \rangle \quad (12)$$

and as a consequence we can estimate the number of charged pions "seen" at this energy in the nucleus to be

$$n_\pi = K \frac{NZ}{A} \frac{m_\pi}{M} \quad (13)$$

The pion contribution gives a sizable effect, since it depends on the average value of the squared charge distribution; on the contrary in photo-reactions or electron scattering the amplitude can be expressed, by means of the Siegert theorem, in terms of the charge density and the positive and negative fluctuations of the charge (due to exchanged pions) cancel out to a large extent. Therefore, in these processes it is necessary to probe much smaller distances to observe deviations from the impulse approximation and there, as already remarked, other effects come into play and blur the picture⁹⁾.

The exchange contribution in the Compton scattering amplitude turns out to be essential in order to satisfy forward dispersion relations¹⁰⁾ which can be evaluated²⁾ using data on photo-absorption⁶⁾ and yield a value of the order of unity for K.

A direct comparison with experiment is not possible in the forward direction because of the presence of the strongly forward-peaked Delbruck scattering, whose theoretical treatment is not yet fully under control¹¹⁾. Therefore, we restrict our analysis to rather large angles and consider the shaded region in the plane (ω, q) (Fig. 1).

As usual, the hardest step in the quantitative analysis consists in the evaluation of K which is very sensitive to the correlation function F and to the choice of the nuclear potential V^{ex} . Here we assume for K the approximate value of one, obtained from the dispersion relation. It can easily be seen, by direct computation, that the shape of $\rho_{\text{ex}}(R)$ does not sensitively depend upon the details of the dynamics but only on the product of $\rho_p(r_1) \cdot \rho_n(r_2)$. This has been tested by using an internucleon potential (Yukawa form) with mass = 1 and 2 fermi⁻¹, and by explicitly introducing a hard core; for this reason they can be used reliably even if Eqs. (7), (8) and (9) may look a bit simplistic because of the presence of the Pauli correlation factor only.

We can then explicitly evaluate the cross-section using ρ_p , ρ_n and K as input. With ρ_p given by a three-parameter fermi distribution and with the hypothesis $\rho_p = \rho_n$ for $Z = N$ nuclei, we find for O^{16} and Ca^{40} the results given in Figs. 2a and 2b where we plot in semilogarithmic scale both $F_p(q^2)^2$ and $[F_p(q^2) + (N/A)K F_{\text{ex}}(q^2)]^2$, and of Fig. 3 which represents the ratio of the two expressions given above¹²⁾.

Some remarks are necessary:

- 1) The cross-section is strongly affected by the contribution of ρ_{ex} . Since the effective radius of ρ_{ex} , which can be thought of as representing the pion distribution, is less than the proton radius, there is a shift to higher momentum transfer of the first minimum of the diffraction spectrum.
- 2) By inverse Fourier transforming the form factor, one can map out the pion charge distribution. In particular, its extrapolation to the low q region gives a measurement of the forward cross-section enhancement $(1 + K N/A)^2$ and yields, therefore, a measurement of K .
- 3) In the diffractive behaviour minima are not filled in due to the absence of Coulomb distortion. In other words, there is an intrinsic exact momentum resolution of the projectile, which means that here the observed charge form factor is really the Fourier transform of the charge distribution. These pure nuclear, unsmearred diffractive effects, of course facilitate the detection of exchange contributions around the minima.
- 4) From an experimental point of view, Delbruck scattering away from the forward direction is negligible and no problem seems to emerge even in a rather poor resolution experiment with no very precise determination of the final nuclear states. Inelastic charge form factors to nearby lying excited states are known to be depressed by orders of magnitude and we can exclude any excitation of the exchange part below the giant resonance region.

Of course we need a good resolution on q which is equivalent to a knowledge of the energy of the incoming photon. This was the real trouble with the first experiment in this energy region carried out 20 years ago¹³⁾; however, it can be overcome nowadays by the new technique of producing monochromatic high-energy photons with laser backward scattering on electron beams¹⁴⁾.

5) In conclusion, in spite of the rather limited energy and momentum transfer range at our disposal, the effect due to exchanged charged particles can be pinned down in a region where it is dominant.

Experimental results can, therefore, be used to determine the pion charge distribution in nuclei rather accurately, without the problems inherent to electron scattering and photo-reactions.

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- 7) Using the optical theorem and the parametrization for the data of Ref. 6) above 50 MeV $\sigma_A(\omega) = 288 (NZ/A)\omega^{-3/2}$ mb (B. Ziegler, internal note, 19.3.76) we find $\text{Im } f(\omega) \approx Z \cdot 12/\sqrt{\omega}$ mb \cdot MeV. This amounts to a correction of 5-10% in the elastic Compton scattering cross-section.
- 8) In Eq. (3) the possibility of a quadrupole structure of the pion distribution arising in deformed nuclei because of anisotropy of exchange effects³⁾ has been overlooked.
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Figure captions

- Fig. 1 : Energy momentum region allowed in elastic Compton scattering experiment within the validity of the present theoretical treatment.
- Fig. 2 : Plot of the charge form factor $|F_p(q^2)|^2$ (dotted line) and of the expression $|F_p(q^2) + K(N/A) F_{ex}^p(q^2)|^2$ (full line) for O^{16} (Fig. 2a) and Ca^{40} (Fig. 2b) (q in MeV/c).
- Fig. 3 : Ratio of the cross-section with and without exchange effects to the cross-section in the allowed region (q in MeV/c).

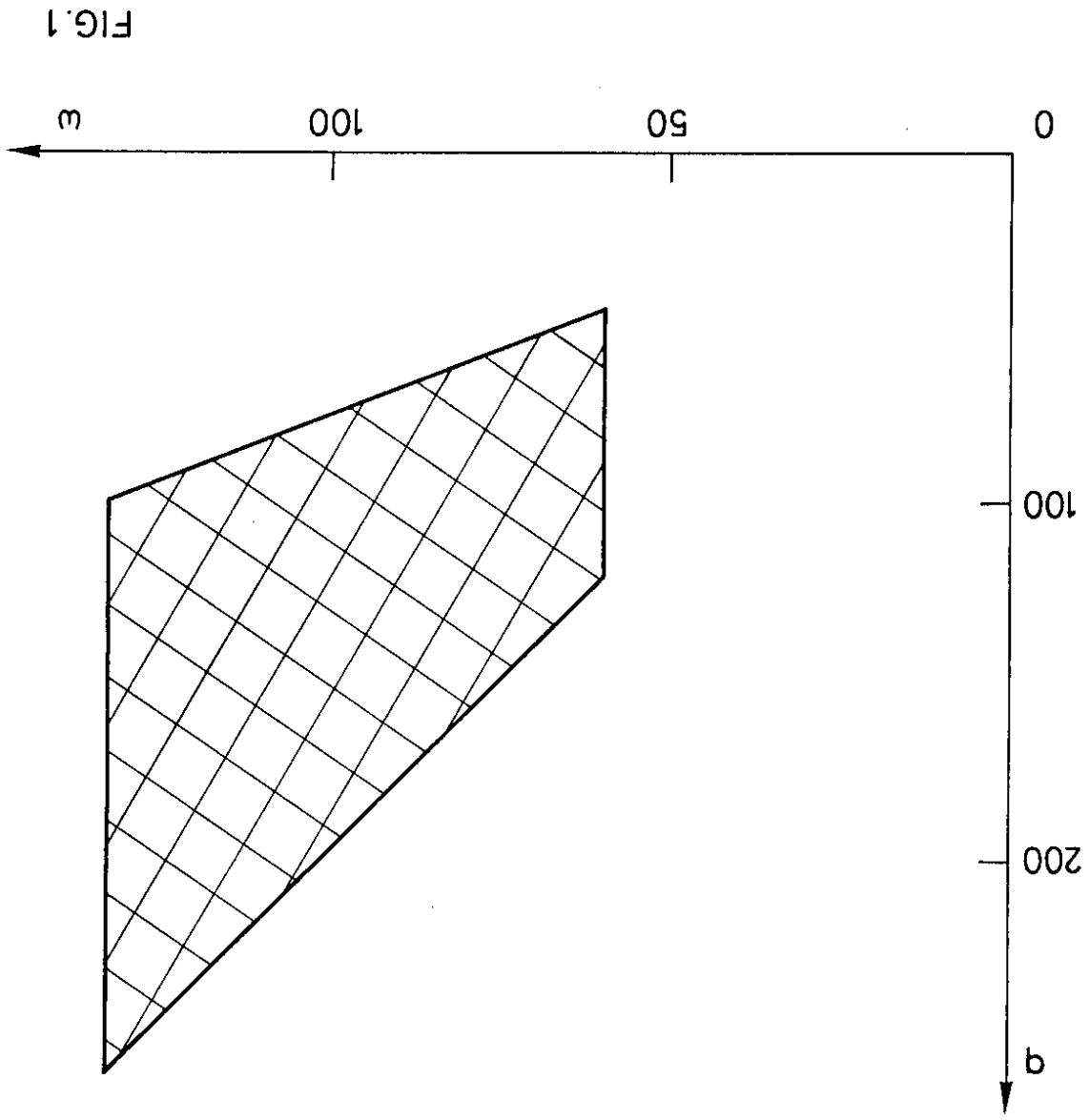


FIG. 1

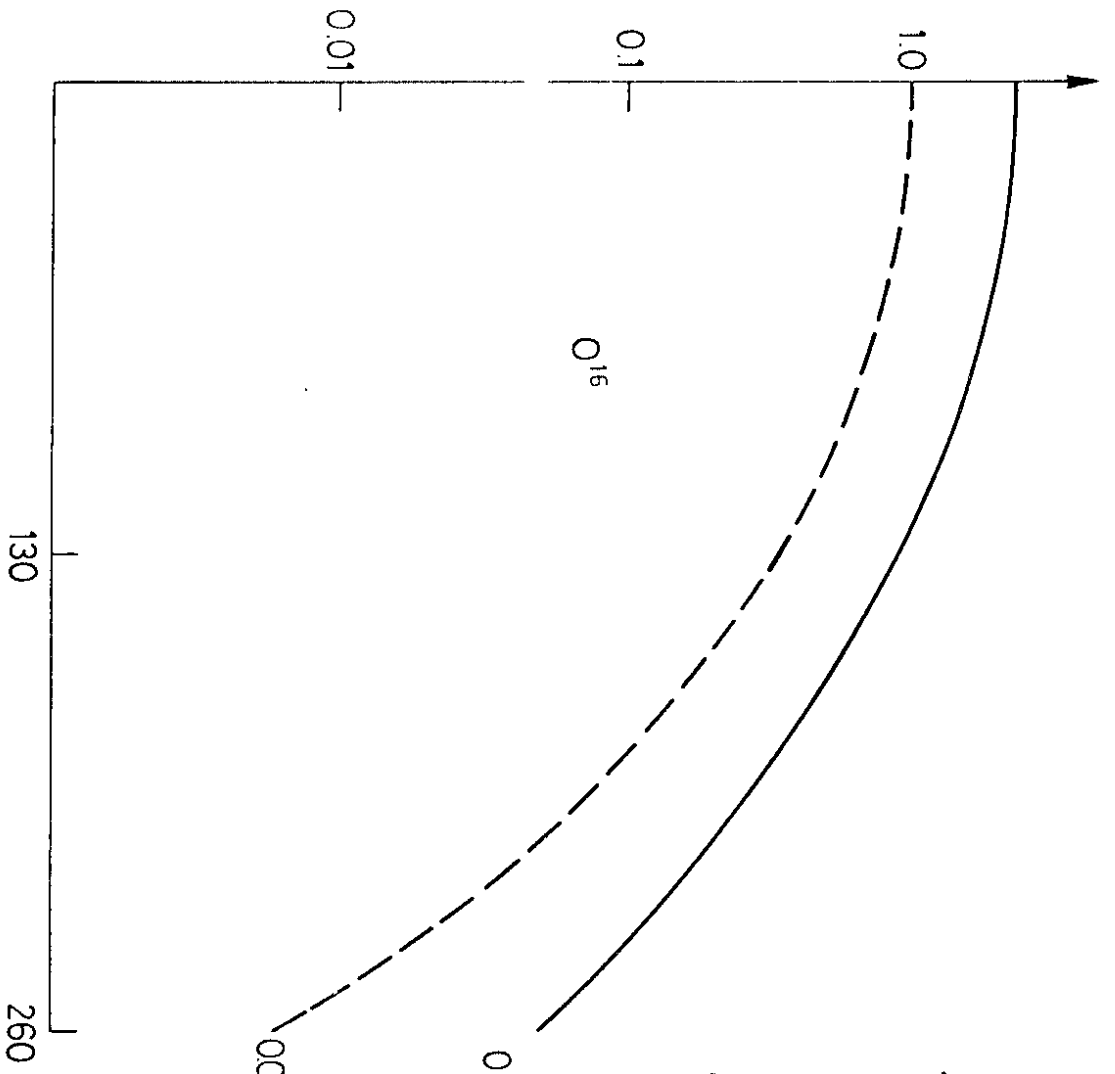


FIG. 2a

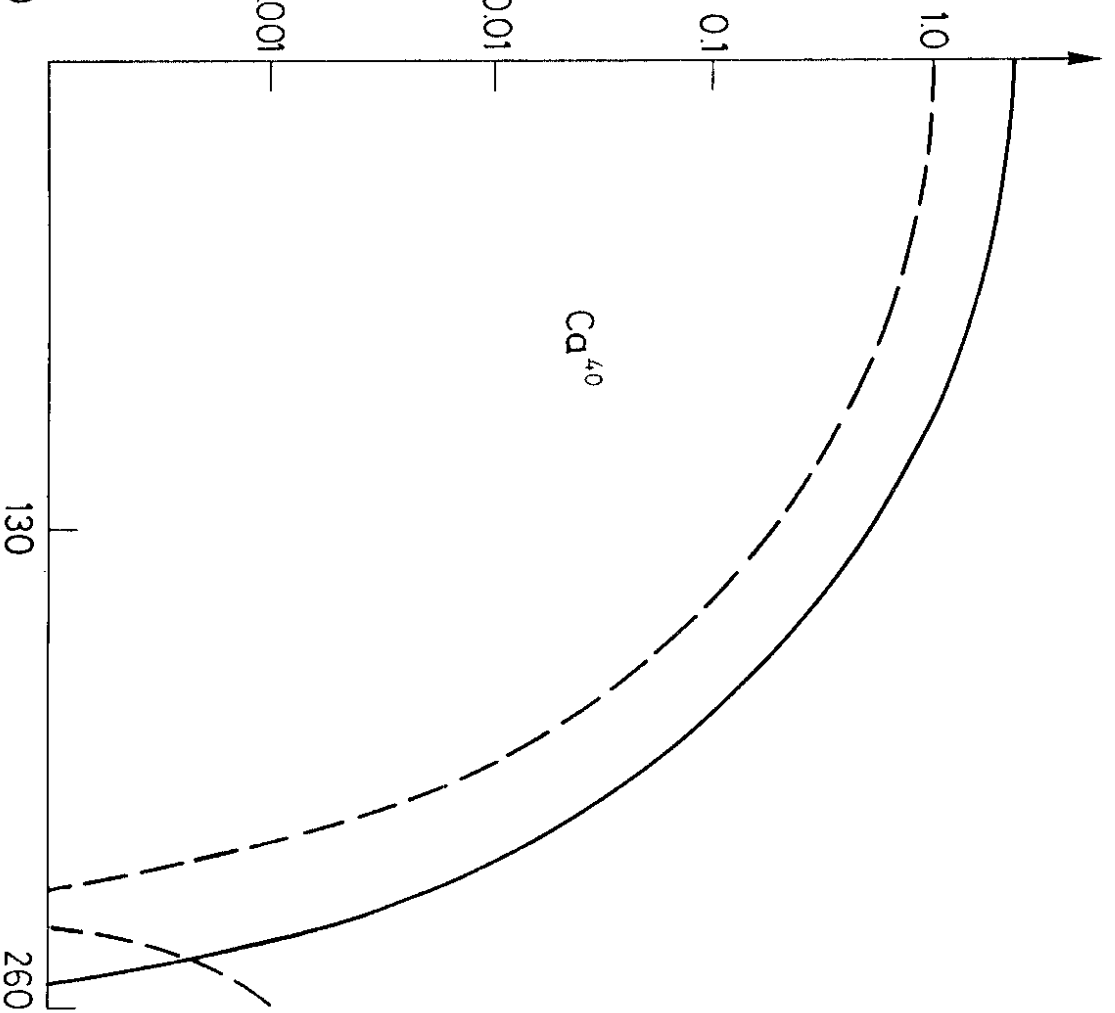


FIG. 2b

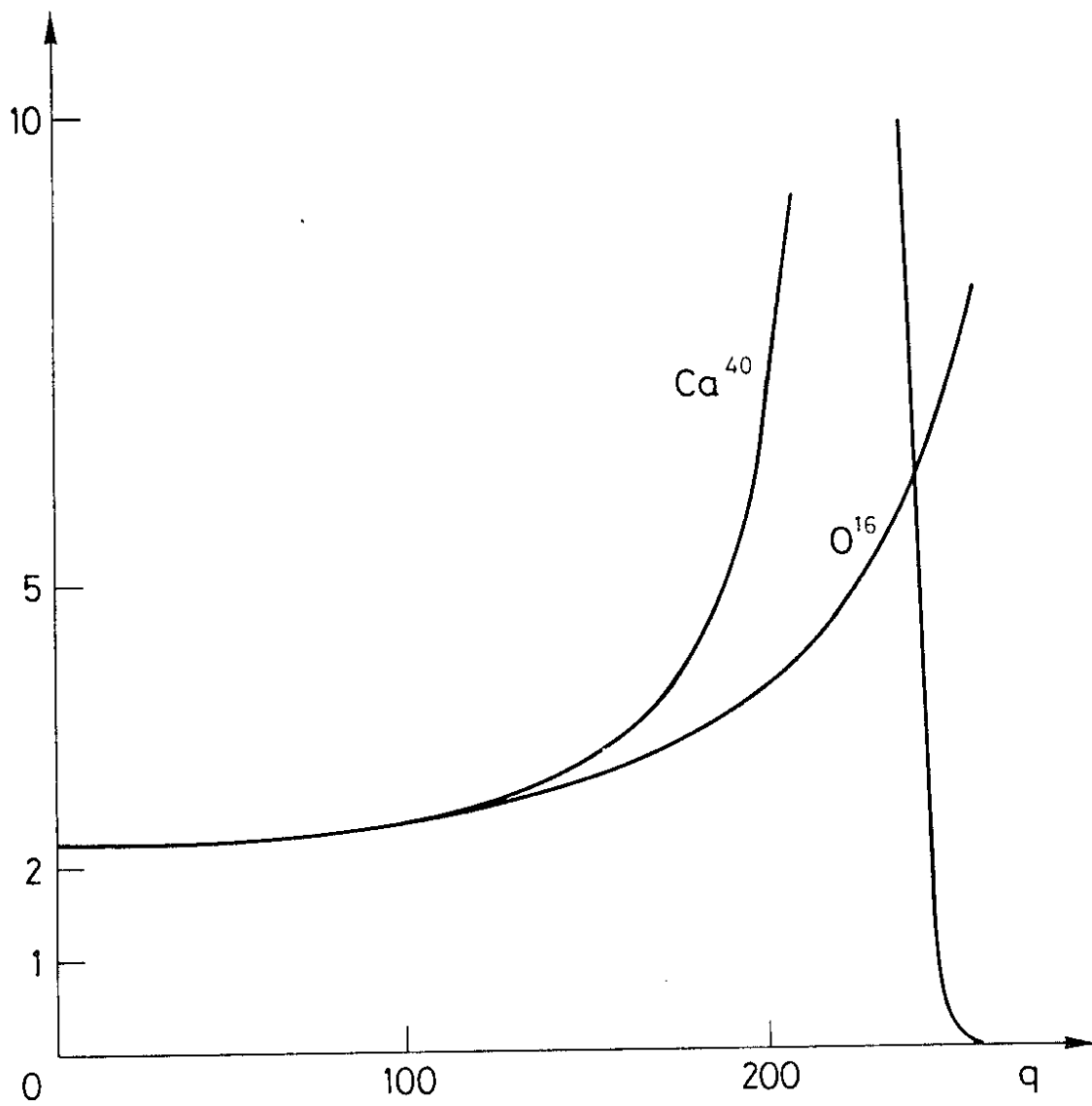


FIG. 3

