

Excitation of Hadronic Matter and H -quantum^{*)}

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The mass levels of mesons and baryon resonances are studied from the view-point of the compositeness of "molecule-type". The model of the density oscillation in the hadronic matter is discussed in connection with the mass levels of mesons, and an estimation of the sound velocity of the density oscillation is given by applying the theory of collective motion. The mass levels of baryon resonances are discussed in relation with Maglić's rule and the theory of intermediate coupling. H -quantum is discussed as being due to a kind of collective motion in the hadronic matter arising from strong correlations which are not taken into account in the collective motion for the density oscillation.

§1. Introduction

The Sakata model¹⁾ of the composite structure of the elementary particles proposed in 1955 has provided us the way to recognize various aspects of the elementary particles as the appearance of the dynamics of the fundamental particles or urbaryons, which lie in strata or levels deeper than those of the elementary particles in the series of infinite strata of matter in Nature each having its own law.²⁾

On the basis of the Sakata model, Ogawa (1959)³⁾ proposed the Ogawa symmetry to treat symmetrically p , n and Λ which were assumed to be the fundamental particles in the original form of the Sakata model. Ikeda, Ogawa and Ohnuki (1959)⁴⁾ then applied a group theoretical method to the Ogawa symmetry, studying the meson octet. Gell-Mann (1964)⁵⁾ and Zweig (1964)⁶⁾ showed that the baryon octet could be treated with such a group theoretical method as in Ref. 4). Many of group-theoretical studies on mesons and baryons or baryon resonances have been done until now by making use of groups of various kinds.

These studies have been, however, mainly in the direction to seek the characteristics of the symmetry properties among the fundamental particles or ur-baryons, without entering into the dynamics of the fundamental particles or ur-baryons. In other words, these studies have been performed from the side of the kinematics, remaining in the classification of the experimental data with the use of the phenomenological mass formula or selection rules.

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As regards the point of view from the side of the dynamics of the fundamental particles or ur-baryons, on the other hand, the so-called non-relativistic or independent quark model has been considered by many authors. In connection with the theory of baryon structure by Taketani and Fujimoto (1965),⁷⁾ the non-relativistic quark model has certain difficulties, upon which discussions are given in a separate paper of the present authors⁸⁾ together with a review of criticisms⁹⁾ against reasonings for it.

As is summarized and discussed in the first paper in Part I of the present issue of this journal, phenomena of multiple production of particles in hadronic interactions in the region of incident energy of the order of $10^1 \sim 10^3$ GeV are closely connected with the production of H -quanta due to Hasegawa (1961).^{10a)} Hasegawa (1963)^{10b)} also observed that there is a phenomenological resemblance between H -quantum and the composite system of a nucleon and an anti-nucleon, and the mass of H -quantum was determined from the phenomenological resemblance of the decay process of H -quantum to the annihilation process of the system of a proton and an anti-proton.

One of the philosophical bases of the Sakata model is the analogy on the theory of atomic nucleus which assumes the nucleus as being a composite system of p and n . Nobody at present has a doubt that an atomic nucleus is composed of p and n . As was pointed out by Taketani (1963),¹¹⁾ however, one knows at the same time that the nuclear statistical model, which assumes a nucleus as a simple aggregate of p and n , and the single particle model, which emphasizes individualities of p and n in a nucleus, are effective in some respects, but the reliance only upon one of them will lead one to deadlocks. It is necessitated to regard the atomic nucleus as a liquid drop in some cases and to emphasize the existence of α -particles as constituent in other cases. One will need other nuclear models, too.

Tati (1969)¹²⁾ proposed an idea to understand the H -quantum as the excited state of the hadronic matter composed of superstrongly interacting many basic particles, which Tati (1966)¹³⁾ supposed to be constituents of ur-baryons, too. About this Tati's idea, one of the present authors (M.T.) has pointed out, in the course of discussions with Tati, that it is important to study the excited states of hadrons by means of the concept of the hadronic matter, thereby getting a clue to the nature of the hadronic matter. At the same time, M.T. pointed out also that it is necessary to study the possibility that the H -quantum arises as an accumulative point of the meson levels, beyond which the excitation of the hadronic matter will have other characteristics different from that of the excitation of the hadronic matter giving rise to the meson levels. In Tati (1970),¹²⁾ this thought of M.T. has been treated as the surface oscillations of the hadronic matter for hadron levels and the internal vibrations in the interior part of the hadronic matter for H -quanta.

As was discussed in Taketani and Fujimoto (1965),⁷⁾ and as has been studied in Nagasaki and Taketani (1973)^{8),14)} in connection with the problem of the hard core in nuclear force and the angular distributions of p - p elastic scattering at high energies, the phenomena of hadronic interactions in the region of incident laboratory energy up to at least a few or several ten GeV strongly suggest us that in this energy region the structure of mesons is excited while the structure of baryons remains in a deeper stratum than that of the structure of mesons. In other words, these phenomena support the possibility pointed out by Taketani (1964)¹⁵⁾ that mesons are like molecules if baryons composed of ur-baryons are considered on the analogy of atoms, to which the names of the compositeness of "atom-type" and that of "molecule-type" were given by Sakata (1964).¹⁶⁾ This is in accordance with the properties mentioned above of the H -quantum.

It seems necessary, therefore, to investigate models of the structure of mesons in which both the excitation of the hadronic matter and the compositeness of "molecule-type" are taken into consideration. The aim of the present paper is to show such a model along the line described in our previous papers.^{17),18)} In §2, the mass levels of mesons are studied with a model in which they are assumed to arise from the density oscillation of the hadronic matter. In §3, the excitation of the hadronic matter is discussed as due to the phonon mode of collective excitation of pairs of a nucleon and anti-nucleon. Taking into account the residual interaction among three and four phonons, the H -quantum is then discussed as due to a kind of collective motion of phonons. In §4, the mass levels of baryon resonances are discussed in connection with Maglič's rule¹⁹⁾ and the theory of intermediate coupling. Section 5 is devoted to discussions of the SH- and heavy quanta observed by Japan-Brazil Emulsion Chamber Collaboration.

§2. Mass levels of mesons

According to Refs.7) and 8), we suppose that the structure of mesons extends over a spatial region with the radius a more or less equal to the nucleon Compton wave length: $a \approx 1/M_N$, M_N being the nucleon mass and $\hbar=c=1$. Let us consider therefore the excitation of the hadronic matter with the radius a . As is mentioned in §1, we shall now discuss the density oscillation of the hadronic matter in order to study the mass levels of mesons.

In the original form of the Sakata model, a pion is composed of pair of a nucleon and an anti-nucleon. The field theoretical bound state problem of the composition of the pion out of pair of a nucleon and an anti-nucleon is complex one, though there have been certain studies with the use of, for example, the chain approximation by Maki (1956).²⁰⁾ The problem of the binding of the hadronic matter, whose excitation is supposed here to give

rise to the mass levels of mesons, is therefore expected to be complex, too. As in the case of the nuclear matter, however, there is a possibility that the excitation of the hadronic matter may be treated by simpler methods, while leaving the problem of its binding to be determined later. We thus assume that the pion is the ground state of the hadronic matter, and consider the density oscillation of the hadronic matter as giving rise to the excitation of the hadronic matter.

Woeste (1952)²¹⁾ studied the density oscillation of compressible nucleus. For the convenience of our present discussion, the essentials of Ref. 21) are summarized in the appendix, rewriting some of the equations in Ref. 21) in suitable forms to our purpose. Rewriting the result given in Ref. 21), we know that the energy ω_ℓ of the density oscillation with the angular momentum ℓ is given in terms of the sound velocity \tilde{c} by the relation

$$\omega_\ell = (\tilde{c}/a)x_\ell, \quad (1)$$

where x_ℓ is a solution of the equation

$$\ell/x_\ell^2 - j_{\ell+1}(x_\ell)/x_\ell j_\ell(x_\ell) = \{\xi^2(\ell-1)(\ell+2)\}^{-1}. \quad (2)$$

Here, j_ℓ is the spherical Bessel function of ℓ -th order, and

$$\xi = (\varepsilon_s/\rho_0 a)^{1/2}/\tilde{c}, \quad (3)$$

ρ_0 and ε_s being the average mass density and the surface energy per unit area of the matter under consideration, respectively.

ξ represents the ratio of the velocity of the surface wave to the sound velocity, because $(\varepsilon_s/\rho_0 a)^{1/2}$ gives us the measure for the velocity of the surface wave.¹⁷⁾ The familiar expression for the incompressible matter can be obtained from (2), by putting $\tilde{c} \rightarrow \infty$.²¹⁾ That is, rewriting (2) as

$$\ell/\omega_\ell^2 - (a/\tilde{c})j_{\ell+1}(x_\ell)/\omega_\ell j_\ell(x_\ell) = (\rho_0 a^3/\varepsilon_s)/(\ell-1)(\ell+2), \quad (4)$$

we have the result in the limit $\tilde{c} \rightarrow \infty$ that

$$\omega_\ell = \{(\varepsilon_s/\rho_0 a^3)\ell(\ell-1)(\ell+2)\}^{1/2}. \quad (5)$$

This has been used in Tati.¹²⁾ In the case of the incompressible matter, the oscillations with $\ell=0$ and 1 cannot be excited on account of the constancy of the density and center-of-mass. In this case ω_0 and ω_1 tend to infinity with \tilde{c} .

In the case of the compressible matter, that is in the case of a finite \tilde{c} , the oscillations with $\ell=0$ and 1 can be excited with finite values of ω_0 and ω_1 . In this case, if ε_s is not small so that $\xi \gtrsim 1$, ω_0 and ω_1 are comparable with ω_2 , ω_3 , and so on. Physically, this is because the surface wave propagates faster than the sound wave. On the other hand, if ε_s is small so that

$\xi \ll 1$, ω_2 , ω_3 , etc. are small compared with ω_0 and ω_1 . These oscillations with $\ell \geq 2$ having very small energies arise from the non-spherical surface deformation which occurs after the sound wave has propagated repeatedly. We suppose, therefore, that on account of some instability, the non-linear effect, for example, these oscillations accompanying the non-spherical surface deformation could not be allowed, as, for example, in the case of heavy enough nuclei which are unstable for fission through the non-spherical deformation with small ω_2 , ω_3 and ω_4 because of the reduction in effective surface tension due to the Coulomb force. It should be noted that for the oscillations with $\ell=0$ and $\ell=1$, the spherical form of the surface is maintained, and also that ω_1 does not depend on ϵ_s and the dependence of ω_0 on ϵ_s is weak.

In making the comparison of the result of our present model with the existing mass levels of mesons, we assume that the intrinsic spin and parity of the mesons are given by the original form of the Sakata model, just as the angular momenta of the odd nucleons in A. Bohr's collective model of the nucleus²²⁾ are coupled to that of the collective motion. On assuming so, there are still two, or more, possibilities of making the comparison, depending on the degree of taking the view point of the symmetry theory, as has been noted in Ref. 17), since, although the oscillation in general of the hadronic matter will naturally carry the charge, strangeness and intrinsic spin, we have not entered into such complexity in our present model.

One way is to take all the mesons together, without taking notice of the symmetry theory. Figure 1 shows the correspondence between the theoretical and experimental mass levels of mesons for this case. In the calculation of the meson mass m_b in our model, we have put

$$m_b = m_\pi + n\omega_\ell^{(k)}, \quad (6)$$

where $\ell=1$ and 2 , $n=1, 2, 3, \dots$, and $\omega_\ell^{(k)}$ is the energy of the k -th normal oscillation with ℓ . For $\ell=0$, we have $\omega_0^{(k)} \cong k\omega_0^{(1)}$ when $\xi \ll 1$. For the state of two phonons with $\ell=1$, the total orbital angular momentum is 0 or 2. In Fig. 1, we have put $a=1/M_N$ and adjusted \tilde{z} so as to fit to the mass of η meson with $\ell=0$, $n=1$ and $k=1$, since η meson is in a 1S_0 state. The value of \tilde{z} thus obtained is $\tilde{z}=0.14$, that is, the sound velocity is about one seventh of the light velocity.

Another way of making the comparison is to take mesons separately along the line of the symmetry theory. In this case, one considers that π , η and K mesons are in ground states degenerate in principle. Figure 2 shows an example of the comparison of such kind, in which ρ meson in a 3S_1 state is fitted by the one phonon state with $\ell=0$. In the case of Fig. 2 we have $\tilde{z}=0.22$. The separate comparisons for η mesons and K mesons gives us also values of \tilde{z} around 0.15.

In any way, \tilde{z} cannot be near to unity if we take, as the smallest excitation

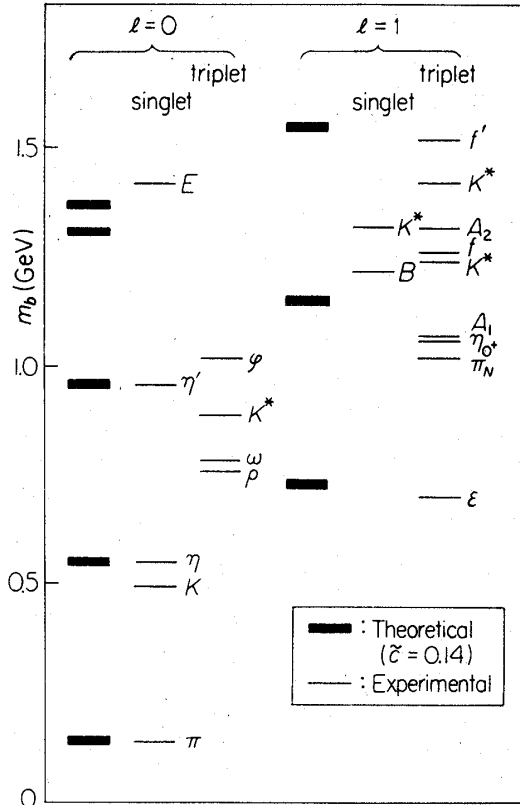


Fig. 1. Correspondence between the theoretical and experimental mass levels of mesons.

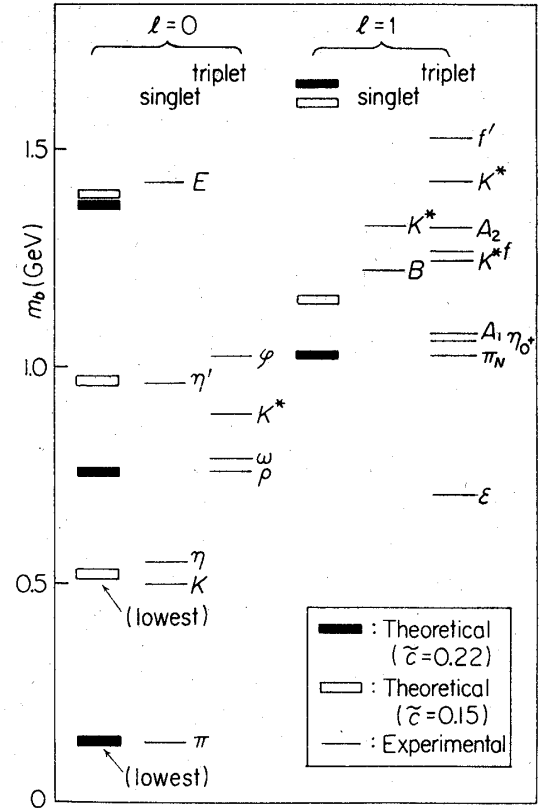


Fig. 2. Correspondence between the theoretical and experimental mass levels of mesons.

energy, several hundred MeV at most, together with $a=1/M_N$. From the discussions given above, we see that the sound velocity in the hadronic matter is

$$\tilde{z} \approx 0.2, \quad (7)$$

in order to get the general trend of the mass levels of mesons.

In our present model, m_π is left to be determined. In this connection, it may be interesting to see $m_\pi \approx \omega_0/2$, the zero point oscillation energy. Under the condition that $\xi \ll 1$ applied above, the mass of the hadronic matter concerned with the density oscillation is not related to the value of $\omega_0^{(k)}$ and $\omega_1^{(k)}$, which are mainly determined by the sound velocity \tilde{z} , as mentioned above. This condition is, when written in another form, $E_s \ll 3 m \tilde{z}^2$, where m is the mass of the hadronic matter concerned with the density oscillation and E_s is the total surface energy. The binding of the hadronic matter itself should determine m_π in relation with m in our model, as has been described at the beginning of the present section.

§3. Collective modes of excitation of hadronic matter and H -quantum

In this section, we shall at first try to make an estimation of the velocity of the sound wave of the density oscillation in the hadronic matter. The quantum theory of density oscillation in an aggregate of many fermions was studied by Tomonaga (1950),²³⁾ and have been formulated by many authors. In this section, we shall make use of the approximational method used in Sawada (1957).²⁴⁾

Let us denote with H_0 and H_{int} the free and interaction Hamiltonians, respectively, of the field of nucleon and anti-nucleon. H_{int} is to represent the fundamental force in the sense of the compositeness of "molecule-type" mentioned in §1. In this section, H_{int} will be assumed to be of the four-fermion type. Let us denote with $\rho'(\mathbf{k})$ such a part of the Fourier component of the field density $\rho(\mathbf{k})$ that is contributed by pairs of a nucleon and an anti-nucleon:

$$\rho'(\mathbf{k}) = (2\pi)^{-3/2} \int d\mathbf{p} (a_{\mathbf{p}+\mathbf{k}/2}^* b_{-\mathbf{p}+\mathbf{k}/2}^* + b_{\mathbf{p}+\mathbf{k}/2} a_{-\mathbf{p}+\mathbf{k}/2}), \quad (8)$$

where $a_{\mathbf{p}}^*$ and $a_{\mathbf{p}}$ are the creation and annihilation operators for the nucleon, and $b_{\mathbf{p}}^*$ and $b_{\mathbf{p}}$ are those for the anti-nucleon. In (8), we have ignored the spin and iso-spin variables in order to make an estimation of the sound velocity of the density oscillation, along the line of the treatment of the density oscillation in our present model in the preceding section.

Let us assume that the ground state Ψ_0 with the energy E_0 is known for the total Hamiltonian $H = H_0 + H_{\text{int}}$:

$$H\Psi_0 = E_0\Psi_0. \quad (9)$$

Let us further assume that H_{int} is divided into two parts,

$$H_{\text{int}} = H_{\text{int}}^{(1)} + H_{\text{int}}^{(2)}, \quad (10)$$

so that we have the commutation relation

$$\begin{aligned} & [a_{\mathbf{p}+\mathbf{k}/2}^* b_{-\mathbf{p}+\mathbf{k}/2}^*, H_{\text{int}}^{(1)}] \\ &= (2\pi)^{-3f} \int d\mathbf{p}' (a_{\mathbf{p}'+\mathbf{k}/2}^* b_{-\mathbf{p}'+\mathbf{k}/2}^* + b_{\mathbf{p}'+\mathbf{k}/2} a_{-\mathbf{p}'+\mathbf{k}/2}), \end{aligned} \quad (11)$$

and

$$\begin{aligned} & [b_{\mathbf{p}+\mathbf{k}/2} a_{-\mathbf{p}+\mathbf{k}/2}, H_{\text{int}}^{(1)}] \\ &= -(2\pi)^{-3f} \int d\mathbf{p}' (a_{\mathbf{p}'+\mathbf{k}/2}^* b_{-\mathbf{p}'+\mathbf{k}/2}^* + b_{\mathbf{p}'+\mathbf{k}/2} a_{-\mathbf{p}'+\mathbf{k}/2}), \end{aligned} \quad (12)$$

where f is the coupling constant of the interaction of four-fermion type. We have then

$$\begin{aligned} & \{H - (E_0 + \varepsilon_{\mathbf{p}+k/2} + \varepsilon_{\mathbf{p}-k/2})\} a_{\mathbf{p}+k/2}^* b_{-\mathbf{p}+k/2}^* \Psi_0 \\ &= -(2\pi)^{-3} f \int d\mathbf{p}' (a_{\mathbf{p}'+k/2}^* b_{-\mathbf{p}'+k/2}^* + b_{\mathbf{p}'+k/2} a_{-\mathbf{p}'+k/2}) \Psi_0 \\ & \quad + [H_{\text{int}}^{(2)}, a_{\mathbf{p}+k/2}^* b_{-\mathbf{p}+k/2}^*] \Psi_0 \end{aligned} \quad (13)$$

and

$$\begin{aligned} & \{H - (E_0 - \varepsilon_{\mathbf{p}+k/2} - \varepsilon_{\mathbf{p}-k/2})\} b_{\mathbf{p}+k/2} a_{-\mathbf{p}+k/2} \Psi_0 \\ &= (2\pi)^{-3} f \int d\mathbf{p}' (a_{\mathbf{p}'+k/2}^* b_{-\mathbf{p}'+k/2}^* + b_{\mathbf{p}'+k/2} a_{-\mathbf{p}'+k/2}) \Psi_0 \\ & \quad + [H_{\text{int}}^{(2)}, b_{\mathbf{p}+k/2} a_{-\mathbf{p}+k/2}] \Psi_0, \end{aligned} \quad (14)$$

where $\varepsilon_{\mathbf{p}}$ is the energy of a nucleon or anti-nucleon with momentum \mathbf{p} :

$$\varepsilon_{\mathbf{p}} = \sqrt{p^2 + M_N^2}. \quad (15)$$

Now, let us denote with Ψ an excited state with energy E of the total Hamiltonian H :

$$H\Psi = E\Psi. \quad (16)$$

Neglecting the last terms in the right-hand sides of (13) and (14), we have from (13), (14) and (16)

$$\begin{aligned} & \langle \Psi | (H - E) a_{\mathbf{p}+k/2}^* b_{-\mathbf{p}+k/2}^* | \Psi_0 \rangle \\ &= (\varepsilon_{\mathbf{p}+k/2} + \varepsilon_{\mathbf{p}-k/2} - \omega_k) \langle \Psi | a_{\mathbf{p}+k/2}^* b_{-\mathbf{p}+k/2}^* | \Psi_0 \rangle \\ & \quad - (2\pi)^{-3} f \int d\mathbf{p}' \langle \Psi | (a_{\mathbf{p}'+k/2}^* b_{-\mathbf{p}'+k/2}^* + b_{\mathbf{p}'+k/2} a_{-\mathbf{p}'+k/2}) | \Psi_0 \rangle \\ &= 0 \end{aligned} \quad (17)$$

and

$$\begin{aligned} & \langle \Psi | (H - E) b_{\mathbf{p}+k/2} a_{-\mathbf{p}+k/2} | \Psi_0 \rangle \\ &= -(\varepsilon_{\mathbf{p}+k/2} + \varepsilon_{\mathbf{p}-k/2} + \omega_k) \langle \Psi | b_{\mathbf{p}+k/2} a_{-\mathbf{p}+k/2} | \Psi_0 \rangle \\ & \quad + (2\pi)^{-3} f \int d\mathbf{p}' \langle \Psi | (a_{\mathbf{p}'+k/2}^* b_{-\mathbf{p}'+k/2}^* + b_{\mathbf{p}'+k/2} a_{-\mathbf{p}'+k/2}) | \Psi_0 \rangle \\ &= 0, \end{aligned} \quad (18)$$

where

$$\omega_k = E - E_0. \quad (19)$$

From (17) and (18) it results that

$$\begin{aligned} & \langle \Psi | (a_{\mathbf{p}+\mathbf{k}/2}^* b_{-\mathbf{p}+\mathbf{k}/2}^* + b_{\mathbf{p}+\mathbf{k}/2} a_{-\mathbf{p}+\mathbf{k}/2}) | \Psi_0 \rangle \\ &= \frac{f}{(2\pi)^3} \left\{ \frac{1}{\varepsilon_{\mathbf{p}+\mathbf{k}/2} + \varepsilon_{\mathbf{p}-\mathbf{k}/2} - \omega_k} + \frac{1}{\varepsilon_{\mathbf{p}+\mathbf{k}/2} + \varepsilon_{\mathbf{p}-\mathbf{k}/2} + \omega_k} \right\} \\ & \quad \times \int d\mathbf{p}' \langle \Psi | (a_{\mathbf{p}'+\mathbf{k}/2}^* b_{-\mathbf{p}'+\mathbf{k}/2}^* + b_{\mathbf{p}'+\mathbf{k}/2} a_{-\mathbf{p}'+\mathbf{k}/2}) | \Psi_0 \rangle, \end{aligned} \quad (20)$$

from which we have the following eigenvalue equation for the excitation energy ω_k in our approximation:

$$1 = \frac{f}{(2\pi)^3} \int d\mathbf{p} \left\{ \frac{1}{\varepsilon_{\mathbf{p}+\mathbf{k}/2} + \varepsilon_{\mathbf{p}-\mathbf{k}/2} - \omega_k} + \frac{1}{\varepsilon_{\mathbf{p}+\mathbf{k}/2} + \varepsilon_{\mathbf{p}-\mathbf{k}/2} + \omega_k} \right\}. \quad (21)$$

The terms $\langle \Psi | [H_{\text{int}}^{(2)}, a_{\mathbf{p}+\mathbf{k}/2}^* b_{-\mathbf{p}+\mathbf{k}/2}^*] | \Psi_0 \rangle$ and $\langle \Psi | [H_{\text{int}}^{(2)}, b_{\mathbf{p}+\mathbf{k}/2} a_{-\mathbf{p}+\mathbf{k}/2}] | \Psi_0 \rangle$ neglected in (17) and (18), respectively, represent the residual interaction in the so-called pair approximation in the theory of many-body problem. A similar eigenvalue equation as (21) was used by Ihara and Hatano (1958),²⁵⁾ who made use of that equation to get approximately the energy of π -meson, $\omega_k \sim \sqrt{k^2 + m_\pi^2}$, by adjusting the value of the coupling constant.

In our present estimation of the sound velocity, we shall make use of (21) to get the following relation between the energy ω_k and the wave number k ,

$$\omega_k = \tilde{c}k, \quad (22)$$

the proportionality constant \tilde{c} being the sound velocity, by adjusting the value of f . To this purpose, we introduce the cutoff momentum Λ into the integration in (21), and adjust the value of f to be

$$(2\pi)^3 f^{-1} = \int_0^{\Lambda} d\mathbf{p} \varepsilon_{\mathbf{p}}^{-1}, \quad (23)$$

which is obtained by putting $k=0$ and $\omega_k=0$ in (21). The value of ω_k is then calculated numerically as a function of k for a given value of Λ . In actual calculation, we have expanded the integrand in the powers of ω_k and k up to ω_k^4 and k^4 .

Figure 3 shows the result of the numerical calculation, in which the variation of $\omega_k/2M_N$ with k/M_N is given for several chosen values Λ/M_N . From Fig. 3, we see that the linear relation (22) between ω_k and k holds approximately well. The value of \tilde{c} thus obtained numerically is almost constant over a wide range of given value of k . For example, in the case $\Lambda/M_N = \sqrt{10}$, \tilde{c} is observed to be constant within a relative variation of about 1% up to $k/M_N \simeq 4.5$. We have also similar results for the cases $\Lambda/M_N = \sqrt{50}$ and 10. The proportionality of ω_k to k is seen to be a little worse in the case $\Lambda/M_N = \sqrt{5}$, giving for the value of \tilde{c} a relative variation of a few percent over the range of given value of k mentioned above.

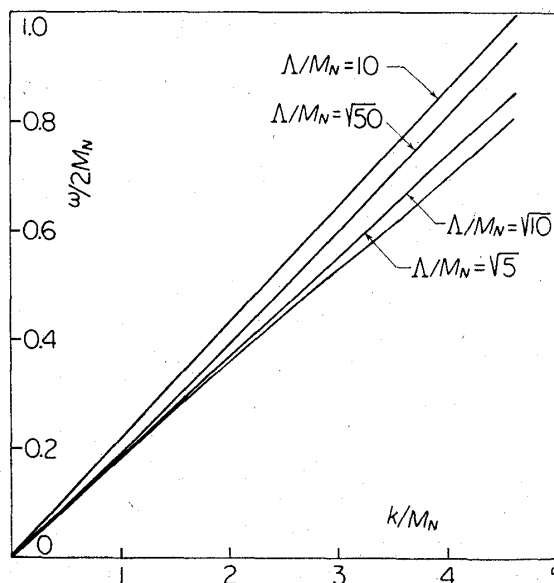


Fig. 3. The energy ω versus the momentum k of the sound wave for several values of the cutoff momentum Λ , in units of the nucleon mass M_N .

Table I shows the value of the sound velocity \bar{z} obtained by taking the average value of \bar{z} over the range of k shown in Fig. 3, for each value of Λ/M_N . In Table I, also shown is the adjusted value of $M_N^2 f/8$ for each case of Λ/M_N . The values of f for the cases $\Lambda/M_N = \sqrt{10}$ and $\sqrt{5}$ are of the same order in their magnitude as the value of the coupling constant taken in the calculation of meson masses based on the Sakata model with the use of the chain approximation.²⁰⁾ As is seen from Table I, the value of \bar{z} obtained here does not depend strongly on the value of Λ/M_N . The values of \bar{z} shown in Table I are about twice larger than that we have had in the preceding section, $\bar{z} \simeq 0.2$. This result of the estimation of \bar{z} in the present section should be said, we think, to be rather satisfactory, if we remember that the value of $\bar{z} \simeq 0.2$ has been deduced in the preceding section by fitting the level scheme given by our present model to the general trend of the existing meson levels, and also that the estimation of \bar{z} given above with the use of the pair approximation for phonon excitation is a simple approximation.

On deriving (17) and (18), we have neglected the contributions from the

Table I. The values of the sound velocity, \bar{z} , and the coupling, $M_N^2 f/8$, for several values of the cutoff momentum, Λ/M_N ; M_N being the nucleon mass.

Λ/M_N	10	$\sqrt{50}$	$\sqrt{10}$	$\sqrt{5}$
\bar{z}	0.42	0.41	0.36	0.35
$M_N^2 f/8$	0.051	0.10	0.57	1.3

terms $\langle \Psi | [H_{\text{int}}^{(2)}, a_{\mathbf{p}+\mathbf{k}/2}^* b_{-\mathbf{p}+\mathbf{k}/2}^*] | \Psi_0 \rangle$ and $\langle \Psi | [H_{\text{int}}^{(2)}, b_{\mathbf{p}+\mathbf{k}/2} a_{-\mathbf{p}+\mathbf{k}/2}] | \Psi_0 \rangle$. As has been mentioned above, these terms represent the residual interaction in the pair approximation for phonon excitation. Some terms in the residual interaction neglected give rise to interactions among three and four phonons. Since higher levels in the level scheme of meson masses correspond to many phonon states in our model, these interactions will play important roles at high excitations.

In order to see the main characteristics of such effects, let us now consider a process in which many mesons with various masses are exchanged, and replace each higher level in our model with an aggregate of bosons each having a mass, μ , of about 0.5 GeV. 0.5 GeV is roughly the average excitation energy to produce one more phonon of the lowest frequency discussed in the preceding section. In accordance with the discussions given above, strong interactions should be supposed to act among three and four bosons with the mass μ .

Nielsen and Olesen (1970)²⁶⁾ have studied the limit of very high-order Feynman diagrams for exchanges of many scalar particles with the non-linear interactions of ϕ^3 and ϕ^4 types, the so-called "fish-net" diagrams. Their result shows that there occur levels having the masses m_n given approximately through the relation

$$m_n^2 \simeq n(\pi\mu^2/q)(3N_3 + 4N_4)/N_3, \quad (24)$$

where n is an integer, q is a simple numerical factor of the order of 1 depending on the geometry of the diagram, and N_3 and N_4 are the numbers of the vertices of the interactions of ϕ^3 and ϕ^4 types, respectively. Applying their result with $N_3/N_4=2$ to our present case, we see that the value $m_1 \simeq 2$ GeV is well compared with the mass of the H -quantum $m_H \simeq 2$ GeV. We see also that m_4 corresponds to $2m_H$, and that m_2 and m_3 are smaller than $2m_H$, so that the levels with masses m_2 and m_4 fall within the spread of m_H , $m_H \simeq (2 \sim 3)$ GeV.

This fact shows that in our present model the H -quantum corresponds to a kind of field theoretical collective mode of the excitation of the hadronic matter. This kind of collective mode is different from that giving rise to low excitations of the hadronic matter, or, in other word, from that assumed in the estimation given above of the sound velocity of the density oscillation in the hadronic matter. It should be noted, however, that in our present model both the kinds of collective modes of excitation of the hadronic matter are considered in the sense of the compositeness of "molecule-type", and the problem of the binding of π -meson itself is left to be solved.

§4. Baryon resonances

In 1966 we have noticed²⁷⁾ the physical implication of Maglić's rule (1966)¹⁹⁾ for the masses M_{N^*} 's of baryon resonances, that are given as

$$M_{N^*} \simeq M_N + m_b, \quad (25)$$

in terms of the masses of nucleon and meson, M_N and m_b respectively. As has been pointed out in Ref. 27), this rule shows that the baryon resonances are nearly zero-energy resonances, and should therefore be understood, for the sake of the field nature of mesons, along the line of the Tomonaga theory (1941)²⁸⁾ of strong or intermediate coupling with the momentum cutoff of the order of M_N , as in the case of the (3, 3) resonance in π^+p scattering.

If one tries to derive the systematics of baryon resonances, of which Maglić's rule gives us a general trend, by applying the theory of potential scattering as was done in Otsuki and Sawada, (1966)²⁹⁾ one will have wave functions with large spatial extension for baryon resonances, on account of the nearly zero binding energy of meson in baryon resonance, as is the case for the wave function of deuteron. The structure of baryon resonance cannot extend, however, beyond more or less the pion Compton wave length. Otherwise, baryon resonances would display some features very different from other phenomena of hadronic interactions.

On applying the Tomonaga theory of intermediate coupling to pion-nucleon scattering, Maki, Sato and Tomonaga (1953)³⁰⁾ divided the meson wave functions into the sets of the bound meson and the scattered meson wave functions. The bound meson wave functions were nothing but the ones that make up the meson cloud around a nucleon, and the scattered meson wave functions were chosen so as to make an orthogonal complete set of wave functions together with the bound meson wave functions. Accordingly, each Fourier component of the meson field was divided into the bound and the scattered parts. They made then an approximation in which such part of the Hamiltonian that includes up to only one scattered meson was retained. The number of the bound mesons was not restricted, on the other hand, and the bound state problem of the meson cloud was solved by applying a variational method of the Hartree type developed by Tomonaga.²⁸⁾

The division of the meson wave functions into the two parts mentioned above was taken because of the fact that the meson cloud of the target nucleon includes in it the Fourier component of the meson field having the same value of momentum as that of the incident meson. The plane wave function of the incident meson is not orthogonal, in the strict sense, to the wave function of mesons in the cloud. In an application of the Tomonaga theory of intermediate coupling to pion-nucleon scattering, Lee and Christian (1954)³¹⁾ treated this problem of the orthogonality a little differently, making use of the variational method also for the scattering.

Let us now consider the simplified Hamiltonian for the meson field,

$$H = H_0 + H_I, \quad (26)$$

with

$$H_0 = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} \quad (27)$$

and

$$H_I = \sum_{\mathbf{k}} (V_{\mathbf{k}} \alpha_{\mathbf{k}} + V_{\mathbf{k}}^+ \alpha_{\mathbf{k}}^+). \quad (28)$$

Here, $\alpha_{\mathbf{k}}^+$ and $\alpha_{\mathbf{k}}$ are creation and annihilation operators for single mesons, respectively, $\omega_{\mathbf{k}}$ is the energy of a meson of mass m_b having momentum \mathbf{k} , $\omega_{\mathbf{k}} = \sqrt{k^2 + m_b^2}$, and $V_{\mathbf{k}}$ is the vertex function of the meson-nucleon interaction, nucleon recoil being ignored. Let us denote with Φ_0 the state of a physical nucleon with the energy,

$$H\Phi_0 = E_0\Phi_0. \quad (29)$$

Then we shall apply the variational principle

$$\delta \langle \Phi | (H - E_0 - \omega_q) | \Phi \rangle = 0, \quad (30)$$

for the scattering problem of the incident meson with momentum \mathbf{q} , taking the form

$$\Phi = \sum_{\mathbf{k}} f(\mathbf{k}) \alpha_{\mathbf{k}}^+ \Phi_0 + \lambda \Phi_0, \quad (31)$$

used by Lee and Christian³¹⁾ for the trial function Φ in (30). In (31), $f(\mathbf{k})$'s are the variational parameters, and λ is the Lagrange multiplier to keep the scattering state orthogonal to the state of the physical nucleon. In fact, carrying out the variation (30) with respect to λ^* , we get

$$\langle \Phi_0 | \Phi \rangle = 0, \quad (32)$$

from which we have

$$\lambda = - \langle \Phi_0 | \sum_{\mathbf{k}} f(\mathbf{k}) \alpha_{\mathbf{k}}^+ | \Phi_0 \rangle. \quad (33)$$

Taking the variation (30) with respect to $f^*(\mathbf{p})$, we get

$$\langle \alpha_{\mathbf{p}}^+ \Phi_0 | (H - E_0 - \omega_q) | \Phi \rangle = 0. \quad (34)$$

Substituting (31) and (33) into (34), we have an integral equation for $f(\mathbf{p})$, which can be solved by putting

$$f(\mathbf{p}) = \delta(\mathbf{p} - \mathbf{q}) + \text{outgoing part}. \quad (35)$$

The kernel in the integral equation includes terms such as $\langle \Phi_0 | V_{\mathbf{q}} \alpha_{\mathbf{p}}^+ | \Phi_0 \rangle$,

$\langle \Phi_0 | \alpha_p^+ \alpha_{p'} | \Phi_0 \rangle$, etc. These can be evaluated by making use of the solution Φ_0 to (29) obtained by applying the Tomonaga theory of intermediate coupling. Those parts of the kernel such as $\langle \Phi_0 | \alpha_p^+ \alpha_{p'} | \Phi_0 \rangle$ do not include the coupling constant explicitly, though Φ_0 does implicitly, and arise from the orthogonality condition (32). In fact, in the lowest order in such kernels only, one has a T -matrix proportional to $\langle \Phi_0 | \alpha_p \alpha_q^+ | \Phi_0 \rangle$. The scattering due to the kernels of such kind represents the "potential scattering" in Maki, Sato and Tomonaga.³⁰⁾

Wick (1955)³²⁾ discussed the orthogonality problem mentioned above by constructing an incident meson wave packet such as

$$\psi = \sum_{\mathbf{k}} g(\mathbf{k}) e^{-i\omega_{\mathbf{k}} t} \alpha_{\mathbf{k}}^+ \Phi_0, \quad (36)$$

which does not overlap with the meson cloud initially, and showed that the scattering problem of the wave packet can be approximated by the scattering problem in stationary state for which one puts

$$(H - E_0 - \omega_q) \Phi = 0 \quad (37)$$

and

$$\Phi = \alpha_q^+ \Phi_0 + \text{outgoing part.} \quad (38)$$

This Wick's method was used in Chew and Low (1956).³³⁾

The meson wave functions used in the variational calculation of Φ_0 in the Tomonaga theory of intermediate coupling are confined in a region smaller than more or less the meson Compton wave length. The overlapping integral of the incident plane wave function with the meson wave functions in the meson cloud may be small for long incident wave length. For such a case, one may ignore the "potential scattering".

Now, one sees that if the orthogonal condition (32) is ignored, that is, if the second term of Φ given by (31) is ignored in (34), one has instead of (34),

$$(H - E_0 - \omega_q) \Phi = 0, \quad (39)$$

with Φ of same form as (38), by noting that $\alpha_p^+ \Phi_0$'s form a complete set in the one scattering meson approximation. Thus, one sees that the differences in the treatment of the orthogonality give rise to the different forms of equations in Maki, Sato and Tomonaga (1953),³⁰⁾ Lee and Christian (1954)³¹⁾ and Chew-Low (1956),³³⁾ but the essential point is common to these works.

Friedman, Lee and Christian (1955)³⁴⁾ applied the variational method to the meson-nucleon scattering for the case of symmetric pseudoscalar mesons. They found that the experimentally observed P -wave phase shifts can be explained by taking the coupling constant $f^2 = 0.712$ and the cutoff momentum for the meson-nucleon interaction $k_{\max} = 6.13 m_\pi$. They calculated the renormalized coupling constant f_r defined as

$$f_r/f = \langle \Phi_{0, \rho'} | \sigma_i \tau_\alpha | \Phi_{0, \rho} \rangle / \langle \Phi_{\rho'}^{(0)} | \sigma_i \tau_\alpha | \Phi_{\rho}^{(0)} \rangle, \quad (40)$$

where $\Phi_{0, \rho}$, and $\Phi_{\rho}^{(0)}$ represent the physical and bare nucleon states, respectively, with spin-isospin index ρ , and σ_i and τ_α are components of the nucleon spin and isospin, respectively, by applying the Tomonaga theory of intermediate coupling to get $\Phi_{0, \rho}$. The value of f_r obtained by them was $f_r^2 = 0.105$. This value of f_r is well compared with the value of the coupling constant obtained in the theory of nuclear force.³⁵⁾

Chew and Low³³⁾ applied a dispersion theoretic method for the formal solution to (37), and evaluated the renormalized coupling constant as $f_r^2 = 0.08$ from the experimentally observed energy dependence of the phase shifts or the cross section. Salzman and Salzman³⁶⁾ solved numerically the formal solution in Chew and Low³³⁾ by an iterative method to obtain the phase shifts. The numerical solution obtained by the iterative method is dependent on the cutoff momentum k_{\max} for the meson-nucleon interaction. They obtained a result compatible with the experiment for the case $f_r^2 = 0.08$ and $k_{\max} = 7 m_\pi$. This result is essentially the same as Friedman, Lee and Christian's result mentioned above.

The discussions given above thus shows that the (3, 3) resonance in π^+p scattering can be explained in terms of the Tomonaga theory of intermediate coupling with the cutoff momentum for the meson-nucleon interaction of the order of M_N .

From the view-point of the compositeness of "molecule-type" for the structure of mesons,⁸⁾ the cutoff momentum of the order of M_N for the meson-nucleon interaction is not a conventional one, since in it the structure of meson is supposed to have a size of the order of the nucleon Compton wave length $1/M_N$.

With the cutoff momentum of the order of M_N for the meson-nucleon interaction, the neglect of the nucleon recoil in the Tomonaga theory of intermediate coupling is not a great defect. This fact is important in the application of the Tomonaga theory of intermediate coupling to scattering of heavier mesons in order to understand Maglić's rule.

Now, the zero energy resonances as given by Maglić's rule in meson-nucleon scattering can be said, seen from other side, to be the zero binding energy isobars of the nucleon. In fact, Lee and Christian³¹⁾ found numerically for the case of charged scalar meson that the same value of the coupling constant as is required to get an isobar with the excitation energy equal to the meson mass gives rise to the zero energy resonance in the scattering.

With the cutoff momentum of the order of M_N , the effect of the nucleon recoil can be taken into account approximately in the calculation of the excitation energy of a nucleon isobar. That is, with the non-relativistic approximation for the energy of the recoil nucleon, the energy of a state in which n mesons are present is given by³⁷⁾

$$M_N + \sum_{i=1}^n \omega_{k_i} + (\sum_{i=1}^n \mathbf{k}_i)^2 / 2M_N,$$

and if we neglect the cross product $(\mathbf{k}_i \cdot \mathbf{k}_j)$ by noting that n mesons are more or less at random in their directions, this is approximated by

$$M_N + \sum_{i=1}^n (\omega_{k_i} + k_i^2 / 2M_N).$$

The substitution of $\omega_k = \sqrt{m_b^2 + k^2} + k^2 / 2M_N$ for $\omega_k = \sqrt{m_b^2 + k^2}$ may cause a slight change in numerical result for the value of the coupling constant required in getting a nucleon isobar of zero binding energy with the use of the Tomonaga theory of intermediate coupling, but it is expected that the essential point concerned is not altered by this substitution.

From these discussions given above in this section, we see that Maglić's rule (25) representing the general characteristics of the experimentally observed mass levels of baryon resonances can be explained, in its essential point, by the meson field theory in which the cutoff momentum for the meson-nucleon interaction nearly equal to the nucleon mass is used, as was noted by us in 1966.¹⁹⁾ The use of this cutoff momentum for the meson-nucleon interaction nearly equal to the nucleon mass is in accordance with the view-point of the compositeness of "molecule-type" taken in the present paper, in which mesons are considered to be composed out of baryons and to have spatial dimensions of the order of the nucleon Compton wave length. This view-point is in sharp contrast to current quark models in which mesons, baryons and baryon resonances are treated in a single way, as is discussed in a separate paper of us.⁸⁾

§5. Discussion and conclusion

In the previous sections, we have studied the mass levels of mesons and baryon resonances from the view-point of the compositeness of "molecule-type". In this view-point, the structure of baryons are considered to lie on a level deeper than that of the structure of mesons in the strata of the structure of nature. The structure of baryons will be governed by the dynamics of ur-baryons or fundamental particles more directly.

Super strong interactions may be present among ur-baryons or fundamental particles. The interactions between baryon and anti-baryon fields to compose mesons are expected to be very strong, too. Such super or very strong interactions will give rise to some matter-like feature for the dynamics of mesons and baryons.

Thus, we have studied the mass levels of mesons with the model of density oscillation in the hadronic matter, supposing that the binding of π -meson itself is governed by more complex dynamics. Then, we have applied the approximation of pair excitation in the theory of collective motion to make

an estimation of the sound velocity of the density oscillation. That is, we have considered that the mass levels of mesons are due to a kind of collective motion in the hadronic matter.

The H -quantum is studied, on the other hand, as the one due to strong correlations among mesons arising from the residual interactions which are neglected in the collective motion mentioned just above. This means that we have considered that the H -quantum is due to another kind of collective motion in the hadronic matter.

When the excitation of the hadronic matter is so high that the binding of π -meson itself breaks down, it is expected that there occur phenomena of new kinds. We think that SH -quantum which decays into several H -quantum may be related to the "boiling" of π -meson. In this connection, it is interesting to note the event "Centaurus" observed by Japan-Brazil Emulsion Chamber Collaboration in cosmic ray study in which a heavy quantum decays into directly into many nucleon pairs.³⁹⁾ This event may also be related to the "boiling" of π -meson which is considered in the present model to be composed of nucleon and anti-nucleon fields.

Appendix

As was done in Woeste (1952),²¹⁾ the density oscillation in a matter can be treated by means of the velocity potential Φ :

$$V = \text{grad } \Phi. \quad (\text{A1})$$

Denoting by ρ the mass density, the continuity equation becomes

$$\text{div}(\rho V) + \partial\rho/\partial t = 0. \quad (\text{A2})$$

From (A1) and (A2), one has

$$\rho \Delta \Phi + (\text{grad } \rho \cdot \text{grad } \Phi) + \partial\rho/\partial t = 0.$$

With the linear approximation that $d\rho \ll \rho$, this reduces to

$$\Delta \Phi + (\partial\rho/\partial t)/\rho_0 = 0, \quad (\text{A3})$$

where ρ_0 is the average density. Assuming the sound velocity \bar{c} is given by the relation

$$\bar{c}^2 = \partial p / \partial \rho, \quad (\text{A4})$$

in terms of the pressure p and density, we have from (A1) and the equation of motion

$$\rho dV/dt = -\text{grad } p,$$

that

$$\text{grad}(\partial\Phi/\partial t + \tilde{c}^2\rho/\rho_0) = 0, \quad (\text{A5})$$

where the linear approximation is used to replace dV/dt with $\partial V/\partial t$. From (A3) and (A5), we have the wave equation:

$$\Delta\rho - \partial^2\rho/\tilde{c}^2\partial t^2 = 0, \quad (\text{A6})$$

from which we have the steady state solution

$$\rho = \rho_0.$$

When the density oscillates in time as

$$\rho = \rho_0 \{1 + \sum_l D_l(t) j_l(kr) P_l(\cos\theta)\}, \quad (\text{A7})$$

where j_l and P_l are respectively the spherical Bessel and Legendre functions and k is the wave number, with the deformation of the surface given by

$$R = a \{1 + \sum_l b_l(t) P_l(\cos\theta)\}, \quad (\text{A8})$$

in terms of the change of the radius R from a , we have the relation

$$D_l j_l(ka) = b_l \xi^2 (\ell - 1)(\ell - 2), \quad (\text{A9})$$

where

$$\xi = \sqrt{\epsilon_s/\rho_0 a}/\tilde{c}, \quad (\text{A10})$$

and ϵ_s is the surface energy per unit area. (A9) results from the change of the pressure at the surface:

$$\begin{aligned} p(R) - p_0 &= \tilde{c}^2 \{\rho(R) - \rho_0\} \\ &= (\epsilon_s/a) \sum_l b_l (\ell - 1)(\ell + 2) P_l(\cos\theta). \end{aligned}$$

The substitution of (A7) into (A3) gives us the general solution for Φ :

$$\Phi = \sum_l \{\Phi_l(r) r^\ell + (\partial D_l/\partial t) j_l(kr)/k^2\} P_l(\cos\theta), \quad (\text{A11})$$

where Φ_l should be determined by the relation

$$(\partial\Phi/\partial r)_a = \partial R/\partial t, \quad (\text{A12})$$

from which one has Φ_l in terms of $\partial b_l/\partial t$. (A9), (A11) and (A12), together with the relation

$$\partial\Phi/\partial t + \tilde{c}^2\rho(R)/\rho_0 = \text{const}, \quad (\text{A13})$$

which follows from (A5), suffice to get the equation of harmonic oscillation

of $b_i(t)$, from which one has (1) and (2) in the text, as was shown in detail by Woeste.²¹⁾

References

- 1) S. Sakata, Reported at the annual meeting of the Physical Society of Japan, Oct. 1955; Prog. Theor. Phys. **16** (1956), 686.
- 2) c.f. the papers of S. Sakata and of M. Taketani, compiled in Prog. Theor. Phys. Suppl. No. 50 (1971), under the title "Philosophical and Methodological Problems in Physics".
- 3) S. Ogawa, Prog. Theor. Phys. **21** (1959), 209.
- 4) M. Ikeda, S. Ogawa and Y. Ohnuki, Prog. Theor. Phys. **22** (1959), 715.
- 5) M. Gell-Mann, Phys. Letters **8** (1964), 214.
- 6) G. Zweig, CERN Preprints TH401, 412 (1964).
- 7) M. Taketani and Y. Fujimoto, Prog. Theor. Phys. Suppl. Extra Number—Commemoration Issue for the 30th Anniversary of the Meson Theory—(1965), 651.
- 8) M. Nagasaki and M. Taketani, the preceding paper in the present issue of Prog. Theor. Phys. Suppl.
- 9) M. Nagasaki, Prog. Theor. Phys. **37** (1967), 437; **39** (1968), 848, 854.
c.f. also M. Taketani, S. Nakamura, Y. Fujimoto, M. Nagasaki, S. Furuichi and S. Sato, Prog. Theor. Phys. Suppl. Nos. 37 & 38 (1966), 440.
- 10a) S. Hasegawa, Prog. Theor. Phys. **26** (1961), 151.
- 10b) S. Hasegawa, Prog. Theor. Phys. **29** (1963), 128.
- 11) M. Taketani, reported at the Symposium on Models and Structures of Elementary Particles (March 1963 at Hiroshima); included in "*Soryushi no Honshitsu*" (Essence of Elementary Particles, Iwanami Co. Ltd., 1963, in Japanese), pp. 329~333, and compiled in Prog. Theor. Phys. Suppl. No. 50 (1971), 214, on being translated into English.
- 12) T. Tati, Prog. Theor. Phys. **43** (1970), 1596.
c.f. also Tati's paper in the present issue of Prog. Theor. Phys. Suppl.
- 13) T. Tati, Prog. Theor. Phys. Suppl. Nos. 37 & 38 (1966), 153.
- 14) M. Nagasaki and M. Taketani, Prog. Theor. Phys. **49** (1973), 1768, 2145.
- 15) M. Taketani, reported at the Symposium on Structure of Elementary Particles held at Kobe on Dec. 2nd, 1964.
- 16) S. Sakata, reported at the Symposium on Comparison of High Energy Physics in Accelerator and Cosmic Ray Energy Regions held at Kyoto on Dec. 25th, 1964.
- 17) M. Nagasaki and M. Taketani, Prog. Theor. Phys. **44** (1970), 1112.
- 18) M. Nagasaki and M. Taketani, Prog. Theor. Phys. **49** (1973), 369.
- 19) B. C. Maglič, Nuovo Cim. **45** (1966), 949.
- 20) Z. Maki, Prog. Theor. Phys. **16** (1956), 444.
c.f. also the review article: Y. Ohnuki, Z. Maki and H. Yamamoto, Prog. Theor. Phys. Suppl. No. 19 (1961), 90.
- 21) K. Woeste, Z. Phys. **133** (1952), 370.
- 22) A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **26** (1952), 14.
- 23) S. Tomonaga, Prog. Theor. Phys. **5** (1950), 544; c.f. also **13** (1955), 467, 482.
- 24) K. Sawada, Phys. Rev. **106** (1957), 372.
- 25) C. Ihara and S. Hatano, Prog. Theor. Phys. **20** (1958), 356.
- 26) H. B. Nielsen and P. Olesen, Phys. Letters **32B** (1970), 203.
P. Olesen, Nucl. Phys. **B29** (1971), 77.
- 27) M. Taketani, S. Nakamura, Y. Fujimoto, M. Nagasaki, S. Furuichi and S. Sato, Prog. Theor. Phys. Suppl. Nos. 37 & 38 (1966), 440.
- 28) c.f. S. Tomonaga's papers written in 1941~1948 compiled in Prog. Theor. Phys. Suppl. No. 2 (1955).
- 29) S. Otsuki and S. Sawada, Prog. Theor. Phys. **36** (1966), 666.

- Y. Fukushima, S. Otsuki, S. Sawada and F. Toyoda, *Prog. Theor. Phys.* **37** (1967), 964.
- 30) Z. Maki, M. Sato and S. Tomonaga, *Prog. Theor. Phys.* **9** (1953), 607; this is included also in *Prog. Theor. Phys. Suppl. No. 2* (1955).
- 31) T. D. Lee and R. Christian, *Phys. Rev.* **94** (1954), 1760.
- 32) G. C. Wick, *Rev. Mod. Phys.* **27** (1955), 339.
- 33) G. F. Chew and F. E. Low, *Phys. Rev.* **101** (1956), 1570.
- 34) M. H. Friedman, T. D. Lee and R. Christian, *Phys. Rev.* **100** (1955), 1494.
- 35) c.f. M. Taketani, Introductory papers to *Prog. Theor. Phys. Suppl. No. 3* (1956) and No. 39 (1967).
- 36) G. Salzman and F. Salzman, *Phys. Rev.* **108** (1957), 1619.
- 37) P. T. Matthews and A. Salam, *Phys. Rev.* **86** (1951), 715.
- 38) T. D. Lee and D. Pines, *Phys. Rev.* **92** (1953), 883.
- 39) Japan-Brazil Emulsion Chamber Collaboration, *Proceeding of the International Conference on Cosmic Ray*, Danver, Aug., 1973; c.f. also the first paper in Part I of the present issue of *Prog. Theor. Phys. Suppl.*