Dr.1900-3

PPPL-2044

PPPL--2044

DE84 002364

EXCITATION OF INTERNAL KINK MODES BY TRAPPED ENERGETIC BEAM IONS

By

L. Chen, R.B. White, and M.N. Rosenbluth

OCTOBER 33

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

EXCITATION OF INTERNAL KINK MODES BY TRAPPED ENERGETIC BEAM IONS

Liu Chen and R. B. White

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

and

M. N. Rosenbluth

Institute of Fusion Studies, University of Texas

Austin, Texas 78712

ABSTRACT

Energetic trapped particles are shown to have a destabilizing effect on the internal kink mode in tokamaks. The plasma pressure threshold for the mode is lowered by the particles. The growth rate is near the ideal magnetohydrodynamic value, but the frequency is comparable to the trapped particle precession frequency. A model for the instability cycle gives stability properties, associated particle losses, and neutron emissivity consistent with the "fishbone" events observed in PDX.

In recent poloidal divertor experiments (PDX) with high-power nearly perpendicular beam injection, bursts of large-amplitude magnetohydrodynamic (MHD) fluctuations, dubbed "fishbones" from the characteristic signature on the Mirnov coils, have been observed.[1,2] These "fishbone" bursts are found to be correlated with significant losses of energetic beam ions and thus have serious implications for the beam-heating efficiencies and the achievable β values in tokamaks.

Detailed experimental measurements have identified the mode structure of the "fishbone" as an $m = 1$, $n = 1$ mode with additional $m > 2$ components, supposedly due to the finite $\varepsilon\beta_p$ toroidal-coupling effects. (Here m and n are, respectively, poloidal and toroidal mode numbers, $\varepsilon = a/R$ is the inverse aspect ratio, and β_p is the poloidal beta.) The plasma pressure threshold for the mode is consistent with that of the internal kink mode. The most crucial feature is that all components rotate toroidally with a frequency comparable to the precession frequency of the trapped beam ions. This resonance feature indicates that proper understanding of both the stability and the beam loss mechanisms require a kinetic treatment of the plasma dynamics.

In this Letter, we employ a gyrokinetic description[3,4] for the trapped beam ions and demonstrate that the internal kink mode can be excited at a lower threshold than that of the ideal MHD prediction, with a frequency given by the precession frequency. A model for the "fishbone" cycle gives MHD amplitudes, particle losses, and neutron emissivity in close agreement with those observed in PDX.

We consider a large-aspect-ratio tokamak plasma consisting of core (c) and hot (h) components. For the purpose of formal orderings, we use $\varepsilon = a/R << 1$ as the small parameter. Since we are interested in the parameter range of the first stability boundary of the internal kink mode,[5] we order

$\beta_{pc} \sim O(1)$ and, for simplicity, $\beta_{ph} \sim O(\varepsilon)$. Temperatures are ordered as $T_c(\sim 1\ \text{keV})/T_h(\sim 50\ \text{keV}) \sim O(\varepsilon^2)$, which implies $n_h/n_c \sim O(\varepsilon^3)$ and, hence, overall charge neutrality may be assumed. We also have, for PDX parameters, $\omega \sim \bar{\omega}_{dh} \sim 6 \times 10^4 \gg \omega_{*c}, \omega_{dc}, \omega_A = v_A/qR \sim 2 \times 10^6$ and thus $|\omega/\omega_A| \sim |\bar{\omega}_{dh}/\omega_A| \sim O(\varepsilon^2)$; similar to the usual internal kink ordering.[5] Here $\bar{\omega}_{dh}$ is the toroidal precession frequency of the trapped hot particles, and ω_* and ω_d denote, respectively, diamagnetic and magnetic drift frequencies.

Consistent with the above orderings, we adopt the ideal MHD description for the core plasma. For the hot component, however, we employ the gyrokinetic description, neglecting the finite Larmor radius correction. To derive the corresponding normal mode equation, we first sum up the collisionless equations of motion for each species and obtain

$$-\omega^2 \rho_m \underset{\sim}{\xi} = \frac{1}{c}\left(\delta \underset{\sim}{j} \times \underset{\sim}{B} + \underset{\sim}{j} \times \delta \underset{\sim}{B}\right) - \underset{\sim}{\nabla} \delta P_c - \underset{\sim}{\nabla} \cdot \delta \underset{\approx}{P}_h \quad , \tag{1}$$

where $\underset{\sim}{\xi}$ is the usual fluid displacement vector. In Eq. (1), noting that $n_h/n_c \sim O(\varepsilon^3)$, we have $\rho_m = n_{oi} m_i$. In addition, the following ideal MHD relations hold: $\delta P_c = -[\underset{\sim}{\xi} \cdot \underset{\sim}{\nabla} P_c + \gamma P_c(\underset{\sim}{\nabla} \cdot \underset{\sim}{\xi})]$, $\delta \underset{\sim}{E}_\perp = i\omega \underset{\sim}{\xi} \times \underset{\sim}{B}/c$, $\delta E_\parallel = 0$, $\delta \underset{\sim}{B} = \underset{\sim}{\nabla} \times (\underset{\sim}{\xi} \times \underset{\sim}{B})$, and $\delta \underset{\sim}{J} = c \underset{\sim}{\nabla} \times \delta \underset{\sim}{B}/4\pi$. The perturbed distribution of the hot component, δF_h, is given by[3,4]

$$\delta F_h = \frac{e}{m}\left(\delta\phi \frac{\partial}{\partial E} - \frac{\mu}{\omega_c} \frac{\delta B_\parallel}{c} \frac{\partial}{\partial \mu}\right) F_{oh} + \delta H_h \quad , \tag{2}$$

and

$$\left[v_\parallel \frac{\partial}{\partial \ell} - i(\omega - \omega_{dh})\right] \delta H_h = i \frac{e}{m} Q\ \delta\psi \ , \tag{3}$$

where $E = v^2/2$, $\mu = v_\perp^2/2B$, ω_c is the cyclotron frequency, $\partial/\partial\ell \equiv \underset{\sim}{e}_\parallel \cdot \underset{\sim}{\nabla}$, $\delta\psi = \delta\phi - v_\parallel \delta A_\parallel/c + v_\perp^2 \delta B_\parallel/2\omega_c c$, $Q = (\omega\partial/\partial E + \hat{\omega}_{*h})\ F_{oh}$; $\hat{\omega}_{*h} \equiv -(i/\omega_c)(\underset{\sim}{e}_\parallel \times \underset{\sim}{\nabla}\ln F_{oh})\cdot\underset{\sim}{\nabla}$, $\omega_{dh} = -i\underset{\sim}{v}_{dh}\cdot\underset{\sim}{\nabla}$, $\underset{\sim}{v}_{dh}$ is the magnetic drift velocity, and $\delta\phi$ and $\delta A_\parallel$ are related to $\underset{\sim}{\xi}$ by $c\underset{\sim}{\nabla}\delta\phi = -i\omega\underset{\sim}{\xi} \times \underset{\sim}{B}$ and $\omega\delta A_\parallel/c = -i\partial\delta\phi/\partial\ell$. Noting that the frequencies are much smaller than the hot-particle transit and bounce frequencies, Eq. (3) can be solved readily for both trapped (t) and untrapped (u) particles. We find that $\delta H_{h,u} = -eQ\delta\phi/m\omega$ and $\delta H_{h,t} = -eQ\delta\phi/m\omega + \delta G_{h,t}$, where $\delta G_{h,t} = 2QE\bar{J}/(\omega - \bar{\omega}_{dh})$. $\bar{A} \equiv (\oint A d\ell/|v_\parallel|)/(\oint d\ell/|v_\parallel|)$ denotes bounce averaging, and $J = (\alpha B/2)\underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}_\perp - (1 - 3\alpha B/2)\ \underset{\sim}{\xi}_\perp \cdot \underset{\sim}{\kappa}$, with $\alpha = \mu/E$ and $\underset{\sim}{\kappa} = \partial\underset{\sim}{e}_\parallel/\partial\ell$. Substituting δH into Eq. (2), we have $\delta\underset{\approx}{P}_h$ given by $\delta\underset{\approx}{P}_h = -\underset{\sim}{\xi}_\perp \cdot \underset{\sim}{\nabla}[P_\perp \underset{\approx}{I} + (P_\parallel - P_\perp)\underset{\sim}{e}_\parallel \underset{\sim}{e}_\parallel]_h + \delta\hat{P}_\perp \underset{\approx}{I} + (\delta\hat{P}_\parallel - \delta\hat{P}_\perp)\underset{\sim}{e}_\parallel \underset{\sim}{e}_\parallel$, where

$$\left\{ \begin{array}{c} \delta\hat{P}_\perp \\ \\ \delta\hat{P}_\parallel \end{array} \right\} = 2^{7/2}\pi m_h B \int_{B_{max}^{-1}}^{B^{-1}} d\alpha (1 - \alpha B)^{1/2} \int_0^\infty dE\ \frac{E^{5/2} Q}{\omega - \bar{\omega}_{dh}}\ \bar{J} \left\{ \begin{array}{c} \frac{\alpha B}{2(1-\alpha B)} \\ \\ 1 \end{array} \right\} \qquad (4)$$

correspond to kinetic contributions due to the trapped energetic particles. Substituting $\delta\underset{\approx}{P}_h$ into Eq. (1), we have a complete normal mode equation in terms of $\underset{\sim}{\xi}$.

To analyze the stability properties, we shall derive a dispersion relation variationally. First, we obtain the following dispersion functional by performing $\int d^3\underset{\sim}{x}\ \underset{\sim}{\xi}^*\cdot$ on Eq. (1) and assuming a fixed conducting boundary, $D[\underset{\sim}{\xi}] = \delta W_{MHD} + \delta W_k + \delta I$, where, with $P \equiv P_c + (P_\perp + P_\parallel)_h/2$,

$$\delta W_{MHD} = \frac{1}{2} \int d^3\underset{\sim}{x} \left\{ \frac{|\delta\underset{\sim}{B}_\perp|^2}{4\pi} - \frac{j_\parallel}{c} (\underset{\sim}{\xi}^*_\perp \times \underset{\sim}{e}_\parallel) \cdot \delta\underset{\sim}{B}_\perp - 2(\underset{\sim}{\xi}_\perp \cdot \underset{\sim}{\nabla}P)(\underset{\sim}{\xi}^*_\perp \cdot \underset{\sim}{\kappa}) \right.$$

$$\left. + B^2 |\underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}_\perp + 2\underset{\sim}{\xi}_\perp \cdot \underset{\sim}{\kappa}|^2 + \gamma P_c |\underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}|^2 \right\} , \qquad (5)$$

$$\delta W_k = -2^{7/2}\pi^2 m_h \int RBrdr \int_{B_{max}^{-1}}^{B_{min}^{-1}} d\alpha \int_\theta^\infty dE\ E^{5/2} K_b \bar{J}^* \frac{Q}{\omega - \bar{\omega}_{dh}}\ \bar{J} , \qquad (6)$$

$K_b = \oint(d\theta/2\pi)(1 - \alpha B)^{-1/2}$, and $\delta I = -\frac{1}{2}\omega^2\int d^3\underset{\sim}{x}\ \rho_m|\underset{\sim}{\xi}|^2$ is the inertial term. Note that in the high- and low-frequency limits (with respect to ω_{*h} and $\bar{\omega}_{dh}$), δW_k reduces, respectively, to that of the collisionless[6,7] and the low-frequency kinetic energy principles[8-10] as it should. To apply the variational method, we have, for the present orderings, $\delta W_k^{(2)} \sim (\beta_{h,t}/\varepsilon)[B^2|\underset{\sim}{\xi}/R|^2V] \sim \varepsilon^2(B^2|\underset{\sim}{\xi}/R|^2V) \sim \delta I^{(2)}$. Here, V is the volume, superscripts denote the orderings, and we have noted, in ordering δI the existence of an inertial singular layer with a width $\Delta_A \sim (\omega/\omega_A)a \sim \varepsilon^2 a$ at $q(r_s) \equiv rB_t/RB_p = 1$. The variational scheme then is to find a trial function, $\underset{\sim}{\xi}_t$, which minimizes D to $O(\varepsilon^3)$ or smaller. Since both δW_{MHD} and δI [assuming $|Im\omega^2|/|Re\omega^2| \sim O(\varepsilon)$ i.e., near marginal stability in the present case] are variational, this minimizing procedure is identical to that of ideal MHD.[5] Let $D(\xi_t)$ be $D_e + D_s$ where D_e and D_s are the contributions from outside and inside the singular layer, respectively. For the case of circular cross sections, we have, for $|r - r_s| >> \Delta_A$ i.e., outside the singular layer, we have $\underset{\sim}{\xi}_t^e$ as given by Bussac <u>et al.</u>[5] We then obtain $D_e \equiv D[\underset{\sim}{\xi}_t^e]$ as $D_e = \delta W_{MHD}^{(2)}[\underset{\sim}{\xi}_t^e] + \delta W_K^{(2)}[\underset{\sim}{\xi}_t^e] + O(\varepsilon^4)$, where, for n = 1,

$$\frac{\delta W_{MHD}^{(2)}}{2\pi R_o} = \frac{\pi}{4}\left(\frac{r_s^2}{R_o^2}B_o\right)^2|\xi_{ro}|^2\delta\hat{W}_T \equiv |\xi_{ro}|^2\left(\frac{r_s B_o}{2R_o}\right)^2\delta\hat{W}_f \tag{7}$$

with $\delta\hat{W}_T$ given in Ref. 5 and, to $O(\varepsilon^3)$,

$$\frac{\delta W_K^{(2)}}{2\pi R_o} \cong \pi^2 m_h 2^{3/2}\frac{|\xi_{ro}|^2}{R_o^2}\int_o^{r_s} r\,dr\int_{1-r/R}^{1+r/R} d(\alpha B)\int_o^\infty dE\ E^{5/2}\frac{K_2^2}{K_b}\left[\frac{Q}{\bar{\omega}_{dh}-\omega}\right]_{1,1}$$

$$\equiv |\xi_{ro}|^2\left(\frac{r_s B_o}{2R_o}\right)^2\delta\hat{W}_K \quad , \tag{8}$$

$K_2 = \oint(d\theta/2\pi)\cos\theta(1 - \alpha B)^{-1/2}$, (1,1) refers to m = 1, n = 1, $B = B_o(1 - r\cos\theta/R)$, and $\Delta q = 1 - q(0) \sim O(\varepsilon)$ is assumed. Note that, assuming a parabolic q profile,[5] we have $\delta\hat{W}_f = 3\pi\Delta q r_s^2(13/144 - \beta_{ps}^2)/R_o^2$ with $\beta_{ps} = -(R_o/r_s^2)^2 \int_o^{r_s} r^2\beta' dr$.

Near the singular q = 1 surface, we have $|x| \equiv |r - r_s| \sim |\Delta_A|$ and the Euler equation for ξ_t^s is

$$\frac{d}{dx}\left(3\omega^2 - |k_\parallel'|^2 V_A^2 x^2\right)\frac{d\xi_{rt}^s}{dx} = 0 \quad ; \tag{9}$$

here, $|k_\parallel'| = q_s'/R_o$. This equation can be solved readily and ξ_{rt}^s matched to ξ_{rt}^e using the causality condition. It is then straightforward to show

$$D_s = \delta W_{MHD}^{(2)}[\xi_t^s] + \delta I^{(2)}[\xi_t^s] + O(\varepsilon^4)$$

$$= 2\pi R_o(B_o r_s/2R_o)^2|\xi_{ro}|^2(-i\omega/\hat{\omega}_A) + O(\varepsilon^4) \quad , \tag{10}$$

with $\hat{\omega}_A \equiv V_A/(3^{1/2}R_o\hat{s})$ and $\hat{s} = r_s q_s'$. Combining D_e and D_s then yields the following dispersion relation:

$$-i\omega/\hat{\omega}_A + \delta\hat{W}_f + \delta\hat{W}_K = 0 \quad . \tag{11}$$

Here we emphasize that the terms in Eq. (11) are all formally of the same order.

Some qualitative features of Eq. (11) are worth noting. First, without the trapped-particle term, $\delta\hat{W}_K$, we recover the ideal MHD results which predict instability for $\delta\hat{W}_f < 0$. Marginal stability occurs at $\omega = 0$. Within the present orderings, the inclusion of $\delta\hat{W}_K$ has little effect on the marginal stability condition at $\omega = 0$, but it has the most interesting effect of introducing an additional branch of solutions to the dispersion relation. In contrast to the ideal-MHD branch, this trapped-particle induced branch can become unstable at $\omega = \omega_r$ and $\delta\hat{W}_f > 0$. This point can easily be seen by substituting into $\delta\hat{W}_K$ a mono-energetic, single magnetic moment distribution F_{oh}. We then find a thresholdless unstable solution with $\omega_r \simeq \bar{\omega}_{dh}$ and ω_i increasing with $\langle\beta_{h,t}\rangle$ (the average trapped-particle β within the $q = 1$ surface) and $\omega_{*h}/\bar{\omega}_{dh} > 0$. One physical explanation of this new instability mechanism is that, for the internal kink mode with $\omega = \omega_r \lesssim \bar{\omega}_{dh}$, the core-plasma MHD mode is positively dissipated due to the Alfvén resonance at the $\omega_r \approx k_\parallel v_A$ singular surface. The hot trapped particles, however, have a precession mode which is, in character, either negative-energy or negative-dissipation due to the wave-particle resonance. The instability is thus the result of coupling between these two modes. The threshold condition then corresponds to the nature of the precession mode.

In order to make detailed comparisons with the PDX experiments, in principle, Eq. (11) should be solved numerically, employing realistic equilibria, $\delta\hat{W}_f$ and F_{oh}. Many interesting features, however, can be derived by assuming the following model distribution function for the slowing-down beam ions; $F_{oh} = c_o E^{-3/2}\delta(\alpha - \alpha_o)$ for $0 \leq E \leq E_m$, where $c_o(r) = P_h(r)\theta_b/(\pi^2 K_{bo} m_h B_o 2^{3/2} E_m)$, $K_{bo} = K_b(\alpha = \alpha_o)$, and θ_b is the magnetic turning point. The corresponding dispersion relation is then given by

$$- i\Omega(\bar{\omega}_{dm}/\hat{\omega}_A) + \delta\hat{W}_{fc} + \langle\beta_{h,t}\hat{I}_o\rangle\Omega\ln(1 - 1/\Omega) = 0 \quad , \tag{12}$$

where $\bar{\omega}_{dm} \equiv \bar{\omega}_{dh}(E = E_m)$, $\Omega = \omega/\bar{\omega}_{dm}$, $\langle y\rangle \equiv (2/r_s^2)\int_o^{r_s} r dr (y)$,

$$\hat{I}_o = (1/2)(\theta_b/\pi K_{bo})[\alpha_o I'(\alpha_o) + I(\alpha_o)\hat{\omega}_{*h}/\hat{\omega}_{dh}] \quad , \tag{13}$$

$$I(\alpha_o) = (K_2^2/K_b)(\alpha = \alpha_o) = (2R_o/r)^{1/2}[2E(k_o^2) - K(k_o^2)]^2/\pi K(k_o^2) \quad , \tag{14}$$

$K_{bo} = (2R_o/r)^{1/2}K(k_o^2)/\pi$, $k_o^2 = (1 + r/R_o - \alpha_o B_o)R_o/2r$, $E(k_o^2)$, and $K(k_o^2)$ being the complete elliptic integrals, $\hat{\omega}_* = (d\ell nP_{h,t}/dr)/r\omega_c$, $\hat{\omega}_{dh} = -[2E(k_o^2)/K(k_o^2) - 1]/rR\omega_c$, and $\delta\hat{W}_{fc}$ corresponds to $\delta\hat{W}_f$ with only the core-plasma pressure contribution. Simple analysis of Eq. (12) then reveals that, even for $\delta\hat{W}_{fc} > 0$, the internal kink mode is destabilized if $\beta_{h,t}$ exceeds a critical value,

$$\langle\beta_{h,t}\hat{I}_o\rangle > \langle\beta_{h,t}\hat{I}_o\rangle_{crit} \approx \bar{\omega}_{dm}/\pi\hat{\omega}_A \quad . \tag{15}$$

Noting that $\hat{I}_o \sim O(1/\varepsilon)$, we then have $\langle\beta_{h,t}\rangle_{crit} \sim O(\varepsilon\bar{\omega}_{dm}/\hat{\omega}_A)$. Near marginal stability, we have $\Omega_r \approx 1/2$ and $\Omega_r \approx 1$ for, respectively, $\pi\hat{\omega}_A\delta\hat{W}_{fc}/\bar{\omega}_{dm} << 1$ and $>> 1$, with the maximum growth rate occurring at $\Omega_r \approx 1/2$. On the other hand, below the critical value instabilities occur for $\delta\hat{W}_{fc} < 0$ i.e., the ideal MHD stability condition. For a typical PDX operating regime, we have $\delta\hat{W}_{fc} > 0$ and thus only the trapped-particle induced internal kink modes are predicted to be unstable for $\langle\beta_{h,t}\rangle \gtrsim \langle\beta_{h,t}\rangle_{crit}$ which is typically $\lesssim O(10^{-2})$ and is consistent with the observations.[1,2] Furthermore, taking $1 - q(0) = O(10^{-1})$, we find $\pi\delta\hat{W}_{fc}\hat{\omega}_A/\bar{\omega}_{dm} \lesssim O(1)$ and, hence, $\omega_r \gtrsim \bar{\omega}_{dm}/2$, and the growth rate

$$\omega_i \approx \hat{\omega}_A(\pi^2/4)(\langle\beta_{h,t}\hat{I}_o\rangle - \langle\beta_{h,t}\hat{I}_o\rangle_{crit}) \tag{16}$$

tends to be of the same order as the usual ideal MHD growth rate. Note that the theoretically predicted $\bar{\omega}_{dm} \gtrsim \omega_r \gtrsim \bar{\omega}_{dm}/2$ is also consistent with the experimental observations.

The beam loss process due to the beam-ion induced internal kink mode has already been considered.[11] The perturbed radial motion of the resonant trapped particles is secular and leads to efficient particle ejection in a toroidally beacon-like ejection pattern for particles with $\bar{\omega}_{dh} \approx \omega_r$. Thus the energy range of the ejected particles is expected to be between $E_{inj}/2$ and E_{inj}.

We can now model the full "fishbone" cycle. Neglecting variations of the core plasma component, the internal kink mode is destabilized by the trapped particles within the $q = 1$ surface with growth rate given by Eq. (16). Assuming the trapped particles to be uniformly distributed within the $q = 1$ surface, we then have, for the amplitude of the kink mode ($A = \delta B_r/B$),

$$\frac{dA}{dt} = A\Gamma\left(\beta_h - \beta_{crit}\right) \tag{17}$$

with $\Gamma = \hat{\omega}_A(\pi^2/4)\langle\hat{I}_o\rangle$. This equation for the mode has been used in Monte Carlo simulations using the Hamiltonian formalism of Ref. 11. These simulations will be reported in a future publication, but the essential results can be reproduced by replacing the particle loss mechanism with a simple model equation. Beam loss is linearly proportional to the mode amplitude A and takes the form of secular outward drift of those trapped particles in resonance with the mode. Noting the loss occurs on a time scale much shorter than the beam deposition time, the rate of particle loss through the $q = 1$ surface is approximately constant until a significant fraction of the particles are lost. Thus

$$\frac{d\beta_h}{dt} = D - AZ\beta_{max}\theta\left(\beta_h - \beta_{min}\right) \quad , \tag{18}$$

where D is the net deposition rate of trapped particles within the $q = 1$ surface, and Z is a measure of the particle loss rate. The Heaviside θ function reflects the fact that only a certain fraction f of the trapped particles can be ejected. The simulation results support this model. The fraction f is typically less than 50%, because the $n = 1$ character of the mode implies that only half of the resonant ($\bar{\omega}_{dh} \approx \omega_r$) particles are toroidally distributed so that the mode produces outward motion. An examination of Eqs.

(17), (18) in the (β_h, A) plane, and in particular the symmetry of these equations about β_{crit}, leads to the result that the motion is periodic with $\beta_{max} = \beta_{crit}/(1 - f/2)$, and $\beta_{min} = (1 - f)\beta_{max}$.

We illustrate the solution of Eqs. (17) and (18) for a PDX case with $B = 10$ kG, $r_s = a/2$, and 4 MW of near perpendicular 50 keV neutral beam injection. This gives $D \approx 2\ sec^{-1}$. The beam ejection efficiency has been obtained for this case with Monte Carlo simulations,[11] giving $Z \approx 2.5 \times 10^6\ sec^{-1}$ and $f = 0.4$. For these parameters we have $\tilde{\omega}_A \approx 1.2 \times 10^6\ sec^{-1}$. Using the expression for $\hat{I}_o$ following Eq. (13), $n = 5 \times 10^{13}$, $\theta_b \approx \pi/4$ and $k_o^2 \approx \sin^2(\theta_b/2) \ll 1$ we find $\hat{I}_o \approx 1$, and thus $\Gamma \approx 2.8 \times 10^6\ sec^{-1}$, and from Eq. (15) $\beta_{crit} \approx 0.01$. The solution to Eqs. (17) and (18) for these parameters is shown in Fig. 1, to be compared with Fig. 1 of Ref. 1. We have multiplied A(t) by the factor $\cos(\tilde{\omega}_{dh}t)$ to produce the Mirnov signal which would be given by the rotating mode. The function $\beta_h(t)$ gives the magnitude of the neutron emission. The fishbone period is dominated by the relatively long period of increasing β_h, with negligible kink mode amplitude. Thus, $\tau_{fb} \approx \Delta\beta_h/D = f\ \beta_{crit}/D[1-(f/2)]$, which is about 2.5 msec in this case. The ~30% variation of β_h, determined by the beam loss, and its time dependence are in good agreement with the observed variation of neutron emissivity. The maximum value of A is consistent with the typical observed values of the Mirnov loop signals. Near $A = A_{max}$ the behavior of A is given by $A \approx A_{max} \exp(-\Gamma\beta_{max}Z\ A_{max}\ t^2/2)$ and $\beta_{max}Z\ A_{max} = \Gamma(\beta_{max} - \beta_{crit})^2/2$ so the width of the "fishbone" burst is $\Delta t = 4/\Gamma(\beta_{max} - \beta_{crit})$, about half a millisecond in this case, also in agreement with the experimental results. The form of the solution depends only weakly on Γ, Z, and β_{crit} as long as Γ, $Z \gg D$.

In summary, we have shown that energetic trapped particles can destabilize the internal kink mode at a plasma pressure threshold lower than that predicted by the ideal MHD theory. *This trapped-particle* induced instability has a real frequency comparable to the trapped-particle toroidal precession frequency and a growth rate of the order of the ideal MHD value. Beam particles are efficiently ejected in a toroidal beacon-like pattern. A simple model for the coupled kink mode and trapped particle system produces a time dependence for these quantities in good agreement with experimental results. A Monte Carlo simulation of the trapped partice population interacting with a kink mode governed by Eq. (17) produces similar results, and will be reported in a future publication. Finally, we remark that since the instability mechanism is of sufficiently general nature, it may be desirable to extend our theoretical calculations to other regimes such as $\beta_p \sim O(\varepsilon^{-1})$, radio-frequency heated plasmas, alpha-particle effects, and so on. In this respect, we note that the ballooning-mode analogue has been discussed by Rosenbluth et al.[12]

ACKNOWLEDGMENTS

The authors would like to acknowledge useful discussions with H. L. Park, J. W. Van Dam, and many members of the Plasma Physics Laboratory, in particular, R. Goldston, K. McGuire, R. M. Kulsrud, J. Manickam, D. A Monticello, C. R. Oberman, F. W. Perkins Jr., and P. H. Rutherford.

This work was supported by U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073 and DE-FG05-80ET-53088.

REFERENCES

[1]K. McGuire et al., Phys. Rev. Lett. 50, 891 (1983).

[2]D. Johnson et al., in Proceedings of the Ninth International Conference on Plasma Physics and Controlled Nuclear Fusion Research (Baltimore, 1982).

[3]T. M. Antonsen, Jr. and B. Lane, Phys. Fluids 23, 1205 (1980).

[4]P. J. Catto, W. M. Tang, and D. E. Baldwin, Plasma Phys. 23, 639 (1981).

[5]M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. 35, 1638 (1975).

[6]M. D. Kruskal and C. R. Oberman, Phys. Fluids 1, 275 (1958).

[7]M. N. Rosenbluth and N. Rostoker, Phys. Fluids 2, 23 (1959).

[8]J. W. Van Dam, M. N. Rosenbluth, and Y. C. Lee, Phys. Fluids 25, 1349 (1982).

[9]T. M. Antonsen, Jr., B. Lane, and J. J. Ramos, Phys. Fluids 24, 1465 (1981).

[10]T. M. Antonsen, Jr. and Y. C. Lee, Phys. Fluids 25, 132 (1982).

[11]R. B. White et al., Phys. Fluids, October 1983.

[12]M. N. Rosenbluth, S. T. Tsai, J. W. Van Dam, and M. G. Engquist, IFSR # 98, University of Texas at Austin, Phys. Rev. Lett. (to be published).

FIGURE CAPTION

FIG. 1. The kink mode amplitude, $A(t)\cos(\bar{\omega}_{dh}t)$ and the beam particle beta, $\beta_h(t)$, vs time as obtained from Eqs. (17) and (18), for PDX parameters.

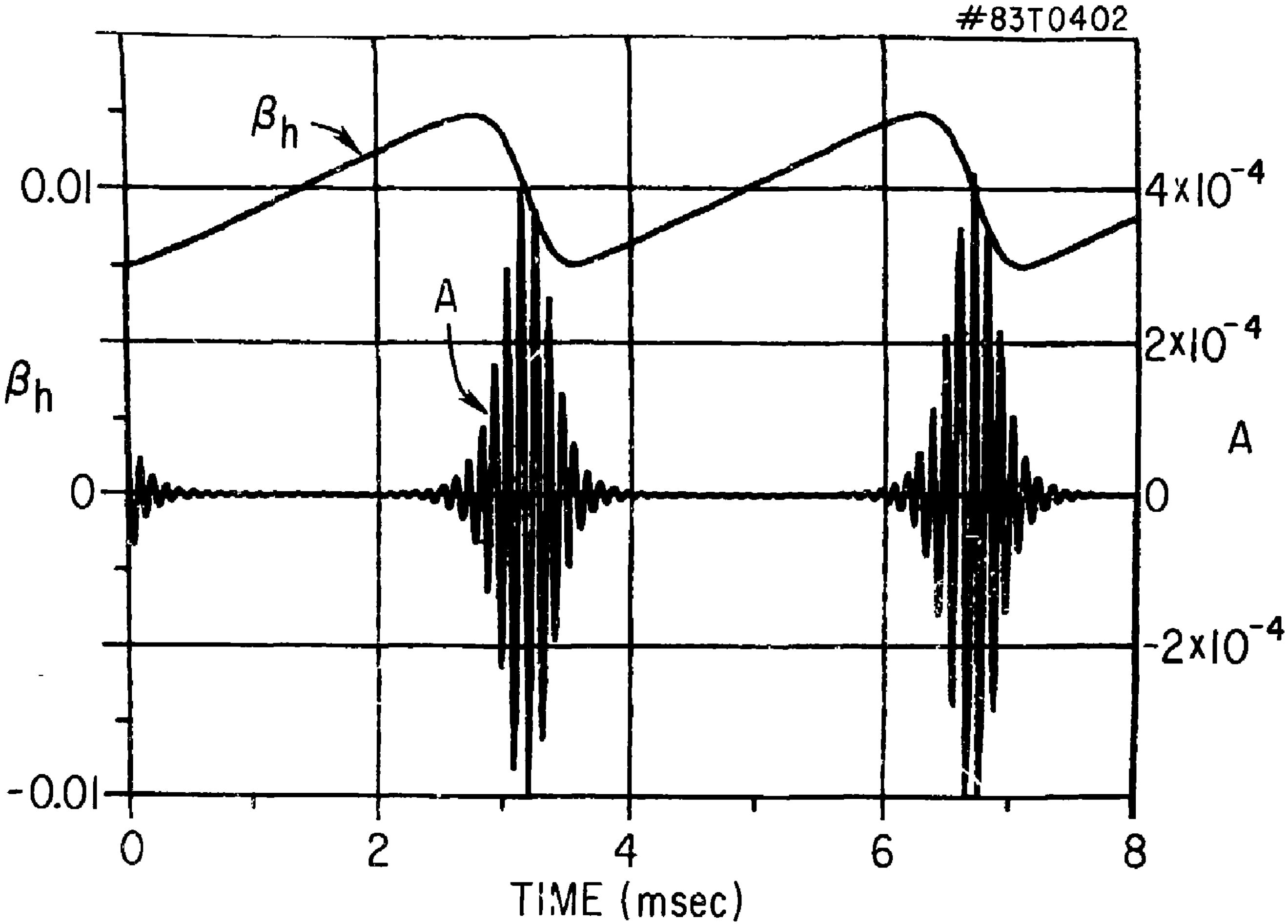
#83T0402
β_h
A
0.01
0
-0.01
4×10^{-4}
2×10^{-4}
0
-2×10^{-4}
0
2
4
6
8
TIME (msec)

Fig. 1

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

Plasma Res Lab, Austra Nat'l Univ, AUSTRALIA
Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
Prof. I.R. Jones, Flinders Univ., AUSTRALIA
Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
Prof. F. Cap, Inst Theo Phys, AUSTRIA
Prof. Frank Verheest, Inst theoretische, BELGIUM
Dr. D. Palumbo, Dg XII Fusion Prog, BELGIUM
Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
Dr. C.R. James, Univ of Alberta, CANADA
Prof. J. Teichmann, Univ of Montreal, CANADA
Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
Prof. S.R. Sreenivasan, University of Calgary, CANADA
Prof. Tudor W. Johnston, INRS-Energie, CANADA
Dr. Hannes Barnard, Univ British Columbia, CANADA
Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
Zhengwu Li, SW Inst Physics, CHINA
Library, Tsing Hua University, CHINA
Librarian, Institute of Physics, CHINA
Inst Plasma Phys, Academia Sinica, CHINA
Dr. Peter Lukac, Komenskeho Univ, CZECHOSLOVAKIA
The Librarian, Culham Laboratory, ENGLAND
Prof. Schatzman, Observatoire de Nice, FRANCE
J. Radet, CEN-BP6, FRANCE
AM Dupas Library, AM Dupas Library, FRANCE
Dr. Tom Mual, Academy Bibliographic, HONG KONG
Preprint Library, Cent Res Inst Phys, HUNGARY
Dr. S.K. Trehan, Panjab University, INDIA
Dr. Indra, Mohan Lal Das, Banaras Hindu Univ, INDIA
Dr. L.K. Chavda, South Gujarat Univ, INDIA
Dr. R.K. Chhajlani, Var Ruchi Marg, INDIA
P. Kaw, Physical Research Lab, INDIA
Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
Prof. S. Cuperman, Tel Aviv University, ISRAEL
Prof. G. Rostagni, Univ Di Padova, ITALY
Librarian, Int'l Ctr Theo Phys, ITALY
Miss Clelia De Palo, Assoc EURATOM-CNEN, ITALY
Biblioteca, del CNR EURATOM, ITALY
Dr. H. Yamato, Toshiba Res & Dev, JAPAN
Prof. M. Yoshikawa, JAERI, Tokai Res Est, JAPAN
Prof. T. Uchida, University of Tokyo, JAPAN
Research Info Center, Nagoya University, JAPAN
Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
Prof. Sigeru Mori, JAERI, JAPAN
Library, Kyoto University, JAPAN
Prof. Ichiro Kawakami, Nihon Univ, JAPAN
Prof. Satoshi Itoh, Kyushu University, JAPAN
Tech Info Division, Korea Atomic Energy, KOREA
Dr. R. England, Civdad Universitaria, MEXICO
Bibliotheek, Fom-Inst Voor Plasma, NETHERLANDS
Prof. B.S. Liley, University of Waikato, NEW ZEALAND
Dr. Suresh C. Sharma, Univ of Calabar, NIGERIA
Prof. J.A.C. Cabral, Inst Superior Tech, PORTUGAL
Dr. Octavian Petrus, ALI CUZA University, ROMANIA
Prof. M.A. Hellberg, University of Natal, SO AFRICA
Dr. Johan de Villiers, Atomic Energy Bd, SO AFRICA
Fusion Div. Library, JEN, SPAIN
Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
Dr. Lennart Stenflo, University of UMEA, SWEDEN
Library, Royal Inst Tech, SWEDEN
Dr. Erik T. Karlson, Uppsala Universitet, SWEDEN
Centre de Recherchesen, Ecole Polytech Fed, SWITZERLAND
Dr. W.L. Weise, Nat'l Bur Stand, USA
Dr. W.M. Stacey, Georg Inst Tech, USA
Dr. S.T. Wu, Univ Alabama, USA
Prof. Norman L. Oleson, Univ S Florida, USA
Dr. Benjamin Ma, Iowa State Univ, USA
Prof. Magne Kristiansen, Texas Tech Univ, USA
Dr. Raymond Askew, Auburn Univ, USA
Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
Dr. D.D. Ryutov, Siberian Acad Sci, USSR
Dr. G.A. Eliseev, Kurchatov Institute, USSR
Dr. V.A. Glukhikh, Inst Electro-Physical, USSR
Institute Gen. Physics, USSR
Prof. T.J. Boyd, Univ College N Wales, WALES
Dr. K. Schindler, Ruhr Universitat, W. GERMANY
Nuclear Res Estab, Julich Ltd, W. GERMANY
Librarian, Max-Planck Institut, W. GERMANY
Dr. H.J. Kaeppler, University Stuttgart, W. GERMANY
Bibliothek, Inst Plasmaforschung, W. GERMANY