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Exclusive Production of Higher Generation Hadrons and Form Factor Zeros in Quantum Chromodynamics*

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ABSTRACT

Perturbative QCD predictions are given for the e^+e^- pair production of vector and pseudoscalar hadrons which are bound states of heavy quarks. A striking result is the prediction of an exact zero above threshold for the production of helicity-zero hadron pairs.

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One of the important testing grounds of the perturbative aspects of Quantum Chromodynamics (QCD) is exclusive processes at moderate to large momentum transfer. By use of the factorization theorem for exclusive processes and evolution equations for distribution amplitudes, the leading scaling behavior and helicity dependence of form factors and hadron scattering amplitudes can be predicted.[1] In some cases, notably $\gamma\gamma \to \pi^+\pi^-$ and K^+K^-, predictions for the normalization and angular behavior of the cross sections can also be made without explicit information on the nature of the bound state wavefunctions.[2] Recent measurements of the normalization and scaling behavior are, in fact, in good agreement with the QCD predictions.[3] In most cases, however, detailed predictions for exclusive processes require knowledge of the nonperturbative structure of the hadrons as summarized by the valence quark distribution amplitudes $\phi_H(x_i, Q)$ of the hadrons. For example, by imposing constraints from QCD sum rules, Chernyak and Zhitnitsky have constructed nucleon distribution amplitudes which account for the sign and normalization as well as the scaling behavior of the proton and neutron magnetic form factors at $-q^2 > 10\ GeV^2$.[4]

In this paper we point out that exclusive pair production of heavy hadrons $|Q_1\overline{Q}_2\rangle$, $|Q_1Q_2Q_3\rangle$ consisting of higher generation quarks ($Q_i = t, b, c$, and possibly s) can be reliably predicted within the framework of perturbative QCD, since the required wavefunction input is essentially determined from nonrelativistic considerations. The results can be applied to e^+e^- annihilation, $\gamma\gamma$ annihilation, and W and Z decay into higher generation pairs.[5] The normalization, angular dependence and helicity structure can be predicted away from threshold, allowing a detailed study of the basic elements of heavy quark hadronization.

A particularly striking feature of the QCD predictions is the existence of a zero in the form factor and e^+e^- annihilation cross section for zero-helicity hadron pair production at the specific timelike value $q^2/4M_H^2 = m_h/2m_\ell$ where m_h and m_ℓ are the heavier and lighter quark masses, respectively. This zero reflects the destructive interference between the spin-dependent and spin-independent (Coulomb exchange) couplings of the gluon in QCD. In fact, all pseudoscalar meson form factors are predicted in QCD to reverse sign from spacelike to timelike asymptotic momentum transfer because of their essentially monopole form. For $m_h > 2m_\ell$ we shall show that the form factor zero occurs in the physical region.

To leading order in $1/q^2$, the production amplitude for hadron pair production is given by the factorized form

$$\mathcal{M}_{H\overline{H}} = \int [dx_i] \int [dy_j]\, \phi_H^\dagger(x_i, \widetilde{q}^2)\, \phi_{\overline{H}}^\dagger(y_j, \widetilde{q}^2)\, T_H(x_i, y_j; q^2, \theta_{CM}) \tag{1}$$

where $[dx_i] = \delta\left(\sum_{k=1}^n x_k - 1\right) \prod_{k=1}^n dx_k$ and $n = 2, 3$ is the number of quarks in the valence Fock state. The scale $\widetilde{q}^2$ is set from higher order calculations, but it reflects the minimum momentum transfer in the process.[1] The main dynamical dependence of the form factor is controlled by the hard scattering amplitude T_H which is computed by replacing each hadron by collinear constituents $P_i^\mu = x_i P_H^\mu$. Since the collinear divergences are summed in ϕ_H, T_H can be systematically computed as a perturbation expansion in $\alpha_s(q^2)$.[6]

The distribution amplitude

$$\phi_H(x_i, q^2) = \int^{k_\perp^2 < |q^2|} [d^2 k_{\perp i}] \psi^{(q^2)}(x_i, \vec{k}_{\perp i}) \tag{2}$$

with

$$[d^2 k_{\perp i}] = 2(2\pi)^3 \delta\left(\sum_{j=1}^n k_{\perp j}\right) \prod_{i=1}^n \frac{d^2 k_{\perp i}}{2(2\pi)^3}$$

is computed from the valence wavefunction of the hadron at equal time $\tau = t + z$ on the light cone and gives the probability amplitude for the constituents with light-cone momentum fraction $x_i = (k_i^0 + k_i^z)/(P_H^0 + P_H^z)$ to combine into the hadron with relative transverse momentum up to the scale Q^2. For the case of heavy quark bound states, we shall assume that the constituents are sufficiently non-relativistic that gluon emission, higher Fock states, and retardation of the effective potential can be neglected. The quark distributions are then controlled by a simple nonrelativistic wavefunction, which we take in the model form:

$$\psi_M(x_i, \vec{k}_{\perp i}) = \frac{C}{x_1^2 x_2^2 \left[M_H^2 - \frac{\vec{k}_{\perp 1}^2 + m_1^2}{x_1} - \frac{\vec{k}_{\perp 2}^2 + m_2^2}{x_2} \right]^2} \tag{3}$$

We choose this form since it coincides with the usual Schrödinger- Coulomb wavefunction in the nonrelativistic limit for hydrogenic atoms and has the correct large momentum behavior induced from the spin- independent gluon couplings. The wavefunction is peaked at the mass ratio $x_i = m_i/M_H$:

$$\left(x_i - \frac{m_i}{M_H} \right)^2 \sim \frac{\langle k_z^2 \rangle}{M_H^2} \tag{4}$$

where $\langle k_z^2 \rangle$ is evaluated in the rest frame. Normalizing the wavefunction to unit probability gives

$$C^2 = 128\pi \left(\langle v^2 \rangle \right)^{5/2} m_r^5 (m_1 + m_2) \tag{5}$$

where $\langle v^2 \rangle$ is the mean square relative velocity and $m_r = m_1 m_2/(m_1 + m_2)$ is the reduced mass. The corresponding distribution amplitude is

$$
\begin{aligned}
\phi(x_i) &= \frac{C}{16\pi^2} \frac{1}{[x_1 x_2 M_H^2 - x_2 m_1^2 - x_1 m_2^2]} \\
&\cong \frac{1}{\sqrt{2\pi}} \frac{\gamma^{3/2}}{M_H^{1/2}} \, \delta\left(x_1 - \frac{m_1}{m_1 + m_2}\right) .
\end{aligned} \tag{6}
$$

It is easy to see from the structure of T_H for $e^+e^- \to M\overline{M}$ that the spectator quark pair is produced with momentum transfer squared $q^2 x_s y_s = 4m_s^2$. Thus heavy hadron pair production is dominated by diagrams in which the primary coupling of the virtual photon is to the heavier quark pair. The perturbative predictions are thus expected to be accurate even near threshold to leading order in $\alpha_s(4m_\ell^2)$ where m_ℓ is the mass of lighter quark in the meson.

We have computed the leading order e^+e^- production helicity amplitudes for higher generation meson ($\lambda = 0, \pm 1$) and baryon ($\lambda = \pm 1/2, \pm 3/2$) pairs from Eq. (1) as a function of q^2 and the quark masses. The Lorentz and gauge invariant form factors for meson pair production are defined by the electromagnetic coupling of the meson as shown in Table I. The analysis is simplified by using the peaked form of the distribution amplitude, Eq. (6). From the calculation of helicity amplitudes, we found that $V_2(q^2) = M_H\, T(q^2)$. Therefore, we use the following notation:

$$F_{0,0}(q^2) = S(q^2)$$

$$F_{1,1}(q^2) = V_1(q^2)$$

$$F_{0,1}(q^2) = M_H \; T(q^2) = V_2(q^2) \; .$$

In the case of meson pairs the (unpolarized) e^+e^- annihilation cross section has the general form

$$4\pi \frac{d\sigma}{d\Omega}(e^+e^- \to M_\lambda \overline{M}_{\overline{\lambda}}) = \frac{3}{4}\beta\sigma_{e^+e^-\to\mu^+\mu^-}\left[\frac{1}{2}\beta^2\sin^2\theta\right.$$

$$\times\left[|F_{0,0}(q^2)|^2 + \frac{1}{(1-\beta^2)^2}\left\{(3-2\beta^2+3\beta^4)|F_{1,1}(q^2)|^2\right.\right.$$

$$\left.\left.- 4(1+\beta^2)\ \mathrm{Re}\,(F_{1,1}(q^2)F^*_{0,1}(q^2)) + 4|F_{0,1}(q^2)|^2\right\}\right] \tag{7}$$

$$\left.+\frac{3\beta^2}{2(1-\beta^2)}\,(1+\cos^2\theta)|F_{0,1}(q^2)|^2\right]$$

where $q^2 = s = 4M_H^2\bar{q}^2$ and the meson velocity is $\beta = 1 - \frac{4M_H^2}{q^2}$. The production form factors have the general form

$$F_{\lambda\overline{\lambda}} = \frac{\langle v^2\rangle^2}{(\bar{q}^2)^2}\,(A_{\lambda\overline{\lambda}} + \bar{q}^2 B_{\lambda\overline{\lambda}}) \tag{8}$$

where A and B reflect the Coulomb-like and transverse gluon couplings, respectively. The results to leading order in α_s are given in Table II. In general A and B have a slow logarithmic dependence due to the q^2-evolution of the distribution amplitudes. The form factor zero for the case of pseudoscalar pair production reflects the numerator structure of the T_H amplitude,

$$\text{Numerator} \sim e_1\left(\bar{q}^2 - \frac{m_1^2}{4M_H^2}\,\frac{1}{x_2y_1} - \frac{m_2^2}{4M_H^2}\,\frac{x_1}{x_2^2y_2}\right)\,. \tag{9}$$

For the peaked wavefunction,

$$F^M_{0,0}(q^2) \propto \frac{1}{(\bar{q}^2)^2}\left\{e_1\left(\bar{q}^2 - \frac{m_1}{2m_2}\right) + e_2\left(\bar{q}^2 - \frac{m_2}{2m_1}\right)\,\frac{m_2^2}{m_1^2}\right\}\,. \tag{10}$$

If m_1 is much greater than m_2 then the e_1 is dominant and changes sign at $q^2/4M_H^2 = m_1/2m_2$. The contribution of the e_2 term and higher order contributions are small and nearly constant in the region where the e_1 term changes sign; such contributions can displace slightly but not remove the form factor zero.

These results also hold in quantum electrodynamics; e.g. pair production of muonium ($\mu - e$) atoms in e_+e_- annihilation. Gauge theory predicts a zero for paramuonium production at $\bar{q}^2 = m_\mu/2m_e$.

These explicit results for form factors also shows that the onset of the leading power-law scaling of a form factor is controlled by the ratio of the A and B terms in Eq. (8); i.e., when the transverse contributions exceed the Coulomb mass-dominated contributions. The Coulomb contribution to the form factor can also be computed directly from the convolution of the initial and final wavefunctions. Thus, contrary to the claim of Ref 7, there are no extra factors of $\alpha_s(q^2)$ which suppress the "hard" versus nonperturbative contributions.

The form factors for the heavy hadrons are normalized by the constraint that the Coulomb contribution to the form factor equals the total hadronic charge at $q^2 = 0$. Further, by the correspondence principle, the form factor should agree with the standard non-relativistic calculation at small momentum transfer. All of these constraints are satisfied by the form

$$F^M_{0,0}(q^2) = e_1 \, \frac{16\gamma^4}{(q^2+\gamma^2)^2} \left(\frac{M_H^2}{m_2^2}\right)^2 \left(1 - \frac{q^2}{4M_H^2}\,\frac{2m_2}{m_1}\right) + 1 \leftrightarrow 2 \ . \qquad (11)$$

At large q^2 the form factor also agrees with the standard QCD prediction

$$F^M_{(0,0)} = e_1 \, \frac{16\pi\alpha_s f_M^2}{9q^2} \left(\frac{M_H^2}{m_2^2}\right) + (1 \leftrightarrow 2) \ , \quad \frac{f_M}{2\sqrt{3}} = \int_0^1 dx \, \phi(x, Q) \qquad (12)$$

where $f_M = (6\gamma^3/\pi M_H)^{1/2}$ is the meson decay constant. Predictions for various heavy mesons are shown in the figures. The results for the cross sections are given in units of R using the $\mu^+\mu^-$ rate as reference. The basic unknown is $\gamma^2 = v^2 m_r^2$ which sets the scale for capture into the wavefunction in relative transverse momentum. The same probability amplitude enters the normalization of the inclusive production of heavy hadrons in heavy quark hadronization.

Although the mass of the strange quark may be too low to trust these predictions, we also give predictions for $F\overline{F}$ production since the predicted zero appears in a domain accessible to present storage rings.

At low relative velocity of the hadron pair one also expects resonance contributions to the form factors. For these heavy systems such resonances could be related to $qq\bar{q}\bar{q}$ bound states. From Watson's theorem, one expects any resonance structure to introduce a final-state phase factor, but not destroy the zero of the underlying QCD prediction.

We have also performed analogous calculations of the baryon form factor, retaining the constituent mass structure. The numerator structure for spin 1/2 baryons has the form

$$A + B\bar{q}^2 + C\bar{q}^4 \ . \tag{13}$$

Thus it is possible to have two form factor zeros; e.g. at spacelike and timelike values of q^2.

Although the measurements are difficult and require large luminosity, the observation of the striking zero structure predicted by QCD would provide a unique test of the theory and its applicability to exclusive processes. As we have shown, the onset of leading power behavior is controlled simply by the mass parameters of the theory; there is no way to postpone the region of validity of the perturbative predictions. The results of this paper may also be applicable to lower mass systems, but in this case the mass parameters for the light quarks are evidently replaced by vacuum condensate and other nonperturbative contributions.[8]

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Table I. Definition of meson form factors. The dashed and solid lines represent the pseudoscalar and vector mesons respectively.

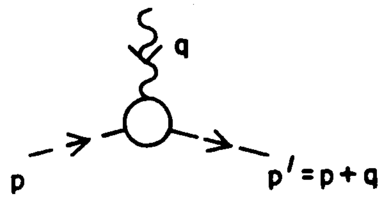
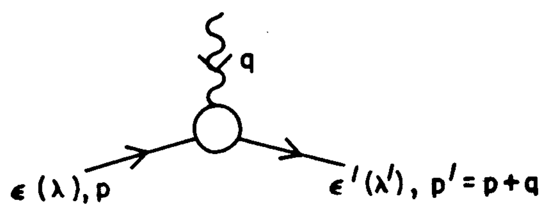
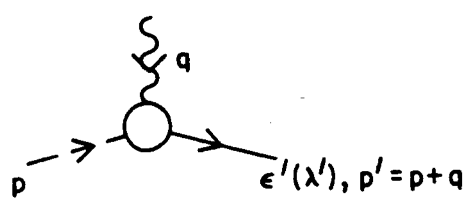
<table>
<tr><th>Vertex Type</th><th>$ie\Gamma_\mu$</th></tr>
<tr><td>
</td><td>$ie\,(p_\mu + p'_\mu)\;\;S(q^2)$</td></tr>
<tr><td>
</td><td>$ie\epsilon(\lambda)\cdot\epsilon^{*\prime}(\lambda')\,(p_\mu + p'_\mu)\,V_1(q^2)$
$-ie\,\{\epsilon(\lambda)\cdot p'\;\epsilon^{*\prime}_\mu(\lambda')$
$+\,\epsilon'^{*}(\lambda')\cdot p\;\epsilon_\mu(\lambda)\}\,V_2(q^2)$</td></tr>
<tr><td>
</td><td>$e\,\epsilon_{\mu\nu\rho\sigma}\,p^\nu\,\epsilon'^{*\rho}(\lambda')\,q^\sigma\,T(q^2)$</td></tr>
</table>

Table II. The results for the meson form factors.

A and B are defined by Eq. (8).

	(0,0)	(0,1)	(1,1)
A	$\frac{e_1 m_1^4 + e_2 m_2^4}{(m_1+m_2)^4}$	$\frac{e_1 m_1^3 + e_2 m_2^3}{(m_1+m_2)^3}$	$\frac{e_1 m_1^4 + e_2 m_2^4}{(m_1+m_2)^4}$
B	$\frac{-2m_1 m_2 (e_1 m_1^2 + e_2 m_2^2)}{(m_1+m_2)^4}$	0	0

FIGURE CAPTION

*Fig.*1. Predictions of

$$R_{M_1 M_2} = \frac{\sigma(e^+e^- \to M_1 M_2)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

for various heavy mesons M_1 and M_2:

(a) $R_{B_C \overline{B}_C}$

(b) $R_{F\overline{F}}$

(c) $R_{\eta_C \ J/\psi}$.

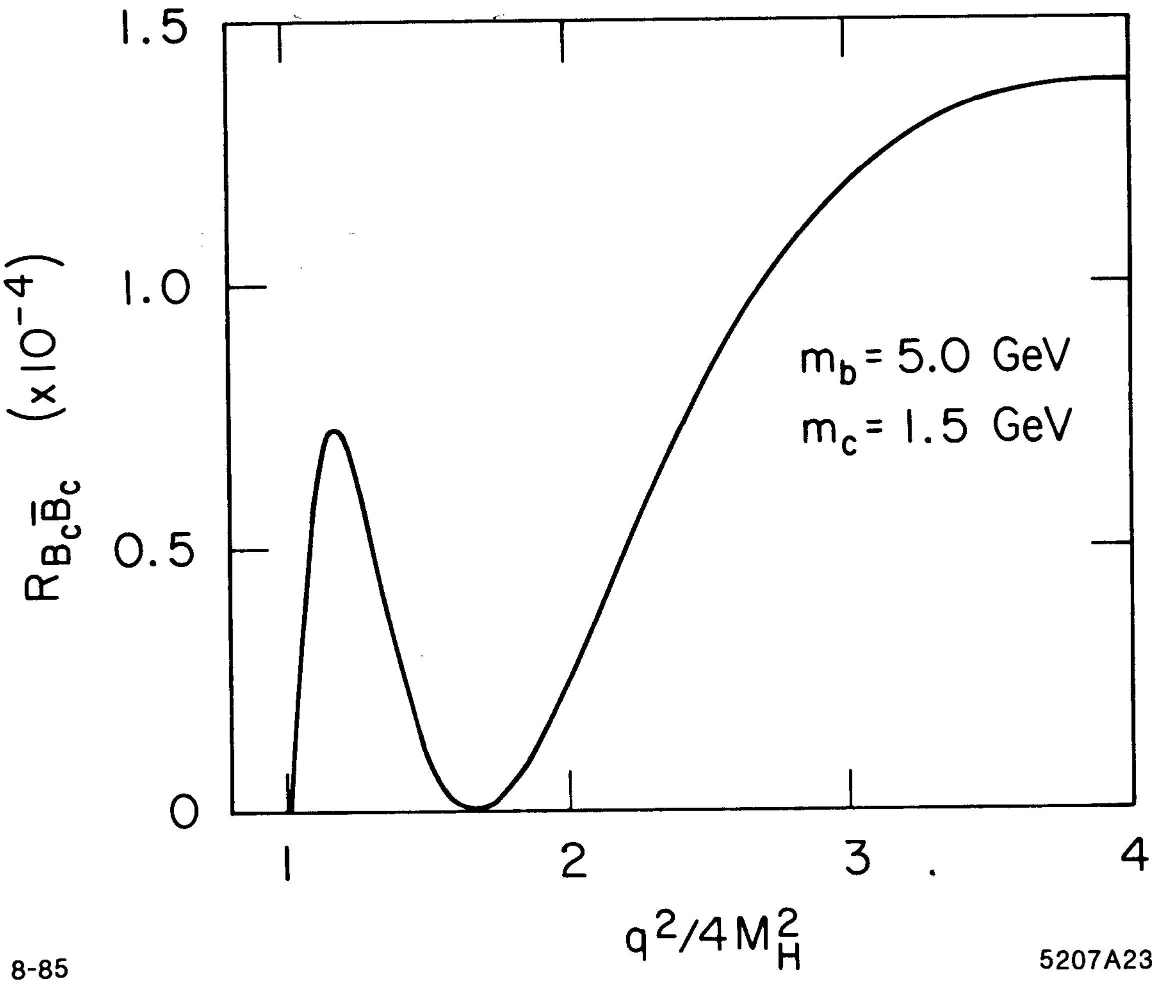

1.5
1.0
0.5
0
$R_{B_c\bar{B}_c}$ $(\times 10^{-4})$
$m_b = 5.0$ GeV
$m_c = 1.5$ GeV
1
2
3
4
$q^2/4M_H^2$
8-85
5207A23

FIGURE 1(A)

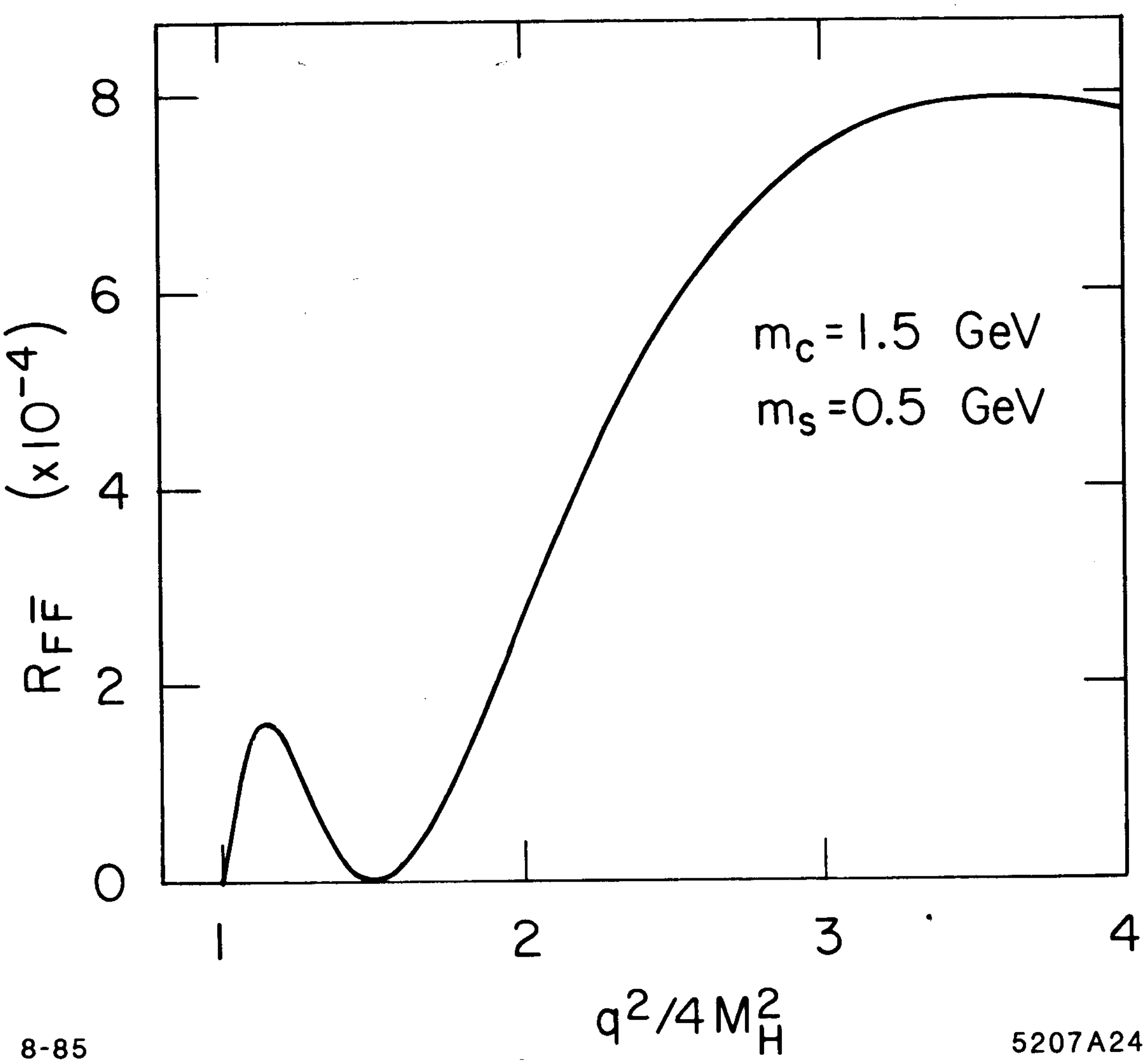


FIGURE 1(B)

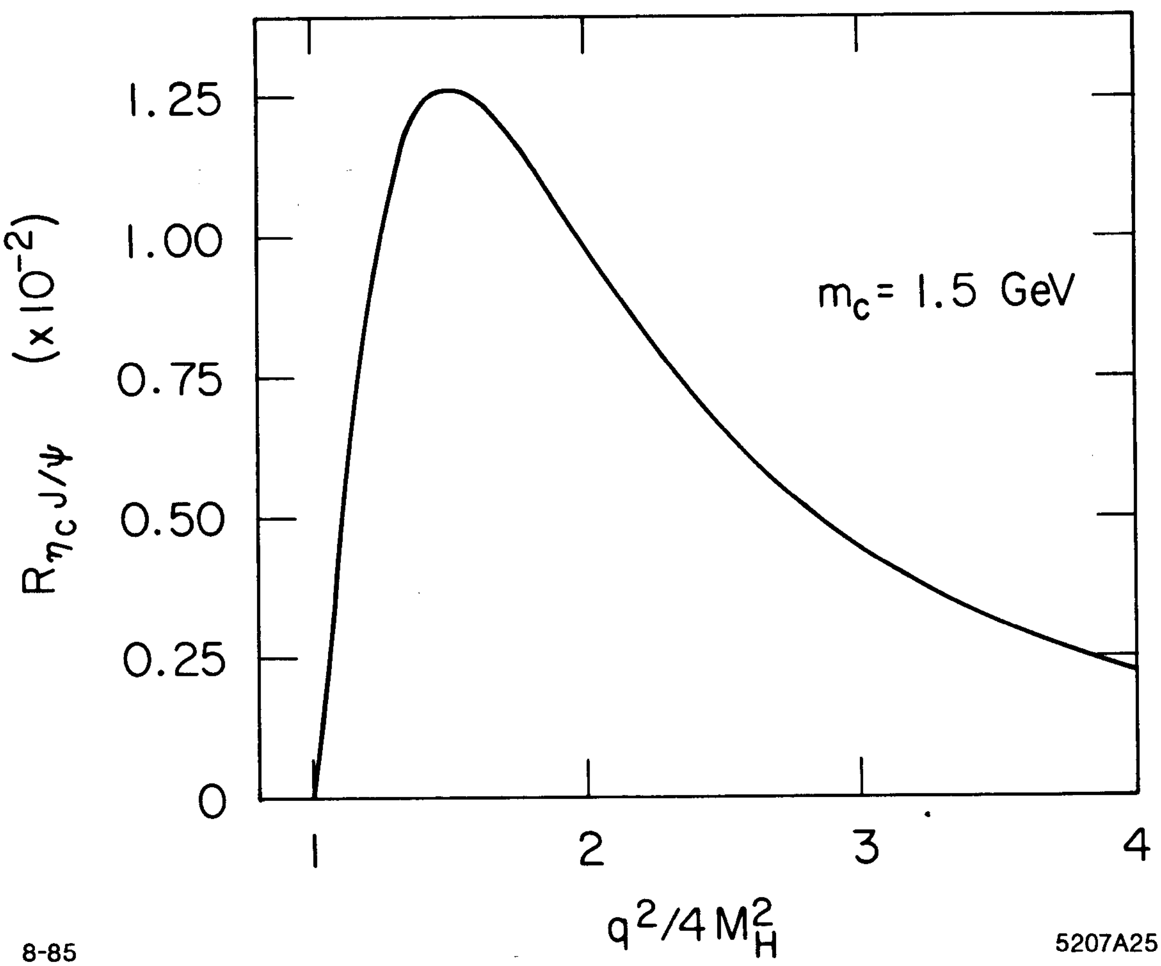

1.25
1.00
0.75
0.50
0.25
0
$R_{\eta_c J/\psi}$ $(\times 10^{-2})$
m_c = 1.5 GeV
1
2
3
4
$q^2/4M_H^2$
8-85
5207A25

FIGURE 1(c)